Is complex number theory free from contradiction?

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Abstract In the paper it is demonstrated that a valid path to a contradiction in complex number theory exists. In the path use is made of Euler's identity and simple trigonometry. Each step can be easily verified.

Keywords Basic complex number theory · Euler's identity · contradiction

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1 Introduction

Complex numbers are at the heart of modern day science. We mention for instance, the analysis of waves with Fourier analysis and quantum theory. Those mathematical theories would see conceptual difficulties when there exists a flaw in the complex numbers. That makes for huge stakes regarding the question raised in this letter. Nevertheless, the presented mahematics delivers a simple proof of a contradiction in complex numbers. Its consequences are for a next step in research of this contradiction.

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2 Simple algebra

Let us start with

$$z_{1/n} = e^{i(\gamma+(1/n))} \tag{1}$$
$$z'_{\sin(1/n)} = (1+\sin(1/n))e^{i\left(\frac{\chi+\pi}{2}\right)}$$

with $n > 0$ together with γ and χ in $\mathbb{R}$. If $\gamma = \gamma_+ = \frac{\chi+\pi}{2}$ then it is possible to have

$$\lim_{n\to\infty} \left(z_{1/n} - z'_{\sin(1/n)}\right)_{\gamma=\gamma_+} = 0 \tag{2}$$

Moreover we can introduce a phase angle $\varphi_{1/n}$ such that $z_{1/n} - z'_{\sin(1/n)} = |z_{1/n} - z'_{\sin(1/n)}|e^{i\varphi_{1/n}}$. There are many paths to go to zero such as in (2). This changes the phase angle with a constant and doesn't make any diffence to the principles of our analysis. Therefore, we concentrate our attention to $z_{1/n} - z'_{\sin(1/n)} = |z_{1/n} - z'_{\sin(1/n)}|e^{i\varphi_{1/n}}$. Now let us write down the separate forms of real and imaginary parts of $z_{1/n} - z'_{\sin(1/n)} = |z_{1/n} - z'_{\sin(1/n)}|e^{i\varphi_{1/n}}$.

$$\cos(\gamma + (1/n)) = -(1+\sin(1/n))\sin\left(\frac{\chi}{2}\right) + |z_{1/n} - z'_{\sin(1/n)}|\cos(\varphi_{1/n}) \tag{3}$$
$$\sin(\gamma + (1/n)) = (1+\sin((1/n)))\cos\left(\frac{\chi}{2}\right) + |z_{1/n} - z'_{\sin(1/n)}|\sin(\varphi_{1/n})$$

In this equation $\cos\left(\frac{\chi+\pi}{2}\right) = -\sin\left(\frac{\chi}{2}\right)$ and $\sin\left(\frac{\chi+\pi}{2}\right) = \cos\left(\frac{\chi}{2}\right)$. Let us establish beforehand that

$$L = \lim_{n\to\infty} \frac{\sin(1/n)}{|z_{1/n} - z'_{\sin(1/n)}|} = \tag{4}$$
$$\lim_{n\to\infty} \frac{\sin(1/n)}{\sqrt{1+(1+\sin(1/n))^2 - 2(1+\sin(1/n))\sin[(1/n)+\pi/2]}}$$

and that $L = \frac{1}{\sqrt{2}}$. The easiest way is to demonstrate $L^2 = \frac{1}{2}$ first and to also carefully note that

$$\lim_{n\to\infty} \sqrt{1+(1+\sin(1/n))^2 - 2(1+\sin(1/n))\sin[(1/n)+\pi/2]} =$$
$$\sqrt{\lim_{n\to\infty}(1+(1+\sin(1/n))^2 - 2(1+\sin(1/n))\sin[(1/n)+\pi/2])} =$$
$$\sqrt{1+(1+0)^2 - 2(1+0)\sin(0+\pi/2)} = \sqrt{1+1-2} = 0$$

