

---

Article

# Two Link's Motion Simulation of Exoskeleton Supporting Leg in 3-D

Andrey Borisov <sup>1</sup>\*

<sup>1</sup> Department of Higher Mathematics, Branch of the "national research UNIVERSITY "MPEI" in Smolensk, Energeticheskiy proezd, house 1, Smolensk, 214013, Russia; borisowandrej@yandex.ru

\* Correspondence: borisowandrej@yandex.ru; Tel.: +79206689126; Borisov A.V.

**Abstract:** A two-link model of exoskeleton with variable-length links for supporting the lower limbs of the human musculoskeletal system is proposed in the article. The researched model differs from the existing ones by the variable-length links, and by the angle calculation method. While in the existing models, the angles are calculated from the regular direction – from vertical, or from horizontal, - in the proposed research they are calculated between the links. As for practical exoskeleton implementation, the proposed method of angle calculation is appropriate to the actual working conditions of the electrical motors with the reduction gears installed in the hinges, which change the angles between the links. The mathematical model in the form of the system of Lagrange differential equations of the second kind is obtained. The obtained mathematical model is examined for existence and uniqueness of the Cauchy solution. The kinematic trajectory of the link motion has been synthesized. The controlling actions required for its implementation have been found.

**Keywords:** exoskeleton; hinge; variable-length link; human musculoskeletal system; angle between links; local system of coordinates; controlling forces and torques

---

## 1. Introduction

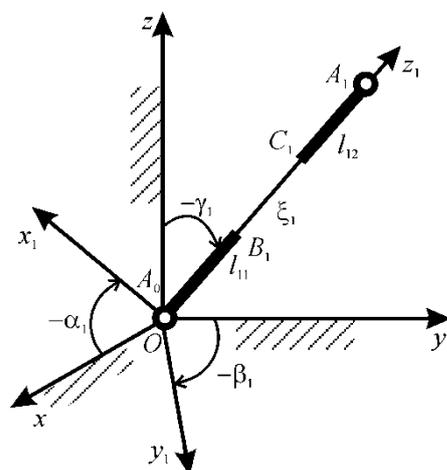
The exoskeletons will have vast scopes of applications after they become comfortable functional devices. They can find applications in military, industry, agriculture, and medicine. By means of exoskeletons it is possible to make the surrounding environment comfortable for the people with impaired motor capabilities and return them to fulfilling lives. In this respect, the exoskeleton not only has a medical role, but also has an important socio-economic value. Owing to exoskeleton the disabled persons could be socialized and returned to active working lives, though previously they had no such an option. Exoskeletons can also be used in rehabilitation and gerontologic centers.

The papers about simulating anthropomorphic mechanisms of various structures usually consider 2-D models and employ angles calculated from the vertical [1-7], or from the horizontal [8-11]. This method of angles calculation is convenient for mathematical simulation of anthropomorphic mechanism motion. It is also suitable for considering a wide range of tasks starting from synthesizing the trajectory of link motion, and ending with the solution of the Cauchy problem with the given control. However, when there is a need of engineering design for a model of anthropomorphic mechanism, for example, for an active exoskeleton with electric motors and gears implementing the relative rotation in hinges for the human musculoskeletal system, the problem relating to angles recalculation arises. The designer needs to calculate the angles between the mechanism links. It is these angles which are changed as a result of electric drive operation. The reasons that lead to a model of exoskeleton with variable-length links are scrutinized in the paper [11]. A 2-D exoskeleton model with angles calculated between the links is proposed in the paper [12]. In the same paper, the model of the variable-length link is also described in detail. The problems of developing exoskeletons are considered

in the papers [13-17]. The functioning exoskeleton models are presented on the websites of the development companies [18-23]. A 3-D model of exoskeleton with angles calculated between the variable-length links is presented in this paper.

## 2. A Model of an Exoskeleton Variable-Length Link with Two Inertial Absolutely Rigid Sections and One Weightless Variable-Length Section in Between

Let's introduce the immobile right-hand Cartesian coordinate system  $Oxyz$ , in which the mechanism motion takes place (Fig. 1). Let's also introduce the mobile local system of coordinates  $Ox_1y_1z_1$ , fixed with the link. We'll direct the mobile axis  $Oz_1$  along the link; the other two axes are introduced based on the right-basis rule. Consider a link model that consists of the two weighty absolutely rigid parts that can perform the relative motion along the  $A_0A_1$  line, passing through its beginning and end (Fig. 1). The spherical hinge, located in the point  $A_0$ , is firmly connected to the supporting surface. The motion of  $C_1A_1$  section relative to the  $A_0B_1$  section along the  $A_0A_1$  direction is performed by the gravity and reaction forces applied by the surface and adjacent rods (not shown on the Fig. 1). This ensures the length change of the section  $B_1C_1$ , and consequently of the entire link  $A_0A_1$ .



