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Review

Lorentz invariance violation tests in astroparticle physics

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Abstract: In this review, we present the latest exclusion limits obtained with astroparticles for Lorentz Invariance Violation (LIV) in the photon sector. We discuss the techniques known as energy-dependent time delay or time lag, subluminal pair production threshold shift, superluminal photon decays and superluminal photon splitting. Perspectives for future results in LIV with the next generation of experiments are also addressed.

Keywords: Lorentz invariance violation; gamma rays; cosmic rays

1. Introduction

Lorentz symmetry has a fundamental role in particle physics and in the theory of relativity. Exploring its limits of validity has been an important motivation for theoretical and experimental studies [1,2]. On the other hand, the formulation of a quantum theory of gravity, one of the main challenges in physics, can motivate Lorentz invariance violation (LIV) [3–5].

LIV phenomena in the photon sector can lead to new measurable effects such as the energy-dependent speed of light, vacuum Cherenkov radiation, photon decays, and also to the modifications of well-known processes, like the photon-pair production [6–10]. These effects predict very singular imprints in astrophysical observations due to the high energies and the long propagation distances [11]. Recently, the community has shown renewed interest in the LIV tests with astroparticles due to the high precision of the new measurements. In this review, we present a summary of such findings and prospects for the next generation of gamma-ray telescopes and ultra-high energy cosmic rays (UHECR) observatories.

First, we discuss the phenomenological approach to the Lorentz violation; then we address the LIV energy-dependent time delay. In the following section, we review the LIV implications in the electron-positron production by a high energy photon interacting, in its propagation from distant sources to the Earth, with the extra-galactic background light (EBL) and the cosmic microwave background (CMB). Next, we review the photon decay and photon splitting processes. Finally, we present a summary table and figures with the most robust exclusion limits to LIV due to the lack of such signatures in astrophysical data and discuss the systematics and possible futures of each technique.

2. Modified dispersion relation for astroparticle tests

The spontaneous symmetry breaking of the Lorentz symmetry in the standard model of particles [1,12] or the introduction of an explicit Lorentz violating term in the SM Lagrangian [13] can induce modifications of dispersion relation (MDR) for particles. A phenomenological generalization of these LIV effects converges on the introduction of a general function of the energy and momentum [11]. Although, there are several forms of MDR for different particles and underlying LIV-theories, some

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of them may lead to similar phenomenology, which have been proven to be useful for LIV tests in extreme environments such as astroparticle scenarios [14]. In this line of thought, a family of MDRs can be addressed by the following expression[1],

$$E_a^2 - p_a^2 = m_a^2 \pm |\delta_{a,n}| p_a^{n+2}, \tag{1}$$

where a stands for the particle type, E_a is its energy, p_a the momentum and m_a, the mass. $\delta_{a,n}$ is the LIV parameter and n, is the leading order of the correction from the underlying theory. In some effective field theories, it is used $\delta_{a,n} = \epsilon_a^{(n)}/M$, where $\epsilon_a^{(n)}$ are the LIV coefficients, and M is the energy scale of the new physics, such as, the Plank energy scale, $E_{\rm Pl} \approx 1.22 \times 10^{28}$ eV, or some Quantum Gravity energy scale, $E_{\rm QG}$. The sign $\pm$ stands for the so-called *superluminal* (+) or *subluminal* (-) dominant phenomena, relative to the speed of light in a LI-vacuum, c. Due to the high energy of the astroparticles involved, p_a, $E_a \gg m_a$ and $p_a \approx E_a$ at first leading order in $\delta_{a,n}$. So that, for photons, Eq. (1) becomes,

$$E_\gamma^2 - p_\gamma^2 = \pm |\delta_{\gamma,n}| E_\gamma^{n+2}, \tag{2}$$

where once again, n is the leading order of the correction from the underlying theory. For simplicity, when $n > 0$, we write the LIV parameter in some LIV energy scale, $E_{LIV}^{(n)} = (\delta_{\gamma,n})^{1/n}$.

