

# Supplement

Included is a sample of all of the Lagrangian solid modeling code used within this study. The core of this code, which is written for a Fortran 90 compiler, is included along with several function files: a bar in tension, a bar in compression, a disk compressed into an elastic steel bar, and the pressure vessel simulation. The code is written so that the core file *spam.f* can be compiled along with the model-building function of choice, and the simulation will run accordingly. The files are:

- **spam.f**: this is the core of the code, and is entirely self contained except for the model function file, which defines the model and specifies the material and simulation properties.
- **tension\_steel.f**: the model function for the bar in tension.
- **compress\_steel.f**: the model function for the bar in compression. It is identical to the tension function except the top and bottom *FixXY* values are reversed so the direction is reversed.
- **ModelSmFixDisk.f**: The model function for a rigid 2D disk compressing into an elastic flat plat.
- **PressVessel.f**: The model function for the 2D pressure vessel, with both steel solid particles and liquid particles under pressure, to demonstrate Lagrangian Fluid Solid Interactions.

In addition, there are several MatLab files used to post-process the data:

- **Analyze\_Data.m**: This file is first run, which reads all of the ASCII data files, and saves it as a single, clean binary MatLab data file *Data.mat*.
- **PlotTension.m**: The file processes the tension plot. It will prompt the model before and after the simulation (exaggerated deflection), as well as calculating the analytical tension stress and transverse deflections, and compares it to the numerical results.

- **PlotComp.m:** Effectively the same as the *PlotTension.m* script, but with small modifications for compression simulations rather than tensile.
- **Plot\_Hertz.m:** The plotting function for the Hertz disk-flat simulation. Various exaggerated deflection plots are generated, as well as calculation of the Hertz analytical stresses to compare to the analytical results.
- **MakeFigs\_PresVes.m:** Plots the 2D pressure vessel model, as well as the relative radial deflections.

To run the simulation, first the user compiles the *spam.f* function with the model function of interest. The machine-language program of the simulation is run, and the code will output the results as various ASCII text files of the simulation results. The user can then go into MatLab and run the *Analyze\_Data.m* script, which reads all of the ASCII data files and outputs a single compressed binary MatLab file (*Data.mat*) of the simulation results. The user can then select the relevant MatLab script to load the MatLab data file and plot and analyze the results of the simulation.

The *Analyze\_Data.m* script will consistently save the results as a binary *Data.mat* file, regardless of the study performed. The saved results are renamed, and the associated MatLab plotting scripts read these different file-names. The scripts are as follows:

- **Tension\_steel.mat**
- **compress\_steel.mat**
- **Hertz.mat**
- **PressVess.mat**

# FORTRAN SOURCE CODE

## spam.f

```
program spam

c -----
implicit none

integer ii,jj,kk,jj0,uu,vv,ctbar(3),ctZ,ctZmod
integer ct,ct2,xx,yy,N,Nsphere,ts,fooint,StudyCt,CenterPt(2)
integer Nts,LoopCT,LoopCT2,stop,start2,stop2,ctCyl,StopWeight
integer xxx,yyy,uv(2),break,breakct,abreak,split,StartStop
integer BSx,cubecyl,ctx(2)
complex i
double precision pi,dx3(3),third,Xloc,Yloc,Zloc,StartDefl,NewDefl
double precision dx00, dxE, dTfactor,dTdxfactor,dTdxfactor2,TTxx
double precision masX,rho_solid,foo3,Xr0,Yr0,Vydt,dt0x,dx0x
double precision deflect, deflectWeight, Rred, Ered, bWeight,mas
double precision h, c02, K_bulk, Vy0,foo,r,dwdx(3),t1,t2
double precision g(3), foo2, foo3vv, dxJ, Arange, dxEj
double precision dt,dt0,Dij(3,3),OMGij(3,3),Sij3(3,3)
double precision dSdt(6),RijAB(3,3),Wij,rho_ij,Radius,RadiusFact
double precision Weight0,Weight,Pweight(3),Vtest(3),Tfract,Efrac
double precision dx,dxd,dE,deltX,deltX2,dxi,a,angleFrac,angle
double precision lambda,mu,Ey,poisson,TT,Time,Bulk_water
double precision Csound,rho0min,dV,gamma,dVdx0
double precision X1(3),X2(3),V1(3),V2(3)
integer, allocatable, dimension(:) :: Mat, FixXY, IsSolid
integer, allocatable, dimension(:) :: WeightXY
integer, allocatable, dimension(:, :) :: Contact,ContactDir
```

```

integer, allocatable, dimension(:, :) :: ContactDirCT
integer, allocatable, dimension(:, :) :: ContactX, ContactDirX
integer, allocatable, dimension(:, :) :: ContactDirCTX
integer, allocatable, dimension(:, :) :: Links
integer, allocatable, dimension(:) :: BreakContact, RadiusCrap
double precision, allocatable, dimension(:, :) :: X, X0, X00
double precision, allocatable, dimension(:, :) :: V, V0, dVdx
double precision, allocatable, dimension(:, :) :: dVdxFct, dVdyFct
double precision, allocatable, dimension(:, :) :: dVdzFct
double precision, allocatable, dimension(:, :) :: V00
double precision, allocatable, dimension(:, :) :: TcomboOut
double precision, allocatable, dimension(:, :, :) :: DSDtN, DSDtN0
double precision, allocatable, dimension(:, :, :) :: Tij, Eij, Eij0
double precision, allocatable, dimension(:, :, :) :: TTij, Tij0
double precision, allocatable, dimension(:, :, :) :: TijCombo
double precision, allocatable, dimension(:) :: rho, rho0, P, P0
double precision, allocatable, dimension(:) :: printVar, Tvm0
double precision, allocatable, dimension(:) :: TimeFct
double precision, allocatable, dimension(:) :: massFct
double precision, allocatable, dimension(:, :) :: D0, Omg0
double precision, allocatable, dimension(:, :) :: VonMiss
double precision, allocatable, dimension(:, :) :: Xout, Yout, Zout
double precision, allocatable, dimension(:, :) :: VoutX, VoutY, VoutZ
double precision, allocatable, dimension(:, :) :: dVdy, Tvm
double precision, allocatable, dimension(:, :) :: Pout, Rout
double precision, allocatable, dimension(:, :) :: DPDT, DPDT0
double precision, allocatable, dimension(:, :) :: T2o
double precision, allocatable, dimension(:, :) :: T11, T12, T13
double precision, allocatable, dimension(:, :) :: T21, T22, T23
double precision, allocatable, dimension(:, :) :: T31, T32, T33
double precision, allocatable, dimension(:, :) :: E22, E11
double precision, allocatable, dimension(:) :: dx0
double precision, allocatable, dimension(:) :: zOffset

```

c  
c  
c

```

-----
These are the universal constants
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```

```

pi = 3.141592653589793 ! set the value of pi
i = (0, 1) ! set the imaginary number i=sqrt(-1)

```

```

g = (/ 0.0, 0.0, 0.0 /)
third=1.0/3.0

c -----
c Declare array sizes
c -----

call ModelBlock(N)

open (unit=1,file="SimSpec.dat",STATUS='OLD',ACTION='READ')
  read (1,*)Ey
  read (1,*)poisson
  read (1,*)rho_solid
  read (1,*)Vtest(1)
  read (1,*)Vtest(2)
  read (1,*)Vtest(3)
  read (1,*)gamma
  read (1,*)Nts
  read (1,*)LoopCT
  read (1,*)StudyCt
  read (1,*)StartDefl
  read (1,*)NewDefl
  read (1,*)dTfactor
  read (1,*)dTdxfactor
  read (1,*)stop
  read (1,*)Bulk_water
close (1)

Vy0 = Vtest(2)
K_bulk = Ey/(3*(1-(2*poisson))) ! bulk modulus of iron
mu=Ey/(2*(1+poisson))
lambda=K_bulk-(2*mu/3)
Ered=2/((1-(poisson**2))/Ey)

Nts=Nts*StudyCt

ALLOCATE (X(N,3))
ALLOCATE (X0(N,3))
ALLOCATE (X00(N,3))
ALLOCATE (V(N,3))

```

```

ALLOCATE (V0 (N, 3))
ALLOCATE (V00 (N, 3))
ALLOCATE (dx0 (N))
ALLOCATE (dVdx (N, 3))
ALLOCATE (dVdxFct (N, Nts))
ALLOCATE (dVdyFct (N, Nts))
ALLOCATE (dVdzFct (N, Nts))
ALLOCATE (Mat (N))
ALLOCATE (FixXY (N))
ALLOCATE (IsSolid (N))
ALLOCATE (rho (N))
ALLOCATE (rho0 (N))
ALLOCATE (massFct (N))
ALLOCATE (P (N))
ALLOCATE (P0 (N))
ALLOCATE (Tij (N, 3, 3))
ALLOCATE (TTij (N, 3, 3))
ALLOCATE (Tij0 (N, 3, 3))
ALLOCATE (TijCombo (N, 3, 3))
ALLOCATE (Eij (N, 3, 3))
ALLOCATE (Eij0 (N, 3, 3))
ALLOCATE (DSDtN (N, 3, 3))
ALLOCATE (DSDtN0 (N, 3, 3))
ALLOCATE (DPDT (N, 3))
ALLOCATE (DPDT0 (N, 3))
ALLOCATE (D0 (N, 6))
ALLOCATE (Omg0 (N, 6))
ALLOCATE (Links (N, N+1))
ALLOCATE (Xout (N, Nts))
ALLOCATE (Yout (N, Nts))
ALLOCATE (Zout (N, Nts))
ALLOCATE (VoutX (N, Nts))
ALLOCATE (VoutY (N, Nts))
ALLOCATE (VoutZ (N, Nts))
ALLOCATE (dVdy (N, Nts))
ALLOCATE (Pout (N, Nts))
ALLOCATE (Rout (N, Nts))
ALLOCATE (TimeFct (Nts))
ALLOCATE (Tvm0 (N))
ALLOCATE (Tvm (N, Nts))

```

```

ALLOCATE (TcomboOut (N,Nts))
ALLOCATE (T11 (N,Nts))
ALLOCATE (T12 (N,Nts))
ALLOCATE (T13 (N,Nts))
ALLOCATE (T21 (N,Nts))
ALLOCATE (T22 (N,Nts))
ALLOCATE (T23 (N,Nts))
ALLOCATE (T31 (N,Nts))
ALLOCATE (T32 (N,Nts))
ALLOCATE (T33 (N,Nts))
ALLOCATE (T22o (N,Nts))
ALLOCATE (E22 (N,Nts))
ALLOCATE (E11 (N,Nts))
ALLOCATE (VonMiss (N,Nts))
ALLOCATE (Contact (N, (N+1)))
ALLOCATE (ContactDir (N, (N+1)))
ALLOCATE (ContactDirCT (N, 3))
ALLOCATE (ContactX (N, (N+1)))
ALLOCATE (ContactDirX (N, (N+1)))
ALLOCATE (ContactDirCTX (N, 3))
ALLOCATE (BreakContact (N))

ALLOCATE (printVar (N))

ALLOCATE (RadiusCrap (N))

```

```

c -----
c Declare size and velocity of particles
c -----

open (unit=1,file="Fixed.dat",STATUS='OLD',ACTION='READ')
do ii=1,N
  read (1,*),FixXY(ii)
enddo
close (1)

open (unit=1,file="Contact.dat",STATUS='OLD',ACTION='READ')
do ii=1,N
  read (1,*),Contact(ii,:)
enddo

```

```

close (1)

open (unit=1,file="ContactDir.dat",STATUS='OLD',ACTION='READ')
do ii=1,N
    read (1,*),ContactDir(ii,:)
enddo
close (1)

open (unit=1,file="ContactDirCT.dat",STATUS='OLD',ACTION='READ')
do ii=1,N
    read (1,*),ContactDirCT(ii,:)
enddo
close (1)

open (unit=1,file="ContactX.dat",STATUS='OLD',ACTION='READ')
do ii=1,N
    read (1,*),ContactX(ii,:)
enddo
close (1)

open (unit=1,file="ContactDirX.dat",STATUS='OLD',ACTION='READ')
do ii=1,N
    read (1,*),ContactDirX(ii,:)
enddo
close (1)

open (unit=1,file="ContactDirCTX.dat",STATUS='OLD',ACTION='READ')
do ii=1,N
    read (1,*),ContactDirCTX(ii,:)
enddo
close (1)

open (unit=1,file="Mass.dat",STATUS='OLD',ACTION='READ')
do ii=1,N
    read (1,*),massFct(ii)
enddo
close (1)

open (unit=1,file="dX.dat",STATUS='OLD',ACTION='READ')
do ii=1,N

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```

        read (1,*),dx0(ii)
    enddo
close (1)

open (unit=1,file="X0.dat",STATUS='OLD',ACTION='READ')
do ii=1,N
    read (1,*),X(ii,:)
enddo
close (1)

open (unit=1,file="V0.dat",STATUS='OLD',ACTION='READ')
do ii=1,N
    read (1,*),V(ii,:)
enddo
close (1)

open (unit=1,file="dVdX.dat",STATUS='OLD',ACTION='READ')
do ii=1,N
    read (1,*),dVdx(ii,:)
enddo
close (1)

open (unit=1,file="rho0.dat",STATUS='OLD',ACTION='READ')
do jj=1,N
    read (1,*),rho0(jj)
enddo
close (1)

open (unit=1,file="rho_init.dat",STATUS='OLD',ACTION='READ')
do jj=1,N
    read (1,*),rho(jj)
enddo
close (1)

open (unit=1,file="IsSolid.dat",STATUS='OLD',ACTION='READ')
do jj=1,N
    read (1,*),IsSolid(jj)
enddo
close (1)

```

```

c -----
c -----

call SpeedSound(Ey,poisson,rho_solid,Csound)
dt0=dTfactor*0.5*(0.25*(2*(MINVAL(dx0)))/Csound)

X00=X
open (unit = 1, file = "Xoriginal.dat")
  do ii=1,N
    write (1,*) ,X00(ii,:)
  enddo
close (1)
Time=0

do ii=1,N
  do jj=1,3
    do kk=1,3
      Tij(ii,jj,kk)=0
      TTij(ii,jj,kk)=0
      Eij(ii,jj,kk)=0
    enddo
    if (Vtest(jj)==0) then
      Tij0(:,jj,jj)=0
    else
      Tij0(:,jj,jj)=Vtest(jj)/(abs(Vtest(jj)))
    endif
  enddo
enddo
Tij0=Tij0*(1e-9)
deflect=0

call cpu_time ( t1 )

do ts=1,Nts
  call cpu_time ( t2 )
  print *,ts,'/',Nts,', Elapsed CPU time = ', (t2 - t1),' seconds

  xxx=((ts-1)*StudyCt/Nts)
  if (xxx==0) then

```

```

    LoopCT2=3*LoopCT
else
    LoopCT2=LoopCT
endif
deflectWeight=MINVAL(dx0)*(StartDefl+(xxx*NewDefl/StudyCt))

do yyy=1,LoopCT2

dt=dTdxfactor*(MINVAL(dx0))/(MAXVAL(abs(V))+(1e-10))
if (abs(deflect)<deflectWeight) then
    dt0x=abs(dTdxfactor*(MINVAL(dx0))/Vtest(2))
else
    dt0x=dt0
endif
if (dt0x<dt) then
    dt=dt0x
endif
if (dt0<dt) then
    dt=dt0
endif
Time=Time+dt

if (ts<=stop) then
    if (abs(deflect)<deflectWeight) then
        deflect=deflect+(Vtest(2)*dt)
    endif
else
    deflect=deflect
endif

call LinkList(N,X,MAXVAL(dx0),Links)
call ContactStress(N,Tij,Contact,ContactDir,Tij0)