**Figure 1.** 3-D model of exoskeleton variable-length link with the introduced absolute and local systems of coordinates.

The lengths of the variable-length link elements are as follows:  $A_0B_1 = l_{11}$ ,  $C_1A_1 = l_{12}$ . The double subscript numbering is related to construction of a multilink exoskeleton model: the first index corresponds to the link number, while the second one corresponds to the number of the weighty section on the link. The variable-length link section  $B_1C_1 = \xi_1(t)$  is considered weightless. It is assumed that the force  $F_1$ , that ensures the required controlled change of the link length, is applied on this section.

Unlike to the papers [1-12], the angles are calculated between the links, i.e. between the axes of the local systems of coordinates counterclockwise from the link with the smaller number to the link with the greater number. As for the first link, the angle is calculated from the axis of the absolute immobile system of coordinates to the axis of the mobile local system of coordinates which is firmly fixed with the first link. On the Fig. 1, the angles  $\alpha_1(t)$ ,  $\beta_1(t)$ ,  $\gamma_1(t)$  between the axes of the immobile and local systems of coordinates are shown in the negative directions.

The position of the weighty section  $C_1A_1$  depends on three parameters and is unequivocally identified by the angles  $\alpha_1(t)$ ,  $\beta_1(t)$  and the variable-length link section  $\xi_1(t)$ . The considered system has three degrees of freedom. We'll designate the controlling torques, applied in the hinge  $A_0$ , as  $M_{1\alpha}$  and  $M_{1\beta}$ . The length change of the link section  $\xi_1(t)$  is controlled by the force  $F_1$ , which is the fourth controlling parameter in the considered system.

The mass of the  $A_0B_1$  rod is equal to  $m_{11}$ . Its moment of inertia relative to the axis passing through mass center perpendicular to the motion plane is equal to  $I_{11}$ . The mass of the  $C_1A_1$  rod is equal to  $m_{12}$ , its moment of inertia relative to the axis passing through its end perpendicular to the motion plane is equal to  $I_{12}$ .

The kinetic energy of the link is the sum of the kinetic energies of the rods  $A_0B_1$  and  $C_1A_1$ :  $T = T_{A_0B_1} + T_{C_1A_1}$ .

$$T = [\dot{\xi}_1^2 m_{12} + (I_{11} + I_{12} + m_{12}(l_{11}^2 + l_{11}l_{12} + 2\xi_1 l_{11} + \xi_1 l_{12} + \xi_1^2))(\dot{\alpha}_1^2 \cos^2 \beta_1 + \dot{\beta}_1^2)]/2. \quad (1)$$

The differential equations of motion composed based on the Lagrange equations of the second kind in the local system of coordinates are as follows:

$$\begin{aligned} & I_{11} + I_{12} + m_{12}((2l_{11} + l_{12})\xi_1 + (l_{11} + l_{12})l_{11} + \xi_1^2)(\ddot{\alpha}_1 \cos^2 \beta_1 - \\ & - 2\dot{\alpha}_1 \dot{\beta}_1 \cos \beta_1 \sin \beta_1) + m_{12} \cos^2 \beta_1 (2l_{11} + l_{12} + 2\xi_1) \dot{\xi}_1 \dot{\alpha}_1 + \\ & + g(m_{11}l_{11} + m_{12}(2l_{11} + l_{12} + 2\xi_1)) \cos \beta_1 = M_{1\alpha}, \end{aligned} \quad (2)$$

$$\begin{aligned} & (I_{11} + I_{12} + m_{12}((2l_{11} + l_{12})\xi_1 + (l_{11} + l_{12})l_{11} + \xi_1^2)) \ddot{\beta}_1 + \\ & + \cos \beta_1 \sin \beta_1 (I_{11} + I_{12} + m_{12}((2l_{11} + l_{12})\xi_1 + (l_{11} + l_{12})l_{11} + \xi_1^2)) \dot{\alpha}_1^2 + \\ & + m_{12}(2l_{11} + l_{12} + 2\xi_1) \dot{\xi}_1 \dot{\beta}_1 - \end{aligned} \quad (3)$$

$$- g(m_{11}l_{11}/2 + m_{12}(l_{11} + l_{12}/2 + \xi_1)) \cos \alpha_1 \sin \beta_1 = M_{1\beta},$$

$$- m_{12}(2l_{11} + l_{12} + 2\xi_1)(\dot{\alpha}_1^2 \cos^2 \beta_1 + \dot{\beta}_1^2)/2 + gm_{12} \cos \alpha_1 \cos \beta_1 + m_{12} \ddot{\xi}_1 = F_1. \quad (4)$$

Thus, the system of differential equations of motion, describing the 3-D model of exoskeleton variable-length link, has been composed.