3. Energy dependent time delay

In a Lorentz invariant regime, photons propagate in vacuum at the same speed independently of its energy, E_γ. However, the Lorentz violating equation (2) has as consequence that photons with different energies propagate with different velocities through the LIV vacuum [15–18]. The LIV effect forecast that photons emitted from an astrophysical source, such as gamma-ray bursts (GRB), active galactic nucleus (AGN), and Blazars, should arrive at different times [5,17,19,20]. The time difference including cosmological corrections is given by

$$\Delta t = \frac{1+n}{2H_0}\left(\frac{\Delta E}{E_{\rm LIV}^{(n)}}\right)^n \int_0^z \frac{(1+z)^n dz}{h(z)}, \qquad h(z) = \sqrt{\Omega_m (1+z)^3 + \Omega_\Lambda}, \tag{3}$$

where H_0 is the Hubble constant in the present time, z is the distance to the source and ΔE is the photon energy difference, $E_{\gamma_2} - E_{\gamma_1}$, where $E_{\gamma_2} > E_{\gamma_1}$. $h(z)$ is the distance element in a expanding universe, and Ω_m and Ω_Λ are the LI cosmological parameters in ΛCDM Cosmology evaluated today [21]. Refs. [15,22] reported earlier studies in this type of test, and Ref. [19] extended it to include corrections due to the expansion to the universe, leading to Eq. (3). Although, there is also a cosmological model independent approach, see for instance Ref. [23,24].

Strong superluminal and subluminal exclusion limits to the Lorentz violation coefficients with $n = 1$ and 2, were reported in [25], using the observations of the GRB090510, while recent limits using the Crab Nebula are reported in [26–28] and the blazar Mrk 501 in [29]. Refs.[22,30,31] present some previous limits studying GRBs. For comparison, we show strong and recent limits in Table 1 and Figure 2. Although this is one of the most common techniques, it depends largely on a good observation of very energetic and unpredictable events, such as GRBs. Moreover, it is common in these studies to assume that the observed photons were emitted simultaneously from the source and the effects of LIV meaningfully dominate the time delay. Recent LIV studies focusing on AGNs [32] and GRBs (see for instance [33] and references therein) point that the intrinsic time delay of the source is significant and needs to be considered in this type of LIV analysis, so that, more detailed studies are worth to be developed in the near future.

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[1] Hereafter, natural unites are used, $c = \hbar = 1$, unless it is explicitly given

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4. Pair-production threshold shift

Gamma rays (γ) propagating from distant sources to Earth interact with the background photons (γ_b) through the pair production process, $\gamma\,\gamma_b \rightarrow \mathrm{e}^+\mathrm{e}^-$. This has the effect of a significant energy attenuation in the gamma-ray energy, E_γ [34]. The minimum energy that a γ_b needs to produce a $\mathrm{e}^+\mathrm{e}^-$ is given by [2] m_e^2/E_γ. However, when LIV is considered [35–38], Equation (1) shifts the energy threshold by

$$E^{\mathrm{th}}_{\gamma_\mathrm{b}} = \frac{m_e^2}{E_\gamma} - \frac{1}{4}\delta_{\gamma,n}\, E_\gamma^{n+1}, \tag{4}$$

where subluminal LIV leads to an increase in the energy threshold of the interaction, while superluminal leads to a decrease. The cumulative effect of this phenomenon may result in measurable changes in the expected attenuation of the gamma-rays flux due to the γ_b [7,39,40].

Equation (4) considers LIV only in photons, however, if LIV is also included for electrons and positrons, $\delta_{\gamma,n} \rightarrow \delta_n^{\mathrm{tot}}$, where δ_n^{tot} is a linear combination of the LIV coefficients of the electrons, positrons, and photons. So that, the LIV phenomenology derived in both cases is preserved. For further discussion, see [41] and references within.

The optical depth including LIV effects can be found using the expression (see for instance [39] and references therein),

$$\tau_\gamma(z,\theta,\eta_b,E_\gamma;n,\delta_n^{\mathrm{tot}}) = \int_0^z \mathrm{d}z \frac{c}{H_0(1+z)h(z)} \int_{-1}^{1} \mathrm{d}(\cos\theta)\frac{1-\cos\theta}{2} \int_{E^{\mathrm{th}}_{\gamma_\mathrm{b}}}^{\infty} \mathrm{d}\epsilon\; \eta_b(\epsilon,z)\; \sigma(E_\gamma,\epsilon,z), \tag{5}$$

where H_0 is the Hubble constant, c is the speed of light in vacuum, $h(z)$ is once again the distance element in a expanding universe, σ is the cross-section of the pair-production process [42], $\theta = [-\pi, +\pi]$ is the angle between particles, η_b is the background photon density and $E^{\mathrm{th}}_{\gamma_b}$ is the background photon energy threshold as given by Equation (4).

