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Article

Polarimetric Quantum-Strong Correlations with Independent Photons on the Poincaré Sphere

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Abstract: Controllable, quantum-strong correlations of polarization states can be implemented with multi-photon independent states. Polarization-based photonic quantum correlations can be traced back to the overlap of the polarization Stokes vectors on the Poincaré sphere between two polarization filters. The quantum Rayleigh scattering prevents a single photon from propagating in a straight line inside a dielectric medium, and it also provides a mechanism for the projective measurement of polarization. Complexities associated with single-photon sources and detectors can be eliminated because the quantum Rayleigh scattering in a dielectric medium destroys entangled photons. Entanglement-free, identical sources and processing devices give rise to correlations rather than these being caused by "quantum nonlocality". These analytic developments were prompted by the vanishing expectation values of the Pauli spin vector for a single photon of maximally entangled photonic Bell states.

Keywords: quantum optics; quantum correlations; polarization correlations; Stokes vectors

1. INTRODUCTION

Single photons and entangled states of single photons underpin the concepts of optical processing of quantum information [1-2]. Integrated photonic platforms and circuits incorporating a large number of sources, devices, processing units, and photodetectors constitute the objective of large-scale research into and implementation of functionalities for a broad range of applications in quantum computing, quantum communication and quantum metrology [1]. "To a large extent, the second quantum revolution we are witnessing these days strongly relies on generation and manipulation of entangled quantum states." [3]

Polarization-entangled photon pairs [1-2] are of particular interest because of the possible manipulation with Pauli spin operators. A particular feature of these polarization-based photonic systems appears to be their high level of correlations between two separately measured sets of one-photon polarizations, which are detected separately and are commonly associated with quantum nonlocality. The design, fabrication, and operation of quantum photonic integrated circuits will substantially benefit from functionalities of "entangled photons" that can be performed with simplified layouts and reduced costs, by using independent, pure states of photons.

It was realized, more than a decade ago, that polarization measurements in the quantum regime "have not been done with single-photon sources." "Some of the experiments have been performed at light levels in the quantum regime, however, and this suggests strongly that the devices will work in the same way given single-photon sources and detectors" [4, p. 264].

Additionally, recent experimental results open up the way for polarization correlations between photons from "two independent sources of polarized optical photons, and detecting the temporal coincidence of pairs of uncorrelated photons which have never been entangled in the apparatus. …. The measurement procedure adopted in the Bell-type experiments yields the polarization relation between the two members of a pair, either

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entangled or not entangled, in their final preparation state" [5]. Indeed, it is straightforward to show - see below - that independent photons from identical sources reproduce or generate the same correlation function on the Poincaré sphere as the "entangled photons", but with the added benefit of externally controlling the correlation, which can significantly improve the performance of data processing devices.

One photon per radiation mode underpins the concept of entangled photons [3] which, apparently, are needed to create a statistical correlation between separately measured quantum events. Yet, the quantum Rayleigh scattering prevents a single photon from propagating in a straight line inside a dielectric medium [6-9]; equally, inside a dielectric medium, the quantum Rayleigh stimulated emission can recapture an absorbed photon as well as coupling photons between two radiation modes, thereby creating groups of photons from individual ones [10-12]. A summary of Rayleigh scattering can be found in Appendix A below.

The quantum Rayleigh spontaneous and stimulated emissions were well documented four decades ago [6-7] when the first experimental results of apparently single photon propagation were incorporated in the theory of quantum optics. Even though the subject was revisited [8] to clearly find that the probability of spontaneous emission increases with the refractive index of the medium, the question of one single photon being scattered by photon-dipole interactions has been completely ignored in the professional literature of quantum optics [13].

