

List of changes :

First of all, we the referees for the time spent to understand and criticize our paper. We agree that some reorganization and some clarifications were needed.

What referee 3 writes is not fair since except for the introduction the whole paper only contains new material. All criticisms of referees 1 and 2 were taken into account.

1. To account for the observation that we should “get to the point” we reorganized the first paragraph of the introduction and added the following sentences

*In this paper, we propose an alternative to anyon based universal quantum computation (UQC) thanks to three-dimensional topology. Our proposal relies on appropriate  $3$ -manifolds whose fundamental group is used for building the magic states for UQC.*

2. To help the reader to enter the subject and understand the motivation of the new paper (referee 1) compared to earlier work of the first author about magic states and IC-OPVMs (referee 2 and 3) we added the second paragraph of the introduction

*Let us remind the context of our work and clarify its motivation. Bravyi & Kitaev introduced the principle of ‘magic state distillation’: universal quantum computation, the possibility to prepare an arbitrary quantum gate, may be realized thanks to the stabilizer formalism (Clifford group unitaries, preparations and measurements) and the ability to prepare an appropriate single qubit non-stabilizer state, called a ‘magic state’ [BravyiKitaev2005]. Then, irrespectively of the dimension of the Hilbert space where the quantum states live, a non-stabilizer pure state was called a magic state [Veitch2014]. An improvement of this concept was carried out in [PlanatRukhsan,PlanatGedik] showing that a magic state could be at the same time a fiducial state for the construction of an informationally complete positive operator-valued measure, or IC-POVM, under the action on it of the Pauli group of the corresponding dimension. Thus UQC in this view happens to be relevant both to such magic states and to IC-POVMs. In [PlanatRukhsan,PlanatGedik], a  $d$ -dimensional magic state follows from the permutation group that organize the cosets of a subgroup  $H$  of index  $d$  of a two-generator free group  $G$ . This is due to the fact that a permutation may be seen as a permutation matrix/gate and that mutually commuting matrices share eigenstates - they are either of the stabilizer type (as elements of the Pauli group) or of the magic type. In the calculation, it is enough to keep magic states that are simultaneously fiducial states for an IC-POVM and we are done. Remarkably, a rich catalog of the magic states relevant to UQC and IC-POVMs can be obtained by selecting  $G$  as the two-letter representation of the modular group  $\Gamma = \text{PSL}(2, \mathbb{Z})$  [PlanatModular]. The next step, developed in this paper, is to relate the choice of the starting group  $G$  to three-dimensional topology. More precisely,  $G$  is taken as the fundamental group*

$\pi_1(S^3 \setminus K)$  of a 3-manifold  $M^3$  defined as the complement of a knot or link  $K$  in the 3-sphere  $S^3$ . A branched covering of degree  $d$  over the selected  $M^3$  has a fundamental group corresponding to a subgroup of index  $d$  of  $\pi_1$  and may be identified as a sub-manifold of  $M^3$ , the one leading to an IC-POVM is a model of UQC. In the specific case of  $\Gamma$ , the knot involved is the left-handed trefoil knot  $T_-$ , as shown in Sec. \ref{trefoil}. More details are provided at the next subsections.

3. To motivate section 4 about Dehn fillings (referee 2), at the end of the introduction of this section 4, we added

*For example, surgeries on the trefoil knot allow to build the most important spherical 3-manifolds - the ones with a finite fundamental group - that are the basis of ADE correspondence. The acronym ADE refers to simply laced Dynkin diagrams that connect apparently different objects such as Lie algebras, binary polyhedral groups, Arnold's theory of catastrophes, Brieskorn spheres and quasicrystals, to mention a few \cite{Sirag2016}.*

4. We accounted for all the minor comments 1 to 22 of the first referee, removed the minor mistake of sectioning (referee 2).
5. We added three references to Veich et al (for UQC), to Magma software, to Jean-Paul Sirag (for the relationship of Dehn fillings and ADE correspondence) and M. Planat et al ( for current new work of the authors that is a following up of the present paper.