Using Equation (5), the resulting mean-free path is given by

$$\lambda = \frac{cz}{H_0\tau_\gamma} = 4040.33[\mathrm{Mpc}] \times \frac{z}{\tau_\gamma}, \tag{6}$$

which significantly changes when LIV is considered, such that, if LIV is subluminal $\lambda_{\mathrm{LI}}/\lambda_{\mathrm{LIV}} < 1$, while it is > 1, when LIV photons are superluminal. That is, subluminal LIV predicts more photons arriving from further distances and sources, while superluminal will significantly reduce the expected gamma rays from distant sources [43,44].

At the TeV gamma-ray energy range, the production of electron-positron pairs is dominated by the interaction with the EBL photons. The cumulative effect of these phenomena forecasts modifications in a source spectra. As an example, Figure 1 shows the gamma-ray attenuation

$$a(E_\gamma, z; \delta_n, n) = \exp\left(-\tau\right), \tag{7}$$

for two different LIV scenarios for $n = 1$, and on the order of magnitude close to the strongest current exclusion limits to $E^{(1)}_{\mathrm{LIV}}$ [40]. For these LIV scales, effects on the spectrum above a few tens of TeV are expected. The subluminal effect predicts a recovery in the spectra at the highest energies, while the superluminal case has the effect of reducing the gamma-ray flux. In this example, we use *Dominguez et al. (2011)* EBL model [45]. Although, using other EBL models such as the ones in [46] and [47], result in similar effects [40].

Earlier tests of this method were reported in Ref. [48,49] by looking at Mrk 501 and Mrk 421, and assuming a drop-off above a few TeV due to intergalactic absorption. A significant improvement

[2] Considering the inelasticity of the process as 1/2

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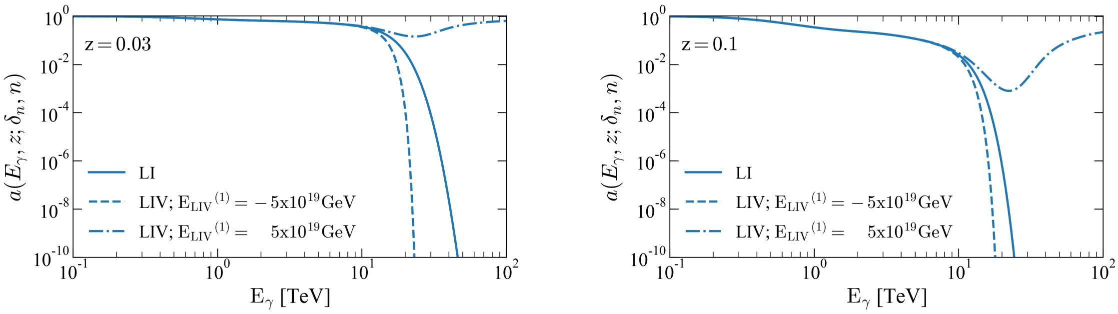

Figure 1. Gamma-ray absorption for LI and LIV scenarios, when LIV is subluminal, there is a recovery in the flux while the superluminal scenario has the effect to reduce the gamma-ray flux. The effect depends on the LIV energy scale, the gamma-ray energy and the redshift to the source, as it can be compared between (a) and (b) for different redshifts but same LIV values.

was presented latter in [7], by testing an extensive set of TeV spectra from blazars and considering uncertainties on the EBL. Although, they found no significant evidence for anomalies, LIV constraints are reported to the linear and quadratic LIV terms. Ref. [29] uses the H.E.S.S. observations of Mrk 501 to improve the LIV exclusion limits. Ref. [40] develops a new analysis procedure to improve the statistical test using an updated TeV gamma-ray data set. Once again, Ref. [40] finds no LIV signatures, and thus stringent exclusion limits are reported with this method. These limits are robust under several tested systematic uncertainties.

As for sensitivity limits, the Cherenkov Telescope Array (CTA) reported prospects to test LIV through the photo pair production threshold shifts. CTA will be capable of detecting gamma rays with energies ranging from about 20 GeV to more than 300 TeV with unprecedented precision in energy and directional reconstruction [50]. This will generate an unparalleled opportunity for this and other LIV tests [51]. Preliminary results from the simulations of CTA observations of the nearby blazar Mrk501 and 1ES 200+209 spectra have shown that CTA will be sensitive to this type of signatures with at least the E_{Pl} for the linear LIV coefficient [52,53].