The assumption that spontaneously emitted, parametrically down-converted individual photons cannot be amplified because of a low level of pump power would, in fact, prevent any sustained emission in the direction of phase-matching condition because of the Rayleigh spontaneous scattering. In a nonlinear crystal pumped, e.g., with a pump wave (p) and for frequency down-converted photons of frequencies $\omega_s + \omega_i = \omega_p$, the gain-providing medium which generates the spontaneous emission, will also amplify the initially single photons, particularly so in the direction of wavevector matching conditions, even for limited space-time overlap. A phase-pulling effect leading to the phase relation $\varphi_s + \varphi_i = \varphi_p + \pi/2$ also occurs, [12] which is capable of countering phase-mismatch. Thus, the commonly assumed one single photon output does not physically happen. At least several photons may be associated with each individual and discrete electronic "click". A group of photons of the same frequency propagating inside a dielectric medium will follow a straight-line because a photon locally absorbed by a dipole, will be recaptured by the other photons in the group through stimulated emission. Nevertheless, sometimes, only one photon may survive the propagation to reach the photodetector.

A common interpretation maintains that: "There is another fundamental issue of nonlocality pertaining to entangled states: the idea that measurements performed in spatially separated locations can affect each other" [3]. However, this interpretation of experimental results as proof of quantum nonlocality does not stand up to physical scrutiny and has been disproved and rebutted from various perspectives [9], [14-19]. The correlation function of "entangled photons" is predicated on a mixed quantum state, or a global wavefunction, which is time- and space- independent. However, an ensemble distribution is built up from instantaneous, single event measurements of photonic beam fronts. The distinction between pure quantum states and mixed quantum states is presented in Section 2 below, in line with long-standing definitions [20].

This article traces the origin of the polarization-based quantum correlation function back to the overlap between the polarization Stokes vectors of the detecting filters, on the measurement Poincaré sphere. After reviewing the shortcomings of entangled states of photons in Section 2, the local measurements leading to statistical distributions of quantum correlations are specified in Section 3 by using polarization states of independent photons. By generating the polarization Stokes vectors through measurements based on Pauli spin operators, the correlation function is obtained in Section 4, with independent photons, in a manner that will reduce the complexities of operational quantum photonic

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systems. Physical aspects that facilitate the design, fabrication, and operations of quantum functionalities are presented in Section 5.

2. THE SHORTCOMINGS OF THE ENTANGLED STATES OF PHOTONS

A particular feature of "entangled photons" is described as follows: "A single particle state vector is not an accurate description of the single particle when it is in an entangled state" [3]. If so, simultaneous measurements of the two polarization-entangled photons are needed in order to determine the correlation function. But a wavefunction collapse is required for the remote influence to take place, namely, one measurement should precede the other. However, in this case one easily finds, as shown in this Section, that the expectation value of the Pauli spin operators – probing the state of polarization of one photon – vanishes, and no polarization can be specified for the correlation or comparison of the distributions of values.

A polarization-entangled photonic state is given in terms of horizontal $|\mathrm{H}\rangle$ and vertical $|\mathrm{V}\rangle$ polarizations by the expression

$$|\psi_{AB}\rangle = \alpha\, |\mathrm{H}\rangle_A\, |\mathrm{H}\rangle_B + \beta\, |\mathrm{V}\rangle_A\, |\mathrm{V}\rangle_B \tag{1}$$

where the indexes A and B refer to the two entangled photons that propagate in opposite directions, and to be detected by spatially separated photodetectors A and B. The normalization condition is $|\alpha|^2 + |\beta|^2 = 1$. The coefficients α and β specify the properties of the source outputs. If only a pair of photons is emitted at any given point in time, then at the level of each individual event and its measurement, the coefficients can only be either $\alpha(t) = 1\ and\ \beta(t) = 0$ or $\alpha(t) = 0\ and\ \beta(t) = 1$. By contrast, the statistical average of the ensemble of measurements would lead to an average of $\overline{\alpha} = \overline{\beta} = \sqrt{0.5}$ resulting in a Bell state $|\psi_{AB}\rangle = (\,|\mathrm{H}\rangle_A\, |\mathrm{H}\rangle_B + \,|\mathrm{V}\rangle_A\, |\mathrm{V}\rangle_B)/\sqrt{2}$. Thus, the Bell state does not reflect the physical reality of *each* single, individual event, and the interpretation becomes counter-intuitive [3]. As a time-independent state describing a statistical ensemble, the Bell state is mathematically equivalent to having all four polarization modes populated simultaneously. It should be noted that the state $|\psi_{AB}\rangle$ does not specify the number of photons it carries, and any manipulation of it involves only its state of polarization.