Contact=ContactX
ContactDir=ContactDirX
ContactDirCT=ContactDirCTX

do jj=1,N
    if (IsSolid(jj)==1) then

```

```

ct=ContactX(jj,1)
do jj0=1,Links(jj,1)
  ii=Links(jj,jj0+1)

  if (ii/=jj) then
    call kernel(X(ii,:),X(jj,:),2*dx0(jj),Wij,dwdx,r)
    do kk=1,3
      dx3(kk)=abs(X(ii,kk)-X(jj,kk))
    enddo
    fooint=MAXLOC(dx3,1)
    dxE=dx0(jj)*(1+Eij(jj,fooint,fooint))
    if (dxE<(dx0(jj)*1.05)) then
      dxE=(dx0(jj)*1.05)
    endif

    if (r<dxE) then
      kk=1
      do uu=2,(ContactX(jj,1)+1)
        vv=ContactX(jj,uu)
        if (ii==vv) then
          kk=0
        endif
      enddo
      if (kk==1) then
        ct=ct+1
        Contact(jj,ct+1)=ii
        ContactDir(jj,ct+1)=fooint
        ContactDirCT(jj,fooint)=ContactDirCT(jj,fooint)+1
        Tij0(jj,fooint,fooint)=-1
      endif
    endif
  endif
enddo
Contact(jj,1)=ct
ContactDir(jj,1)=ct
endif
enddo

```

c

---

c \_\_\_\_\_

c\_\_\_\_\_Calculate the new velocity. \_\_\_\_\_

```
X0=X
V0=V
do xx=1,3
  do ii=1,N
    dxE=dx0(ii)*(1+Eij(ii,xx,xx))
    foo = g(xx)
    mass=massFct(ii)
    if (IsSolid(ii)==1) then

      if (Contact(ii,1)>0) then
        do uu=2,((Contact(ii,1))+1)
          jj=Contact(ii,uu)
          if (ContactDir(ii,uu)==xx) then
            if (uu>(1+ContactX(ii,1))) then
              call Compression(xx,ii,jj,N,X,Eij,Ey,dx0,mass,foo2,a)
            else
              call Tension(xx,ii,jj,N,X,Eij,Ey,dx0,mass,foo,foo2,TT)
            endif
            foo=foo+foo2
          else
            yy=ContactDir(ii,uu)
            call Shear(xx,yy,ii,jj,N,X,Eij,mu,dx0,mass,foo2,TT)
            foo=foo+foo2
          endif
        enddo
      endif
    endif

    if (IsSolid(ii)==1) then
      do jj=1,N
        if (IsSolid(jj)==0) then
          X1=X(ii,:)
          X2=X(jj,:)
          call LJfctSolid(xx,X1,X2,dx0(ii),P(ii),a)
          foo=foo+(a/mass)
        endif
      enddo
    endif
  enddo
enddo
```

```

        endif
    enddo
else
    do jj=1,N
        if (IsSolid(jj)==1) then

            X1=X(ii,:)
            X2=X(jj,:)
            V1=V(ii,:)
            V2=V(jj,:)
            call LJfct(xx,X1,X2,V1,V2,dx0(ii),dt,a)

            foo=foo+a
        endif
    enddo
endif

ct=2
do jj0=1,Links(ii,1)
    jj=Links(ii,jj0+1)
    call kernel(X(ii,:),X(jj,:),2*dx0(ii),Wij,dwdx,r)
    if (IsSolid(jj)==0) then
        foo2=(P(jj)-P(ii))*(dx0(ii)**2)/mass
        foo2=-foo2*((X(ii,xx)-X(jj,xx))/r)
        foo=foo2
    endif
enddo

dVdx0=dVdx(ii,xx)
dVdx(ii,xx) = foo
V(ii,xx)=V(ii,xx)+(dt*(dVdx0+dVdx(ii,xx))/2)
enddo
enddo

X0=X
V00=V
do xx=1,3
    do ii=1,N

```

```

        if (IsSolid(ii)==1) then
            if ((Contact(ii,1))>0) then
                foo=0
                do uu=2, ((Contact(ii,1))+1)
                    jj=Contact(ii,uu)
                    foo=foo+V00(jj,xx)
                enddo
                foo=foo/(Contact(ii,1))
                foo=(foo+(V00(ii,xx)))/2
                V(ii,xx)=foo
            endif
        endif
    enddo
enddo

```

c\_\_\_\_\_Definition of FixXY\_\_\_\_\_

```

c      FixXY(ii)=0 --> Free Particle
c      FixXY(ii)=1 --> Fixed in space
c      FixXY(ii)=2 --> Fixed in X/1-direction
c      FixXY(ii)=3 --> Fixed in Y/2-direction
c      FixXY(ii)=4 --> Fixed in Z/3-direction
c      FixXY(ii)=5 --> Free in X/1-direction only
c      FixXY(ii)=6 --> Free in Y/2-direction only
c      FixXY(ii)=7 --> Free in Z/3-direction only
c      FixXY(ii)=8 --> Follows V_test
c      FixXY(ii)=9 --> Follows -V_test
c      FixXY(ii)=10 --> Follows V_test in the X/1-direction
c      FixXY(ii)=11 --> Follows -V_test in the X/1-direction
c      FixXY(ii)=12 --> Follows V_test in the Y/2-direction
c      FixXY(ii)=13 --> Follows -V_test in the Y/2-direction
c      FixXY(ii)=14 --> Follows V_test in the Z/3-direction
c      FixXY(ii)=15 --> Follows -V_test in the Z/3-direction

```

c\_\_\_\_\_End Definition of FixXY\_\_\_\_\_

```

        foo2=0
c_____Calculate the new location. _____

```

```

do uu=1,3
  do ii=1,N
    if (IsSolid(ii)==1) then
      dV=V(ii,uu)+V0(ii,uu)
      if (dV>(0.1*(dx0(ii))/dt)) then
        dV=0
        V(ii,uu)=0
      elseif (abs(dV)<(1e-6)) then
        dV=0
        V(ii,uu)=0
      endif
    endif

    if (FixXY(ii)==1) then
      X(ii,uu)=X00(ii,uu)
    elseif (FixXY(ii)==2) then
      if (uu==1) then
        X(ii,uu)=X00(ii,uu)
      else
        X(ii,uu)=X(ii,uu)+(dt*dV/2)
      endif
    elseif (FixXY(ii)==3) then
      if (uu==2) then
        X(ii,uu)=X00(ii,uu)
      else
        X(ii,uu)=X(ii,uu)+(dt*dV/2)
      endif
    elseif (FixXY(ii)==4) then
      if (uu==3) then
        X(ii,uu)=X00(ii,uu)
      else
        X(ii,uu)=X(ii,uu)+(dt*dV/2)
      endif
    elseif (FixXY(ii)==5) then
      if (uu==1) then
        X(ii,uu)=X(ii,uu)+(dt*dV/2)
      else
        X(ii,uu)=X00(ii,uu)
      endif
    elseif (FixXY(ii)==6) then
      if (uu==2) then

```



```

        X(ii,uu)=X(ii,uu)+(dt*dV/2)
    else
        X(ii,uu)=X00(ii,uu)
    endif
elseif (FixXY(ii)==7) then
    if (uu==3) then
        X(ii,uu)=X(ii,uu)+(dt*dV/2)
    else
        X(ii,uu)=X00(ii,uu)
    endif
elseif (FixXY(ii)==8) then
    if (uu==2) then
        X(ii,uu)=X00(ii,uu)+deflect
    else
        X(ii,uu)=X00(ii,uu)
    endif
elseif (FixXY(ii)==9) then
    if (uu==2) then
        X(ii,uu)=X00(ii,uu)-deflect
    else
        X(ii,uu)=X00(ii,uu)
    endif
elseif (FixXY(ii)==10) then
    if (uu==1) then
        X(ii,uu)=X00(ii,uu)+deflect
    else
        X(ii,uu)=X(ii,uu)+(dt*dV/2)
    endif
elseif (FixXY(ii)==11) then
    if (uu==1) then
        X(ii,uu)=X00(ii,uu)-deflect
    else
        X(ii,uu)=X(ii,uu)+(dt*dV/2)
    endif
elseif (FixXY(ii)==12) then
    if (uu==2) then
        X(ii,uu)=X00(ii,uu)+deflect
    else
        X(ii,uu)=X(ii,uu)+(dt*dV/2)
    endif

```

```

elseif (FixXY(ii)==13) then
  if (uu==2) then
    X(ii,uu)=X00(ii,uu)-deflect
  else
    X(ii,uu)=X(ii,uu)+(dt*dV/2)
  endif
elseif (FixXY(ii)==14) then
  if (uu==3) then
    X(ii,uu)=X00(ii,uu)+deflect
  else
    X(ii,uu)=X(ii,uu)+(dt*dV/2)
  endif
elseif (FixXY(ii)==15) then
  if (uu==3) then
    X(ii,uu)=X00(ii,uu)-deflect
  else
    X(ii,uu)=X(ii,uu)+(dt*dV/2)
  endif
else
  X(ii,uu)=X(ii,uu)+(dt*dV/2)
endif
else
  X(ii,uu)=X(ii,uu)+(dt*dV/2)
endif
enddo
enddo

```

```

c_____Get new Stress. _____
call GetDensity(X,V,N,dx0,dt,massFct,rho,IsSolid,rho)

```

```

c Calculate the Tensial Stress
Tij=Tij-TTij
do xx=1,3
  do ii=1,N
    mass=massFct(ii)
    if (IsSolid(ii)==1) then

      if (Contact(ii,1)>0) then
        foo=0

```

```

ct2=0
do uu=2, ((Contact(ii,1))+1)
  TTxx=0
  do vv=1,3
    TTxx=TTxx+Tij0(ii,vv,vv)
  enddo

  jj=Contact(ii,uu)
  if (ContactDir(ii,uu)==xx) then
    if (uu>(1+ContactX(ii,1))) then
      call Compression(xx,ii,jj,N,X,Eij,Ey,dx0,mass,a,TT)
    else
      call Tension(xx,ii,jj,N,X,Eij,Ey,dx0,mass,TTxx,a,TT)
    endif
    foo=foo+TT
    ct2=ct2+1
  endif
enddo

if (FixXY(ii)>0) then
  foo=foo/2
endif
if (ct2==0) then
  TTij(ii,xx,xx)=0*TTij(ii,xx,xx)
else
  TTij(ii,xx,xx)=foo/ct2
endif
endif

else
  TTij(ii,xx,xx)=(Bulk_water/3)*((rho(ii)/rho0(ii))-1)
endif
enddo
enddo

```

```

c Calculate the Shear Stress
do vv=1,3
  call GetUV(vv,uv)

```

```

do ii=1,N
  if (IsSolid(ii)==1) then
    foo=0
    do uu=2, ((Contact(ii,1))+1)
      jj=Contact(ii,uu)
      if (ContactDir(ii,uu)==uv(2)) then
        call Shear(uv(1),uv(2),ii,jj,N,X,Eij,mu,dx0,mass,a,T
        foo=foo+TT
      endif
    enddo

    if (FixXY(ii)==0) then
      if (ContactDirCT(ii,uv(2))==0) then
        foo=0
      else
        foo=foo/ContactDirCT(ii,uv(2))
      endif
    else
      foo=foo/2
    endif

    TTij(ii,uv(2),uv(1))=foo
    TTij(ii,uv(1),uv(2))=foo
  endif
enddo

Tij=Tij+TTij

```

c \_\_\_\_\_Get new strain\_\_\_\_\_

```

do kk=1,N
  do ii=1,3
    Eij(kk,ii,ii)=(1/Ey)*(Tij(kk,ii,ii))
    call GetUV(ii,uv)
    foo=(poisson/Ey)*(Tij(kk,uv(1),uv(1)))
    foo=foo+(poisson/Ey)*(Tij(kk,uv(2),uv(2)))
    Eij(kk,ii,ii)=Eij(kk,ii,ii)-foo
  enddo

```

```

do ii=1,3
  do jj=1,3
    if (jj/=ii) then
      Eij(kk,ii,jj)=Tij(kk,ii,jj)/(2*mu)
    endif
  enddo
enddo
enddo

```

c \_\_\_\_\_ Save Data \_\_\_\_\_

```

do ii=1,N
  foo=(Tij(ii,1,1)-Tij(ii,2,2))*2
  foo=foo+(Tij(ii,2,2)-Tij(ii,3,3))*2
  foo=foo+(Tij(ii,1,1)-Tij(ii,3,3))*2
  foo2=(Tij(ii,1,2)**2)+(Tij(ii,2,3)**2)+(Tij(ii,3,1)**2)
  foo=sqrt((foo+(6*foo2))/2)
  Tvm0(ii)=foo
  P(ii)=(Tij(ii,1,1)+Tij(ii,2,2)+Tij(ii,3,3))/3
enddo

TimeFct(ts)=Time
Xout(:,ts)=X(:,1)
Yout(:,ts)=X(:,2)
Zout(:,ts)=X(:,3)
VoutX(:,ts)=V(:,1)
VoutY(:,ts)=V(:,2)
VoutZ(:,ts)=V(:,3)
Rout(:,ts)=rho
Pout(:,ts)=P
T11(:,ts)=Tij(:,1,1)
T12(:,ts)=Tij(:,1,2)
T13(:,ts)=Tij(:,1,3)
T21(:,ts)=Tij(:,2,1)
T22(:,ts)=Tij(:,2,2)
T23(:,ts)=Tij(:,2,3)
T31(:,ts)=Tij(:,3,1)

```

```

T32(:,ts)=Tij(:,3,2)
T33(:,ts)=Tij(:,3,3)
Tvm(:,ts)=Tvm0
E22(:,ts)=Eij(:,2,2)
E11(:,ts)=Eij(:,1,1)
dVdxFct(:,ts)=dVdx(:,1)
dVdyFct(:,ts)=dVdx(:,2)
dVdzFct(:,ts)=dVdx(:,3)

enddo
enddo

call cpu_time ( t2 )
print *,'Elapsed CPU time = ', (t2 - t1),' seconds'

c -----
c  Print data to dat-files
c -----

open (unit = 1, file = "VarDat.dat")
write (1,*)Ey
write (1,*)mu
write (1,*)StudyCt
close (1)

open (unit = 1, file = "X.dat")
do jj=1,Nts
printVar=Xout(:,jj)
write (1,*)printVar
enddo
close(1)

open (unit = 1, file = "Y.dat")
do jj=1,Nts
printVar=Yout(:,jj)
write (1,*)printVar
enddo
close(1)

```

```

open (unit = 1, file = "Z.dat")
do jj=1,Nts
    printVar=Zout(:,jj)
    write (1,*),printVar
enddo
close(1)

open (unit = 1, file = "P.dat")
do jj=1,Nts
    printVar=Pout(:,jj)
    write (1,*),printVar
enddo
close(1)

open (unit = 1, file = "rho.dat")
do jj=1,Nts
    printVar=Rout(:,jj)
    write (1,*),printVar
enddo
close(1)

open (unit = 1, file = "T11.dat")
do jj=1,Nts
    printVar=T11(:,jj)
    write (1,*),printVar
enddo
close(1)

open (unit = 1, file = "T12.dat")
do jj=1,Nts
    printVar=T12(:,jj)
    write (1,*),printVar
enddo
close(1)

open (unit = 1, file = "T13.dat")
do jj=1,Nts
    printVar=T13(:,jj)
    write (1,*),printVar
enddo

```

```

close(1)

open (unit = 1, file = "T21.dat")
do jj=1,Nts
    printVar=T21(:,jj)
    write (1,*),printVar
enddo
close(1)

open (unit = 1, file = "T22.dat")
do jj=1,Nts
    printVar=T22(:,jj)
    write (1,*),printVar
enddo
close(1)

open (unit = 1, file = "T23.dat")
do jj=1,Nts
    printVar=T23(:,jj)
    write (1,*),printVar
enddo
close(1)

open (unit = 1, file = "T31.dat")
do jj=1,Nts
    printVar=T31(:,jj)
    write (1,*),printVar
enddo
close(1)

open (unit = 1, file = "T32.dat")
do jj=1,Nts
    printVar=T32(:,jj)
    write (1,*),printVar
enddo
close(1)

open (unit = 1, file = "T33.dat")
do jj=1,Nts
    printVar=T33(:,jj)

```



```

        write (1,*),printVar
    enddo
close(1)

open (unit = 1, file = "E22.dat")
do jj=1,Nts
    printVar=E22(:,jj)
    write (1,*),printVar
enddo
close(1)

open (unit = 1, file = "E11.dat")
do jj=1,Nts
    printVar=E11(:,jj)
    write (1,*),printVar
enddo
close(1)

open (unit = 1, file = "VonMises.dat")
do jj=1,Nts
    printVar=Tvm(:,jj)
    write (1,*),printVar
enddo
close(1)

open (unit = 1, file = "Vx.dat")
do jj=1,Nts
    printVar=VoutX(:,jj)
    write (1,*),printVar
enddo
close(1)

open (unit = 1, file = "Vy.dat")
do jj=1,Nts
    printVar=VoutY(:,jj)
    write (1,*),printVar
enddo
close(1)

open (unit = 1, file = "Vz.dat")

```

```

do jj=1,Nts
  printVar=VoutZ(:,jj)
  write (1,*) ,printVar
enddo
close(1)

open (unit = 1, file = "Time.dat")
do jj=1,Nts
  printVar=TimeFct(jj)
  write (1,*) ,printVar
enddo
close(1)

```

c -----