The limits discussed in this section, including the preliminary sensitivity limits, are presented in Table 1 and Figure 2 and compared with the limits coming from the other LIV effects presented in this review.

As for the TeV gamma-ray energy window, the evolution line of this method is now well established, and it will grow with the advent of new astrophysical data from the current and the next gamma-ray telescopes.

Ultra high energy photons, on the other hand, provide a different challenge for this method, since no UHE photon events have been observed so far [54]; at the energy range $10^{14.5}$ eV$< E_\gamma < 10^{19}$ eV photon energy range, the photo-production of electron-positron pairs is dominated by the interaction with CMB photons, and at the highest end of the spectrum, $E_\gamma > 10^{19}$eV, the interaction is dominated by photons from the Radio Background. In order to face the dare, Ref. [38,39,55] performed a search for LIV signatures in the propagation of secondary UHE-photons emitted on the propagation of UHECR, by computing the so-called GZK-photon flux on Earth. In addition, [39] considered for the first time in the literature, several ultra-high-energy cosmic rays (UHECR) injection and source distribution models, including the model combination that was shown to best describe the UHECR energy spectrum and composition data [56].

The resulting subluminal LIV effect is the increase in the predicted GZK-photon flux [39,41]. Then, by comparing these results with the most updated upper limits on the integrated photon flux imposed by the Pierre Auger Observatory [54], the LIV-scenarios with $\delta_0 \sim -10^{-20}$, $\delta_1 \sim -10^{-38}$ eV^{-1}, and $\delta_2 \sim -10^{-56}$ eV^{-2} can be excluded, since they predict GZK-photons above the current

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upper limits, in the astrophysical scenario which best describes UHECR data. Recent works have obtained similar limits with other astrophysical scenarios [57]. These limits are several orders of magnitude more restrictive than those imposed using TeV gamma-rays. They strongly rely, however, on astrophysical assumptions about the UHECRs propagation and sources, such as the injected spectra and composition and the redshift evolution. Little is known about the sources of UHECR and while different models can properly describe the spectral and composition data, the LIV limits obtained with each of these models can differ by several orders of magnitude as discussed in Ref. [39]. Therefore, the comparison from results obtained with UHECR and gamma-rays is not straight-forward, given the different assumptions, energies, and systematics involved.

The unique subluminal signature reviewed in this section is quite attractive for LIV tests using gamma-ray spectra and UHE photon limits, however not very promising for the superluminal cases. As it can be seen in the Figure 1, distinguishing between the intrinsic source effects due to LI absorption and superluminal LIV effects may not be trivial and therefore it requires some new techniques and more significant characterization of LIV effects at a level that is currently below the precision of current astrophysical telescopes and models. Besides, as it will be seen in the next sections, there are other more promising channels for superluminal LIV with renewed interest in the community.

5. Photon decay

There are processes forbidden in the LI physics but permitted under LIV which can lead to restrictive scenarios for astroparticles, such as the photon decay. This process is described by $\gamma \to \ell^+\ell^-$, where ℓ stand for any fermion, however, the lightest and most common channel belongs to $\gamma \to e^+e^-$, which is the one we consider below.

Photon decay to an electron-positron pair has been studied in the context of MDR and effective field theories such as the standard model extension (see for instance [6,58,59]) . This process has an energy threshold, which for $E_\gamma \gg m_e$ and for any order n is

$$E_\gamma^{\rm th} \sim \left(\frac{4m_e^2}{\delta_{\gamma,n}}\right)^{1/(n+2)} . \tag{8}$$

Below this threshold, photons behaves as in a LI regime and do not decay. Moreover, if $\delta_{\gamma,n} \to 0$, then $E_\gamma^{\rm th} \to \infty$. The resulting modified decay rates for the photon decay into e^+e^- steadily grow with E_γ, so that, once this process is allowed, it is very fast and effective [6,8,13,59]. Photon decay strongly restricts the possible propagation of LIV photons to very short distances; hence, it is unlikely for a LIV photon to travel astronomical distances, and a direct test of superluminal LIV emerges from any observed high energy cosmic photon observation, and by using Equation (8), $\delta_{\gamma,n} \lesssim 4m_e^2/E_\gamma^{n+2}$.