This state $|\psi_{AB}\rangle$ somehow remains unchanged despite the photons propagating through dielectric media of beam splitters, optical fibers, crystal polarizers, etc., and is used to calculate an ensemble correlation function between the polarization states obtained by setting the linear polarization filters to various angles θ_A and θ_B with respect to a common frame of reference, i. e, $|\mathrm{H}\rangle$ and $|\mathrm{V}\rangle$ at a third location. The quantum evaluation of the correlation function is carried out with two polarization filter operators $\hat{\sigma}_A$ and $\hat{\sigma}_B$ and given by

$$\begin{aligned} E_c(\theta_A;\theta_B) &= \langle\psi_{AB}\,|\,\hat{\sigma}_A \otimes \hat{\sigma}_B\,|\psi_{AB}\rangle = \\ &= cos\ 2\theta_A\ cos\ 2\theta_B + + 2\ \alpha\ \beta\ sin\ 2\theta_A\ \ sin\ 2\ \theta_B \end{aligned} \tag{2}$$

where the Pauli spin operators for the linear polarizations are denoted $\hat{\sigma}_k = sin(\,2\theta_k)\,\hat{\sigma}_1 + cos(2\theta_k)\,\hat{\sigma}_3$ with $k = A$ or B, the angle θ_k specifies the rotation of a linear polarization filter and the projecting Pauli operators are in this case

$$\hat{\sigma}_1 = |\,\mathrm{H}\,\rangle\langle\,\mathrm{V}\,| + |\,\mathrm{V}\,\rangle\langle\,\mathrm{H}\,| \text{ and } \hat{\sigma}_3 = |\mathrm{H}\,\rangle\langle\mathrm{H}\,| - |\mathrm{V}\,\rangle\langle\mathrm{V}\,|\,.$$

The first operator $\hat{\sigma}_1$ leads to the second term of Eq. (2) and corresponds to an interference effect of the two projected input eigenstates onto the measuring eigenmodes, and requires for a non-zero value that the two eigenstates be simultaneously populated, that is, each detecting basis mode of the same polarization analyzer should receive more than one photon. For *each* single event or measurement of the statistical ensemble either $|\mathrm{H}\rangle_A\ and\ |\mathrm{H}\rangle_B$, or $|\mathrm{V}\rangle_A\ and\ |\mathrm{V}\rangle_B$ are present simultaneously, but Eq. (2) requires, for maximum correlation, the simultaneous presence of both possible outputs, that is $\alpha \neq 0$

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and $\beta \neq 0$. This can only happen if all four radiation modes are populated simultaneously, e.g. $\overline{\alpha} = \overline{\beta} = \sqrt{0.5}$. The Pauli operators act on the state of polarization, regardless of the number of photons. In other words, the mixed state of "entangled photons" emulates a source of multiple photons per radiation mode, i.e.$|\psi_{AB}\rangle = (|\mathrm{H}\rangle_A |\mathrm{H}\rangle_B + |\mathrm{V}\rangle_A |\mathrm{V}\rangle_B)/\sqrt{2}$ where the indexes *A* and *B* refer to the two *groups* of photons that propagate in different directions, and to be detected by spatially separated photodetectors *A* and *B*.

If a collapse of the wave function is to take place for entangled photons upon detection of a photon at either location, then the two separate measurements do not coincide as required by Eq. (2). In this case, a local measurement vanishes for the maximally entangled Bell states, e. g. $|\psi_{AB}\rangle = (|\mathrm{H}\rangle_A |\mathrm{H}\rangle_B + |\mathrm{V}\rangle_A |\mathrm{V}\rangle_B)/\sqrt{2}$, that is, $\langle \psi_{AB} | \hat{\sigma}_A \otimes \hat{I}_B |\psi_{AB}\rangle = 0$, with $\hat{I}_B = |\mathrm{H}\rangle\langle\mathrm{H}| + |\mathrm{V}\rangle\langle\mathrm{V}|$ being the identity operator. This leads to a physical contradiction as local experimental outcomes determine the state of polarization to be compared with its pair quantum state. This overlooked feature of maximally entangled Bell states renders them incompatible with the polarimetric measurements carried out to determine the state of polarization of photons, thereby explaining the experimental results of reference [5] which were obtained with independent photons.