### 3. Description of the Model with Two Variable-Length Links and Generalization of the Model on $n$ – link Case

Consider the model of two exoskeleton links with angles calculated between the links and the introduced local systems of coordinates (Fig. 2). The design of each link is similar to the one considered in the previous paragraph (Fig. 1). The bold thick lines show inertial absolutely rigid link sections  $A_0B_1$ ,  $C_1A_1$ ,  $A_1B_2$ ,  $C_2A_2$ . The thin lines show the weightless variable-length sections  $B_1C_1 = \xi_1(t)$ ,  $B_2C_2 = \xi_2(t)$ .

The generalized coordinates describing the mechanism position are the angles between the corresponding axes of coordinates and the lengths of the weightless link sections (Fig. 2):  $\varphi_i$ ,  $\beta_i$ ,  $\gamma_{i-1}$ ,  $\xi_i$  ( $i = 1, 2$ ). The considered system has seven degrees of freedom. Seven independent drives are required to implement the controlled motion – to control each rotation angle, and to control the length change of each link. The drives can be of different types: hydraulic, pneumatic, electric. In this study it is assumed that electric motors are used. To control the angular coordinates, the electric motors are installed with reduction units decreasing the speed and increasing the torques. As for linear coordinates, controlling the change of link length, the electric motor with rack and pinion or screw-type gear is used.

The system of differential equations of motion for the model shown on the Fig. 2 is composed similar to that of the model with one link. However, the obtained equations are so cumbersome that we can't list all of them in this paper. Therefore, the analysis of the equations has been done, their pattern has been identified, and the equations have been presented in the generalized vector-matrix form. The matrix subscripts designate the corresponding generalized coordinate:  $\kappa = 1, 2, 3, 4$ , here 1 corresponds to the generalized coordinate  $\alpha$ , 2 –  $\beta$ , 3 –  $\gamma$ , 4 –  $\xi$ .

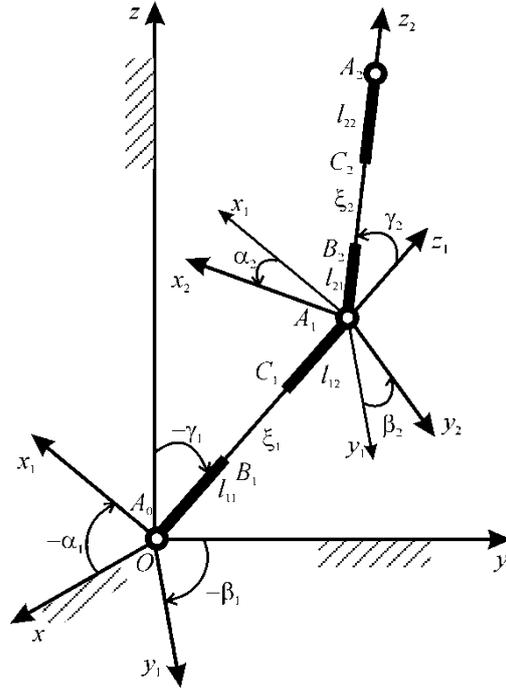


Figure 2. 3-D model of two variable-length exoskeleton links with the angles calculated between the links.

