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Article

Energy and Personality: a bridge between Physics and Psychology

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Abstract: The objective of this paper is to present a mathematical formalism that states a bridge between Physics and Psychology, concretely between analytical dynamics and personality theory in order to open new insights in this theory. In this formalism energy plays a central role. First, the short-term personality dynamics can be measured by the General Factor of Personality (GFP) response to an arbitrary stimulus. This GFP dynamical response is modelled by a stimulus-response model: an integro-differential equation. The bridge between Physics and Psychology is provided when the stimulus-response model can be formulated as a linear second order differential equation and, subsequently, reformulated as a Newtonian equation. This bridge is strengthened when the Newtonian equation is derived from a minimum action principle, obtaining the current Lagrangian and Hamiltonian functions. However, the Hamiltonian is a non-conserved energy. Then, some changes provide a conserved Hamiltonian function: the Ermakov-Lewis energy. This energy is presented, as well as the GFP dynamical response that can be derived from it. An application case is presented: an experimental design in which 28 individuals consumed 26.51 g of alcohol. This experiment provides an ordinal scale for the Ermakov-Lewis energies that predicts the effect of a single dose of alcohol.

Keywords: personality dynamics; general factor of personality; stimulus-response model; minimum action principle; Hamiltonian; Ermakov-Lewis energy

1. Introduction

Can personality be "dynamic", i.e., changing through time, and opposed to an unchanging "structure"? "The term "structure" as applied to personality has come to connote stability and relative permanence of organization as opposed to states in flux or change which have been termed "dynamic" [1] (page 293).

On the one hand, research on personality has been based almost entirely on the study of the subject differences in stable traits, which are temporally invariant and can be slightly influenced by situations. This is known as the personality trait perspective. Although this approach has been fruitful and has shown important results about the personality structure, the dynamic aspects of personality have not been sufficiently considered. On the other hand, the social-cognitive approach considers that situations underlie the human behavior differences, but it does not accept traits as an explanation of behavior. Both approaches have been competitors historically [2]. An integrative approach to personality that takes into account both stable and dynamic aspects is necessary. This approach has to incorporate both traits and states, thereby reconciling both the stable and the dynamic aspects of personality [3].

There are several integrative models of personality, such as the density distribution approach [4] and the recent Whole Trait Theory [2]. This model asserts that individuals differ not only when regarding their average trait level, but also in how their personality states vary. Besides, the network models of personality [5-7] are based on the idea that personality emerges from the connective structure of different elements. Moreover, the cognitive-affective processing system (CAPS) model of personality [8-10] considers the person-situation interactions, and the PersDyn model [3] considers the trajectory of personality states, which is captured by means of three model parameters: baseline, variability, and attractor strength, as well as the temporal order of the states. Finally, the Complex Dynamical Systems model [11] is a dynamical approach that can exhibit a complex and unpredictable behavior (chaos).

Observe that these approaches attempt to build bridges between dynamics, fundamental in Physics, and personality, fundamental in Psychology. In fact, in science there exists a close attempt to connect dissimilar disciplines, even those whose fields of study seem to be greatly distant, for instance, General Systems Theory (GST) proposed by L. von Bertalanffy [12]. The long-term objective of GST is to construct a universal language common to all scientific disciplines, trying to economize inside knowledge representation and searching for its basic principles. However, a realistic way to reach this objective deals with searching general interdisciplinary theories. A way to obtain these theories is stating bridges or "isomorphisms" between disciplines. The importance of the bridges or "isomorphisms" in science was emphasized by L. Ferrer [13], which defined them as translations of theories from a discipline to another one because a given problem can be considered as being similar in both ones or simply by the challenge to open a new theoretical approach.

Therefore, the objective of this paper is to present a bridge between Physics and Psychology, concretely between analytical dynamics and personality theory, playing a main role in this objective the concept of energy. This objective tries to answer the question stated at the beginning of the paper: Can personality be "dynamic", i.e., changing through time, and opposed to an unchanging "structure"? In fact, S. Amigó [14] speculated already about an approach in which the energy conservation was a theoretical advance to explain personality dynamics. However, this paper presents the way to deal with personality energy in a rigorous manner.

Take into account that here the concrete person and situation (stimuli that activate behaviors) is considered to be an only system. Thus, to make these sciences converge, a correspondence is proposed: on the one hand, the one between the potential energy and the trait as capacity or disposition to perform some behavior and, on the other hand, the one between the kinetic energy and the dynamic process of the personality system. Thus, we resort to the laws of Physics, concretely to analytical dynamics, in order to be applied to Psychology. In fact, this approach is not completely new. On a hand, cognition and decision-making, from the mathematical formalism of quantum mechanics [15] is a good example, and, on the other hand, certain psychological mechanisms, such as the action and perception, appeal to the principle of free energy imported from thermodynamics [16,17]. Nevertheless, no unified theory of analytical dynamics and thermodynamics has been accepted as definitive, despite the attempts of I. Prigogine, such as for instance that presented in [18].

The former stage of this paper objective is to state a dynamical approach applied to the personality theory based on the work of S. Amigó [14], who developed the *Unique Trait Personality Theory* (UTPT). The UTPT claims for a single trait to understand the overall human personality. This single trait is substituted subsequently by the equivalent concept of *General Factor of Personality* (GFP) in [19] in order to follow the generally accepted scientific term. In order to measure the GFP, these authors [19] created a validated questionnaire, the *General Factor of Personality Questionnaire* (GFPQ). This questionnaire is a good instrument to measure the GFP as a personality stable trait in a trait-format scale. However, the same authors had previously developed the Five-Adjective Scale of the General Factor of Personality (GFP-FAS) [20,21], which offers the possibility to measure the GFP dynamical or situational response, composed by five adjectives in a state-format scale [19]. The dimensions of both the GFP and the GFP-FAS are those proposed in [22], that is, a *hedonic scale* which units are named *activation units* (*au*). The interval of variation inside this hedonic scale depends on the particular scale of each adjective of the GFP-FAS. For instance, in the application case presented in Section 5 the hedonic scores vary inside the interval [0,25] au.

In the last decade, the dynamics of the GFP as a consequence of one or more stimuli has been developed. Concretely, several works studying the GFP short-term and long-term dynamical response to stimulant drugs (such as caffeine, cocaine or methylphenidate) and depressant drugs such as alcohol have been studied, as well as the equivalent biological bases of personality responses. The works [23,24] provide the references to all these works. Nevertheless they are also presented in Section 2 in order to highlight the changes in the mathematical structure of the stimulus-response model here presented with respect to a precedent one used in some of those previous works.

In the above mentioned former stage works, the stimulus-response model is presented as a difference-differential equation, i.e., as a discrete-delay differential equation, in which the GFP time derivative is the balance of three terms described by literature: the homeostatic, the excitatory and the inhibitory terms (see Section 2 for details). In the referred works the state-format GFP hedonic scale was applied and the dynamical response was an inverted U shape followed by a slight U shape, which evolved under a value called "tonic level", to which the dynamics tends asymptotically, and plays the role of the "attractor strength" described in the PersDyn model [3]. However, in the present work the stimulus-response model is an integral-differential equation or continuous-delay differential equation. This new approach permits to steer the stimulus-response model to the analytical dynamics formalism by the suitable mathematical operations.

Summarizing the former stage: it is a dynamical approach to personality theory that deepens into its biological bases [24] and into the dynamical nature of personality, which focus on both the general and the individual dynamical responses, in opposite to an exclusive statistical static approach.

Starting from that first stage, a bridge between Physics and Psychology can be created, trying to bring to Psychology a fundamental principle of Physics: the energy conservation principle in the context of psychological reactions to external stimuli, which has been shortly sketched in [25]. First of all, the stimulus-response model can be converted from an integro-differential equation to a linear second order differential equation. From this new formulation the analytical dynamics of the stimulus-response model can be developed. On the one hand, the Newtonian formulation, the minimum action principle and the Lagrangian and Hamiltonian functions can be stablished [26]. Note that it is a straightforward way to state the announced bridge or "isomorphism" between Physics and Psychology. In fact, the Hamiltonian function provides a first definition of energy as an addition of a kinetic energy and a potential energy. Concretely, it has the same mathematical structure as the physical problem corresponding to a harmonic oscillator with mass and retrieving parameter depending on time, influenced by an external time-dependent force.