The mixed quantum state $|\psi_{AB}\rangle$ of Eq. (1) is space- and time-independent and considered to be a global state which can be used in any context, anywhere, and at any time. Nevertheless, the Hilbert spaces of the two photons move away from each other and do not spatially overlap, so that any composite Hilbert space is *mathematically* generated by means of a tensor product. Even so, the absence of a Hamiltonian of interaction renders any suggestion of a mutual influence rather questionable [14].

A physical analysis should involve polarimetric Stokes vectors generated at the two separate measuring locations. Photons polarized parallel to the common reference coordinates will pass randomly through polarization filters or analyzers as a result of quantum Rayleigh scatterings of photons [6-9]. These photons will emerge with the same state of polarization as that of the filters, and the corresponding Stokes parameters of the polarization state vector $\vec{s}$ on the Poincaré sphere are calculated as the expectation values of the Pauli spin vector operator $\hat{\sigma} = (\hat{\sigma}_1, \hat{\sigma}_2, \hat{\sigma}_3)$ for the Jones vectors [21] of the filters k = *A*, *B* , i. e. $|u(\theta_k)\rangle = cos\ \theta_k |\mathrm{H}\rangle + sin\ \theta_k |\mathrm{V}\rangle$, resulting in [21]:

$$\vec{s}_k = \langle u(\theta_k)| \hat{\sigma} |u(\theta_k)\rangle \tag{3}$$

The relation connecting the correlation overlap between two polarization state vectors $|u_k\rangle = |u(\theta_k)\rangle$ in the Jones representation and the overlap or correlation of their corresponding Stokes vectors $\vec{s}_k$ on the Poincaré sphere is given [21] by:

$$|\langle u_A | u_B \rangle|^2 = \frac{1}{2}(1 + \vec{s}_A \cdot \vec{s}_B) \tag{4a}$$

$$E_c(\theta_A; \theta_B) = \vec{s}_A \cdot \vec{s}_B = cos\, 2(\theta_A - \theta_B) \tag{4b}$$

to obtain in Eq. (4b) the same correlation function as in Eq. (2) for the corresponding coefficients of "maximally entangled" photons.

In the next Section, independent states of photons will be identified in the Jones representation of polarization states and the overlap of the corresponding Stokes vectors on the Poincaré sphere will explain the quantum correlation as the mathematical overlap of the two polarization filter states in the joint Hilbert space of the measurements.

It is claimed that the presence of two operators in the correlation tensor product results in stronger correlation values for entangled states. Yet, the spatially separate measurements of one photon reaching each polarization filter can be identified specifically by using the identity operators $\hat{I}_k$ for each polarization filter, that is:

$$\begin{aligned} E_c(\theta_A; \theta_B) &= (\langle \psi_{AB} | \hat{\sigma}_A \otimes \hat{I}_B)(\hat{I}_A \otimes \hat{\sigma}_B |\psi_{AB}\rangle) \\ &= \langle \Phi_A | \Phi_B \rangle = \end{aligned} \tag{5}$$

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where the state $|\Phi_k\rangle$ is generated by the operator $\hat{\sigma}_k$. In this way, the correlation function is associated with the overlap, or fidelity, of two state vectors $|\Phi_k\rangle$ displayed on the Poincaré sphere as an inner product. As a consequence, the same correlation function between the polarization filters of the detectors can be generated with independent states of photons as presented in the next Section.