```

open (unit = 1, file = "Fixed.dat")
do jj=1,N
  write (1,*) ,FixXY(jj)
enddo
close (1)

open (unit = 1, file = "Contact.dat")
do jj=1,N
  write (1,*) ,Contact(jj,:)
enddo
close (1)

open (unit = 1, file = "ContactDir.dat")
do jj=1,N
  write (1,*) ,ContactDir(jj,:)
enddo
close (1)

open (unit = 1, file = "ContactDirCT.dat")
do jj=1,N
  write (1,*) ,ContactDirCT(jj,:)
enddo
close (1)

open (unit = 1, file = "ContactX.dat")

```

```

do jj=1,N
    write (1,*) ,ContactX(jj,:)
enddo
close (1)

open (unit = 1, file = "ContactDirX.dat")
do jj=1,N
    write (1,*) ,ContactDirX(jj,:)
enddo
close (1)

open (unit = 1, file = "ContactDirCTX.dat")
do jj=1,N
    write (1,*) ,ContactDirCTX(jj,:)
enddo
close (1)

open (unit = 1, file = "Links.dat")
do jj=1,N
    write (1,*) ,Links(jj,:)
enddo
close (1)

open (unit = 1, file = "Mass.dat")
do jj=1,N
    write (1,*) ,massFct(jj)
enddo
close (1)

open (unit = 1, file = "dX.dat")
do jj=1,N
    write (1,*) ,dx0(jj)
enddo
close (1)

open (unit = 1, file = "X0.dat")
do jj=1,N
    write (1,*) ,X(jj,:)
enddo

```

```

close (1)

open (unit = 1, file = "V0.dat")
do jj=1,N
  write (1,*) ,V(jj,:)
enddo
close (1)

open (unit = 1, file = "dVdX.dat")
do jj=1,N
  write (1,*) ,dVdxFct(jj,:)
enddo
close (1)

open (unit = 1, file = "dVdY.dat")
do jj=1,N
  write (1,*) ,dVdyFct(jj,:)
enddo
close (1)

open (unit = 1, file = "dVdZ.dat")
do jj=1,N
  write (1,*) ,dVdzFct(jj,:)
enddo
close (1)

end program

```

c

---

c

---

```

subroutine GetUV(ii,uv)
-----
implicit none

integer, intent(in) :: ii
integer, intent(out) :: uv(2)
integer jj,ct

```

```

ct=0
do jj=1,3
  if (jj/=ii) then
    ct=ct+1
    uv(ct)=jj
  endif
enddo

END subroutine GetUV

```

c

---

c

```

subroutine FindL(N,h,mu,X,V,P,Tij,DSDtN)
-----
implicit none

integer, intent(in) :: N
double precision, intent(in) :: Tij(N,3,3)
double precision, intent(in) :: X(N,3),V(N,3),h
double precision, intent(in) :: mu
double precision, intent(out) :: DSDtN(N,3,3),P(N)

integer ii,jj, kk, uu, vv
double precision L(3,3),D(3,3),Omg(3,3),r,W,Wtotal
double precision dV,dx,D0(3,3),Omg0(3,3),DSDt(3,3)
double precision foo,foo1,foo2,foo3,foo4,Sij(3,3)
double precision dwdx(3)

do jj=1,N
  Wtotal = 0
  do vv=1,3
    do uu=1,3
      L(uu,vv)=0
    enddo
  enddo
  do ii=1,N
    call kernel(X(jj,:),X(ii,:),h,W,dwdx,r)
    do vv = 1,3

```

```

do uu = 1,3
  dV=(V(ii,uu)-V(jj,uu))
  dx=(X(ii,vv)-X(jj,vv))
  if (dx==0) then
    L(uu,vv)=L(uu,vv)
  else
    L(uu,vv)=L(uu,vv)+(dwdx(vv)*dV/dx)
    Wtotal=Wtotal+W
  endif
enddo
enddo
L=L*0/Wtotal

do vv=1,3
  do uu=1,3
    D(uu,vv)=(L(uu,vv)+L(vv,uu))/2
    Omg(uu,vv)=(L(uu,vv)-L(vv,uu))/2
  enddo
enddo

do vv=1,3
  do uu=1,3
    Sij(uu,vv)=Tij(jj,uu,vv)
    if (uu==vv) then
      Sij(uu,vv)=Sij(uu,vv)-P(jj)
    endif
  enddo
enddo

do vv=1,3
  do uu=1,3
    foo1=0
    foo2=0
    do kk=1,3
      foo1=foo1+(Sij(uu,kk)*Omg(vv,kk))
      foo2=foo2+(Omg(uu,kk)*Sij(kk,vv))
    enddo
    foo3=2*mu*D(uu,vv)
  enddo
enddo

```

```

        if (uu==vv) then
            foo4=(-2*mu/3)*D(uu,vv)
        else
            foo4=0
        endif
        DSDtN(jj,uu,vv)=foo1+foo2+foo3+foo4
    enddo
enddo

```

```

enddo

```

```

END subroutine FindL

```

```

c

```

---

```

    subroutine kernel(x1,x2,h,w,dwdx,r)

```

```

c-----

```

```

c  Subroutine to calculate the smoothing kernel wij and its
c  derivatives dwdxi j.

```

```

c      r      : Distance between particles i and j           [in]
c      dx     : x-, y- and z-distance between i and j       [in]
c      h      : Smoothing length                             [in]
c      w      : Kernel for all interaction pairs             [out]
c      dwdx   : Derivative of kernel with respect to x, y and z [out]

```

```

    implicit none

```

```

    double precision, intent(in) :: x1(3),x2(3),h
    double precision, intent(out) :: w,dwdx(3),r
    integer i, j, d, skf
    double precision q, dw, factor,dx(3),pi,u

```

```

    do i=1,3
        dx(i)=abs(X1(i)-X2(i))
    enddo
    r=(dx(1)**2)+(dx(2)**2)+(dx(3)**2)

```

```

r=sqrt(r)

pi = 3.14159265358979
u=10/(7*pi)
q = r/h
w = 0.e0
do d=1,3
    dwdx(d) = 0.e0
enddo

if (r==0) then
    w=0
    dwdx(1)=0
    dwdx(2)=0
    dwdx(3)=0
else
    if (q<1) then
        w=1+(-1.5*(q**2))+(0.75*(q**3))
        do i=1,3
            dwdx(i)=((9*r/(4*h))-3)*(dx(i)/(h**2))
        enddo
    else
        if (q<2) then
            w=((2-q)**3)/4
            do i=1,3
                dwdx(i)=(-3/(4*h*r))*((2-q)**2)*dx(i)
            enddo
        else
            w=0
            dwdx(1)=0
            dwdx(2)=0
            dwdx(3)=0
        endif
    endif
endif
w=w*u
do i=1,3
    dwdx(i)=u*dwdx(i)
enddo

```



END subroutine kernel

c

---

subroutine GetDPDT(N,X,V,dx,K,massFct,rho,DPDT)

c

---

implicit none

integer, intent(in) :: N  
double precision, intent(in) :: X(N,3),V(N,3)  
double precision, intent(in) :: dx(N),massFct(N)  
double precision, intent(in) :: K,rho(N)  
double precision, intent(out) :: DPDT(N,3)

integer ii,jj, kk  
double precision W0, dwdx(3),foo,dV,r  
double precision h,mass

do jj=1,N  
  h=2\*dx(jj)  
  mass=massFct(jj)  
  do ii=1,N  
    call kernel(X(ii,:),X(jj,:),h,W0,dwdx,r)  
    foo=0  
    do kk=1,3  
      dV=V(ii,kk)-V(jj,kk)  
      foo=-K\*mass\*dV\*dwdx(kk)/rho(ii)  
      DPDT(jj,kk)=DPDT(jj,kk)+foo  
    enddo  
  enddo  
enddo

END subroutine GetDPDT

c

---

subroutine GetDensity0(X,N,dx,mass,rho0,IsSolid,rho)

c

---

implicit none

```

integer, intent(in) :: N, IsSolid(N)
double precision, intent(in) :: X(N,3), rho0(N)
double precision, intent(in) :: dx(N), mass(N)
double precision, intent(out) :: rho(N)

integer ii, jj, kk
double precision W0, r0, fooN, fooD, h, r, dwdx(3)
double precision rhoIn(N)

rhoIn=rho0
do jj=1,N
  fooN=0
c    fooD=0
  h = 3*dx(jj)
  do ii=1,N
    if (IsSolid(ii)==IsSolid(jj)) then
      call kernel(X(ii,:),X(jj,:),h,W0,dwdx,r)
      fooN=fooN+(mass(ii)*W0)
    endif
  enddo
  rho(jj)=((3.14159*4/3)*fooN/(h**3))
enddo

END subroutine GetDensity0

```

```

c -----
subroutine GetDensity(X,V,N,dx,dt,massFct,rho0,IsSolid,rho)
c -----
implicit none

integer, intent(in) :: N, IsSolid(N)
double precision, intent(in) :: X(N,3), V(N,3), rho0(N)
double precision, intent(in) :: dt,dx(N), massFct(N)
double precision, intent(out) :: rho(N)

integer ii, jj, kk
double precision W,r,dwdx(3),foo1,foo2
double precision h,mass,rhoIn(N)

```

```

rhoIn=rho0
do jj=1,N
  h=dx(jj)
  mass=massFct(jj)
  foo1=0
  do ii=1,N
    if (IsSolid(ii)==IsSolid(jj)) then
      call kernel(X(ii,:),X(jj,:),h,W,dwdx,r)
      do kk=1,3
        foo1=foo1-((mass/rhoIn(ii))*V(ii,kk)*dwdx(kk))
      enddo
    else
      foo1=0
    endif
  enddo
  foo2=dt*foo1*rhoIn(jj)
  rho(jj)=rhoIn(jj)+foo2
enddo

```

END subroutine GetDensity

c

---

```

subroutine SpeedSound(YM,poisson,rho0min,Csound)

implicit none

double precision, intent(in) :: YM,poisson,rho0min
double precision, intent(out) :: Csound
double precision CsoundT,CsoundS,G

CsoundT=(YM*(1-poisson))/rho0min
CsoundT=CsoundT/((1+poisson)*(1-(2*poisson)))
CsoundT=sqrt(CsoundT)

G=(3*(1-poisson)/(1+poisson))-1
G=G*YM/(4*(1-(2*poisson)))
CsoundS=sqrt(G/rho0min)

```

```

if (CsoundT>CsoundS) then
  Csound=CsoundT
else
  Csound=CsoundS
endif

end subroutine SpeedSound

```

c

---

```

subroutine LJfct (p,X1,X2,V1,V2,r0,dt,a)

implicit none

integer, intent(in) :: p
double precision, intent(in) :: X1(3),X2(3)
double precision, intent(in) :: V1(3),V2(3)
double precision, intent(in) :: r0,dt
double precision, intent(out) :: a

integer ii
double precision foo,r,r00,coeff,dir,foo2

r=0
do ii=1,3
  r=r+((X1(ii)-X2(ii))**2)
enddo
r=sqrt(r)
if ((abs(X1(p)-X2(p)))==0) then
  dir=0
else
  dir=(X1(p)-X2(p))/abs(X1(p)-X2(p))
endif

if (r>r0) then
  foo=0
else
  coeff=((r0/r)**12)-((r0/r)**4)
  foo=(V1(p)-V2(p))*coeff*dir/dt

```

```

endif
a=foo

end subroutine LJfct

```

c

---

c

---

```

subroutine LJfctSolid(p,X1,X2,r0,P2,f)

implicit none

integer, intent(in) :: p
double precision, intent(in) :: X1(3),X2(3)
double precision, intent(in) :: P2,r0
double precision, intent(out) :: f

integer ii
double precision foo,r,del,trig

r=0
do ii=1,3
    r=r+((X1(ii)-X2(ii))**2)
enddo
del=(X1(p)-X2(p))
r=sqrt(r)
trig=del/r

if (r>r0) then
    foo=0
else
    foo=trig*P2*r0*r0
endif
f=foo

end subroutine LJfctSolid

```

```
subroutine Tension(xx,ii,jj,N,X,E,Ey,dx0,mass,TTxx,a,TT)

implicit none

integer, intent(in) :: xx,ii,jj,N
double precision, intent(in) :: X(N,3),E(N,3,3)
double precision, intent(in) :: Ey,dx0(N),mass,TTxx
double precision, intent(out) :: a,TT

integer ct,uv(2)
double precision foo,dx,strain,Ai,A0,dir
double precision dxE,strainE,a2,TT1,TT2,TTxxR

A0=(dx0(ii))*2
dx=X(jj,xx)-X(ii,xx)
dxE=(1+((E(ii,xx,xx)+E(jj,xx,xx))/2))*dx0(ii)

if (abs(dx)>0) then
    dir=dx/abs(dx)
else
    dir=0
endif

strain=(abs(dx)-dx0(ii))/dx0(ii)
strainE=(abs(dx)-dxE)/dxE

a=0
if (ii==jj) then
    a=0
    TT1=0
else

    call GetUV(xx,uv)
    Ai=A0*(1+E(ii,uv(1),uv(1)))
    Ai=Ai*(1+E(ii,uv(2),uv(2)))
    a=dir*Ai*Ey*strainE/mass
```

```

        if (strain>0) then
            TT1=Ey*strain
        else
            TT1=0
        endif
    endif

    call Compression(xx,ii,jj,N,X,E,Ey,dx0,mass,a2,TT2)

    TTxxR=TTxx
    if (TTxxR>0) then
        TT=TT1
    elseif (TTxxR<0) then
        TT=TT2
    else
        if (strainE<0) then
            TT=TT2
        elseif (strainE>0) then
            TT=TT1
        else
            TT=(TT1+TT2)/2
        endif
    endif

end subroutine Tension

```

c

---

```

subroutine Compression(xx,ii,jj,N,X,E,Ey,dx0,mass,a,TT)

implicit none

integer, intent(in) :: xx,ii,jj,N
double precision, intent(in) :: X(N,3),E(N,3,3)
double precision, intent(in) :: Ey,dx0(N),mass
double precision, intent(out) :: a,TT

integer ct,uv(2)
double precision foo,dx,strain,Ai,A0,dir

```

```

double precision dxE, strainE

A0=(dx0(ii))**2
dx=X(jj,xx)-X(ii,xx)
dxE=(1+((E(ii,xx,xx)+E(jj,xx,xx))/2))*dx0(ii)

if (abs(dx)>0) then
    dir=dx/abs(dx)
else
    dir=0
endif

strain=(abs(dx)-dx0(ii))/dx0(ii)
strainE=(abs(dx)-dxE)/dxE

a=0
if (ii==jj) then
    a=0
    TT=0
else

    call GetUV(xx,uv)
    Ai=A0*(1+E(ii,uv(1),uv(1)))
    Ai=Ai*(1+E(ii,uv(2),uv(2)))
    a=dir*Ai*Ey*strainE/mass

    if (strain<0) then
        TT=Ey*strain
    else
        TT=0
    endif
endif

if (TT>0) then
    TT=0
    a=0
endif

end subroutine Compression

```



```
subroutine Shear(xx,yy,ii,jj,N,X,E,G,dx0,mass,a,TT)

implicit none

integer, intent(in) :: xx,yy,ii,jj,N
double precision, intent(in) :: X(N,3),E(N,3,3)
double precision, intent(in) :: G,dx0(N),mass
double precision, intent(out) :: a,TT

integer ct,uv(2)
double precision delta,gamma,L
double precision foo,Ai,A0,dir

L=dx0(ii)
A0=(dx0(ii))*2
delta=(X(jj,xx)-X(ii,xx))
if (abs(delta)>0) then
    dir=delta/abs(delta)
else
    dir=0
endif
delta=abs(delta)
gamma=delta/L

if (ii==jj) then
    a=0
    TT=0
else
    if (gamma>0) then
        TT=G*gamma
    else
        TT=0
    endif
endif
```