Note that the cause-effect approach given by the integro-differential equation is widened towards the approach given by the minimum action principle, for which the dynamics minimizes a global functional, the action, between to arbitrary times. This new formulation provides an epistemological validation of the stimulus-response model, due to not all second order differential equations can be derived from a minimum action principle. This is the problem known in the scientific literature as the *Inverse Lagrange Problem*, i.e., finding a Lagrangian function that produces a known second order differential equation. Classical works such as [27] show the difficulty of this problem and in many cases the impossibility to find a Lagrangian function for a determined second order differential equation.

However, the found Hamiltonian function is not a conserved amount due to the stimulus time-dependence. Then, the problem can be transformed into a formulation that

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provides the well-known Ermakov-Lewis invariant [28], which can be reinterpreted as an energy invariant [29]. Thus, starting from our second order differential equation that describes the short-term personality dynamics, and following one of the methods presented in [28], an Ermakov-Lewis energy is found, which can be interpreted as a personality energy-invariant.

In order to illustrate some of the possibilities for Personality Theory that this new perspective offers, an application case referred to an experimental design in which 28 individuals consumed 25.51 g of alcohol (data taken from the work [23]) is presented. Here, the effect of a single dose of alcohol is measured by taking into account both perspectives of personality [1] (page 293): the stable or trait perspective, in which the effect of alcohol consumption is measured as the difference between the trait GFP and the base-line of the GFP-FAS, and the flux of change or dynamic perspective, in which the effect of alcohol consumption is measured as the difference between the maximum GFP-FAS reached and its base-line. Once an ordinal scale for the Ermakov-Lewis energies has been stated, both measures correlate positively with this scale. Then, the Ermakov-Lewis energy becomes a significant scalar magnitude in order to measure the effects of an alcohol dose. Besides, the Ermakov-Lewis energy is also an addition of a kinetic and a potential energy. Thus, it can be observed that the potential energy has its maximum at the beginning of the experiment, equaling almost completely the total Ermakov-Lewis energy, and the kinetic energy being almost zero. Then, it can be observed that both energies exchange their dynamics oscillating around the equilibrium state, whose value coincides statistically with the maximum GFP-FAS reached. In conclusion, this experiment provides evidence about that the inference statistics on groups can be applied to the Ermakov-Lewis energy, which becomes a predictor scalar magnitude of a stimulus effect.

About the following sections: Section 2 presents the stimulus-response model as an integro-differential equation and the steps to reach a linear second order differential equation, as well as its subsequent Newtonian form. Section 3 is devoted to the minimum action principle and the Lagrangian and Hamiltonian formulations, presenting the first non-conserved energy. Section 4 provides the way to get the invariant Ermakov-Lewis energy through the suitable changes. Section 5 is devoted to the application case with alcohol and its main results. Section 6 is the conclusions section where some possible future applications are presented.

2. The stimulus-response model and its Newtonian form

The stimulus-response model, presented by first time in [30] is given by the following integro-differential equation:

$$\left.\begin{array}{c} \dot{q}(t) = a\big(b - q(t)\big) + \delta \cdot s(t) \cdot q(t) - \sigma \cdot \int_{t_0}^{t} exp\left(\frac{r-t}{\tau}\right) \cdot s(r) \cdot q(r)dr \\ q(t_0) = q_0 \end{array}\right\} \tag{1}$$

In Eq. (1) the function $s(t)$ represents the time dynamics of an arbitrary stimulus and $q(t)$ the GFP dynamics, while b and q_0 are respectively its tonic level and its initial value. This $q(t)$ dynamics is a balance of three terms, which provide its time derivative:

1. The *homeostatic control* $a\big(b - q(t)\big)$, i.e., the cause of the fast recovering of the tonic level b to which $q(t)$ tends asymptotically; thus parameter a is called as the *homeostatic control power*. The correct interpretation of the tonic level b is important to be stressed: its value is situational and depends on the individual and the kind of stimulus. However, it plays the same role as the “attractor strength” described in the PersDyn model [3].

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2. The *excitation effect* $\delta \cdot s(t) \cdot q(t)$, which tends to increase the GFP; thus parameter δ is called as the *excitation effect power*.
3. The *inhibitor effect* $\sigma \cdot \int_{t_0}^{t} exp\left(\frac{r-t}{\tau}\right) \cdot s(r) \cdot q(r)dr$, which tends to decrease the GFP and is the cause of a continuously delayed recovering with the weight $exp\left(\frac{r-t}{\tau}\right)$; thus σ is called as the *inhibitor effect power* and τ as the *inhibitor effect delay*. This term makes that this stimulus-response model can also be referred as a continuous-delay differential equation.

Such as it happens with parameter *b*, the rest of the model parameters (*a*, δ, *σ* and *τ*) depend on the individual personality or individual biology and on the type of stimulus.

Table 1 presents the dimensions, units and variation intervals of the variables and parameters involved in the stimulus-response model of Eq. (1). Note that the time unit depends on the experimental design; due to it its units are represented as *t* (time). In addition, the stimulus dimension and its corresponding units depend on the stimulus' nature; due to it they are respectively presented as stimulus (*S*) and *s*. In the theoretical application case presented in Section 5 the time units are minutes, and due to the stimulus has a biochemical nature (alcohol dynamics), its dimension and units are respectively *alcohol amount* (*AA*) and *grams* (*g*). The dimension of the GFP is, such as it has been presented in Section 1, that proposed in [22] as *hedonic scale* (*HS*), and the corresponding units as *activation units* (*au*).

Table 1. Dimensions, units and the variation intervals of the variables and parameters involved in the stimulus-response model of Eq. (1).

Variable/Parameter	Symbol	Dimension	Units	Variation interval
Time	t	Time (T)	t	$[t_0, +\infty]$
GFP	$q(t)$	Hedonic scale (HS)	au	[0,25]
Stimulus	$s(t)$	Stimulus (S)	s	$[0, +\infty]$
Homeostatic control power	a	T^{-1}	t^{-1}	$[0, +\infty]$
Tonic level	b	HS	au	[0,50]
Excitation effect power	δ	$S^{-1}\cdot T^{-1}$	$s^{-1}\cdot t^{-1}$	$[0, +\infty]$
Inhibitor effect power	σ	$S^{-1}\cdot T^{-2}$	$s^{-1}\cdot t^{-2}$	$[0, +\infty]$
Inhibitor effect delay	τ	T	t	$[0, +\infty]$

Similar stimulus-response models have been presented in the last 13 years. A first theoretical presentation was the one of [22], in which the excitation effect was defined as $\boldsymbol{\delta \cdot s\,(t)/b}$ and the inhibitor effect had a discrete delay as $\boldsymbol{\sigma \cdot b \cdot s(t-\tau) \cdot q(t-\tau)}$, which convert the model into a difference-differential or discrete-delay differential equation. A generalization toward many doses trying to reproduce the sensitization and habituation effects, as well as the cocaine addiction process, was provided in [31]. The work [32] is a validation of the above mentioned discrete-delay differential equation with one dose of caffeine, showing how the model can be used as an instrument to predict those individuals inclined to the caffeine misuse. The same model was used in the work [33]

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to predict the GFP response as a consequence of methylphenidate use and of the self-regulation therapy described in [14]. In fact, in the work [33] the stimulus-response model also reproduces the c-fos gen dynamical expression, as a fundamental biochemical base of personality. Also the same model was used in [34] to state a dynamical relationship among the Big Five personality factors and the GFP but in the particular case that the inhibitor effect has not a delay, i.e, as $\boldsymbol{\sigma \cdot b \cdot s(t) \cdot q(t)}$ instead of $\boldsymbol{\sigma \cdot b \cdot s(t-\tau) \cdot q(t-\tau)}$. This same simplified approach is used to study the body-mind problem from a dynamical perspective in [35], although the same body-mind problem has been deeply studied by including the delay in the model of [24]. Finally, the above cited work [23] deals with the study of the GFP response to an alcohol dose and how to use the stimulus-response model to predict the effect of a single dose of alcohol.