Ref. [13] reported early exclusion limits for the corresponding $n = 0$ scenario, by considering ~ 20 TeV photons from the Crab Nebula, while Ref. [48] considers ~ 20 TeV observed by CANGAROO [60]. The advent of new data by HEGRA [61], and the observations of the SNR RX J1713.7-394 by H.E.S.S. [62], allowed more restrictive limits due to the absence of photon decay [6,8,63]. Tevatron photons have also been used to test photon decay [59], however, due to the lower photon energies involved when compared with astrophysical photon observations, constraints on photon decay are less restrictive.

The strong effect of photon decay would lead to a hard cutoff at high photon energies in the astrophysical spectra of a given source. The very recent results of gamma-ray energies above 100 TeV by the High Altitude Water Cherenkov (HAWC) observatory [64,65], together with the dedicated search of such hard cutoff in different spectra, results in the stringent constraints combining the Crab and three other sources [66–68].

Prospects to test photon decay through the observation of the SNR RX J1713.7-394 were reported in the science motivation paper for the Southern Gamma-Ray Survey Observatory (SGSO), which will be a next-generation wide field-of-view gamma-ray survey instrument, sensitive to gamma-rays in the energy range from 100 GeV to hundreds of TeV, which can lead to new exclusion limits (Ref. [69,70]).

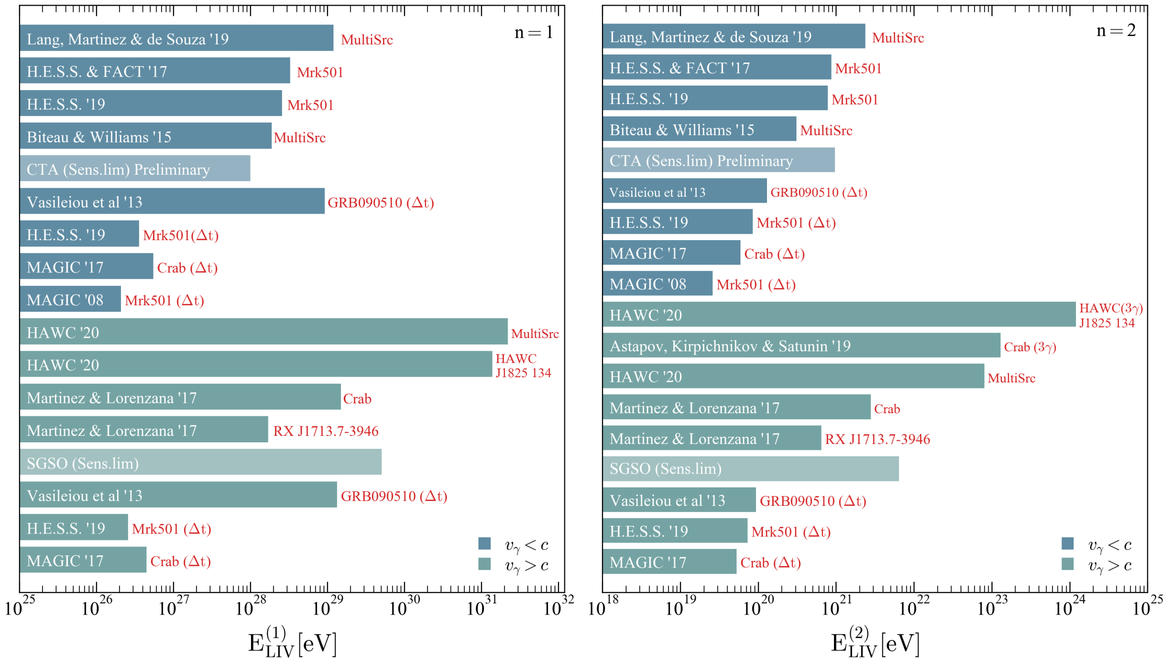

Figure 2. Strong and recent constraints on Lorentz invariance violation coming from the lack of signatures of photon decay, energy-dependent time delay, photon splitting and pair-production threshold shifts in astrophysical data. In the top (blue, $v_\gamma < c$), there are subluminal limits and at the bottom (green, $v_\gamma > c$) superluminal ones. The left and right panels are for the approximation orders $n = 1$ and 2 respectively.

The limits addressed in this section are also included in Table 1 and Figure 2.