```

      if (gamma>0) then
        call GetUV(yy,uv)
        Ai=A0*(1+E(ii,uv(1),uv(1)))
        Ai=Ai*(1+E(ii,uv(2),uv(2)))
        a=dir*Ai*G*gamma/mass
      else
        a=0
      endif

endif

end subroutine Shear

```

c

---

```

subroutine ContactStress(N,TijIn,Contact,ContactDir,TijOut)

implicit none

integer, intent(in) :: N,Contact(N,N+1),ContactDir(N,N+1)
double precision, intent(in) :: TijIn(N,3,3)
double precision, intent(out) :: TijOut(N,3,3)

integer ii,jj,xx,uu,cycle,ContactDirCTx
double precision Tij0(N,3,3),Tij(N,3,3)
double precision foo>Total(3),Avg(3)

Tij0=TijIn

do cycle=1,1000
do ii=1,N
  do xx=1,3
    Avg(xx)=0
    ContactDirCTx=0
    do jj=2,(Contact(ii,1))
      if (ContactDir(ii,jj)==xx) then
        uu=Contact(ii,jj)

```

```

        Avg(xx)=Avg(xx)+Tij0(uu,xx,xx)
        ContactDirCTx=ContactDirCTx+1
    endif
enddo
if (ContactDirCTx==0) then
    Tij(ii,xx,xx)=Tij0(ii,xx,xx)
else
    foo=Avg(xx)/ContactDirCTx
    Tij(ii,xx,xx)=(Tij0(ii,xx,xx)+foo)/2
endif
enddo
enddo
Tij0=Tij
enddo
TijOut=Tij

end subroutine ContactStress

```

c

---

```

subroutine LinkList(N,X,h,Links)

implicit none

integer, intent(in) :: N
double precision, intent(in) :: X(N,3),h
integer, intent(out) :: Links(N,(N+1))

integer ii,jj,xx,uu,ct,fooInt,LinkCT(3),LLloc(N,3)
double precision foo,DistCT(3),Wij,dwdx(3),r

do ii=1,3
    DistCT(ii)=MAXVAL(X(:,ii))-MINVAL(X(:,ii))
    LinkCT(ii)=CEILING(DistCT(ii)/h)
enddo
Links(:,:)=0

do ii=1,N

```

```

do jj=1,3
  foo=(X(ii,jj)-MINVAL(X(:,jj)))/DistCT(jj)
  foo=ceiling(foo*LinkCT(jj))
  if (foo==0) then
    foo=foo+1
  endif
  LLloc(ii,jj)=foo
enddo
enddo

do ii=1,N
  ct=1
  do jj=1,N
    xx=1
    do uu=1,3
      fooInt=abs(LLloc(ii,uu)-LLloc(jj,uu))
      if (fooInt>1) then
        xx=0
      endif
    enddo
    if (xx==1) then
      call kernel(X(ii,:),X(jj,:),h,Wij,dwdx,r)
      if (Wij>0) then
        ct=ct+1
        Links(ii,ct)=jj
      endif
    endif
  enddo
  Links(ii,1)=ct-1
enddo

end subroutine LinkList

```

## tension\_steel.f

```
subroutine ModelBlock(Nout)

implicit none

integer, intent(out) :: Nout
integer N,ctbar(3),Nsphere,ct,ii,jj,kk,uu
integer fooint,split
double precision RadiusFactor,Radius,masX,dx00
double precision angleFrac,angle,Yr0,dx3(3),third
double precision Zloc,Yloc,Xloc,r,pi,rho_solid
integer, allocatable, dimension(:) :: FixXY,IsSolid,BreakContact
integer, allocatable, dimension(:,:) :: Contact,ContactX
integer, allocatable, dimension(:,:) :: ContactDir,ContactDirX
integer, allocatable, dimension(:,:) :: ContactDirCT,ContactDirC
double precision, allocatable, dimension(:,:) :: X,V,dVdx
double precision, allocatable, dimension(:) :: massFct,dx0
double precision, allocatable, dimension(:) :: rho0,rho

open (unit = 1, file = "SimSpec.dat")
  write (1,*),207e9      ! Young's Modulus of Steel
  write (1,*),0.3        ! Poisson's Ratio of Steel
  write (1,*),7800       ! Density of Steel
  write (1,*),0          ! Forced Density in X direction (Vtest)
  write (1,*),1e3        ! Forced Density in Y direction (Vtest)
  write (1,*),0          ! Forced Density in Z direction (Vtest)
  write (1,*),0.3        ! Gamma (blend stress)
  write (1,*),250        ! Number of Recorded Time Steps (Nts)
  write (1,*),6          ! Time Steps between recorded Time Steps
  write (1,*),1          ! Time Steps between recorded Time Steps
  write (1,*),0.18       ! (StartDefl)
  write (1,*),0.05       ! (NewDefl)
  write (1,*),0.25       ! (dTfactor)
  write (1,*),0.01       ! (dTdxfactor)
  write (1,*),250        ! (stop)
  write (1,*),2.15e9     ! (Bulk_water)
close (1)
```

```

ctbar(1)=3      ! 3
ctbar(2)=11     ! 11
ctbar(3)=3      ! 3
masX=0.1
rho_solid=7800

```

```

third=1.0/3.0
dx00=(masX/rho_solid)**third
N=(ctbar(1)*ctbar(2)*ctbar(3))

```

```

c _____
c                      ALLOCATION
c _____

```

```

ALLOCATE (FixXY(N))
ALLOCATE (IsSolid(N))
ALLOCATE (BreakContact(N))
ALLOCATE (X(N,3))
ALLOCATE (V(N,3))
ALLOCATE (dVdx(N,3))
ALLOCATE (rho(N))
ALLOCATE (rho0(N))
ALLOCATE (massFct(N))
ALLOCATE (dx0(N))
ALLOCATE (Contact(N,(N+1)))
ALLOCATE (ContactDir(N,(N+1)))
ALLOCATE (ContactDirCT(N,3))
ALLOCATE (ContactX(N,(N+1)))
ALLOCATE (ContactDirX(N,(N+1)))
ALLOCATE (ContactDirCTX(N,3))

```

```

c _____

```

```

c Set the parameters for the steel bar
ct = 0
Zloc=-dx00
do uu=1,ctbar(3)
  Zloc=Zloc+dx00
  Yloc=-dx00
  do jj=1,ctbar(2)
    Yloc = Yloc + dx00
    Xloc = 0
    do ii=1,ctbar(1)
      ct = ct + 1
      if (jj==ctbar(2)) then
        FixXY(ct)=12
      elseif (jj==1) then
        FixXY(ct)=13
      else
        FixXY(ct)=0
      endif
      Xloc = Xloc + dx00
      X(ct,1) = Xloc
      X(ct,2) = Yloc
      X(ct,3) = Zloc
      V(ct,1) = 0
      V(ct,2) = 0
      V(ct,3) = 0
      dVdX(ct,1) = 0
      dVdX(ct,2) = 0
      dVdX(ct,3) = 0
      rho(ct) = rho_solid
      rho0(ct) = rho_solid
      dx0(ct) = dx00
      IsSolid(ct) = 1
      massFct(ct) = masX
      if (jj==(ctbar(2)/2)) then
        BreakContact(ct)=1
      elseif (jj==(1+(ctbar(2)/2))) then
        BreakContact(ct)=2

```

```

        else
            BreakContact(ct)=0
        endif
    enddo
enddo
enddo

c Set Contact links
do jj=1,N
    if (IsSolid(jj)==1) then
        ct=0
        do ii=1,N
            if (IsSolid(ii)==1) then
                if (ii/=jj) then
                    r=0
                    do kk=1,3
                        dx3(kk)=abs(X(ii,kk)-X(jj,kk))
                        r=r+((X(ii,kk)-X(jj,kk))**2)
                    enddo
                    r=sqrt(r)
                    if (r<(dx0(jj)*1.1)) then
                        split=1
                        if (BreakContact(ii)==1) then
                            if (BreakContact(jj)==2) then
                                split=0
                            endif
                        endif
                        if (BreakContact(ii)==2) then
                            if (BreakContact(jj)==1) then
                                split=0
                            endif
                        endif
                        split=1
                        if (split==1) then
                            ct=ct+1
                            Contact(jj,ct+1)=ii
                            fooint=MAXLOC(dx3,1)
                            ContactDir(jj,ct+1)=fooint
                            ContactDirCT(jj,fooint)=ContactDirCT(jj,fooint)+1
                        endif
                    endif
                endif
            endif
        enddo
    endif
enddo

```



```

endif
endif
endif
endif
enddo
Contact(jj,1)=ct
ContactDir(jj,1)=ct
endif
enddo

ContactX=Contact
ContactDirX=ContactDir
ContactDirCTX=ContactDirCT

```

```

open (unit = 1, file = "Fixed.dat")
do jj=1,N
  write (1,*) ,FixXY(jj)
enddo
close (1)

```

```

open (unit = 1, file = "ContactDirX.dat")
do jj=1,N
  write (1,*) ,ContactDirX(jj,:)
enddo
close (1)

```

```

open (unit = 1, file = "Contact.dat")
do jj=1,N
  write (1,*) ,Contact(jj,:)
enddo
close (1)

```

```

open (unit = 1, file = "ContactDir.dat")
do jj=1,N
  write (1,*) ,ContactDir(jj,:)
enddo
close (1)

```

```

open (unit = 1, file = "ContactDirCT.dat")
do jj=1,N
    write (1,*),ContactDirCT(jj,:)
enddo
close (1)

open (unit = 1, file = "ContactX.dat")
do jj=1,N
    write (1,*),ContactX(jj,:)
enddo
close (1)

open (unit = 1, file = "ContactDirCTX.dat")
do jj=1,N
    write (1,*),ContactDirCTX(jj,:)
enddo
close (1)

open (unit = 1, file = "Mass.dat")
do jj=1,N
    write (1,*),massFct(jj)
enddo
close (1)

open (unit = 1, file = "dX.dat")
do jj=1,N
    write (1,*),dx0(jj)
enddo
close (1)

open (unit = 1, file = "X0.dat")
do jj=1,N
    write (1,*),X(jj,:)
enddo
close (1)

open (unit = 1, file = "V0.dat")
do jj=1,N
    write (1,*),V(jj,:)
enddo

```

```

close (1)

open (unit = 1, file = "dVdX.dat")
do jj=1,N
  write (1,*) ,dVdx(jj,:)
enddo
close (1)

open (unit = 1, file = "rho0.dat")
do jj=1,N
  write (1,*) ,rho0(jj)
enddo
close (1)

open (unit = 1, file = "rho_init.dat")
do jj=1,N
  write (1,*) ,rho(jj)
enddo
close (1)

open (unit = 1, file = "IsSolid.dat")
do jj=1,N
  write (1,*) ,IsSolid(jj)
enddo
close (1)

Nout=N

end subroutine ModelBlock

```

c

---

## compress\_steel.f

```
subroutine ModelBlock(Nout)

implicit none

integer, intent(out) :: Nout
integer N,ctbar(3),Nsphere,ct,ii,jj,kk,uu
integer fooint,split
double precision RadiusFactor,Radius,masX,dx00
double precision angleFrac,angle,Yr0,dx3(3),third
double precision Zloc,Yloc,Xloc,r,pi,rho_solid
integer, allocatable, dimension(:) :: FixXY,IsSolid,BreakContact
integer, allocatable, dimension(:,:) :: Contact,ContactX
integer, allocatable, dimension(:,:) :: ContactDir,ContactDirX
integer, allocatable, dimension(:,:) :: ContactDirCT,ContactDirC
double precision, allocatable, dimension(:,:) :: X,V,dVdx
double precision, allocatable, dimension(:) :: massFct,dx0
double precision, allocatable, dimension(:) :: rho0,rho

open (unit = 1, file = "SimSpec.dat")
  write (1,*),207e9      ! Young's Modulus of Steel
  write (1,*),0.3        ! Poisson's Ratio of Steel
  write (1,*),7800       ! Density of Steel
  write (1,*),0          ! Forced Density in X direction (Vtest)
  write (1,*),-1e3       ! Forced Density in Y direction (Vtest)
  write (1,*),0          ! Forced Density in Z direction (Vtest)
  write (1,*),0.3        ! Gamma (blend stress)
  write (1,*),250        ! Number of Recorded Time Steps (Nts)
  write (1,*),6          ! Time Steps between recorded Time Steps
  write (1,*),1          ! Time Steps between recorded Time Steps
  write (1,*),0.18       ! (StartDefl)
  write (1,*),0.05       ! (NewDefl)
  write (1,*),0.25       ! (dTfactor)
  write (1,*),0.01       ! (dTdxfactor)
  write (1,*),250        ! (stop)
  write (1,*),2.15e9     ! (Bulk_water)
close (1)
```

```

ctbar(1)=3      ! 3
ctbar(2)=11     ! 11
ctbar(3)=3      ! 3
masX=0.1
rho_solid=7800

```

```

third=1.0/3.0
dx00=(masX/rho_solid)**third
N=(ctbar(1)*ctbar(2)*ctbar(3))

```

```

c _____
c                      ALLOCATION
c _____

```

```

ALLOCATE (FixXY(N))
ALLOCATE (IsSolid(N))
ALLOCATE (BreakContact(N))
ALLOCATE (X(N,3))
ALLOCATE (V(N,3))
ALLOCATE (dVdx(N,3))
ALLOCATE (rho(N))
ALLOCATE (rho0(N))
ALLOCATE (massFct(N))
ALLOCATE (dx0(N))
ALLOCATE (Contact(N,(N+1)))
ALLOCATE (ContactDir(N,(N+1)))
ALLOCATE (ContactDirCT(N,3))
ALLOCATE (ContactX(N,(N+1)))
ALLOCATE (ContactDirX(N,(N+1)))
ALLOCATE (ContactDirCTX(N,3))

```

```

c _____

```

```

c Set the parameters for the steel bar
ct = 0
Zloc=-dx00
do uu=1,ctbar(3)
  Zloc=Zloc+dx00
  Yloc=-dx00
  do jj=1,ctbar(2)
    Yloc = Yloc + dx00
    Xloc = 0
    do ii=1,ctbar(1)
      ct = ct + 1
      if (jj==ctbar(2)) then
        FixXY(ct)=12
      elseif (jj==1) then
        FixXY(ct)=13
      else
        FixXY(ct)=0
      endif
      Xloc = Xloc + dx00
      X(ct,1) = Xloc
      X(ct,2) = Yloc
      X(ct,3) = Zloc
      V(ct,1) = 0
      V(ct,2) = 0
      V(ct,3) = 0
      dVdX(ct,1) = 0
      dVdX(ct,2) = 0
      dVdX(ct,3) = 0
      rho(ct) = rho_solid
      rho0(ct) = rho_solid
      dx0(ct) = dx00
      IsSolid(ct) = 1
      massFct(ct) = masX
      if (jj==(ctbar(2)/2)) then
        BreakContact(ct)=1
      elseif (jj==(1+(ctbar(2)/2))) then
        BreakContact(ct)=2
      else
        BreakContact(ct)=0
      endif
    enddo
  enddo
enddo

```

```

endif
enddo
enddo
enddo

c Set Contact links
do jj=1,N
  if (IsSolid(jj)==1) then
    ct=0
    do ii=1,N
      if (IsSolid(ii)==1) then
        if (ii/=jj) then
          r=0
          do kk=1,3
            dx3(kk)=abs(X(ii, kk)-X(jj, kk))
            r=r+((X(ii, kk)-X(jj, kk))**2)
          enddo
          r=sqrt(r)
          if (r<(dx0(jj)*1.1)) then
            split=1
            if (BreakContact(ii)==1) then
              if (BreakContact(jj)==2) then
                split=0
              endif
            endif
            if (BreakContact(ii)==2) then
              if (BreakContact(jj)==1) then
                split=0
              endif
            endif
            split=1
            if (split==1) then
              ct=ct+1
              Contact(jj, ct+1)=ii
              fooint=MAXLOC(dx3,1)
              ContactDir(jj, ct+1)=fooint
              ContactDirCT(jj, fooint)=ContactDirCT(jj, fooint)+1
            endif
          endif
        endif
      endif
    enddo
  endif
enddo

```