The stimulus-response model of Eq. (1) presents the novelty that the excitation effect is proportional to the stimulus and to the GFP, not only to the stimulus, and that the tonic level is present neither in the excitation effect nor in the inhibitor effect. It was used as a theoretical advance by first time in [30] under the hypothesis that being more nonlinear is synonymous of being more adaptable to different responses. This hypothesis was confirmed in the experimental design presented in [36] because Eq. (1) can also reproduce the dynamical happiness and depression responses. In addition, this new approach permits to steer the stimulus-response model to the analytical dynamics formalism by the suitable mathematical operations that are developed in the next paragraphs and sections.

In the following, in order to get a Newtonian formulation for Eq. (1) in this section and a Lagrangian function in Section 3, a second order differential equation equivalent to Eq. (1) is needed. To do this, firstly the time derivative is taken in Eq. (1):

$$\begin{aligned}\ddot{q}(t) &= \big(-a+\delta\cdot s(t)\big)\dot{q}(t)+\delta\cdot\dot{s}(t)\cdot q(t) - \\ &-\sigma\frac{d}{dt}\left(exp\left(-\frac{t}{\tau}\right)\int_{t_0}^{t} exp\left(\frac{r}{\tau}\right)\cdot s(r)\cdot q(r)\,dr\right) = \\ &= \big(-a+\delta\cdot s(t)\big)\dot{q}(t)+\delta\cdot\dot{s}(t)\cdot q(t) + \\ &+\left(\frac{\sigma}{\tau}\right)\int_{t_0}^{t} exp\left(\frac{r-t}{\tau}\right)\cdot s(r)\cdot q(r)dr-\sigma\cdot s(t)\cdot q(t)\end{aligned} \quad (2)$$

Subsequently the term $exp\left(-\frac{t}{\tau}\right)\int_{t_0}^{t} exp\left(\frac{r}{\tau}\right)\cdot s(r)\cdot q(r)\,dr$ is isolated from Eq. (1) and substituted in Eq. (2), which after reorganization can be written with its initial conditions as:

$$\left.\begin{array}{c}\ddot{q}(t)+\gamma(t)\cdot\dot{q}(t)+v(t)\cdot q(t)=\frac{a\cdot b}{\tau}\\ q(t_0)=q_0\\ \dot{q}(t_0)=a(b-q_0)+\delta\cdot s_0\cdot q_0\end{array}\right\} \quad (3)$$

In Eq. (3) s_0 is the stimulus' value in the initial time $t=t_0$ and:

$$v(t)=\frac{a}{\tau}+\left(\sigma-\frac{\delta}{\tau}\right)s(t)-\delta\cdot\dot{s}(t) \quad (4)$$

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$$\gamma(t) = a + \frac{1}{\tau} - \delta \cdot s(t) \tag{5}$$

However, to reach a convenient structure to obtain the Newtonian formulation, Eq (3) must be rewritten in its Sturm-Liouville canonical form by multiplying it by the factor:

$$u(t) = u_0 \cdot exp\left(\int_{t_0}^{t} \gamma(r)\, dr\right) = u_0 \cdot exp\left(\left(a + \frac{1}{\tau}\right)(t - t_0) - \delta \int_{t_0}^{t} s(r) dr\right) \tag{6}$$

In Eq. (6) u_0 is an undetermined constant. Thus, Eq. (5) becomes:

$$\left.\begin{array}{c} \frac{d}{dt}\left(u(t) \cdot \dot{q}(t)\right) + u(t) \cdot v(t) \cdot q(t) = u(t) \cdot a \cdot b/\tau \\ q(t_0) = q_0 \\ \dot{q}(t_0) = a(b - q_0) + \delta \cdot s_0 \cdot q_0 \end{array}\right\} \tag{7}$$

Note that Eq. (7) is a version of the stimulus-response model equivalent to the second Newton's law of dynamics [26] given by Eq. (1). In fact, following this Newton's law, if the momentum is defined as $p(t) = u(t) \cdot \dot{q}(t)$, Eq. (7) can be rewritten as:

$$\left.\begin{array}{c} \frac{dp}{dt} = \frac{d}{dt}\left(u(t) \cdot \dot{q}(t)\right) = -u(t) \cdot v(t) \cdot q(t) + u(t) \cdot a \cdot b/\tau \\ p(t_0) = u(t_0) \cdot \dot{q}(t_0) \end{array}\right\} \tag{8}$$

Eq. (8) is a first bridge stated in this paper between Physics and Psychology. In fact, this is an epistemological approach that must be emphasized: from the initial stimulus-response model that holds a causal-effect principle, the Newtonian causal-effect approach of Eq. (8) puts the optics on the forces that produce changes in the momentum dynamics. More concretely Eq. (8) is equivalent to that of a harmonic oscillator with a time-dependent mass $u(t)$, which is non-dimensional, and with a time-dependent retrieving parameter $v(t)$ with T^{-2} dimensions, and subjected to the external force $u(t) \cdot a \cdot b/\tau$ with $HS{\cdot}T^{-2}$ dimensions (see Table 1). The difference with respect to the physical problem is that, here, the retrieving parameter $v(t)$ can take an arbitrary sign during its evolution, while in Physics it is always positive.

3. The minimum action principle and the Lagrangian and Hamiltonian functions

Let us briefly summarize the minimum action principle. In Physics the action S is written as [26]:

$$S = \int_{t_1}^{t_2} L(t, q, \dot{q}) dt \tag{9}$$

In Eq. (9) $t_2 > t_1$ are two arbitrary time instants and $L(t, q, \dot{q})$ is the Lagrangian function, which has the dimensions of energy; thus the action S has the dimensions of energy by time. The minimum action principle asserts that, from all the possible trajec-

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tories between t_1 and t_2, the trajectory that minimizes the action is that holds the Euler-Lagrange equation [26]:

$$\frac{d}{dt}\left(\frac{\partial L}{\partial \dot{q}}\right) = \frac{\partial L}{\partial q} \tag{10}$$

Note that if the Lagrangian L would depend on more variables Eq. (10) would hold for every one of these variables. In our case the only variable of the formalism is q that represents the GFP dynamics. By the visual inspection of Eq. (7) it is easy to deduce that the Lagrangian that holds Eq. (10) is:

$$L(t,q,\dot{q}) = \left(\frac{1}{2}\right)u(t)\cdot\dot{q}^2 - \left(\frac{1}{2}\right)u(t)\cdot v(t)\cdot q^2 + u(t)(a\cdot b/\tau)\cdot q \tag{11}$$

Observe that the aspect of the Lagrangian is the most common case in Physics: it is a subtraction between a kinetic energy $T(t,\dot{q}) = \left(\frac{1}{2}\right)u(t)\cdot\dot{q}^2$ and a potential energy $V(t,q) = \left(\frac{1}{2}\right)u(t)\cdot v(t)\cdot q^2 - u(t)(a\cdot b/\tau)\cdot q$, such that:

$$L(t,q,\dot{q}) = T(t,\dot{q}) - V(t,q) \tag{12}$$

The epistemological consequences of this new approach are still more radical than the ones provided by Newton's equation in Eq. (8): the causal-effect principles that take place in every instant in Eq. (1) or Eq. (8) are reinterpreted from the minimum action principle that minimizes the action globally between two arbitrary time instants, i.e., for all the set of possible trajectories between these two arbitrary time instants. In fact, the minimum action principle is a way to validate epistemologically the presented formalism due to not all second order differential equations such as Eq. (8) can be deduced from a minimum action principle. This is the problem known in the scientific literature as the *Inverse Lagrange Problem*; that is, finding a Lagrangian that produces a known second order differential equation from the Euler-Lagrange equations. Classical works such as [27] show the difficulty to find a Lagrangian. In a similar case to that of Eq. (8), the problem reduces itself to find the potential energy $V(t,q)$.