$$\begin{aligned}
 & A_{\kappa}(\alpha, \beta, \gamma, \xi) \ddot{\alpha} + B_{\kappa}(\alpha, \beta, \gamma, \xi) \ddot{\beta} + C_{\kappa}(\alpha, \beta, \gamma, \xi) \ddot{\gamma} + D_{\kappa}(\alpha, \beta, \gamma, \xi) \ddot{\xi} + \\
 & + E_{\kappa}(\alpha, \beta, \gamma, \xi) \dot{\alpha} \dot{\alpha} + F_{\kappa}(\alpha, \beta, \gamma, \xi) \dot{\beta} \dot{\beta} + G_{\kappa}(\alpha, \beta, \gamma, \xi) \dot{\gamma} \dot{\gamma} + 2H_{\kappa}(\alpha, \beta, \gamma, \xi) \dot{\alpha} \dot{\xi} + \\
 & + 2K_{\kappa}(\alpha, \beta, \gamma, \xi) \dot{\beta} \dot{\xi} + 2L_{\kappa}(\alpha, \beta, \gamma, \xi) \dot{\gamma} \dot{\xi} + 2N_{\kappa}(\alpha, \beta, \gamma, \xi) \dot{\alpha} \dot{\beta} + 2P_{\kappa}(\alpha, \beta, \gamma, \xi) \dot{\alpha} \dot{\gamma} + \\
 & + 2Q_{\kappa}(\alpha, \beta, \gamma, \xi) \dot{\beta} \dot{\gamma} + gR_{\kappa}(\alpha, \beta, \gamma, \xi) = M_{\kappa}(\alpha, \beta, \gamma, \xi),
 \end{aligned} \quad (5)$$

here:  $\alpha, \beta, \gamma$  – the angular generalized coordinates of the concentrated masses;  $\alpha = (\alpha_1, \dots, \alpha_n)^T$ ,  $\beta = (\beta_1, \dots, \beta_n)^T$ ,  $\gamma = (\gamma_1, \dots, \gamma_{n-1}, 0)^T$  – the vectors of the angles between the axes of the coordinate systems;  $\xi$  – the generalized coordinates related to length change of the links;  $\xi = (\xi_1, \dots, \xi_n)^T$  – the vector of the variable-length links lengths;  $\dot{\alpha}, \dot{\beta}, \dot{\gamma}$  – the vectors of angular velocities;  $\ddot{\alpha}, \ddot{\beta}, \ddot{\gamma}$  – the vectors of angular accelerations;  $\dot{A} = \text{diag}(\dot{\alpha}_1, \dots, \dot{\alpha}_n)$ ,  $\dot{B} = \text{diag}(\dot{\beta}_1, \dots, \dot{\beta}_n)$ ,  $\dot{\Gamma} = \text{diag}(\dot{\gamma}_1, \dots, \dot{\gamma}_n)$  – the diagonal matrices;  $M_{\kappa}$  – the vectors of generalized forces;  $A_{\kappa}(\alpha, \beta, \gamma, \xi)$ ,  $B_{\kappa}(\alpha, \beta, \gamma, \xi)$ ,  $C_{\kappa}(\alpha, \beta, \gamma, \xi)$ ,  $D_{\kappa}(\alpha, \beta, \gamma, \xi)$ ,  $E_{\kappa}(\alpha, \beta, \gamma, \xi)$ ,  $F_{\kappa}(\alpha, \beta, \gamma, \xi)$ ,  $G_{\kappa}(\alpha, \beta, \gamma, \xi)$ ,  $H_{\kappa}(\alpha, \beta, \gamma, \xi)$ ,  $K_{\kappa}(\alpha, \beta, \gamma, \xi)$ ,  $L_{\kappa}(\alpha, \beta, \gamma, \xi)$ ,  $N_{\kappa}(\alpha, \beta, \gamma, \xi)$ ,  $P_{\kappa}(\alpha, \beta, \gamma, \xi)$ ,  $Q_{\kappa}(\alpha, \beta, \gamma, \xi)$  – the matrices describing the mechanism structure;  $R_{\kappa}(\alpha, \beta, \gamma, \xi)$  – the matrices specified by the gravity moments.

This system can be applied to a model with any finite number of variable-length links similar to that as it was done in the papers [8-10], in which the high-speed recurrent and matrix algorithms of composing the system of differential equations of motion were developed. The maximal link number used for building an exoskeleton is limited only by the computing power, since the equations' complexity grows non-linear with the system geometry.

#### 4. Examination of the Obtained Mathematical Model in the Form of the System of Differential Equations of Motion for Solution Existence and Its Uniqueness

Let's undertake the analytical study of the considered exoskeleton model. Considering the ballistic motions, we can assume that the right part of the first equation in the system (2) equals to zero. We also assume the right part of the second equation in the system (3) be potential. Therefore, an energy integral exists for the considered model [7]:

$$E = T + \Pi =$$

$$= [\dot{\xi}_1^2 m_{12} + (I_{11} + I_{12} + m_{12}(l_{11}^2 + l_{11}l_{12} + 2\xi_1 l_{11} + \xi_1 l_{12} + \xi_1^2))(\dot{\alpha}_1^2 \cos^2 \beta_1 + \dot{\beta}_1^2)]/2 + \quad (6)$$

$$+ g(m_{11}l_{11}/2 + m_{12}(l_{11} + l_{12}/2 + \xi_1))\cos\alpha_1\sin\beta_1 = h.$$