```

        endif
    endif
enddo
    Contact(jj,1)=ct
    ContactDir(jj,1)=ct
endif
enddo

```

```

ContactX=Contact
ContactDirX=ContactDir
ContactDirCTX=ContactDirCT

```

```

open (unit = 1, file = "Fixed.dat")
do jj=1,N
    write (1,*) ,FixXY(jj)
enddo
close (1)

```

```

open (unit = 1, file = "ContactDirX.dat")
do jj=1,N
    write (1,*) ,ContactDirX(jj,:)
enddo
close (1)

```

```

open (unit = 1, file = "Contact.dat")
do jj=1,N
    write (1,*) ,Contact(jj,:)
enddo
close (1)

```

```

open (unit = 1, file = "ContactDir.dat")
do jj=1,N
    write (1,*) ,ContactDir(jj,:)
enddo
close (1)

```

```

open (unit = 1, file = "ContactDirCT.dat")
do jj=1,N

```



```

        write (1,*),ContactDirCT(jj,:)
    enddo
close (1)

open (unit = 1, file = "ContactX.dat")
do jj=1,N
    write (1,*),ContactX(jj,:)
enddo
close (1)

open (unit = 1, file = "ContactDirCTX.dat")
do jj=1,N
    write (1,*),ContactDirCTX(jj,:)
enddo
close (1)

open (unit = 1, file = "Mass.dat")
do jj=1,N
    write (1,*),massFct(jj)
enddo
close (1)

open (unit = 1, file = "dX.dat")
do jj=1,N
    write (1,*),dx0(jj)
enddo
close (1)

open (unit = 1, file = "X0.dat")
do jj=1,N
    write (1,*),X(jj,:)
enddo
close (1)

open (unit = 1, file = "V0.dat")
do jj=1,N
    write (1,*),V(jj,:)
enddo
close (1)

```

```

open (unit = 1, file = "dVdX.dat")
do jj=1,N
    write (1,*),dVdx(jj,:)
enddo
close (1)

open (unit = 1, file = "rho0.dat")
do jj=1,N
    write (1,*),rho0(jj)
enddo
close (1)

open (unit = 1, file = "rho_init.dat")
do jj=1,N
    write (1,*),rho(jj)
enddo
close (1)

open (unit = 1, file = "IsSolid.dat")
do jj=1,N
    write (1,*),IsSolid(jj)
enddo
close (1)

Nout=N

end subroutine ModelBlock

```

c

---

## ModelSmFixDisk.f

```
subroutine ModelBlock(Nout)

implicit none

integer, intent(out) :: Nout
integer N,ctbar(3),Nsphere,ct,ii,jj,kk,DiskRat
integer fooint
double precision RadiusFactor,Radius,masX,masXdisk,dx0x,dx0xDisk
double precision angleFrac,angle,Yr0,dx3(3)
double precision Zloc,Zloc2,Yloc,Xloc,r,mass,pi,rho_solid
integer, allocatable, dimension(:) :: FixXY,IsSolid
integer, allocatable, dimension(:,:) :: Contact,ContactX
integer, allocatable, dimension(:,:) :: ContactDir,ContactDirX
integer, allocatable, dimension(:,:) :: ContactDirCT,ContactDirC
double precision, allocatable, dimension(:,:) :: X,V,dVdx
double precision, allocatable, dimension(:) :: massFct,dx0,rho0
```

c      Start the Specification Parameters

```
open (unit = 1, file = "SimSpec.dat")
  write (1,*),207e9            ! Young's Modulus of Steel
  write (1,*),0.3            ! Poisson's Ratio of Steel
  write (1,*),7800            ! Density of Steel
  write (1,*),0.0            ! Forced Density in X direction (Vtest)
  write (1,*),100.0          ! Forced Density in Y direction (Vtest)
  write (1,*),0.0            ! Forced Density in Z direction (Vtest)
  write (1,*),0.3            ! Gamma (blend stress)
  write (1,*),100            ! Number of Recorded Time Steps (Nts)
  write (1,*),1            ! Time Steps between recorded Time Steps
  write (1,*),1            ! Time Steps between recorded Time Steps
  write (1,*),5*2.54e-6      ! (StartDefl)
  write (1,*),5*2.54e-6      ! (NewDefl)
  write (1,*),0.05           ! (dTfactor)
  write (1,*),0.005          ! (dTdxfactor)
  write (1,*),15            ! (stop)
```

```

write (1,*),2.15e9      ! (Bulk_water)
close (1)

```

```

ctbar(1)=101
ctbar(2)=10
ctbar(3)=1
Radius=20.0*(2.54e-4)
DiskRat=3
dx0x=0.25*(Radius/20.0)
pi=ACOS(-1.0)
rho_solid=7800

```

```

if ((ctbar(1)*dx0x)>(2*Radius)) then
  angleFrac=pi
else
  angleFrac=2*ASIN((ctbar(1)*dx0x)/(2*Radius))
  angleFrac=angleFrac/2
endif

```

```

dx0xDisk=dx0x/DiskRat
masX=(dx0x**3)*rho_solid
masXdisk=(dx0xDisk**3)*rho_solid

```

```

Nsphere=(angleFrac*Radius/dx0xDisk)
N=((Nsphere*DiskRat)+(ctbar(1)*ctbar(2)))*ctbar(3)

```

```

c _____
c               ALLOCATION
c _____

```

```

ALLOCATE (FixXY(N))
ALLOCATE (IsSolid(N))
ALLOCATE (X(N,3))
ALLOCATE (V(N,3))
ALLOCATE (dVdx(N,3))
ALLOCATE (rho0(N))

```

```

ALLOCATE (massFct (N))
ALLOCATE (dx0 (N))
ALLOCATE (Contact (N, (N+1)))
ALLOCATE (ContactDir (N, (N+1)))
ALLOCATE (ContactDirCT (N, 3))
ALLOCATE (ContactX (N, (N+1)))
ALLOCATE (ContactDirX (N, (N+1)))
ALLOCATE (ContactDirCTX (N, 3))

```

c

---

c Set the parameters for the steel bar

```

Yr0=(ctbar(2)*dx0x)+Radius
ct=0
Zloc=-dx0x
do kk=1,ctbar(3)
  Zloc=Zloc+dx0x
  Yloc=-dx0x
  do jj=1,ctbar(2)
    Yloc=Yloc+dx0x
    Xloc=-((dx0x*ctbar(1))/2)-(dx0x/2)
    do ii=1,ctbar(1)
      Xloc=Xloc+dx0x
      ct=ct+1
      if (jj==1) then
        FixXY(ct)=1
      else
        FixXY(ct)=0
      endif
      X(ct,1) = Xloc
      X(ct,2) = Yloc
      X(ct,3) = Zloc
      V(ct,1) = 0
      V(ct,2) = 0
      V(ct,3) = 0
      dVdX(ct,1) = 0
    enddo
  enddo
enddo

```

```

        dVdX(ct,2) = 0
        dVdX(ct,3) = 0
        rho0(ct) = rho_solid
        massFct(ct) = masX
        dx0(ct) = (massFct(ct)/rho_solid)**0.333333
        IsSolid(ct) = 1
    enddo
enddo

Zloc2=Zloc-(dx0xDisk*(DiskRat+1)/2)
do ii=1,DiskRat
    Zloc2=Zloc2+dx0xDisk
    angle=(angleFrac/2)+(angleFrac/(2*Nsphere))
    do jj=1,Nsphere
        angle=angle-(angleFrac/Nsphere)
        Xloc=-(SIN(angle))*Radius
        Yloc=Yr0-((COS(angle))*Radius)
        ct=ct+1
        FixXY(ct)=8
        X(ct,1) = Xloc
        X(ct,2) = Yloc
        X(ct,3) = Zloc2
        V(ct,1) = 0
        V(ct,2) = 0
        V(ct,3) = 0
        dVdX(ct,1) = 0
        dVdX(ct,2) = 0
        dVdX(ct,3) = 0
        rho0(ct) = rho_solid
        massFct(ct) = masXdisk
        dx0(ct) = (massFct(ct)/rho_solid)**0.333333
        IsSolid(ct) = 1
    enddo
enddo
enddo
print *,N,ct

```

```

c Set Contact links
do jj=1,N

```

```

if (IsSolid(jj)==1) then
  ct=0
  do ii=1,N
    if (IsSolid(ii)==1) then
      if (ii/=jj) then
        r=0
        do kk=1,3
          dx3(kk)=abs(X(ii,kk)-X(jj,kk))
          r=r+((X(ii,kk)-X(jj,kk))**2)
        enddo
        r=sqrt(r)
        if (r<(dx0(jj)*1.1)) then
          if (FixXY(jj)==FixXY(ii)) then
            ct=ct+1
            Contact(jj,ct+1)=ii
            fooint=MAXLOC(dx3,1)
            ContactDir(jj,ct+1)=fooint
            ContactDirCT(jj,fooint)=ContactDirCT(jj,fooint)+1
          endif
        endif
      endif
    endif
  enddo
  Contact(jj,1)=ct
  ContactDir(jj,1)=ct
endif
enddo

ContactX=Contact
ContactDirX=ContactDir
ContactDirCTX=ContactDirCT

open (unit = 1, file = "Fixed.dat")
do jj=1,N
  write (1,*) ,FixXY(jj)
enddo
close (1)

open (unit = 1, file = "ContactDirX.dat")

```

```

do jj=1,N
    write (1,*) ,ContactDirX(jj,:)
enddo
close (1)

open (unit = 1, file = "Contact.dat")
do jj=1,N
    write (1,*) ,Contact(jj,:)
enddo
close (1)

open (unit = 1, file = "ContactDir.dat")
do jj=1,N
    write (1,*) ,ContactDir(jj,:)
enddo
close (1)

open (unit = 1, file = "ContactDirCT.dat")
do jj=1,N
    write (1,*) ,ContactDirCT(jj,:)
enddo
close (1)

open (unit = 1, file = "ContactX.dat")
do jj=1,N
    write (1,*) ,ContactX(jj,:)
enddo
close (1)

open (unit = 1, file = "ContactDirCTX.dat")
do jj=1,N
    write (1,*) ,ContactDirCTX(jj,:)
enddo
close (1)

open (unit = 1, file = "Mass.dat")
do jj=1,N
    write (1,*) ,massFct(jj)
enddo
close (1)

```



```

open (unit = 1, file = "dX.dat")
do jj=1,N
  write (1,*) ,dx0(jj)
enddo
close (1)

open (unit = 1, file = "X0.dat")
do jj=1,N
  write (1,*) ,X(jj,:)
enddo
close (1)

open (unit = 1, file = "V0.dat")
do jj=1,N
  write (1,*) ,V(jj,:)
enddo
close (1)

open (unit = 1, file = "dVdX.dat")
do jj=1,N
  write (1,*) ,dVdx(jj,:)
enddo
close (1)

open (unit = 1, file = "rho0.dat")
do jj=1,N
  write (1,*) ,rho0(jj)
enddo
close (1)

open (unit = 1, file = "rho_init.dat")
do jj=1,N
  write (1,*) ,rho0(jj)
enddo
close (1)

open (unit = 1, file = "IsSolid.dat")
do jj=1,N
  write (1,*) ,IsSolid(jj)

```

```
enddo  
close (1)  
  
Nout=N  
  
end subroutine ModelBlock
```

c

---

## PressVessel.f

```
subroutine ModelBlock(Nout)

implicit none

integer, intent(out) :: Nout
integer N,ctPV(3),Nsphere,ct,ii,jj,kk,uu
integer fooint,split
double precision RadiusFactor,Radius,masX,massL,dx00,Bulk_water
double precision angleFrac,angle,Yr0,dx3(3),third,RR,pi,P
double precision Zloc,Yloc,Xloc,rho_solid,rho_liquid,rho_liquidC
integer, allocatable, dimension(:) :: FixXY,IsSolid,BreakContact
integer, allocatable, dimension(:,:) :: Contact,ContactX
integer, allocatable, dimension(:,:) :: ContactDir,ContactDirX
integer, allocatable, dimension(:,:) :: ContactDirCT,ContactDirC
double precision, allocatable, dimension(:,:) :: X,V,dVdx
double precision, allocatable, dimension(:) :: dx0,massFct
double precision, allocatable, dimension(:) :: rho0,rho

open (unit = 1, file = "SimSpec.dat")
  write (1,*),207e9      ! Young's Modulus of Steel
  write (1,*),0.3        ! Poisson's Ratio of Steel
  write (1,*),7800       ! Density of Steel
  write (1,*),0.0        ! Forced Density in X direction (Vtest)
  write (1,*),0.0        ! Forced Density in Y direction (Vtest)
  write (1,*),0.0        ! Forced Density in Z direction (Vtest)
  write (1,*),0.3        ! Gamma (blend stress)
  write (1,*),1000       ! Number of Recorded Time Steps (Nts)
  write (1,*),1          ! Time Steps between recorded Time Steps
  write (1,*),1          ! Time Steps between recorded Time Steps
  write (1,*),0.18       ! (StartDefl)
  write (1,*),0.05       ! (NewDefl)
  write (1,*),0.05       ! (dTfactor)
  write (1,*),0.01       ! (dTdxfactor)
  write (1,*),15         ! (stop)
  write (1,*),2.15e9     ! (Bulk_water)
close (1)
```

```

ctPV(1)=25      ! 12
ctPV(2)=20      ! 9
ctPV(3)=1       ! 1

masX=0.1
P=101135*1
rho_solid=7800
rho_liquid=1000
Bulk_water=2.15e9

rho_liquidC=rho_liquid*((P/Bulk_water)+1)
print *,rho_liquidC

third=1.0/3.0
dx00=((masX/rho_solid)**third)
massL=rho_liquidC*(dx00**3)

N=0
Xloc=(-ctPV(1)*dx00)-(dx00/2)
do ii=1,(2*ctPV(1))
Xloc=Xloc+dx00
Yloc=(-ctPV(1)*dx00)-(dx00/2)
do jj=1,(2*ctPV(1))
Yloc=Yloc+dx00
RR=SQRT((Xloc**2)+(Yloc**2))
if (RR<(ctPV(1)*dx00)) then
N=N+1
endif
enddo
enddo
N=N*ctPV(3)
print *,N

```

```

c _____
c                      ALLOCATION
c _____

```

```

ALLOCATE (FixXY(N))
ALLOCATE (IsSolid(N))
ALLOCATE (BreakContact(N))
ALLOCATE (X(N,3))
ALLOCATE (V(N,3))
ALLOCATE (dVdx(N,3))
ALLOCATE (rho(N))
ALLOCATE (rho0(N))
ALLOCATE (massFct(N))
ALLOCATE (dx0(N))
ALLOCATE (Contact(N,(N+1)))
ALLOCATE (ContactDir(N,(N+1)))
ALLOCATE (ContactDirCT(N,3))
ALLOCATE (ContactX(N,(N+1)))
ALLOCATE (ContactDirX(N,(N+1)))
ALLOCATE (ContactDirCTX(N,3))

```

c

---

```

print *,N

```

c Set the parameters for the steel bar

```

ct=0
Zloc=-dx00
do uu=1,(ctPV(3))
  Zloc=Zloc+dx00
  Xloc=(-ctPV(1)*dx00)-(dx00/2)
  do ii=1,(2*ctPV(1))
    Xloc=Xloc+dx00
    Yloc=(-ctPV(1)*dx00)-(dx00/2)
    do jj=1,(2*ctPV(1))
      Yloc=Yloc+dx00
      RR=SQRT((Xloc**2)+(Yloc**2))
      if (RR<(ctPV(1)*dx00)) then
        ct = ct + 1
        if (RR<(ctPV(2)*dx00)) then
          IsSolid(ct) = 0
          massFct(ct) = massL

```