Advancing in the bridge between Physics and Psychology, the momentum p and the Hamiltonian can be defined from the Lagrangian as [26]:

$$p = \frac{\partial L}{\partial \dot{q}} = u(t)\cdot\dot{q} \tag{13}$$

$$H(t,q,p) = \frac{\partial L}{\partial \dot{q}}\dot{q} - L(t,q,\dot{q}) =$$

$$= \left(\frac{1}{2}\right)\left(\frac{p^2}{u(t)}\right) + \left(\frac{1}{2}\right)u(t)\cdot v(t)\cdot q^2 - u(t)(a\cdot b/\tau)\cdot q \tag{14}$$

Note that Eq. (14) is actually a kind of energy in the physical sense because it can be rewritten as:

$$H(t,q,p) = T(t,p) + V(t,q) \tag{15}$$

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where $T(t,p) = \left(\frac{1}{2}\right)\left(\frac{p^2}{u(t)}\right)$ is the kinetic energy and $V(t,q) = \left(\frac{1}{2}\right)u(t)\cdot v(t)\cdot q^2 - u(t)(a\cdot b/\tau)\cdot q$ is the potential energy. In the context of the formalism here presented, the three energies, H, T and V, have the dimensions of HS^2·T^{-2} (see Table 1). However, $H(t,q,p)$ it is not an invariant energy due to it is explicitly time-dependent [26]. However, it is possible to get an invariant energy, known in the scientific literature as an Ermakov-Lewis invariant, with the suitable changes. This is the goal of the following section.

4. Getting the invariant Ermakov-Lewis energy

The Ray and Reid's work [28] provides several methods to get invariants related to Eq. (3); these are known as Ermakov-Lewis invariants (note that a collection of invariants can be got). Here we follow what we think is the most intuitive Ray and Reid's method, which works directly on Eq. (3). First of all, the change $Q(t) = f(t)\cdot q(t)$, with unknown $f(t)$, is essayed in Eq. (3). This change provides that the term multiplying to $\dot{Q}(t)$ can be removed when $f(t) = \sqrt{u(t)}$; thus $u(t)$ is non-dimensional, and the $\dot{Q}(t)$ variable has the same dimensions as $Q(t)$, i.e., HS dimensions (see Table 1). Therefore the so-called as normal form of a second order differential equation is obtained:

$$\ddot{Q}(t) + \Omega(t)\cdot Q(t) = (a\cdot b/\tau)\sqrt{u(t)} \tag{16}$$

where

$$Q(t) = \sqrt{u(t)}\cdot q(t) \tag{17}$$

$$\Omega(t) = v(t) - \left(\left(\frac{1}{2}\right)\left(\frac{\ddot{u}(t)}{u(t)}\right) - \left(\frac{1}{4}\right)\left(\frac{\dot{u}^2(t)}{u^2(t)}\right)\right) \tag{18}$$

That is, the mentioned change reduces Eq. (3) to Eq. (16), which is the equation of a harmonic oscillator with frequency $\Omega(t)$ and with dimensions T^{-2} (see Table 1), subjected to an external force $(a\cdot b/\tau)\sqrt{u(t)}$ with HS·T^{-2} dimensions (see Table 1). Two new consecutive changes are needed now: the first change on the dependent variable of Eq. (16) as $x(t) = Q(t)/C(t) + A(t)$, and the second change on the independent variable $\varphi = \int_{t_0}^{t}\frac{dr}{C^2(r)}$, where $C(t)$ and $A(t)$ are undetermined auxiliary functions by the moment. Observe that if $x(t)$ has to conserve the same dimensions as $q(t)$ and $Q(t)$ then $A(t)$ has to have these dimensions, i.e., HS, and $C(t)$ has to be non-dimensional (see Table 1). These changes provide:

$$\ddot{x}(\varphi) + C^3(t)\left(\ddot{C}(t) + \Omega(t)\cdot C(t)\right)\cdot x(\varphi) +$$

$$+C^3(t)\left(-\ddot{C}(t)\cdot A(t) - 2\dot{C}(t)\cdot\dot{A}(t) - C(t)\cdot\ddot{A}(t) - \Omega(t)\cdot C(t)\cdot A(t) - (a\cdot b/\tau)\sqrt{u(t)}\right) = 0 \tag{19}$$

where $\ddot{x}(\varphi) = \frac{d^2x(\varphi)}{d\varphi^2}$. In order to Eq. (19) becomes an equation with constant parameters, we force it to hold:

$$\ddot{C}(t) + \Omega(t)\cdot C(t) = \frac{k}{C^3(t)} \tag{20}$$

$$\ddot{A}(t) + 2\left(\frac{\dot{C}(t)}{C(t)}\right)\dot{A}(t) + k\cdot\frac{A(t)}{C^4(t)} + (a\cdot b/\tau)\left(\frac{\sqrt{u(t)}}{C(t)}\right) = 0 \tag{21}$$

where k is an undetermined constant. Note that the $\varphi = \int_{t_0}^{t}\frac{dr}{C^2(r)}$ variable has dimensions of time (T), thus it could be interpreted as an intrinsic time of the dynamics that arises as a consequence of the stimulus.

In addition, taking into account Eqs (20) and (21), Eq. (19) becomes:

$$\ddot{x}(\varphi) + k\cdot x(\varphi) = 0 \tag{22}$$

The Lagrangian, momentum and Hamiltonian corresponding to Eq. (22), through the corresponding Euler-Lagrange equations, are:

$$L_x(\varphi, x, \dot{x}) = \left(\frac{1}{2}\right)\dot{x}^2 - \left(\frac{k}{2}\right)x^2 \tag{23}$$

$$p_x = \frac{\partial L}{\partial \dot{x}} = \dot{x} \tag{24}$$

$$E = H_x(\varphi, x, p_x) = \frac{\partial L}{\partial \dot{x}}\dot{x} - L_x(\varphi, x, \dot{x}) = \left(\frac{1}{2}\right)p_x^2 + \left(\frac{k}{2}\right)x^2 \tag{25}$$

Note that the Hamiltonian H_x is explicitly time-independent (in which the time is φ), therefore it is an invariant energy [26], and this is the reason to be also called as E. In fact, undoing the changes proposed above, this energy can be expressed in terms of the original variables, arising as the known Ermakov-Lewis invariant [28]:

$$E = T_e + V_e = \left(\frac{1}{2}\right)\left(\sqrt{u(t)}\cdot C(t)\cdot\dot{q} + C^2(t)\cdot\dot{A}(t) + \left(\left(\frac{1}{2}\right)C(t)\left(\frac{\dot{u}(t)}{\sqrt{u(t)}}\right) - \sqrt{u(t)}\cdot\dot{C}(t)\right)q\right)^2 + \left(\frac{k}{2}\right)\left(\left(\frac{\sqrt{u(t)}}{C(t)}\right)q + A(t)\right)^2 \tag{26}$$

Observe that also Eq. (26) is an addition of a kinetic energy $T_e = \left(\frac{1}{2}\right)\left(\sqrt{u(t)}\cdot C(t)\cdot\dot{q} + C^2(t)\cdot\dot{A}(t) + \left(\left(\frac{1}{2}\right)C(t)\left(\frac{\dot{u}(t)}{\sqrt{u(t)}}\right) - \sqrt{u(t)}\cdot\dot{C}(t)\right)q\right)^2$ and a potential energy $V_e = \left(\frac{k}{2}\right)\left(\left(\frac{\sqrt{u(t)}}{C(t)}\right)q + A(t)\right)^2$. Thus, such as also T. Padmanabhan emphasizes [29], the Ermakov-Lewis invariant E of Eq. (26) is an invariant energy. In our context, its dimensions are also $HS^2{\cdot}T^{-2}$ (see Table 1). However, although Te is a kinetic energy in Eq. (25) with respect to the x variable, it is not a "pure kinetic energy" in Eq. (26), such as the kinetic energy of Eq. (15), because it contains the q variable in addition to its derivative $\dot{q}$. Nevertheless, it will be referred as a kinetic energy from now onwards. Note in addition that $C(t)$ and $A(t)$ must hold Eqs. (20) and (21) and that, Eq. (22) can be solved analytically:

$$x(\varphi) = \begin{cases} k_1\varphi + k_2 : k = 0 \\ k_1\cdot sin\left(\sqrt{k}\cdot\varphi\right) + k_2\cdot cos\left(\sqrt{k}\cdot\varphi\right) : k > 0 \\ k_1\cdot exp\left(\sqrt{-k}\cdot\varphi\right) + k_2\cdot exp\left(-\sqrt{-k}\cdot\varphi\right) : k < 0 \end{cases} \tag{27}$$