From this we can derive:

$$[\dot{\xi}_1^2 m_{12} + (I_{11} + I_{12} + m_{12}(l_{11}^2 + l_{11}l_{12} + 2\xi_1 l_{11} + \xi_1 l_{12} + \xi_1^2))(\dot{\alpha}_1^2 \cos^2 \beta_1 + \dot{\beta}_1^2)]/2 = \quad (7)$$

$$= 2(h - g(m_{11}l_{11}/2 + m_{12}(l_{11} + l_{12}/2 + \xi_1))\cos\alpha_1\sin\beta_1).$$

The left part of the equation (7) is positive-definite. The right part of the equation (7) depends on trigonometric function and accepts all of its possible values from a limited enclosed domain. Therefore, it is bounded below by a constant  $H$ :

$$[\dot{\xi}_1^2 m_{12} + (I_{11} + I_{12} + m_{12}(l_{11}^2 + l_{11}l_{12} + 2\xi_1 l_{11} + \xi_1 l_{12} + \xi_1^2))(\dot{\alpha}_1^2 \cos^2 \beta_1 + \dot{\beta}_1^2)]/2 \leq H. \quad (8)$$

Thus, the Cauchy solution for the system of differential equations of motion (2) exists under any initial conditions; it is unique and infinitely continued [7].

The obtained result can be generalized for a model with  $n$  mobile links. In this way, it has been demonstrated that the Cauchy problem for a mechanism with  $n$  mobile links has a solution under any initial conditions; it is unique and infinitely continued [7].

### 5. Controlling the 3-D Two-Link Exoskeleton Model by the Analytically Specified Kinematic Properties of the Motion

In some cases, the exoskeleton control should be done based on a certain algorithm [12-23]. For example this is required for people with impaired human musculoskeletal system, for muscle training to rehabilitate the human motor activity, or in sports, when a work-out is done in exoskeleton or in parts of it.

Let's use the exoskeleton control method based on the analytically specified kinematic properties of the motion, proposed for a 2-D model, having modified them for a 3-D case.

$$\alpha_1(t) = \pi/2 + j_1 j_4 \sin[f_1 - (1 - \cos[2\pi t/T])\pi/2],$$

$$\alpha_2(t) = \pi/2 + j_2 j_4 \cos[f_2 - (1 - \cos[2\pi t/T])\pi/2],$$

$$\beta_1(t) = \pi/2 + j_1 \sin[f_1 - (1 - \cos[2\pi t/T])\pi/2],$$

$$\beta_2(t) = \pi/2 + j_2 \cos[f_2 - (1 - \cos[2\pi t/T])\pi/2], \quad (9)$$

$$\gamma_1(t) = j_3 \cos[2\pi t/T],$$

$$l_1(t) = l_{11} + l_{12} + \xi_1^* \cdot (1 + l \cdot \sin[2\pi t/T]),$$

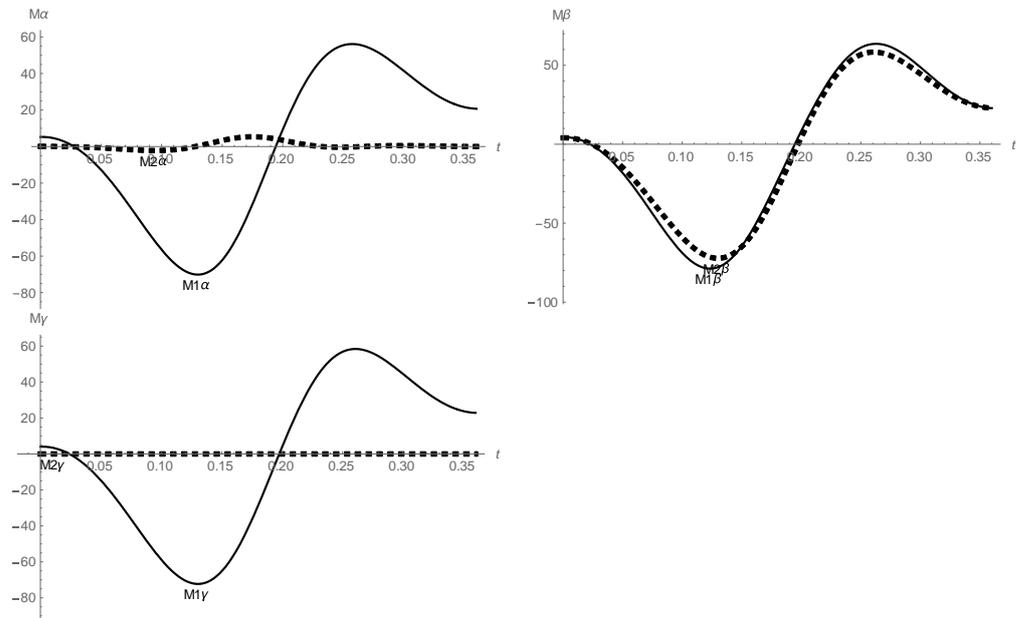
$$l_2(t) = l_{21} + l_{22} + \xi_2^* \cdot (1 + l \cdot \sin[2\pi t/T]).$$