Therefore, the rule of l'Hopital can be applied. In addition,

$$L' = \lim_{n\to\infty} \frac{\cos(1/n) - 1}{\sin(1/n)} = 0 \tag{5}$$

Subsequently, focus the attention to the $\cos(\gamma + \frac{1}{n})$ of (3) in the first place. After some rewriting and taking $\gamma = \gamma_+$ we can obtain from the first equation of (3)

$$-\sin\left(\frac{\chi}{2}\right)\left(\frac{\cos(1/n)-1}{\sin(1/n)}\right)\frac{\sin(1/n)}{|z_{1/n}-z'_{\sin(1/n)}|} - \cos\left(\frac{\chi}{2}\right)\frac{\sin(1/n)}{|z_{1/n}-z'_{\sin(1/n)}|} = \tag{6}$$

$$-\sin\left(\frac{\chi}{2}\right)\frac{\sin(1/n)}{|z_{1/n}-z'_{\sin(1/n)}|} + \cos(\varphi_{1/n})$$

Let us write $\lim_{n\rightarrow\infty}\varphi_{1/n} = \varphi$. With increasing n, the circle around zero in the complex plane shrinks. In each of these concentric circles with shrinking radius, $\varphi_{1/n}$ exists. If a reader thinks that the limit $\lim_{n\rightarrow\infty}\varphi_{1/n} = \varphi$ is non-existent, then where in this process, i.e. for which $N \in \mathbf{N}$ such that $n \geq N$, does $\varphi_{1/n}$ no longer exists?

With, $L' = 0$ and $L = \frac{1}{\sqrt{2}}$ it then follows that

$$\cos(\varphi) = \frac{1}{\sqrt{2}}\left(\sin\left(\frac{\chi}{2}\right) - \cos\left(\frac{\chi}{2}\right)\right) \tag{7}$$

Along similar lines and using $L' = 0$ and $L = \frac{1}{\sqrt{2}}$ we can obtain from the second equation of (3) that

$$\cos\left(\frac{\chi}{2}\right)\left(\frac{\cos(1/n)-1}{\sin(1/n)}\right)\frac{\sin(1/n)}{|z_{1/n}-z'_{\sin(1/n)}|} - \sin\left(\frac{\chi}{2}\right)\frac{\sin(1/n)}{|z_{1/n}-z'_{\sin(1/n)}|} = \tag{8}$$

$$\cos\left(\frac{\chi}{2}\right)\frac{\sin(1/n)}{|z_{1/n}-z'_{\sin(1/n)}|} + \sin(\varphi_{1/n})$$

Therefore,

$$\sin(\varphi) = -\frac{1}{\sqrt{2}}\left(\sin\left(\frac{\chi}{2}\right) + \cos\left(\frac{\chi}{2}\right)\right) \tag{9}$$

2.1 The case $\chi/2 = \pi/3$

Let us assume that $\chi = 2\pi/3$. Moreover, let us restrict the interval of the limit phase angle φ, with,$-\pi \leq \varphi \leq \pi$. Then, $\sin(\chi/2) = \frac{\sqrt{3}}{2} \approx 0.866$ and $\cos(\chi/2) = 1/2 = 0.500$. From equations (7) and (9) we then obtain

$$\cos(\varphi) = \frac{1}{\sqrt{2}}\left(\frac{\sqrt{3}}{2} - \frac{1}{2}\right) \approx 0.259 \tag{10}$$

$$\sin(\varphi) = -\frac{1}{\sqrt{2}}\left(\frac{\sqrt{3}}{2} + \frac{1}{2}\right) \approx -0.966$$

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Following the path of the angular analysis this gives

$$\cos(\varphi) + \sin(\varphi) = -\frac{1}{\sqrt{2}} \tag{11}$$
$$\cos(\varphi) - \sin(\varphi) = \frac{\sqrt{3}}{\sqrt{2}}$$

In addition, we also have

$$\cos(\varphi)\sin(\varphi) = \frac{1}{\sqrt{2}}\left(\frac{\sqrt{3}}{2} - \frac{1}{2}\right)\left(-\frac{1}{\sqrt{2}}\right)\left(\frac{\sqrt{3}}{2} + \frac{1}{2}\right) = \tag{12}$$
$$-\frac{1}{2}\left(\frac{3}{4} - \frac{1}{4}\right) = \left(-\frac{1}{2}\right) \times \frac{1}{2} = -\frac{1}{4}$$