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Undoing again in Eq. (27) the changes proposed before and assuming that the $A(t)$ and $C(t)$ auxiliary variables hold Eqs. (20) and (21), the final $q(t)$ expression is obtained:

$$q(t) = \begin{cases} \left(\frac{C(t)}{\sqrt{u(t)}}\right) \cdot \left(-A(t) + k_1 \int_{t_0}^{t} \frac{dr}{C^2(r)} + k_2\right) : k = 0 \\ \left(\frac{C(t)}{\sqrt{u(t)}}\right)\left(-A(t) + k_1 \cdot sin\left(\sqrt{k} \int_{t_0}^{t} \frac{dr}{C^2(r)}\right) + k_2 \cdot cos\left(\sqrt{k} \int_{t_0}^{t} \frac{dr}{C^2(r)}\right)\right) : k > 0 \\ \left(\frac{C(t)}{\sqrt{u(t)}}\right)\left(-A(t) + k_1 \cdot exp\left(\sqrt{-k} \int_{t_0}^{t} \frac{dr}{C^2(r)}\right) + k_2 \cdot exp\left(-\sqrt{-k} \int_{t_0}^{t} \frac{dr}{C^2(r)}\right)\right) : k < 0 \end{cases} \quad (28)$$

Note that some problems must still be solved: (a) the initial conditions for $A(t)$ and $C(t)$ to solve analytically or numerically Eqs. (20) and (21); (b) the suitable choice for $q(t)$ in Eq. (28); and (c) the values of k_1 and k_2 parameters, as well as the value of parameter k. In the following, these problems shall try to be solved with a goal of universality, i.e., to be independent of the individual or of the type of stimulus.

First of all, in order to choose the initial conditions for $A(t)$ and $C(t)$, the following assumptions in $t = t_0$ in Eq. (26) are done: $u_0 = 1$, $C_0 = 1$, $A_0 = 0\,\text{au}$, $\dot{A}_0 = 0\,\text{au} \cdot t^{-1}$ and $\frac{1}{2} C_0 \left(\dot{u}_0 / \sqrt{u_0}\right) - \sqrt{u_0} \cdot \dot{C}_0 = 0$, which provide $\dot{C}_0 = (1/2)\dot{u}_0 t^{-1} = (1/2)(a + 1/\tau - \delta \cdot s_0)\, t^{-1}$, and also provide the initial value of the Ermakov-Lewis energy:

$$E = E_0 = \left(\frac{1}{2}\right) \dot{q}_0^2 + \left(\frac{k}{2}\right) q_0^2\, \text{au}^2 \cdot \text{t}^{-2} = \left(\frac{1}{2}\right) \left(a(b - q_0) + \delta \cdot s_0 \cdot q_0\right)^2 + \left(\frac{k}{2}\right) q_0^2\, au^2 \cdot t^{-2} \quad (29)$$

Note that Eq. (29) is a classical addition of kinetic and potential energy, whose value is conserved for all the GFP evolution period as a consequence of a stimulus.

In addition the choice of $q(t)$ in (28) is clear: the $k > 0$ case. The case $k = 0$ has the unstable term $k_1 \int_{t_0}^{t} dr/C^2(r)$, and the $k < 0$ case has the unstable term $k_1 \cdot exp\left(\sqrt{-k} \int_{t_0}^{t} dr/C^2(r)\right)$. Once the case $k > 0$ has been chosen as the stable one, the comparison of Eq. (28) in $t = t_0$ with the initial values in Eq. (7) provides that $k_1 = \dot{q}_0/\sqrt{k}$ and $k_2 = q_0$ with $\dot{q}_0 = a(b - q_0) + \delta \cdot s_0 \cdot q_0$. Observe that finally one parameter is non-fixed. The preferred option is taking k_1 as the free parameter due to the k parameter (with dimensions T^{-2}) can be considered in future studies as a measure of the resistance of the individual to change its personality (as compared with a harmonic oscillator in Physics). Then $k = \frac{\dot{q}_0^2}{k_1^2}$.

Then, the conclusion is that the Ermakov-Lewis energy of Eq. (26) can be written as:

$$E = \left(\frac{1}{2}\right) \dot{q}_0^2 + \left(\frac{k}{2}\right) q_0^2 = T_e + V_e =$$
$$\left(\frac{1}{2}\right)\left(\sqrt{u(t)} \cdot C(t) \cdot \dot{q} + C^2(t) \cdot \dot{A}(t) + \left(\left(\frac{1}{2}\right) C(t) \left(\frac{\dot{u}(t)}{\sqrt{u(t)}}\right) - \sqrt{u(t)} \cdot \dot{C}(t)\right) q\right)^2 +$$
$$+ \left(\frac{1}{2}\right) \left(\frac{\dot{q}_0^2}{k_1^2}\right) \left(\left(\frac{\sqrt{u(t)}}{C(t)}\right) q + A(t)\right)^2 \quad (30)$$

Moreover, the $q(t)$ dynamics is written as:

$$q(t) = \frac{C(t)}{\sqrt{u(t)}}\left(-A(t) + k_1 \cdot sin\left(\frac{\dot{q}_0^2}{k_1^2} \int_{t_0}^{t} \frac{dr}{C^2(r)}\right) + q_0 \cdot cos\left(\frac{\dot{q}_0^2}{k_1^2} \int_{t_0}^{t} \frac{dr}{C^2(r)}\right)\right) \quad (31)$$

Note in Eqs. (30) and (31) that $\dot{q}_0 = a(b - q_0) + \delta \cdot s_0 \cdot q_0$, and that k_1 is a free but positive-valued parameter.

5. An application case: a stimulus given by a dose of alcohol

In order to illustrate some of the possibilities for Personality Theory that this new perspective offers, an application case referred to an experimental design in which 28 individuals consumed 26.51 g of alcohol (data taken from the work [23]) is used to study the personality dynamics as a consequence of a single dose of alcohol consumption. This application case allows observing the following contributions of this theoretical approach inside psychological processes:

1. It permits to obtain the personality invariant Ermakov-Lewis energy generated as a consequence of a stimulus applied to an individual, as an amount of characteristic energy corresponding to the dynamical process.

2. It permits to measure the effect of a stimulus through the Ermakov-Lewis energy.

3. It permits to obtain the dynamics of the kinetic and potential energies that define the personality invariant Ermakov-Lewis energy, as well as its relationship with the tonic level or attractor of the GFP dynamical response to a stimulus.

However, before detailing these contributions, some previous results about the stimulus dynamics are necessary to be presented. In fact, observe that all the theoretical background developed in the previous sections has a general application because it is valid for an arbitrary stimulus $s(t)$. Natural hypotheses for the mathematical structure of the stimulus are that $s(t)$ be zero in a determined time $t > t_0$ or that $s(t) \to 0$ as $t \to +\infty$. These are of course ideal cases because other uncontrolled stimuli can simultaneously have some influence on the individual personality. These ideal cases are similar to those of some problems in Physics, for instance, a motion problem when a friction force is not considered because it is negligible. This is the key feature: the negligibility of other stimuli in the sense that they must stay statistically hidden in the GFP evolution caused by the studied stimulus; on the contrary, the effects of the studied stimulus on personality would be clearly observable.

In the experiment here considered, the outcomes of the 28 alcohol consumers are calibrated for Eq. (1), and the average parameter values are taken as representative of the consumer group. Thus, the effect on the individual personality hides the situational effects of other stimuli [24,33,36].