```

        rho0(ct)=rho_liquid
        rho(ct)=rho_liquidC
    else
        IsSolid(ct) = 1
        massFct(ct) = masX
        rho0(ct)=rho_solid
        rho(ct)=rho_solid
    endif
    FixXY(ct)=0
    X(ct,1) = Xloc
    X(ct,2) = Yloc
    X(ct,3) = Zloc

    endif
enddo
enddo
enddo

dx0 = dx0 + (dx00)

```

```

c Set Contact links
do jj=1,N
    if (IsSolid(jj)==1) then
        ct=0
        do ii=1,N
            if (IsSolid(ii)==1) then
                if (ii/=jj) then
                    RR=0
                    do kk=1,3
                        dx3(kk)=abs(X(ii,kk)-X(jj,kk))
                        RR=RR+((X(ii,kk)-X(jj,kk))**2)
                    enddo
                    RR=sqrt(RR)
                    if (RR<(dx0(jj)*1.1)) then
                        ct=ct+1
                        Contact(jj,ct+1)=ii
                        fooint=MAXLOC(dx3,1)
                        ContactDir(jj,ct+1)=fooint
                        ContactDirCT(jj,fooint)=ContactDirCT(jj,fooint)+1
                    endif
                endif
            endif
        enddo
    endif
enddo

```

```

                endif
            endif
        endif
    enddo
    Contact(jj,1)=ct
    ContactDir(jj,1)=ct
endif
enddo

ContactX=Contact
ContactDirX=ContactDir
ContactDirCTX=ContactDirCT

open (unit = 1, file = "Fixed.dat")
do jj=1,N
    write (1,*) ,FixXY(jj)
enddo
close (1)

open (unit = 1, file = "ContactDirX.dat")
do jj=1,N
    write (1,*) ,ContactDirX(jj,:)
enddo
close (1)

open (unit = 1, file = "Contact.dat")
do jj=1,N
    write (1,*) ,Contact(jj,:)
enddo
close (1)

open (unit = 1, file = "ContactDir.dat")
do jj=1,N
    write (1,*) ,ContactDir(jj,:)
enddo
close (1)

open (unit = 1, file = "ContactDirCT.dat")

```

```

do jj=1,N
    write (1,*) ,ContactDirCT(jj,:)
enddo
close (1)

open (unit = 1, file = "ContactX.dat")
do jj=1,N
    write (1,*) ,ContactX(jj,:)
enddo
close (1)

open (unit = 1, file = "ContactDirCTX.dat")
do jj=1,N
    write (1,*) ,ContactDirCTX(jj,:)
enddo
close (1)

open (unit = 1, file = "Mass.dat")
do jj=1,N
    write (1,*) ,massFct(jj)
enddo
close (1)

open (unit = 1, file = "dX.dat")
do jj=1,N
    write (1,*) ,dx0(jj)
enddo
close (1)

open (unit = 1, file = "X0.dat")
do jj=1,N
    write (1,*) ,X(jj,:)
enddo
close (1)

open (unit = 1, file = "V0.dat")
do jj=1,N
    write (1,*) ,V(jj,:)
enddo
close (1)

```



```

open (unit = 1, file = "dVdX.dat")
do jj=1,N
  write (1,*),dVdx(jj,:)
enddo
close (1)

open (unit = 1, file = "rho0.dat")
do jj=1,N
  write (1,*),rho0(jj)
enddo
close (1)

open (unit = 1, file = "rho_init.dat")
do jj=1,N
  write (1,*),rho(jj)
enddo
close (1)

open (unit = 1, file = "IsSolid.dat")
do jj=1,N
  write (1,*),IsSolid(jj)
enddo
close (1)

Nout=N

end subroutine ModelBlock

```

c

---

# MATLAB SOURCE CODE

## Analyze\_Data.m

```
clear
close all
tic

Position=[200 200 900 800];
TitleSz=24; LblSz=20; AxSz=16;
NL=sprintf('\n');

raw=load('VarDat.dat');
Ey=raw(1);
G=raw(2);
StudyCt=raw(3);

V0=0;

X0=load('X0.dat');
Xdat=load('X.dat');
Ydat=load('Y.dat');
Zdat=load('Z.dat');
Vx=load('Vx.dat');
Vy=load('Vy.dat');
dVdx=load('dVdX.dat');
dVdy=load('dVdY.dat');
dVdz=load('dVdZ.dat');
P=load('P.dat');
rho=load('rho.dat');
Fixed=load('Fixed.dat');
```

```

E11=load('E11.dat');
E22=load('E22.dat');
dx=load('dX.dat');
T11=load('T11.dat');
T21=load('T21.dat');
T31=load('T31.dat');
T12=load('T12.dat');
T22=load('T22.dat');
T32=load('T32.dat');
T13=load('T13.dat');
T23=load('T23.dat');
T33=load('T13.dat');
IsSolid=load('IsSolid.dat');
Mass=load('Mass.dat');
rho0=load('rho0.dat');
rho_init=load('rho_init.dat');
TimeFct=load('Time.dat');

Cylrng=find(Fixed==8);
R1=max(X0(Cylrng,2))-min(X0(Cylrng,2))+(mean(dx)/2);

foo=size(Xdat);
ct=foo(1); stop=ct; a=ct;
N=foo(2); clear foo
if stop>ct
    stop=ct;
end
plotCt=5; Range=(1:plotCt)*floor(ct/plotCt);
Vy(:,1)=-V0; Vy(:,N)=V0;
Poisson=(Ey/(2*G))-1;

Ered=1/((1-(Poisson^2))/Ey);

Xdat2=Xdat;
Ydat2=Ydat;
Zdat2=Zdat;
for ii=1:N
    Xdat2(:,ii)=Xdat2(:,ii)-(X0(ii,1));
    Ydat2(:,ii)=Ydat2(:,ii)-(X0(ii,2));
    Zdat2(:,ii)=Zdat2(:,ii)-(X0(ii,3));

```

```

end

Xdat3=Xdat2;
Ydat3=Ydat2;
Zdat3=Zdat2;
for ii=1:ct
    for jj=1:N
        Xdat3(ii,jj)=Xdat3(ii,jj)-mean(Xdat2(ii,:));
        Ydat3(ii,jj)=Ydat3(ii,jj)-mean(Ydat2(ii,:));
        Zdat3(ii,jj)=Zdat3(ii,jj)-mean(Zdat2(ii,:));
    end
end

NL=(['\n']);
%dtplot=load('Time.dat');

Vr=sqrt((Vx.^2)+(Vy.^2));
Xr=sqrt((Xdat.^2)+(Ydat.^2));
Xr2=Xr;
for ii=1:ct
    Xr2=Xr(ii,:)-Xr(1,:);
end

save Data

toc

```

## PlotTension.m

```
clear
close all
tic

Position=[200 200 900 800];
TitleSz=24; LblSz=20; AxSz=16;
NL=sprintf('\n');

% load Data
load Tension_steel

X=Xdat(end,:);
Y=Ydat(end,:);
Z=Zdat(end,:);

Xdat2=Xdat;
Ydat2=Ydat;
Zdat2=Zdat;
for ii=1:N
    Xdat2(:,ii)=Xdat2(:,ii)-((Xdat2(1,ii)));
    Ydat2(:,ii)=Ydat2(:,ii)-((Ydat2(1,ii)));
    Zdat2(:,ii)=Zdat2(:,ii)-((Zdat2(1,ii)));
end

Poisson=(Ey/(2*G))-1;

exagY=1;
exag=2e1;
exagstr=' (20) ';
matstr=' Steel';

Xe=Xdat(1,:)+(Xdat2(end,:)*exag);
Ye=Ydat(1,:)+(Ydat2(end,:)*exagY);
Ze=Zdat(1,:)+(Zdat2(end,:)*exag);

dtplot=(TimeFct(:,1)); dt=max(diff(TimeFct)); dx=median(dx);
height=max(X0(:,2))-min(X0(:,2))+dx;
```

```

width=max(X0(:,1))-min(X0(:,1))+dx;

ff=find(X0(:,2)==max(X0(:,2)));
Tcalc=Ey*max(Ydat2(:,:))'/height;
Tavg=max(abs(T22(:,ff))');

YY=zeros(1,N);
for ii=2:(N-1)
    YY(ii)=(abs(Y(ii)-Y(ii-1))-dx)+(abs(Y(ii)-Y(ii+1))-dx);
    YY(ii)=YY(ii)*Ey/dx;
end
ratio=max(T22').*((min(T22').^-1)); ratio=ratio-1;

PoissonError=100*abs((Poisson/(max(abs(Xdat2(ct,:)))/width)/(max(abs(
TensionError = 100*abs((Tcalc(ct)/Tavg(ct))-1);

MaxStress=abs(Tavg(ct));

dxmat=(max(X0(:,2))-min(X0(:,2)))/20;
axismat=[(min(Xe)-dxmat) (max(Xe)+dxmat) (min(Ye)-dxmat) (max(Ye)+dxmat)
TCstr='Tension';
MaxStressStr=[num2str(MaxStress*(1e-6)) ' MPa'];
TitleStr=[matstr ' - Max Stress = ' MaxStressStr 10];

figure(1)
set(gcf,'Position',Position)
set(0,'DefaultAxesFontSize',AxSz,'DefaultAxesFontWeight','b')
hold off
for ii=1:N
    if Fixed(ii)==0
        plot(Xe(ii),Ye(ii),'ko','LineWidth',4)
        hold on
    else
        plot(Xe(ii),Ye(ii),'kx','LineWidth',4)
        hold on
    end
end

```

```

end
axis(axismat)
title([TitleStr 'Exaggerated ' exagstr TCstr],'fontsize',TitleSz,'fontw
xlabel('X','fontsize',LblSz,'fontweight','b');
ylabel('Y','fontsize',LblSz,'fontweight','b');

figure(2)
set(gcf,'Position',Position)
set(0,'DefaultAxesFontSize',AxSz,'DefaultAxesFontWeight','b')
hold off
for ii=1:N
    if Fixed(ii)==0
        plot(Xdat(1,ii),Ydat(1,ii),'ko','LineWidth',4)
        hold on
    else
        plot(Xdat(1,ii),Ydat(1,ii),'kx','LineWidth',4)
        hold on
    end
end
axis(axismat)
title([TitleStr 'Before ' TCstr],'fontsize',TitleSz,'fontweight','b');
xlabel('X','fontsize',LblSz,'fontweight','b');
ylabel('Y','fontsize',LblSz,'fontweight','b');

display(' ');
display(['Poisson Error = ' num2str(PoissonError) '%']);
display(['Tension Error = ' num2str(TensionError) '%']);
display(' ');

toc

```

## PlotComp.m

```
clear
close all
tic

Position=[200 200 900 800];
TitleSz=24; LblSz=20; AxSz=16;
NL=sprintf('\n');

% load Data
load compress_steel

X=Xdat(end,:);
Y=Ydat(end,:);
Z=Zdat(end,:);

Xdat2=Xdat;
Ydat2=Ydat;
Zdat2=Zdat;
for ii=1:N
    Xdat2(:,ii)=Xdat2(:,ii)-((Xdat2(1,ii)));
    Ydat2(:,ii)=Ydat2(:,ii)-((Ydat2(1,ii)));
    Zdat2(:,ii)=Zdat2(:,ii)-((Zdat2(1,ii)));
end

Poisson=(Ey/(2*G))-1;

exagY=1;
exag=2e1;
exagstr=' (20) ';
matstr=' Steel';

Xe=Xdat(1,:)+(Xdat2(end,:)*exag);
Ye=Ydat(1,:)+(Ydat2(end,:)*exagY);
Ze=Zdat(1,:)+(Zdat2(end,:)*exag);

dtplot=(TimeFct(:,1)); dt=max(diff(TimeFct)); dx=median(dx);
height=max(X0(:,2))-min(X0(:,2))+dx;
```



```

width=max(X0(:,1))-min(X0(:,1))+dx;

ff=find(X0(:,2)==max(X0(:,2)));
Tcalc=Ey*max(Ydat2(:,:))'/height;
Tavg=max(abs(T22(:,ff))');

YY=zeros(1,N);
for ii=2:(N-1)
    YY(ii)=(abs(Y(ii)-Y(ii-1))-dx)+(abs(Y(ii)-Y(ii+1))-dx);
    YY(ii)=YY(ii)*Ey/dx;
end
ratio=max(T22').*((min(T22').^-1)); ratio=ratio-1;

PoissonError=100*abs((Poisson/(max(abs(Xdat2(ct,:)))/width)/(max(abs(
TensionError = 100*abs((Tcalc(ct)/Tavg(ct))-1);

MaxStress=abs(Tavg(ct));

dxmat=(max(X0(:,2))-min(X0(:,2)))/20;
axismat=[(min(Xe)-dxmat) (max(Xe)+dxmat) (min(Ye)-dxmat) (max(Ye)+dxmat)

TCstr='Compression';
MaxStressStr=[num2str(MaxStress*(1e-6)) ' MPa'];

TitleStr=[matstr ' - Max Stress = ' MaxStressStr 10];

figure(1)
set(gcf,'Position',Position)
set(0,'DefaultAxesFontSize',AxSz,'DefaultAxesFontWeight','b')
hold off
for ii=1:N
    if Fixed(ii)==0
        plot(Xe(ii),Ye(ii),'ko','LineWidth',4)
        hold on
    else
        plot(Xe(ii),Ye(ii),'kx','LineWidth',4)
        hold on

```

```

        end
    end
    axis(axismat)
    title([TitleStr 'Exaggerated ' exagstr TCstr],'fontsize',TitleSz,'fontw
xlabel('X','fontsize',LblSz,'fontweight','b');
ylabel('Y','fontsize',LblSz,'fontweight','b');

figure(2)
set(gcf,'Position',Position)
set(0,'DefaultAxesFontSize',AxSz,'DefaultAxesFontWeight','b')
hold off
for ii=1:N
    if Fixed(ii)==0
        plot(Xdat(1,ii),Ydat(1,ii),'ko','LineWidth',4)
        hold on
    else
        plot(Xdat(1,ii),Ydat(1,ii),'kx','LineWidth',4)
        hold on
    end
end
axis(axismat)
title([TitleStr 'Before ' TCstr],'fontsize',TitleSz,'fontweight','b');
xlabel('X','fontsize',LblSz,'fontweight','b');
ylabel('Y','fontsize',LblSz,'fontweight','b');

display(' ');
display(['Poisson Error = ' num2str(PoissonError) '%']);
display(['Tension Error = ' num2str(TensionError) '%']);
display(' ');

toc

```

## Plot\_Hertz.m

```
clear

load Hertz

X=Xdat; Y=Ydat; Z=Zdat;
[ct N]=size(X);

rng=(1:101)+(101*9);

for ii=[1 5:5:ct]
    figure(1)
    plot(X(ii,rng),Y(ii,rng),'*')
    title([num2str(ii) '/' num2str(ct)]);
    pause(0.1)
end

figure(2)
plot(Xdat(1,:),Ydat(1,:),'*')
```

## MakeFigs\_PresVes.m

```
clear all
close all

load PressVess

Position=[200 200 900 600];
TitleSz=24; LblSz=20; AxSz=16;
MkFig=0; % Set to 1 to save JPEG figures
Res='-r600';
NL=sprintf('\n');

aa=find(IsSolid==1); bb=find(IsSolid==0);
TimeFct=mean(TimeFct');

del=1.10;
axismat=[(del*min(X0(:,1))) (del*max(X0(:,1))) (del*min(X0(:,2))) (del*max(X0(:,2)))];

ha=figure(1);
set(gcf,'Position',Position)
set(0,'DefaultAxesFontSize',AxSz,'DefaultAxesFontWeight','b')
plot(X0(aa,1),X0(aa,2),'*',X0(bb,1),X0(bb,2),'o','Linewidth',2)
axis(axismat);
%legend('Hertz','SPAM','Location','Best');
title('Before Simulation','fontsize',LblSz,'fontweight','b');
ylabel('Y (m)','fontsize',LblSz,'fontweight','b')
xlabel('X (m)','fontsize',LblSz,'fontweight','b');
if MkFig==1
    print(ha,'-djpeg','Start.jpeg',Res)
    close all
end

ha=figure(2);
set(gcf,'Position',Position)
set(0,'DefaultAxesFontSize',AxSz,'DefaultAxesFontWeight','b')
plot(Xdat(end,aa),Ydat(end,aa),'*',Xdat(end,bb),Ydat(end,bb),'o','Linewidth',2)
axis(axismat);
%legend('Hertz','SPAM','Location','Best');
```