6. Photon splitting

Another process forbidden in LI physics which might be allowed, if superluminal LIV is considered, is the photon splitting, $\gamma \to 3\gamma$ [10,71]. In contrast with the photon decay, this process does not involve a threshold and only happens for $n = 2$. Its rate, however, depends on the energy and there is a probability for a photon not splitting on its way to Earth,

$$P = e^{-L_{\rm source}/\langle L_{\gamma\to 3\gamma}\rangle}, \tag{9}$$

where $L_{\rm source}$ is the distance between the source and Earth, and $\langle L_{\gamma\to 3\gamma}\rangle$ is the mean free path for the photon splitting in the quadratic approximation calculated in Refs. [10,72], which is given by

$$\langle L_{\gamma\to 3\gamma}\rangle \approx 16\ \text{Mpc} \left(\frac{E^{(2)}_{\rm LIV}}{10^{14}\ \text{GeV}}\right)^{10} \left(\frac{E_\gamma}{40\ \text{TeV}}\right)^{-19} . \tag{10}$$

This would lead to a suppression in the spectrum. Ref. [72] used the absence of such suppression in the Crab Nebula spectrum measured by HEGRA, while Ref. [10] used Tibet to impose limits on the effect. The dedicated search for such a signature in the spectra by the HAWC Collaboration improved these limits using the results of eHWC J1825-134 [66]. These limits are also presented in Table 1 and Figure 2.

7. Final remarks

In this work, we have summarized the most common and recent techniques used to search for signatures of Lorentz Invariance Violation in the photon sector in astrophysical data as well as the latest limits imposed using them, see Table 1 and Figure 2.

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Type	$\lvert\delta_0\rvert$ 10^{-17}	$E_{\rm LIV}^{(1)}$ 10^{28}eV	$E_{\rm LIV}^{(2)}$ 10^{21}eV	Bound	Source	Reference
Limit	-	12.08	2.38	PP (−)	MultiSrc	Lang, Martínez, and de Souza (2019) [40]
Limit	-	3.3	0.87	PP (−)	Mrk501	H.E.S.S. and FACT (2017) [73]
Limit	-	2.6	0.78	PP (−)	Mrk 501	H.E.S.S. (2019) [29]
Limit	-	1.9	0.31	PP (−)	MultiSrc	Biteau and Williams (2015) [7]
Limit†	~ 0.001	$\sim 10^{10}$	$\sim 10^{7}$	PP (−)	UHECR	Lang, Martínez, and de Souza (2018) [39]
Sens.lim.	-	∼1.22	∼0.97	PP (−)	-	CTA Consortium (2019) [52,53]
Limit	-	9.3	0.13	∆t (−)	GRB090510	Vasileiou et al. (2013) [25]
Limit	-	0.055	0.059	∆t (−)	Crab	MAGIC Collaboration (2017) [27]
Limit	-	0.036	0.085	∆t (−)	Mrk 501	H.E.S.S. (2019) [29]
Limit	-	0.021	0.026	∆t (−)	Mrk 501	MAGIC Collaboration (2008) [74]
Limit	-	-	1200	3γ (+)	2HWC J1825-134	HAWC Collaboration (2020) [66]
Limit	-	-	410	3γ (+)	Crab (Tibet)	Satunin (2019) [10]
Limit	-	-	130	3γ (+)	Crab (HEGRA)	Astapov, Kirpichnikov, and Satunin (2019) [72]
Limit	1.29	2220	80	PD (+)	MultiSrc	HAWC Collaboration (2020) [66]
Limit	-	15	2.8	PD (+)	Crab (HEGRA)	Martínez and Lorenzana (2017) [8]
Limit	-	1.7	0.65	PD (+)	RX J1713.7-3946 (H.E.S.S.)	Martínez and Lorenzana (2017) [8]
Limit	6×10^{5}	-	-	PD (+)	Tevatron	A. Hohensee *et al* (2016) [59]
Limit	40	-	-	PD (+)	Crab (HEGRA)	Schreck (2013) [63]
Limit	50	-	-	PD (+)	Crab (CANGAROO)	Stecker and Glashow (2001) [48]
Limit	180	-	-	PD (+)	RX J1713.7-3946 (H.E.S.S.)	Klinkhamer & Schreck (2008) [6]
Limit	300	-	-	PD (+)	Crab (Themistocle)	Coleman & Glashow (1997) [13]
Sens.lim.	-	$\sim 10^{2}$	~ 10	PD (+)	-	SGSO Alliance [69,70]
Limit	-	13.4	0.09	∆t (+)	GRB090510	Vasileiou et al. (2013) [25]
Limit	-	0.026	0.073	∆t (+)	Mrk 501	H.E.S.S. (2019) [29]
Limit	-	0.045	0.053	∆t (+)	Crab	MAGIC Collaboration (2017) [27]