To obtain the simplest mathematical structure of alcohol dynamics $s(t)$ a two level pharmacokinetics model [37] is considered:

$$\left.\begin{array}{c} \frac{dm(t)}{dt} = -\alpha \cdot m(t) \\ m(t_0) = M \end{array}\right\} \tag{32}$$

$$\left.\begin{array}{c} \frac{ds(t)}{dt} = \alpha \cdot m(t) - \beta \cdot s(t) \\ s(t_0) = s_0 \end{array}\right\} \tag{33}$$

In Eq. (32) $m(t)$ represents the evolution of the ingested alcohol before entering in the organism's plasma and metabolizing system, being M the alcohol initial amount and being α the alcohol assimilation rate. In Eq. (33) the $s(t)$ variable represents the alcohol amount in organism, assuming that its initial value is s_0, i.e., the neither metabolized nor excreted alcohol of a possible previous consumption, and β is the alcohol elimination rate. The coupled differential equations system of Eqs. (32) and (33) can be integrated producing:

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$$s(t) = s_0 \cdot exp(-\beta \cdot t) + \begin{cases} \left(\alpha \cdot \frac{M}{\beta-\alpha}\right)\left(exp(-\alpha \cdot t) - exp(-\beta \cdot t)\right): \alpha \neq \beta \\ \alpha \cdot M \cdot t \cdot exp(-\alpha \cdot t): \alpha = \beta \end{cases} \quad (34)$$

In the considered experiment the subjects consumed 26.51 g of alcohol, and their GFP was measured every 7 minutes during 126 minutes, with the 5 adjectives scale of the GFP-FAS in the hedonic scale [20-22], inside the interval [0,25] au, i.e., each adjective is scored inside the interval [0,5] in the hedonic scale, and the initial condition (base-line) q_0 was also measured before consumption. In order to calibrate the model of Eq. (1), the assimilation and elimination rates values for alcohol vary inside the following confidence intervals: $\alpha \in [0.00118, 0.0205]\ min^{-1}$ and $\beta \in [0.00462, 0.00533]\ min^{-1}$ (95% confidence), calculated from the outcomes of [38,39]. The results of the model calibration provide the optimal parameter values for each individual and, from them and the corresponding initial values the Ermakov-Lewis energies can be computed by Eq. (29).

From a psychological point of view, the Ermakov-Lewis energy represents the individual dynamics as a direct and more exact way because, being an individual invariant (or an individual conserved energy) it increases the reliability criterion of the dynamical response to a stimulus. Thus, the first step is to present the Ermakov-Lewis energies (E) of the 28 alcohol consumers by their percentiles (PC) in Table 2.

Table 2. Percentiles (PC) and Ermakov-Lewis energies (E)

PC	E ($au^2 \cdot min^{-2}$)
5	0.0110474
10	0.0310201
15	0.2843064
20	1.0410622
25	2.7886561
30	3.8237451
35	4.5159857
40	5.4501270
45	6.0233622
50	8.5371192
55	9.7771215
60	12.9138017
65	18.3775677
70	23.4749812
75	26.3534685
80	27.6097661
85	56.9425103
90	718.4536519
95	44179.1148819

In addition, Table 2 permits to consider the Ermakov-Lewis energy as an ordinal variable, and then, to classify its values into three categories by the 33th and 66th percentiles, sorting them from the lesser to the greater category by their scores. Now, the relationship between the Ermakov-Lewis energy and the effect of an alcohol dose can be stated. To do this, let DIFtrait and DIFmax be respectively the difference between the trait and the initial value (base-line), and the GFP-FAS maximum reached and the initial value (base-line). Both variables are normally distributed: DIFtrait has a 0.138 Kolmogorov-Smirnov test outcome with a 1.88 signification level, and DIFmax has a 0.11 Kolmogorov-Smirnov test outcome with a 0.2 signification level. In addition, both variables can

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also be classified into three categories by the 33th and 66th percentiles, sorting them as well from the lesser to the greater categories by their scores.

On a hand, the relationship between the ordinal Ermakov-Lewis energy and the effect of an alcohol dose from a stable perspective can be set up with the DIFtrait ordinal variable and, on the other hand, the effect of an alcohol dose from a dynamic perspective can be set up with the DIFmax ordinal variable. Table 3 shows both relationships with a Gamma test. Both correlations are positive and significant. In addition, these results also show that: (a) the greater Ermakov-Lewis energy the greater maximum GFP-FAS score; (b) the lower initial GFP-FAS score the greater Ermakov-Lewis energy will be involved in the dynamical process.

Table 3. Gamma coefficients and statistical significance beween the Ermakov-Lewis energy (E) and the DIFtrait and DIFmax variables.

	E	
	Gamma	**Sig.**
DIFtrait	0.477	0.033
DIFmax	0.520	0.013

This relationship is strengthened by correlating the categorized Ermakov-Lewis energy as a dependent variable with the non-categorized DIFtrait and DIFmax variables by the Eta and the Square Eta tests. See Table 4 in which the differences between the GFP initial condition (or base-line) and the GFP maximum reached (DIFmax), as well as the differences between the GFP initial condition (or base-line) and the GFP trait (DIFtrait), predict a moderated proportion of the Ermakov-Lewis energy variance. Concretely, DIFtrait predicts a 47% of the Ermakov-Lewis energy variance, while DIFmax predicts a 41% of the Ermakov-Lewis energy variance.

Table 4. Eta and Square Eta correlations between the categorized Ermakov-Lewis energy (E), as dependent variable, and the non-categorized DIFtrait and DIFmax variables.

	E	
	Eta	**Square Eta**
DIFtrait	0.687	0.47
DIFmax	0.642	0.41

In order to reproduce the representative GFP dynamics of the consumer group, the stimulus-response model of Eq. (1) has been calibrated for the mean scores of the 28 individuals. Table 5 presents the outcomes of the optimal parameter values, as well as the Ermakov-Lewis energy and its initial kinetic and potential energies. Observe that the initial value of alcohol in plasma is $s_0 = 0$ g, i.e., the individuals have not consumed alcohol from very long before.

Table 5. Optimal parameter values and the Ermakov-Lewis energy and its initial kinetic and potential energies for the mean scores of the 28 individuals.

Parameter name	**Symbol**	**Values with units**
Initial GFP	q_0	13.4 au
Initial stimulus	s_0	0 g
Alcohol initial amount	M	26.51 g
Alcohol assimilation rate	α	0.011 min^{-1}
Alcohol elimination rate	β	0.004 min^{-1}
Homeostatic control power	a	0.059 min^{-1}
Tonic level	b	19,636 au

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Excitation effect power	δ	0.0009 $g^{-1}\cdot min^{-1}$
Inhibitor effect power	σ	0.0001 $g^{-1}\cdot min^{-2}$
Inhibitor effect delay	τ	28.774 min
Ermakov-Lewis energy	E	12.576 $au^2\cdot min^{-2}$
Initial kinetic energy	T_0	0.0697 $au^2\cdot min^{-2}$
Initial potential energy	V_0	12.506 $au^2\cdot min^{-2}$

The presented computations and figures have been done with MATHEMATICA. Figure 1 presents the evolution of the stimulus s(t) given by Eq. (34), i.e., the evolution of the alcohol present in the organism, during a period four times the period of the experiment (126 min). Observe its trend to zero as $t \to +\infty$.

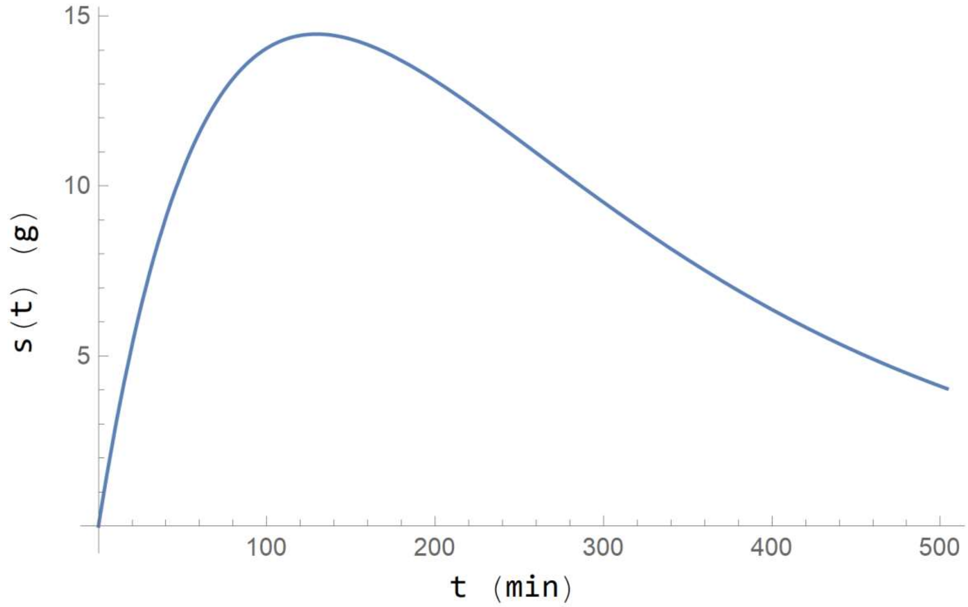

Figure 1. Evolution of the alcohol present in the organism in a period four times the period of the experiment (126 min).