here:  $T$  – the walking period,  $j_i$  and  $f_i$  – the walk parameters,  $l_i^*$  – the initial lengths of non-deformed link,  $l$  – the coefficient of link change. The initial lengths of non-deformed links are as follows:  $l_1^* = 0.385$  m,  $l_2^* = 0.477$  m. These lengths are distributed on the link as follows:  $l_{11} = 0.15$  m,  $\xi_1^* = 0.085$  m,  $l_{21} = 0.2$  m,  $\xi_2^* = 0.077$  m,  $l_{i1} = l_{i2}$  ( $i = 1, 2$ ). The masses of the links are  $m_1 = 2.91$  kg,  $m_2 = 8.93$  kg. They are equally distributed between the two weighty absolutely rigid link sections, i.e.  $m_{i1} = m_{i2} = m_i / 2$  ( $i = 1, 2$ ). The moments of inertial of the weighty link sections for the links relative to the axis passing through the bottom point of the weighty link part are as follows:  $I_{11} = 0.011$  kg·m<sup>2</sup>,  $I_{21} = 0.060$  kg·m<sup>2</sup>,  $I_{i1} = I_{i2}$  ( $i = 1, 2$ ). The acceleration due to gravity is  $g = 9.81$  m/s<sup>2</sup>. The period of the single-support step phase, i.e. the half of the walking period is  $t_k = 0.36$  s.

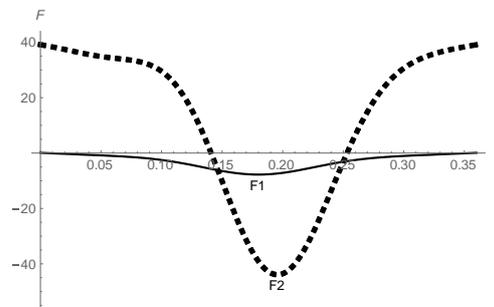
The link length change coefficient is  $l = 0.25$ . The walk parameters are as follows:  $j_1 = j_2 = 0.25$ ,  $j_3 = 0.279$ ,  $j_4 = 0.1$ ,  $f_1 = \pi/2$ ,  $f_2 = 0.687$ .

The analytical expressions (9) and the values of the corresponding walk parameters are selected in order to synthesize the periodic anthropomorphic walk.

The listed curves of the controlling torques (Fig. 3) and the lengthwise forces (Fig. 4) have been obtained by solving the algebraic system (5) for the model presented on the Fig. 2 with the motion kinematics specified as per (9).