Table 1. Strong and recent astrophysical LIV limits. $\lvert\delta_0\rvert$ are upper limits while $E_{\rm LIV}^{(n)}$ are lower limits. PP stands for pair production, ∆t for energy-dependent time delay, (3γ) for photons splitting, and PD for photon decay. In addition, (+) indicates $\delta_{\gamma,n} > 0$, while (−) is for $\delta_{\gamma,n} < 0$.
† Limits from the astrophysical scenario which best describes UHECR data.

In the energy dependent time delay searches, the main source of uncertainty is the emission time of photons with different energies. The time delay in the emission of photons with different energy must be smaller than the time delay caused by LIV propagation. Therefore, a better understanding of the mechanism involved in the emission of photons may lead to a more robust limit. From the experimental perspective, the detection of time delays depends on the detection of flares, only possible with fast slewing telescopes and wide field-of-view satellites.

For the photon pair-production of propagating gamma-rays, the main sources of uncertainties are the EBL distribution and the intrinsic spectral shape. The current systematics are discussed in Ref. [40]. New EBL measurements are foreseen using the propagation of TeV photons [75]. The key experimental future is the extension of the detectable energy range to the highest possible energies. Energy spectra with energy above a few TeV might show a very characteristic and almost background free signal.

Improved limits for both the time delay and the interaction threshold shift are expected within the next years with the advent of the new generation of imaging air-Cherenkov telescopes (IACTs), leaded by the construction of the Cherekov Telescope Array. Better sensitivity and wider energy range will be

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achieved and, thus, more frequent and energetic GRBs are expected as well as new measurements of TeV spectra, with more statistics, larger maximum detected energies and farther sources.

For the UHE photons, on the other hand, the hypothesis about the UHECR source injection, composition and distribution play a determining role as discussed in Ref. [39]. In the upcoming years, LIV results will become slightly better due to more restrictive upper limits on the flux coming from longer exposure time. However, the main improvement would come from a better understanding of the UHECR composition which is aimed for the next phase of the present UHECR observatories [76].

On the superluminal cases, limits on the photon splitting are usually less restrictive than those obtained via photon decay. The photon splitting results in a change of the spectrum and its effects could only be seen in a study involving the whole spectrum. The photon decay, on the other hand, is an abrupt effect and just the most energetic photon is enough to impose limits. Also, for the photon decay, no astrophysical assumptions about the source or the propagation are made and only the systematics on the energy estimation influence the resulting limits. For this reason, these limits are more robust and less model dependent. Improvement in these limits depend on the detection of more energetic photons, which may be possible with future experiments such as the SGSO [69,70].

In summary, astrophysics has proven to be a crucial tool in the study of Lorentz invariance violation with stringent limits recently imposed by several works. This has lead to an increase in the interest of the community in this subject and to great expectations for the following years, with the development of new analysis techniques, improvement of data, due to better statistics and better understanding of systematics and new generation of experiments.

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Abbreviations

The following abbreviations are used in this manuscript:

LI	Lorentz invariance
LIV	Lorentz invariance violation
EBL	Extragalactic background bight
CMB	Cosmic microwave background
GRB	Gamma-Ray Bursts
AGN	Active Galactic Nucleus
CTA	Cherenkov Telescope Array
H.E.S.S.	High Energy Stereoscopic System
MAGIC	Major Atmospheric Gamma-ray Imaging Cherenkov
SGSO	Southern Gamma-Ray Survey Observatory
HAWC	Hight Altitud Water Cherenkov
GZK	Kenneth Greisen, Vadim Kuzmin and Georgiy Zatsepin
UHECR	Ultra High Energy Cosmic Ray
UHE	Ultra High Energy
IACT	Imaging Air Cherenkov Telescopes
PP	Photon pair production
PD	Photon decay
(3γ)	Photon splitting into three photons
Δt	Energy-dependent time delay

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