Figure 2 presents the mean scores of the 28 consumers and the calibrated curve obtained with the stimulus-response model given by the optimal parameter values given by Table 5. The evolution period is restricted to the one of the experiment (126 min). The GFP initial condition score is 13.29 au and the maximum score reached is 19.07 au. The determination coefficient obtained in the calibration is R^2=0.986 and an Anderson-Darling test for the residuals provide that they distribute normally as N(0, 0.23) with a statistic of 0.14 and a signification level of 0.99, i.e., it is demonstrated that the residuals are white noise.

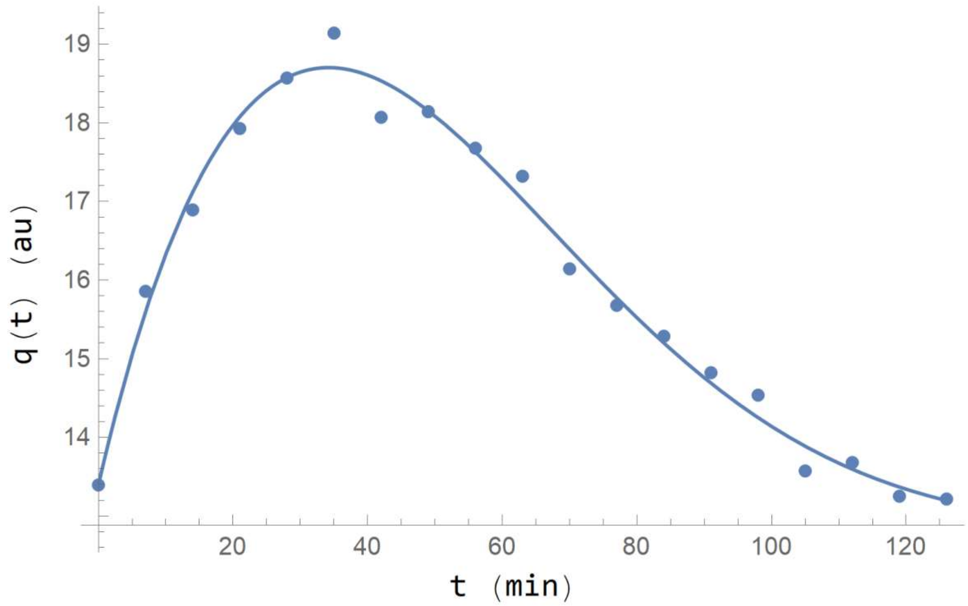

Figure 2. Evolution of the GFP or $q(t)$ for the 28 individual mean scores (dots) and the calibrated curve (line), in the period of the experiment (126 min).

Besides, Figure 3 shows the projection of the same curve for four times the period of the experiment jointly with its tonic level or attractor b=19,636 au. Note the trend of the GFP response to this value as $t \to +\infty$. Note also that this representative individual of the group reproduces clearly the response pattern pointed out by the literature as a consequence of alcohol consumption [23], i.e., the inverted-U shape GFP response with a recovering period under its initial value and an asymptotic convergence to the tonic level value as $t \to +\infty$, coinciding with the phases of the dynamical response described by the PersDyn model [3].

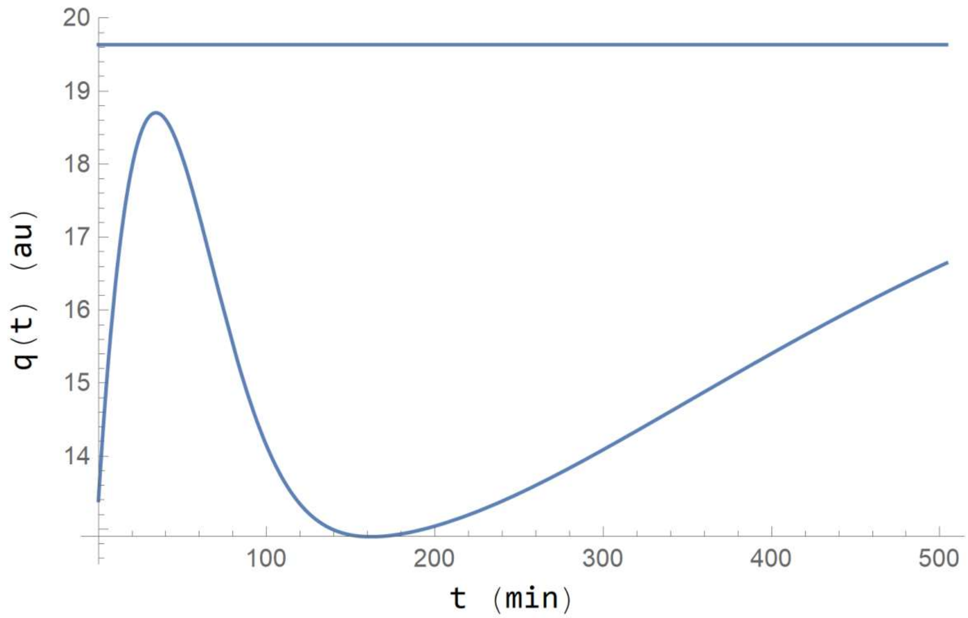

Figure 3. Evolution of the GFP or *q(t)* for the calibrated curve and the tonic level b=19,636 au (upper line), in a period four times the one of the experiment, i.e., 4·126 min.

To compute the evolution of the Ermakov-Lewis energy and the kinetic and potential energies given by Eq. (30), the $A(t)$ and $C(t)$ auxiliary variables dynamics have been solved numerically by Eqs. (20) and (21) with the initial values provided in Section 4, taking into account that the free parameter value k_1 has been chosen as $k_1 = 1\, au$. Then $k = \left(\frac{\dot{q}_0}{k_1}\right)^2 = (a(b - q_0) + \delta \cdot s_0 \cdot q_0)^2$, i.e., $k = 0.0016\ \mathrm{min}^{-2}$ (note that $s_0 = 0$ g).

Figure 4 presents the evolution of the Ermakov-Lewis energy with value $E = \left(\frac{1}{2}\right) \dot{q}_0^2 + \left(\frac{k}{2}\right) q_0^2 = 12.576\ \mathrm{au}^2 \cdot \mathrm{min}^{-2}$ (constant), jointly with its kinetic and potential energies.

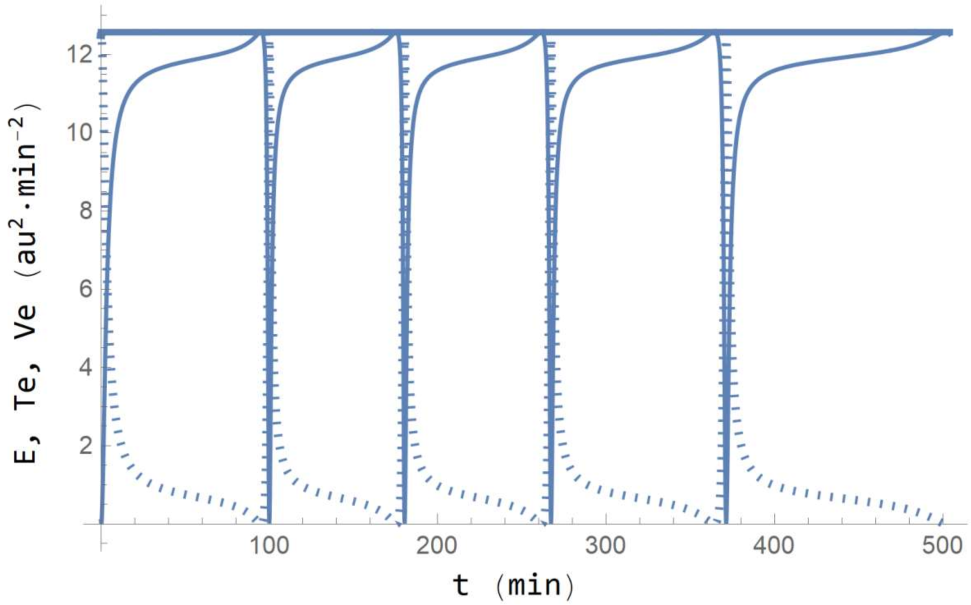

Figure 4. Ermakov-Lewis energy (E, upper straight line), kinetic energy (Te, first increasing line), and potential energy (Ve, first decreasing dotted line), versus time.