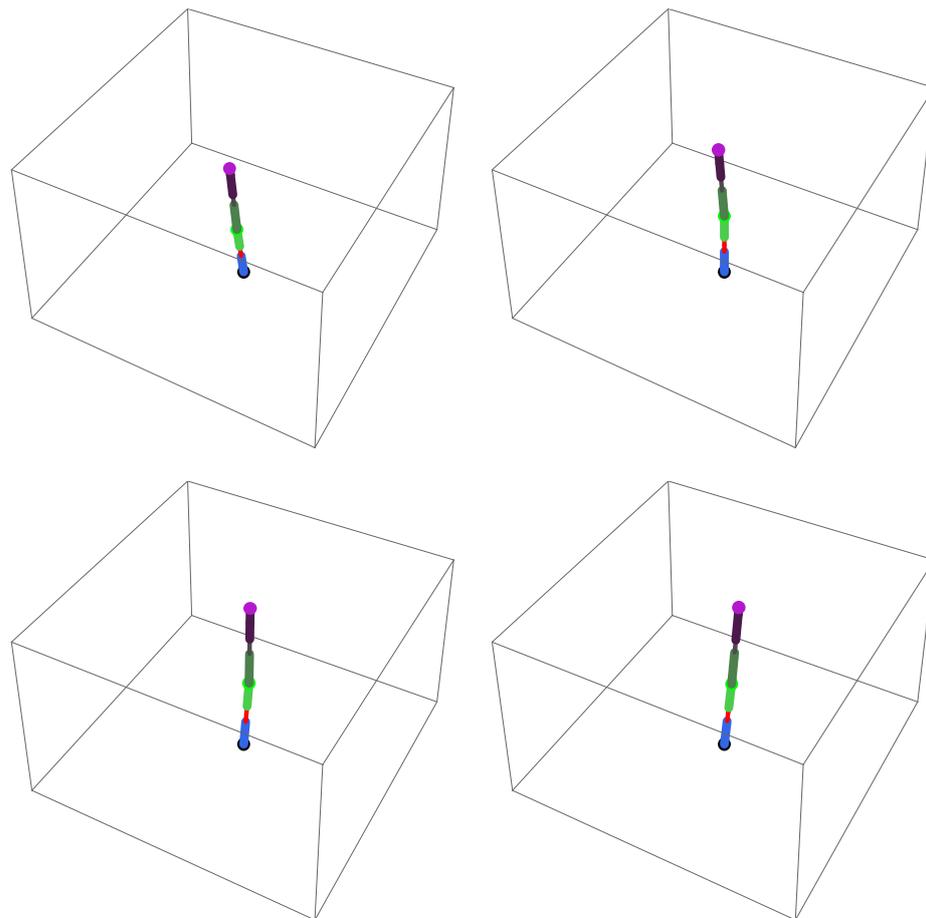
**Figure 3.** The curves representing the controlling torques in the hinges of the two-link exoskeleton as functions of time.



**Figure 4.** The curves representing the lengthwise forces applied to the exoskeleton links as functions of time.

The obtained relationships for controlling torques and lengthwise forces as functions of time allow to assess the required motors and reduction gears, rack and pinion or screw-type gears to make sure they can implement the theoretically calculated torques and forces respectively.

The kinogram frames of the pictographic animated visualization of motion with the kinematics specified by (9) are shown on the Fig. 5. The kinogram frames simulate the motion of the shin and the hip of the supporting leg in the single-support motion phase.



**Figure 5.** The kinogram frames of the exoskeleton motion.

Thus, the problem of synthesizing the controlling actions for the given motion of the two-link exoskeleton is solved. The prospects of this approach for simulating the exoskeleton motion are demonstrated.

## 6. Conclusions

In this way, the model of two links of variable lengths and angles calculated between the links is described. Subsequently, adding links to this model one can build an exoskeleton model with any given number of links. This allows developing exoskeletons with the specified functionality.

The theoretical significance of the results obtained in this study consists of the newly developed variable-length link models, and of the new angles calculation method, which is applied when dealing with electric motors.

The practical significance of the obtained results lies in the possibility of developing active exoskeletons for various purposes, medical prosthetic devices, anthropomorphic robots, spacesuits. These active exoskeletons take into account the human anthropomorphic specifics. The implementation of these active exoskeletons will make a significant contribution into development of various industry sectors, healthcare, sports, and armed forces.

The obtained results can be used for constructing the human-machine system in the form of 3-D mechanism with controlled change of link lengths. This system can be used as a primary or an additional means of transportation, as well as for performing other heavy-duty jobs in order to decrease the loads on the human musculoskeletal system. Furthermore, the developed mechanism model could be applied to compensate the motions of the people with the musculoskeletal system impairments. It allows restoring or altogether overriding the motor activity of the human musculoskeletal system. It can also

be used in healthcare for rehabilitation purposes, muscle training, restoring neural pathways, for example by post-stroke patients. It can be used as an auxiliary device that enhances the human power capabilities in industry when carrying heavy objects, since it increases the payload, stamina, and protects the human musculoskeletal system from injuries. The mechanism of controlled length change of the links can be applied in healthcare centers when nursing immobile patients, as well as by surgeons during prolonged surgery. It can also be applied in industry, agriculture, armed forces and in all other domains where personnel is expected to carry heavy weights or stay in the same position for a long time. The developed model can be used for building exoskeletons for children with impaired or injured musculoskeletal system, for example with spinal muscular atrophy, infantile cerebral paralysis etc.