3. QUANTUM CORELATIONS OF INDEPENDENT PHOTONS

The correlation function E_c for the detection of two photons A and B of the state $|\psi_{AB}\rangle$ of Eq. (1) is defined as the sum of averaged products of any two eigenvalues +1 or −1 assigned to eigenstates $|x(\theta)\rangle$ and $|\,y\,(\theta)\rangle$, respectively, involving the probabilities of their coincident detections, i.e. P_{++}; P_{--}; P_{-+}; P_{+-} for various settings θ_A and θ_B of the polarization filters:

$$E_c(\theta_A;\theta_B) \equiv P_{++}(\theta_A;\theta_B) + P_{--}(\theta_A;\theta_B) - P_{+-}(\theta_A;\theta_B) - P_{-+}(\theta_A;\theta_B) \tag{6}$$

The probabilities are linked experimentally to the counts $N_{i,j}$ of coincident photons through the equality $N_{ij}(\theta_A;\theta_B) = P_{ij}(\theta_A;\theta_B)\, N_{tot}$, where $i\,, j\ = +\,; -$ and N_{tot} is the total number of coincident photons. In the case of independent statistical events at the two spatially separated detectors, the joint probability becomes the product of the independent probabilities, that is $P_{ij}(\theta_A;\theta_B) = P_i(\theta_A)\, P_j(\theta_B)$. As a simple example, let us consider the detecting filter's polarization eigenstates, for $k = A;\, B$, in the reference frame of coordinates which lies in the measurement Hilbert space, common to the two locations, that is:

$$|x(\theta_k)\rangle = cos\theta_k|x\rangle + sin\theta_k|y\rangle \tag{7a}$$

$$|y(\theta_k)\rangle = -sin\theta_k|x\rangle + cos\theta_k|y\rangle \tag{7b}$$

rotated from the reference states $|x\rangle$ and $|y\rangle$ by an angle θ_k.

For the same photon state of linear polarization rotated by an angle φ from the reference coordinates, that is:

$$|\,\Psi\,(\varphi)\rangle = cos\,\varphi\ |x\rangle + \sin\varphi\,|y\rangle \tag{8}$$

reaching both detectors, the correlation function is derived in the remainder of this Section. The case of different input rotation angles will be derived in Section 4.

The equality of Eq. (6) can be rewritten, for independent statistics, as:

$$\begin{aligned} E_c(\theta_A;\theta_B) &= [P_+\,(\theta_A) - P_-\,(\theta_A)][P_+\,(\theta_B) - P_-\,(\theta_B)] = \\ &= P\,(\theta_A) \otimes P\,(\theta_B) \end{aligned} \tag{9}$$

with the vectorial structure of $P\,(\theta_k) = (P_+\,(\theta_k)\,; -P_-\,(\theta_k)\,)$, the dyadic or tensor product is a shorthand notation for the direct product of the two $P\,(\theta_k)$ vectors. The tensor product is commonly used in quantum mechanics to point out that two different Hilbert spaces may have different systems of coordinates. Otherwise, the direct product is more practical.

From Eqs. (7) the projection operators for the two measuring eigenstates are

$$\hat{P}_+(\theta) = |x(\theta)\rangle\langle x(\theta)| \tag{10a}$$

$$\hat{P}_-(\theta) = |y(\theta)\rangle\langle y(\theta)| \tag{10b}$$

The polarization observable $\hat{\sigma}(\theta_k)$ in Eq. (9) has the form:

$$\hat{\sigma}(\theta_k) = \hat{P}_+(\theta_k) - \hat{P}_-(\theta_k) = \sin(\,2\theta_k)\,\hat{\sigma}_1 + cos(2\theta_k)\,\hat{\sigma}_3 \tag{11}$$

the projecting Pauli operators being $\hat{\sigma}_1 = |x\rangle\langle y| + |y\rangle\langle x|$ and $\hat{\sigma}_3 = |x\rangle\langle x| - |y\rangle\langle y|$. The angle θ_k of a rotated polarization filter is set in the Jones representation relative to a measurement basis of reference or generic eigenstates in the measurement Hilbert space of $\mathcal{H} = \mathcal{H}_A \otimes \mathcal{H}_B$.