```

title('After Simulation','fontsize',LblSz,'fontweight','b');
ylabel('Y (m)','fontsize',LblSz,'fontweight','b')
xlabel('X (m)','fontsize',LblSz,'fontweight','b');
if MkFig==1
    print(ha,'-djpeg','Final.jpeg',Res)
    close all
end

Xr2=Xr;
for ii=1:length(Xr2)
    Xr2(:,ii)=Xr(:,ii)-Xr(1,ii);
end

ha=figure(3);
set(gcf,'Position',Position)
set(0,'DefaultAxesFontSize',AxSz,'DefaultAxesFontWeight','b')
plot(TimeFct*(1e6),-Xr2(:,bb)*(1e9),'Linewidth',2)
title('Radial Displacement - Liquid','fontsize',LblSz,'fontweight','b')
ylabel(['\delta' ' R (nm)'],'fontsize',LblSz,'fontweight','b')
xlabel(['Time (' '\mu' ' s)'],'fontsize',LblSz,'fontweight','b');
if MkFig==1
    print(ha,'-djpeg','Rfct_Liquid.jpeg',Res)
    close all
end

ha=figure(4);
set(gcf,'Position',Position)
set(0,'DefaultAxesFontSize',AxSz,'DefaultAxesFontWeight','b')
plot(TimeFct*(1e6),-Xr2(:,aa)*(1e9),'Linewidth',2)
title('Radial Displacement - Solid','fontsize',LblSz,'fontweight','b')
ylabel(['\delta' ' R (nm)'],'fontsize',LblSz,'fontweight','b')
xlabel(['Time (' '\mu' ' s)'],'fontsize',LblSz,'fontweight','b');
if MkFig==1
    print(ha,'-djpeg','Rfct_Solid.jpeg',Res)
    close all
end

```

# **Hertz Study Results**

This report discusses a detailed description of all of the simulation results when studying the Hertz Contact Stress of a Disk and Flat plat in compressive contact.

**About the Study:**

- The purpose of this study is to better verify and validate the Smoothed Particle Applied Mechanics (SPAM) model as it applies to Hertzian contact-mechanics.
- While the simulation is capable of 3D studies, only a 2D layer of particles are studied.
- The model will be represented by an inelastic disk being in contact with a elastic flat plate.
- The fixed 2D disk will be comprised of the same Lagrangian particles as the flat plate.
- The disk will be forced down at a user-specified velocity and then stay in place for a specified number of time steps.
- The plate will rest on a boundary of fixed solid particles beneath it.
- This effort will ensure that the deflection length is less than one tenth the disk radius.

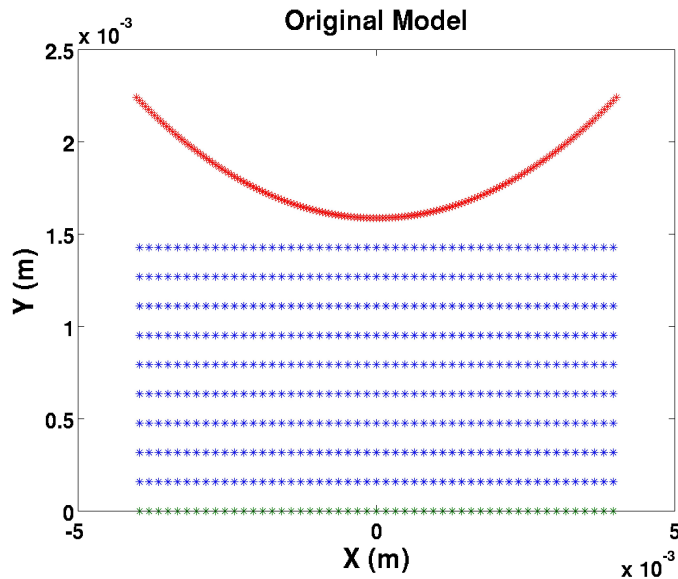
**Hertzian Equations for Comparison of SPAM:**

- $P_{max} = |T_{22}|$  for the Top Center Particle
- $\delta = |Y(t) - Y(0)|$  for the Top Center Particle
- $a = R \sin(\cos^{-1}(\frac{R-\delta}{R}))$ 
  - R = Radius of Disk
  - a = half length of (theoretical) contact area
- Reduced Modulus:
  - $\frac{1}{E} = \frac{1-\nu_1^2}{E_{y,1}} + \frac{1-\nu_2^2}{E_{y,2}}$ 
    - $E_y$  = Young's Modulus of Elasticity (Pa)
    - $\nu$  = Poisson's Ration
    - 1 and 2 represent the parameters of the disk and plate respectively
  - The disk is assumed to be rigid and inelastic, and thus is assumed to have a Young's Modulus of infinity. Based on the assumptions taken for the disk radius, though it is safe to assume the disk is rigid, and thus the Young's Modulus is infinite, therefore:
    - $E = \frac{E_y}{1-\nu^2}$
- $Weight_N (N/m) = \frac{a^2 \pi E}{4 R}$ 
  - This is the normalized weight, or the weight per unit length of the disk
- The calculated maximum stress is found via:
  - $Max Stress_{calculated} = \frac{2 Weight_N}{\pi a}$
- Simulated Weight
  - $Weight = \sum_N (T_{22} * dx^2)$
  - Taken for all the fixed particles at the bottom of the flat disk
- The simulated weight is compared to a calculated total weight based on the simulated (with SPAM) maximum stress at the center of the top of the flat.
  - $Max Stress_{SPAM} = \frac{Weight}{\pi a dx} \rightarrow Weight = \pi a dx (Max Stress_{SPAM})$



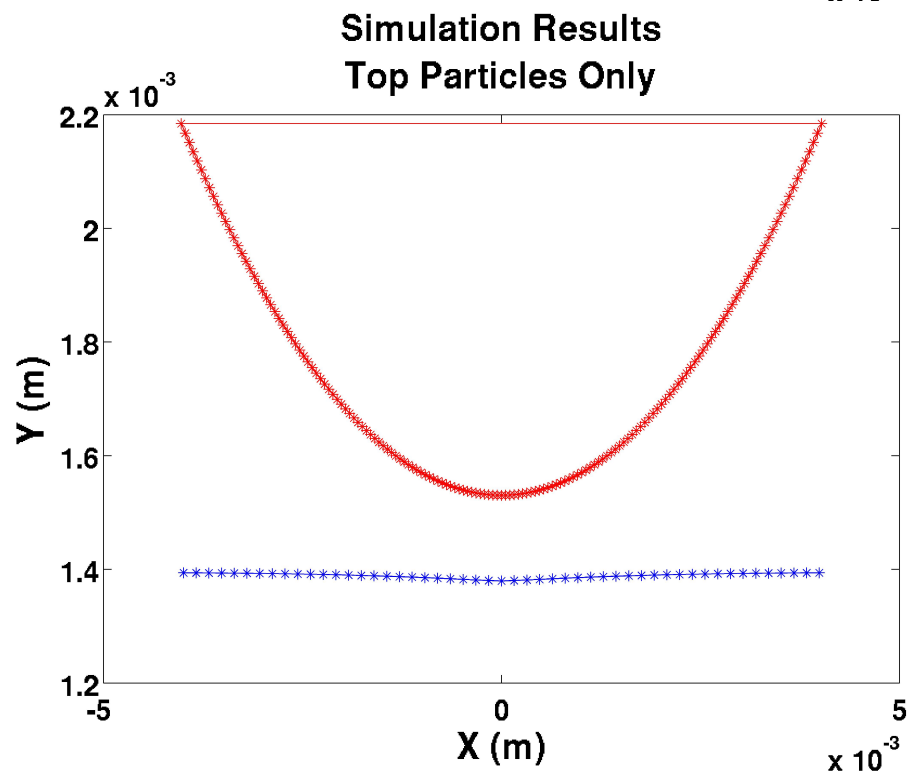
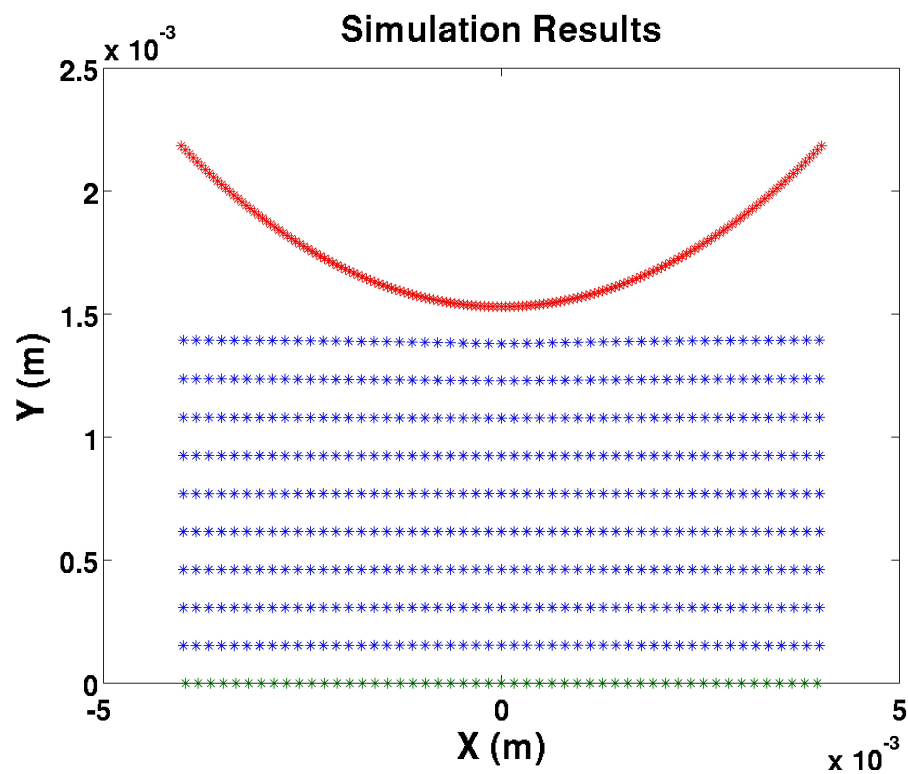
**Model:**

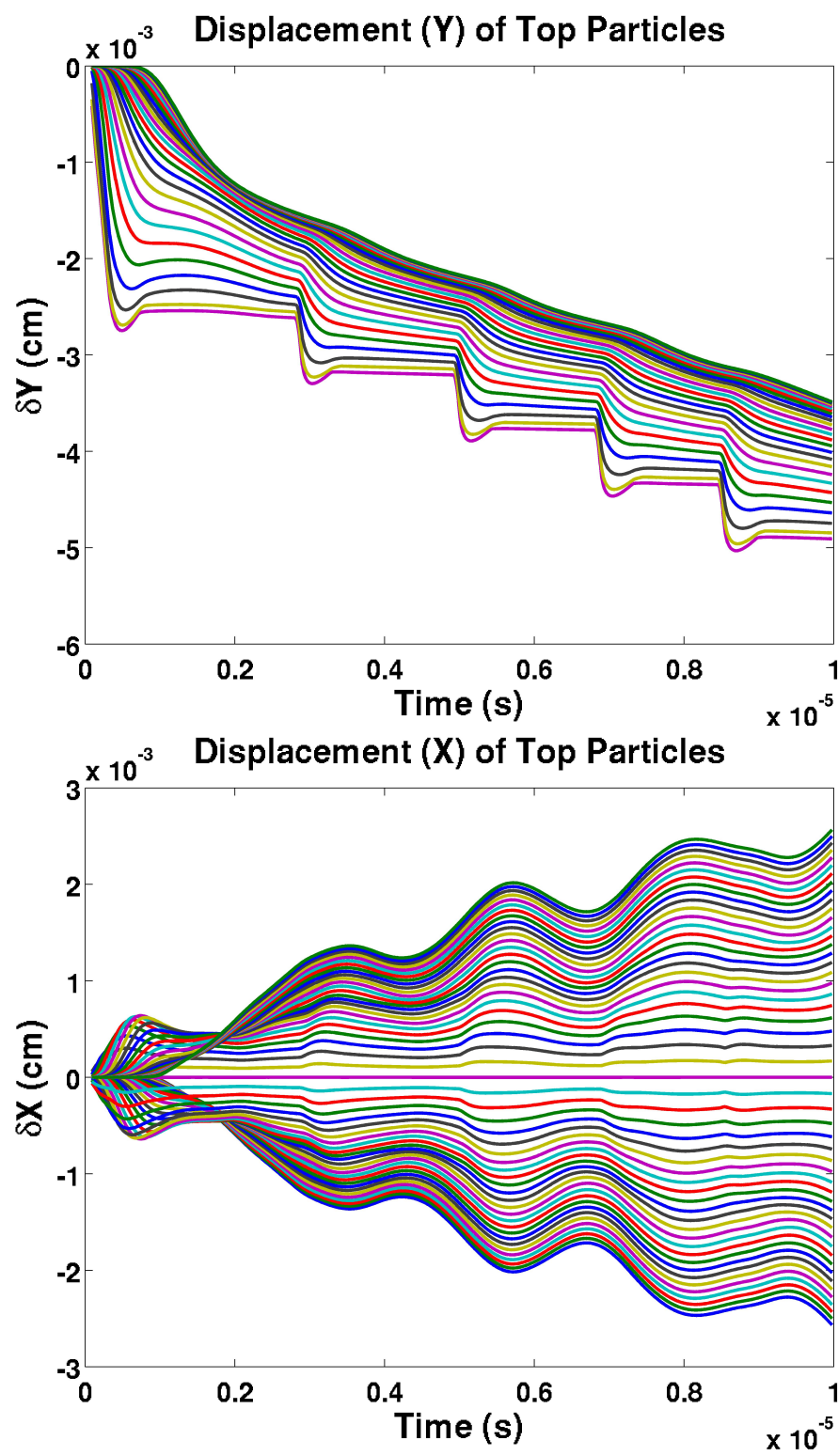
- Steel: Young's Modulus at 207 GPa, Poisson's Ratio at 0.3
- Color Code:
  - Red: Rigid Disk
  - Blue: elastic solid
  - Green = Fixed boundary

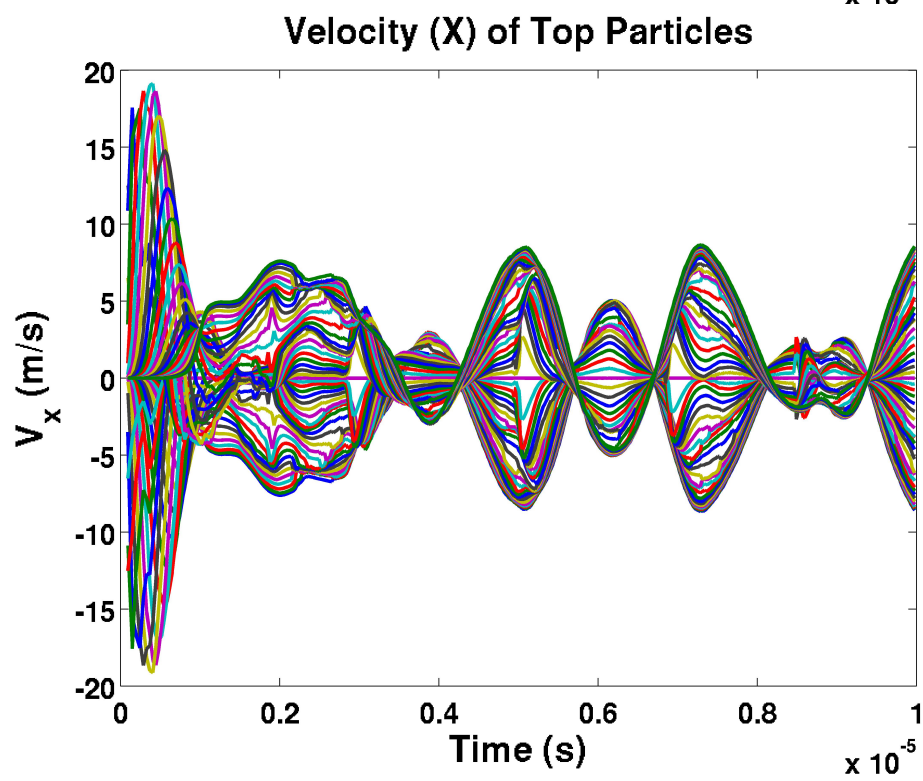
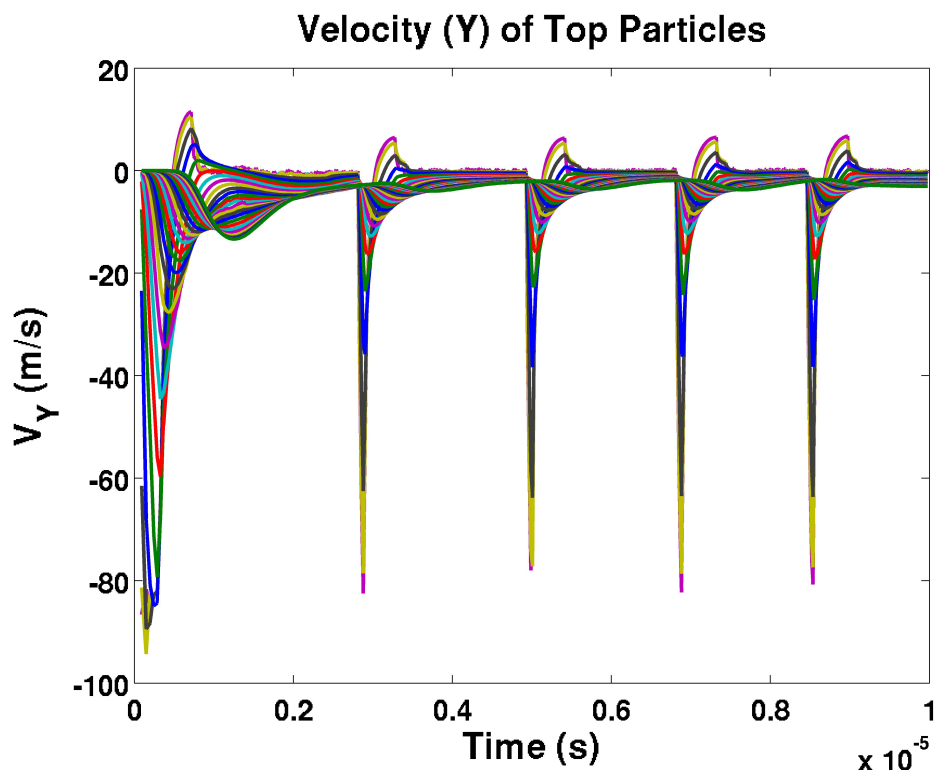
**Results of Simulation:**

- Calculated Contact Area Half-Length ( $a$ ) = 1.6501 meters
  - $a / R = 8.78\%$
- Simulated Deflection: -0.0125 inch
- Stress (MPa) Tensor of Top Center Particle ( $T_{ij}$ ):
 

0	6.4147	0
6.4147	-4.0477	0
0	0	0
- Pressure – Error = 5.359%
  - Simulated (SPAM) Max Pressure, at Top Center Particle = 10.0525 GPa
  - Calculated (Hertz) Max Pressure, at Top Center Particle = 9.9896 GPa
- Weight – Error = 2.2053%
  - Simulated (SPAM) Max Pressure, at Top Center Particle = 5.9842 kN
  - Calculated (Hertz) Max Pressure, at Top Center Particle = 5.8552 kN







R = 20 meters, L = 12.25 m, H = 2.5 m, dX = 0.25 m

51 \* 10 Particles

<b>Disk Deflection (m)</b>	<b>Contact Length (m)</b>	<b>Weight – SPAM (GN)</b>	<b>Weight – Hertz (GN)</b>	<b>Max T22 – SPAM (GPa)</b>	<b>Max T22 – Hertz (GPa)</b>
0.0412	1.2828	6.5983	7.3621	7.3071	7.2952
0.0505	1.4206	8.9462	8.7679	7.8587	8.0784
0.0595	1.5420	11.1197	10.5022	8.6718	8.7690
0.0684	1.6532	13.0201	12.4286	9.5720	9.4015
0.0773	1.7566	14.8473	14.5197	10.5241	9.9897

<b>Disk Deflection (m)</b>	<b>% Error (Weight)</b>	<b>% Error (Max T22)</b>
0.0412	10.3750	0.1624
0.0505	2.0330	2.7202
0.0595	5.8792	1.1079
0.0684	4.7594	1.8128
0.0773	2.2565	5.3490

R = 30 meters, L = 18.375 m, H = 3.75 m, dX = 0.375 m

51 \* 10 Particles

<b>Disk Deflection (m)</b>	<b>Contact Length (m)</b>	<b>Weight – SPAM (GN)</b>	<b>Weight – Hertz (GN)</b>	<b>Max T22 – SPAM (GPa)</b>	<b>Max T22 – Hertz (GPa)</b>
0.0618	1.9242	14.8477	16.5667	7.3080	7.2952
0.0758	2.1308	20.1331	19.7297	7.8595	8.0783
0.0893	2.3130	25.0249	23.6305	8.6721	8.7689
0.1027	2.4798	29.3003	27.9638	9.5721	9.4013
0.1159	2.6349	33.4118	32.6699	10.5244	9.9895

<b>Disk Deflection (m)</b>	<b>% Error (Weight)</b>	<b>% Error (Max T22)</b>
0.0618	10.3760	0.1758
0.0758	2.0448	2.7087
0.0893	5.9011	1.1036
0.1027	4.7793	1.8172
0.1159	2.2708	5.3543

R = 40 meters, L = 24.5 m, H = 5.0 m, dX = 0.50 m

51 \* 10 Particles

<b>Disk Deflection (m)</b>	<b>Contact Length (m)</b>	<b>Weight – SPAM (GN)</b>	<b>Weight – Hertz (GN)</b>	<b>Max T22 – SPAM (GPa)</b>	<b>Max T22 – Hertz (GPa)</b>
0.0824	2.5657	26.3945	29.4497	7.3074	7.2952
0.1010	2.8411	35.7889	35.0722	7.8588	8.0784
0.1191	3.0840	44.4844	42.0097	8.6720	8.7689
0.1369	3.3063	52.0851	49.7116	9.5717	9.4013