Note in Figure 4 that Te and Ve exchange their dynamics while the Ermakov-Lewis energy (*E*) keeps constant, in fact this exchange continues for four full blocks. But, what do these blocks mean? To answer this question note that the time interval of the first block ends at approximately the 98th min but it corresponds with the score of 13.54 au. Comparing with the physical problem of the harmonic oscillator, it is equivalent to the case when the mass runs half period. In addition, in t=0 min ($q_0 = 13.39\, au$), the kinetic energy is practically zero, Te=0.0697 au^2·min^{-2}, and the potential energy, Ve=12.506 au^2·min^{-2}, is practically equal to the Ermakov-Lewis energy, E=12.576 au^2·min^{-2}. Thus, after 98 min, these values are recovered, with a score of 13.54 au, in which Te is again practically zero and Ve is practically *E*. Observe that the GFP score is almost the same and represents the extremes of a harmonic oscillator.

The conclusion is that the GFP maximum score reached, 19.07 au, is identified as the equilibrium point. In other words, the GFP maximum scored reached is the value to which the oscillator tends to return. But this value is similar to the one of the tonic level *b*=19,636 au. However, this is not a coincidence. Let *b*2 be the categorized variable of the 28 *b* parameter values. If a gamma test is done between *b*2 variable and the reached maximum GFP-FAS variable (categorized), the statistic obtained is γ = 0.722, with a signification level p<0.001. Thus, statistically the GFP maximum scored reached and the tonic level or attractor are closely related.

Therefore, a general conclusion is that, from the theoretical and practical points of view, the Ermakov-Lewis energy represents a central instrument to study personality

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and its dynamics as a consequence of a stimulus, as well as an advanced tool to confirm the predictions of the UTPT [14,22].

5. Conclusions and future work

The here presented finding about a bridge or "isomorphism" between Physics and Psychology, concretely between analytical dynamics and personality theory, must be emphasized. The conclusion is that we can apply the energy conservation principle of Physics to obtain the state-level personality dynamics produced by some environmental stimuli; in fact we can consider some psychological mechanisms as analogous to those of Physics.

In order to reach these results, on the one hand, the stimulus-response model, that has an original integro-differential formulation, has been reformulated as a Newtonian equation of a harmonic oscillator with one external force acting on it. In addition, the generalized mass and the retrieving force of this harmonic oscillator, as well as the referred external force, depend on time. Thus, the problem of the personality dynamics links with Physics in a natural but complex way due to the time-dependencies of the Newtonian formulation on the stimulus-response model.

On the other hand, the link between analytical dynamics and personality theory goes farther due to the Newtonian formulation of the stimulus-response model can be deduced from a minimum action principle through the Euler-Lagrange equations. This approach has an important epistemological consequence, that is, in the same way that the natural laws of dynamics studied by Physics, the personality dynamics can be studied from a principle that puts its optics in the minimization of a global magnitude such as action is. Moreover, the minimum action principle permits to state the Lagrangian and the Hamiltonian functions for the personality dynamics. The Hamiltonian function is the most important one because it arises as an addition of a kinetic energy and a potential energy, such as it happens in many cases in Physics. Therefore, the link between analytical dynamics and personality theory is still more strengthened.

However, the complexity of the Newtonian formulation of the stimulus-response model is translated into the Hamiltonian function, becoming an explicit time-dependent function and, as a consequence, not being a conserved magnitude. Nevertheless, from approximately the last fifty years this problem has been studied in Physics, and the researches have provided a way to get a conserved energy: the Ermakov-Lewis energy. Following similar steps of these researches, an Ermakov-Lewis energy can be also get for personality dynamics and a new way to obtain results of this dynamics has been able to be presented. In fact, an application case for a concrete stimulus, alcohol, has been also presented, in order to demonstrate that the Ermakov-Lewis energy and the related formalism can be handled mathematically. Therefore, the link between analytical dynamics (Physics) and personality theory (Psychology) has been brought beyond.

Besides, the application case has confirmed the relationship between the kinetic and potential energies of the Ermakov-Lewis energy and the GFP dynamical response, concretely, the relationship between the potential energy and the capacity of an individual to react to a stimulus, as well as the relationship between the stable personality and the kinetic energy. In fact, the invariant Ermakov-Lewis energy has been demonstrated to be of central importance to better understand the dynamic response to a stimulus: this characteristic energy amount can be used in inferential statistics, with the sense that a dynamics can be reduced to a representative scalar, obtaining a relationship between the Ermakov-Lewis energy and the effect of a stimulus such as an alcohol dose, as well as the relationship between the potential and kinetic energies and the tonic level or dynamics' attractor.

On the other hand, the inspiration obtained from the application cases of the Ermakov-Lewis energy in Physics should be considered. See for instance the work [40] for these applications. However, the most important application for the authors is the one

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related with the quantum approach, for instance, that considered in the work [41]. In this approach, the tonic level as asymptotic state in Eq. (1) is not considered, and the quantization rules are applied on the Hamiltonian of Eq. (14). Then, a time-dependent Schrödinger equation arises, from which the wave function can be solved analytically in a similar way that it has been provided for the Ermakov-Lewis energy. The wave function provides the quantum version of the Hamilton equations deduced by D. Bohm & B. J. Hiley [42] from the Schrödinger equation, which are stochastic, and from which quantized trajectories and bifurcations can be studied. Thus, multiple GFP dynamical response patterns and asymptotic states can arise. Therefore, the authors' hypothesis is that this approach could provide: (a) a way to study the normal and the disorder dynamical patterns of personality; (b) how a bifurcation can steer, as a consequence of a stimulus, from a normal pattern of personality to a disordered one. Then, those sudden changes that many times are observed in personality theory could have a mathematical explanation.

About the limitations of this work, it must be also emphasized that the personality dynamics given by Eq. (1) is restricted to the short-term response to a single stimulus. A future research should deal with a stimulus-response model that considers a long-term response such as the one provided in [31] where addiction and recovering is considered as a consequence of a series of cocaine consumptions, with different doses and consumption frequency patterns. Of course, this approach would be more realistic, because the processes of sensitization and habituation are observed even in the second dose consumption. However, the study here presented has been developed in the same way as the study [22] that considers a single dose of a stimulant drug. In other words: Science normally walks from easier to more complex approaches.

Other possible future research works can follow different lines. On the one hand, experimental designs to observe the energy of personality in relation to its biological bases should be set up. That is, something similar to, for instance, those devoted to model the biological responses to a stimulus in relation with personality [24,33,35].

This article cannot be ended without expressing that, in addition to the methodological and practical applications that the proposal here presented suggests, the finding of a conserved energy in psychological processes, specifically in the dynamics of situational personality, assumes something more profound, and it is that there exists a temporal symmetry in the psychological mechanisms. If the corresponding Ermakov-Lewis Lagrangian is symmetric respect to time, as it is in our case, according to the Noether's theorem there must be a conserved quantity, and we have proved it for energy. Symmetry is considered as the deepest unifying concept of Physics in recent times, being understood as the ultimate foundation of physical laws, and here it has been verified that it can also be the foundation of psychological processes. In fact, the Noether's theorem reproduces the same results for the Ermakov-Lewis energy as the dynamical approach here presented, such as Ray & Reid [28] also deduce.

In general, the authors consider that the finding here presented is a theoretical progress in personality theory, from which different applications have to be found in the future. In fact, when a coherent set of ideas such as those presented in this approach has been so successful in Physics from Newton to the present days, it should be developed until its last theoretical and experimental consequences.

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