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By combining Eqs. (8), (9) and (11), the correlation function is evaluated for $P_+(\theta_k) - P_-(\theta_k) = \langle\hat{\sigma}(\theta_k)\rangle$ to be:

$$E_c(\theta_A;\theta_B) = \langle\Psi\,|\,\hat{\sigma}(\theta_A)|\,\Psi\rangle\langle\Psi\,|\,\hat{\sigma}(\theta_B)|\,\Psi\rangle = \\ = cos\,2(\theta_A-\varphi)\;cos\,2\,(\theta_B-\varphi) \tag{12}$$

The correlation function for $\theta_A = 0, \theta_B = \theta\; and\; \varphi = 0\; or\; \pi/2$ in Eq. (12) becomes $E_c(0;\theta) = cos\,2\,\theta$, which is the result for entangled states of photons [9], [14-15]. With only one state of polarization being populated in Eq. (8), this example points to the correlation between the polarization analyzers as the source of experimentally detected correlations, as opposed to an assumed quantum nonlocality. The corresponding classical derivation is presented in Appendix B below, in terms of Stokes vectors overlapping on the same Poincaré sphere, suggesting the possibility of implementing quantum-strong correlation functions with reduced complexities of multi-photon systems.

The correlation function is a *numerical* calculation as opposed to a physical interaction. Thus, the numerical comparison of the data sets is carried out at a third location *C* where the reference system of coordinates is located for comparison or correlation calculations of the two sets of measured data, and does not require physical overlap of the observables whose operators are aligned with the system of coordinates of the measurement Hilbert space onto which the detected state vectors are mapped. In this case, the correlation operator $\hat{C} = \hat{\sigma}_A \otimes \hat{\sigma}_B$ of Eq. (2) can be reduced to [21; Eq. (A6)]:

$$\hat{C} = (\boldsymbol{a}\cdot\hat{\sigma})(\boldsymbol{b}\cdot\hat{\sigma}) = \boldsymbol{a}\cdot\boldsymbol{b}\hat{I} + i(\boldsymbol{a}\times\boldsymbol{b})\cdot\hat{\sigma} \tag{13}$$

where the polarization vectors $\boldsymbol{a}$ and $\boldsymbol{b}$ identify the orientation of the detecting polarization filters in the Stokes representation, and $\hat{\sigma} = (\hat{\sigma}_1, \hat{\sigma}_2, \hat{\sigma}_3)$ is the Pauli spin vector (with $\hat{\sigma}_2 = i\,\hat{\sigma}_1\,\hat{\sigma}_3$). The presence of the identity operator in Eq. (13) implies that, when the last term vanishes for a linear polarization state, the correlation function is determined by the orientations of the polarization filters. This can be easily done with independent and linearly polarized states of Eq. (8), because of a zero-expectation value for $\hat{\sigma}_2$, namely $\langle\Psi\,|\,\hat{\sigma}_2\,|\,\Psi\rangle = 0$, which implies that the commutator relation $\langle\Psi\,|\,[\hat{\sigma}_3,\hat{\sigma}_1]\,|\,\Psi\rangle = 0$ also vanishes for the state of Eq. (8).

It is often said that "In quantum mechanics, two physical quantities represented by non-commuting observables cannot be measured simultaneously with arbitrary precision. Whenever we measure one observable, we influence the state in such a way that the measurement outcomes for the other observable is disturbed " [3]. "…Entanglement-assisted nonlocal correlations and uncertainty are two aspects of the same phenomenon, imprinted in the algebra of quantum mechanics…" [3]. However, it is often ignored that the Heisenberg uncertainty relation involves a set of quantum wave functions. If the product of the two operators results in anther operator for which the expectation value vanishes, regardless of the order of the operators, then the lower limit of the uncertainty is zero. This is compatible with the fact that the alleged "entanglement-assisted nonlocal correlations" are not supported by a Hamiltonian of interactions, nor do the Hilbert spaces of the two photons overlap, as they propagate away from each other [14].