And so

$$\sin(2\varphi) = 2\cos(\varphi)\sin(\varphi) = -\frac{1}{2} \tag{13}$$
$$\cos(2\varphi) = \cos^2(\varphi) - \sin^2(\varphi) = -\frac{\sqrt{3}}{2}$$

Therefore, with $-2\pi \leq 2\varphi \leq 2\pi$ and both cos and sin negative in (13), we are allowed to set $2\varphi = \pi + \frac{\pi}{6} = \frac{7\pi}{6}$, with *both* $\sin(2\varphi) = -\frac{1}{2}$ *and* $\cos(2\varphi) = -\frac{\sqrt{3}}{2}$. Hence, $\varphi = \frac{7\pi}{12}$ and the φ is in the interval $-\pi \leq \varphi \leq \pi$. But $\varphi = \frac{7\pi}{12}$ gives

$$\cos(\varphi) = \cos\left(\frac{7\pi}{12}\right) \approx -0.259 \tag{14}$$
$$\sin(\varphi) = \sin\left(\frac{7\pi}{12}\right) \approx 0.966$$

And this is *in contradiction* with (10). Further, when we select $2\varphi = -\pi + \frac{\pi}{6} = -\frac{5\pi}{6}$ it is $-2\pi \leq 2\varphi \leq 2\pi$. Then, for $\varphi = -\frac{5\pi}{12}$ in the required interval of φ, there is no contradiction.

3 Conclusion & discussion

With valid mathematical steps two unequal phase angles, φ_1 and φ_2 can be derived from a problem in complex number theory. In our example we showed that one of those phase angles gives rise to a contradictory result. This proofs, unexpectedly perhaps, a valid path to a contradiction in complex number theory (CNT). Obviously there will be scepticism regarding a discovery of a contradiction in the complex numbers.

The reader must note that there is a difference between what is presented here and aleged made-simple equivalents. This remark can be illustrated by noting that in the first place if one

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does not know there might be a contradictory phase angle like $\phi_1 = 7\pi/12$, one could compute such a phase angle from a problem like the one presented here and happily employ $\phi_1 = 7\pi/12$ because one ignores the possibility of a contradictory phase angle in CNT. Furthermore a trivial example like: let's start with $x = -1$ and look at x^2 then there is one contradictory and one valid x presented. This is *not* the same thing as was presented in the present paper. Without knowledge of the present paper there is *no* correct solution primed. But when starting with $x = -1$ there *is* a correct solution primed. This made-simple example is therefore not at all demonstrtaed to be equivalent to what is presented here.

Moreover, the "it is only a matter of a phase $\exp(\imath\pi)$" is incorrect either. For, we have *two* different phase angles. If a phase factor $\exp(\imath\pi)$ repairs the inconsistency for $\varphi = \frac{7\pi}{12}$ this change creates it for $\varphi = -\frac{5\pi}{12}$.

Scepticism is a good way to advance in the right direction in mathematics and logic. But this scepticism must be fair. It is meaningless if the following questions remain unanswered:

- Is there a mistake in mathematics & in logic in *this* paper? If so where in terms: page, formla & fact. If the latter cannot be accomplished, a rejection is not based on the content of *this* paper.
- Is Wittgenstein right when he [1] asks to first find out how much harm a contradiction does to mathematics?
- Is there a philosophical reason [2] to accept certain absurdities and reject others?

To the author's mind, if a theory is consistent then a valid path to a contradiction should not exist. More details concerning the mathematics are provided in a draft [3].

Declarations

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References

1. L. WITTGENSTEIN, Bemerkungen über die Grundlagen der Mathematik/Remarks on the foundations of mathematics, p56, Blackwell 1956.
2. J. MIZE, Bringing the mathematician into the equation, *Academia Letters*, doi: 10.20935/AL824, 2021.
3. H. GEURDES, Is complex number theory free from contradiction?, viXra:2108.0066v1, 2021.