0.1546	3.5132	59.3943	58.0780	10.5241	9.9896

<b>Disk Deflection (m)</b>	<b>% Error (Weight)</b>	<b>% Error (Max T22)</b>
0.0824	10.3741	0.1670
0.1010	2.0437	2.7179
0.1191	5.8907	1.1050
0.1369	4.7745	1.8127
0.1546	2.2665	5.3505

R = 50 meters, L = 30.625 m, H = 6.25 m, dX = 0.625 m

51 \* 10 Particles

<b>Disk Deflection (m)</b>	<b>Contact Length (m)</b>	<b>Weight – SPAM (GN)</b>	<b>Weight – Hertz (GN)</b>	<b>Max T22 – SPAM (GPa)</b>	<b>Max T22 – Hertz (GPa)</b>
0.1030	3.2070	41.2442	46.0171	7.3078	7.2951
0.1263	3.5513	55.9236	54.8013	7.8590	8.0783
0.1488	3.8549	69.5117	65.6398	8.6720	8.7689
0.1711	4.1329	81.3883	77.6770	9.5720	9.4013
0.1932	4.3916	92.8097	90.7467	10.5240	9.9896

<b>Disk Deflection (m)</b>	<b>% Error (Weight)</b>	<b>% Error (Max T22)</b>
0.1030	10.3718	0.1730
0.1263	2.0480	2.7142
0.1488	5.8988	1.1050
0.1711	4.7779	1.8153
0.1932	2.2735	5.3501

R = 0.2 inches, L = 0.1275 inches, H = 0.025 inches, dX = 0.0025 inches

51 \* 10 Particles

<b>Disk Deflection (mm)</b>	<b>Contact Length (mm)</b>	<b>Weight – SPAM (N)</b>	<b>Weight – Hertz (N)</b>	<b>Max T22 – SPAM (GPa)</b>	<b>Max T22 – Hertz (GPa)</b>
0.0105	0.3258	425.3947	475.1323	7.3094	7.2953
0.0128	0.3608	576.8719	565.7163	7.8591	8.0785
0.0151	0.3917	716.9652	677.5850	8.6720	8.7690
0.0174	0.4199	839.5070	801.8216	9.5718	9.4014
0.0196	0.4462	957.3682	936.7987	10.5245	9.9897

<b>Disk Deflection (mm)</b>	<b>% Error (Weight)</b>	<b>% Error (Max T22)</b>
0.0105	10.4682	0.1937
0.0128	1.9719	2.7160
0.0151	5.8118	1.1066
0.0174	4.7000	1.8124
0.0196	2.1957	5.3537

R = 0.3 inches, L = 0.1913 inches, H = 0.375 inches, dX = 0.0038 inches

51 \* 10 Particles

<b>Disk Deflection (mm)</b>	<b>Contact Length (mm)</b>	<b>Weight – SPAM (kN)</b>	<b>Weight – Hertz (kN)</b>	<b>Max T22 – SPAM (GPa)</b>	<b>Max T22 – Hertz (GPa)</b>
0.0157	0.4888	0.9572	1.0689	7.3087	7.2952
0.0192	0.5412	1.2980	1.2728	7.8590	8.0785
0.0227	0.5875	1.6132	1.5246	8.6721	8.7691
0.0261	0.6299	1.8889	1.8042	9.5721	9.4014
0.0294	0.6693	2.1541	2.1079	10.5250	9.9898

<b>Disk Deflection (mm)</b>	<b>% Error (Weight)</b>	<b>% Error (Max T22)</b>
0.0157	10.4565	0.1853
0.0192	1.9760	2.7177
0.0227	5.8130	1.1058
0.0261	4.6973	1.8153
0.0294	2.1928	5.3577

R = 0.4 inches, L = 0.2550 inches, H = 0.5 inches, dX = 0.005 inches

51 \* 10 Particles

<b>Disk Deflection (mm)</b>	<b>Contact Length (mm)</b>	<b>Weight – SPAM (kN)</b>	<b>Weight – Hertz (kN)</b>	<b>Max T22 – SPAM (GPa)</b>	<b>Max T22 – Hertz (GPa)</b>
0.0209	0.6517	1.7066	1.9001	7.3076	7.2955
0.0257	0.7216	2.3128	2.2632	7.8605	8.0784
0.0302	0.7833	2.8727	2.7115	8.6760	8.7687
0.0348	0.8398	3.3629	3.2089	9.5768	9.4012
0.0393	0.8923	3.8344	3.7489	10.5296	9.9894

<b>Disk Deflection (mm)</b>	<b>% Error (Weight)</b>	<b>% Error (Max T22)</b>
0.0209	10.1859	0.1663
0.0257	2.1896	2.6967
0.0302	5.9464	1.0570
0.0348	4.7990	1.8678
0.0393	2.2794	5.4081

R = 0.5 inches, L = 0.3188 inches, H = 0.625 inches, dX = 0.0063 inches

51 \* 10 Particles

<b>Disk Deflection (mm)</b>	<b>Contact Length (mm)</b>	<b>Weight – SPAM (kN)</b>	<b>Weight – Hertz (kN)</b>	<b>Max T22 – SPAM (GPa)</b>	<b>Max T22 – Hertz (GPa)</b>
0.0262	0.8146	2.6590	2.9695	7.3092	7.2952
0.0321	0.9021	3.6060	3.5358	7.8593	8.0785
0.0378	0.9792	4.4818	4.2352	8.6726	8.7690
0.0435	1.0498	5.2477	5.0117	9.5723	9.4015
0.0491	1.1155	5.9843	5.8552	10.5250	9.9896

<b>Disk Deflection (mm)</b>	<b>% Error (Weight)</b>	<b>% Error (Max T22)</b>
0.0262	10.4552	0.1921
0.0321	1.9856	2.7127
0.0378	5.8221	1.1000
0.0435	4.7098	1.8169
0.0491	2.2053	5.3590



R = 0.2 inches, L = 0.25 inches, H = 0.025 inches, dX = 0.0025 inches

101 \* 10 Particles

<b>Disk Deflection (mm)</b>	<b>Contact Length (mm)</b>	<b>Weight – SPAM (kN)</b>	<b>Weight – Hertz (kN)</b>	<b>Max T22 – SPAM (GPa)</b>	<b>Max T22 – Hertz (GPa)</b>
0.0102	0.3224	0.4040	0.5156	8.0164	7.2189
0.0124	0.3551	0.6047	0.6508	9.1878	7.9494
0.0146	0.3852	0.7897	0.7912	10.2975	8.6234
0.0168	0.4129	0.9484	0.9431	11.4494	9.2449
0.0190	0.4385	1.0611	1.1126	12.7192	9.8172

<b>Disk Deflection (mm)</b>	<b>% Error (Weight)</b>	<b>% Error (Max T22)</b>
0.0102	21.6538	11.0477
0.0124	7.0836	15.5788
0.0146	0.1923	19.4141
0.0168	0.5628	23.8462
0.0190	4.6256	29.5609

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R = 0.3 inches, L = 0.375 inches, H = 0.0375 inches, dX = 0.00375 inches

101 \* 10 Particles

<b>Disk Deflection (mm)</b>	<b>Contact Length (mm)</b>	<b>Weight – SPAM (kN)</b>	<b>Weight – Hertz (kN)</b>	<b>Max T22 – SPAM (GPa)</b>	<b>Max T22 – Hertz (GPa)</b>
0.0154	0.4836	0.9090	1.1601	8.0157	7.2188
0.0186	0.5326	1.3602	1.4642	9.1877	7.9493
0.0219	0.5777	1.7765	1.7801	10.2968	8.6233
0.0252	0.6194	2.1337	2.1220	11.4491	9.2449
0.0284	0.6577	2.3872	2.5033	12.7189	9.8172

<b>Disk Deflection (mm)</b>	<b>% Error (Weight)</b>	<b>% Error (Max T22)</b>
0.0154	21.6425	11.0386
0.0186	7.1023	15.5783
0.0219	0.2067	19.4061
0.0252	0.5482	23.8431
0.0284	4.6363	29.5575

R = 0.4 inches, L = 0.5 inches, H = 0.05 inches, dX = 0.005 inches

101 \* 10 Particles

<b>Disk Deflection (mm)</b>	<b>Contact Length (mm)</b>	<b>Weight – SPAM (kN)</b>	<b>Weight – Hertz (kN)</b>	<b>Max T22 – SPAM (GPa)</b>	<b>Max T22 – Hertz (GPa)</b>
0.0205	0.6449	1.6161	2.0623	8.0156	7.2188
0.0248	0.7101	2.4182	2.6031	9.1875	7.9494
0.0292	0.7703	3.1581	3.1648	10.2971	8.6234
0.0336	0.8258	3.7930	3.7726	11.4494	9.2449
0.0379	0.8770	4.2438	4.4502	12.7185	9.8172

<b>Disk Deflection (mm)</b>	<b>% Error (Weight)</b>	<b>% Error (Max T22)</b>
0.0205	21.6366	11.0375
0.0248	7.0994	15.5758
0.0292	0.2121	19.4090
0.0336	0.5425	23.8448
0.0379	4.6367	29.5541

R = 0.5 inches, L = 0.625 inches, H = 0.0625 inches, dX = 0.00625 inches

101 \* 10 Particles

<b>Disk Deflection (mm)</b>	<b>Contact Length (mm)</b>	<b>Weight – SPAM (kN)</b>	<b>Weight – Hertz (kN)</b>	<b>Max T22 – SPAM (GPa)</b>	<b>Max T22 – Hertz (GPa)</b>
0.0256	0.8061	2.5255	3.2234	8.0182	7.2187
0.0311	0.8877	3.8447	4.0651	9.1820	7.9499
0.0366	0.9629	4.9896	4.9451	10.2974	8.6234
0.0420	1.0322	5.9783	5.8977	11.4560	9.2443
0.0474	1.0961	6.6832	6.9584	12.7286	9.8165

<b>Disk Deflection (mm)</b>	<b>% Error (Weight)</b>	<b>% Error (Max T22)</b>
0.0256	21.6513	11.0749
0.0311	5.4199	15.4984
0.0366	0.8983	19.4129
0.0420	1.3672	23.9251
0.0474	3.9553	29.6661