## References

1. Beletsky B.B. Biped walking: the model problems of dynamics and control. – M.: *Nauka*, **1984** – 288 p. (in Russian)
2. Berbyuk V.E. Dynamics and optimization of robotic systems. - Kiev: *Naukova Dumka*, **1989**. – 192 p. (in Russian).
3. Chernousko F.L. Methods of control of nonlinear mechanical systems. / F.L. Chernousko, I.M. Ananievsky, S.A. Reshmin. – M.: *Fizmatlit*, **2006**. – 328 p. (in Russian).
4. Golubev Yu, Melkumova E. (2018) Two-legged Walking Robot Prescribed Motion on a Rough Cylinder // *AIP Conference Proceedings* 1959, 030009. Published by the American Institute of Physics; doi: 10.1063/1.5034589
5. Vukobratovitch M. Marching anthropomorphic mechanisms. – M.: *Mir*, **1976**. – 541 p. (in Russian)
6. Pavlovsky B. E. About developments of marching machines // *Preprints of IPM named after M.V. Keldysh*. – **2013**. - №101. – 32 p. URL:<http://library.keldysh.ru/preprint.asp?id=2013-101>
7. Formalsky A.M. Motion of anthropomorphic mechanisms – Moscow. : *Nauka*, **1982**. – 368 P. (in Russian).
8. Borisov A.V., Rozenblat G.M. Modeling the Dynamics of an Exoskeleton with Control Torques in the Joints and a Variable Length of the Links Using the Recurrent Method for Constructing Differential Equations of Motion // ISSN 1064-2307, *Journal of Computer and Systems Sciences International*, **2018**, Vol. 57, No. 2, pp. 319–347.
9. Borisov A.V., Rozenblat G.M. Matrix method of constructing the differential equations of motion of an exoskeleton and its control // *Journal of Applied Mathematics and Mechanics* 81 (2017). PP. 351-359.
10. Borisov A.V. Two-Dimensional And Three-Dimensional Models Of Anthropomorphic Robot And Exoskeleton With Links Of Variable Length. // *Proceedings of 24th International Conference “MECHANIKA 2019”*. **17 May 2019** Kaunas University of Technology, Lithuania. PP. 26-39.
11. Borisov A V, Chigarev A V. (2020) The Causes of a Change in The Length of a Person’s Link and Their Consideration When Creating an Exoskeleton. // *Biomedical Journal of Scientific and Technical Research*. ISSN: 2574-1241. Vol. 25, Iss. 1. – P. 18769-18771. DOI: 10.26717/BJSTR.2020.25.004137
12. Borisov A.V., Konchina L.V., Kulikova M.G., Maslova K.S. Simulation of Human Motion in Protective Passive Exoskeleton with Spring Components of Two Kinds // *The Issues of the Military Equipment. Science and Technology Magazine. Technical means for counter-terrorism*. Vol 16. Iss. 9-10 (147-148). – **2020**, pp. 23-31.
13. Piña-Martínez E., Rodríguez-Leal E. (2015) Inverse Modeling of Human Knee Joint Based on Geometry and Vision Systems for Exoskeleton Applications // *Mathematical Problems in Engineering*. Vol. **2015**, Article ID 145734, 14 pages <http://dx.doi.org/10.1155/2015/145734>
14. A robotic exoskeleton for overground gait rehabilitation (2013) / M. Bortole, A. del Ama, E. Rocon, J. C. Moreno, F. Brunetti, J. L. Pons // *Proceedings of the IEEE International Conference on Robotics and Automation (ICRA '13)*, May **2013**. P. 3356-3361.
15. Hassan M., Kadone H., Suzuki K., Sankai Y. (2012) Exoskeleton robot control based on cane and body joint synergies // *Proceedings of the 25th IEEE/RSJ International Conference on Robotics and Intelligent Systems (IROS '12)*, October **2012**. P. 1609-1614.
16. Tsukahara A., Hasegawa Y., Eguchi K., Sankai Y. (2015) Restoration of gait for spinal cord injury patients using HAL with intention estimator for preferable swing speed // *IEEE Transactions on Neural Systems and Rehabilitation Engineering*. V. 23, №. 2. P. 308-318.
17. Tsukahara A., Hasegawa Y., Sankai Y. (2011) Gait support for complete spinal cord injury patient by synchronized leg-swing with HAL // *Proceedings of the IEEE/RSJ International Conference on Intelligent Robots and Systems (IROS '11)*, September **2011**. P. 1737-1742.
18. <https://exoatlet.ru/>
19. <http://www.indego.com/indego/us/en/home>
20. <https://rewalk.com/>
21. <https://www.cyberdyne.jp/english/products/HAL/index.html>
22. <https://eksobionics.com/>
23. <http://raytheon.mediaroom.com/index.php?s=43&item=1652>