4. QUANTUM CORRELATIONS WITH ARBITRARY INDEPENDENT PHOTONS ON THE POINCARÉ SHPERE

In order to emphasize the role played by independent states of photons, these states $|\,\psi\rangle_k$ will be expanded in terms of the polarization eigenstates of the reference system of coordinates that will also define the joint Poincaré sphere. The states are, with $k = A$ or B:

$$|\,\psi_k\rangle = cos\,\varphi_k\,|x\rangle + sin\,\varphi_k\,|y\rangle \tag{14}$$

for two different angles φ_A and φ_B, relative to the x – axis of reference in the measurement-related Hilbert space onto which the detected states are projected by the measuring detectors *A* and *B*, respectively.

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The polarization operator $\hat{\sigma}$ projects the incoming states onto the measurement Hilbert space for comparison of the two separate data sets. The polarization measurement operators of Eq. (11) produce the output states

$$|\Phi_k\rangle = \sin(2\theta_k)\,\hat{\sigma}_1\,|\,\psi_k\,\rangle + cos(2\theta_k)\,\hat{\sigma}_3\,|\,\psi_k\,\rangle \tag{15}$$

which analogously to the overlapping inner product of the last line of Eq. (5), lead to the correlation function of

$$E_c = \langle\Phi_A\,|\,\Phi_B\rangle = cos[2\,(\theta_A - \theta_B) - (\varphi_A - \varphi_B)] \tag{16}$$

Recalling that the phases φ_k are set in the Jones representation, this result is consistent with Eq. (4) linking the overlap of the Jones vectors to the correlation of the corresponding Stokes vectors on the Poincaré sphere where the angle $2\varphi_k$ applies, that is:

$$\begin{gathered} E_c = \vec{s}_1 \cdot \vec{s}_2 = 2\,cos^2(\triangle\,\phi) - 1 = \cos 2\,(\triangle\,\phi) \\ \triangle\,\phi = \theta_A - \theta_B - (\varphi_A - \varphi_B) \end{gathered} \tag{17}$$

The quantum correlation function of Eq. (17) between two independent states of polarized photons is equivalent to the overlap of their Stokes vectors on the joint Poincaré sphere of the measurement Hilbert space. Quantum-strong correlation are possible with independent states of photons because the source of the correlation is the polarization states of the detecting filters or analyzers, making any claim of quantum nonlocality unnecessary.

As the same correlation functions are derived for independent and single qubits generated through quantum Rayleigh conversion of photons as for entangled photons, it follows that the violations of any type of relevant Bell inequalities will also take place in the same way. It is the similarity between two systems operating under similar conditions, which gives rise to correlations of output polarization states as opposed to a hypothetical collapse of an entangled wave-function. The correlations result from similar, if not identical, distributions of polarization states between experimental setups as opposed to what is conceptually believed to be a non-local quantum effect which has an unspecified nature but is being pursued because of historical reasons.

Once the same correlation functions are derived using only states of polarizations emitted spontaneously by the quantum Rayleigh conversion of photons, no other physical processes are required to explain the experimental results.

Let us now consider a few characteristics associated with local realism [22] of quantum measurements in the context of quantum Rayleigh conversion of photons:

1. *Locality* of measurements is supported by the use of single and independent photonic qubits to explain the experimental results of apparently enhanced correlations of outcomes;
2. *Randomness* of experimental parameters stems from the quantum Rayleigh spontaneous emission that generates the projection from the polarization state $|x\rangle$ of the input photons to the rotated polarization state $|\,\psi_k\,\rangle = cos\,\varphi_k\,|x\rangle + sin\,\varphi_k\,|y\rangle$ of Eq. (14); and,
3. *Realism* of values carried by the detected photons is indicated by the physical effect of the measuring operators on the detected photons in quantum states of Eq. (14). As the expectation values of the product operator $\langle\hat{\sigma}_1\hat{\sigma}_3\rangle$ are found to vanish for the pure states of Eq. (14) projected onto the measurement Hilbert space, i.e. $\langle\psi(\varphi)|\hat{\sigma}_3\hat{\sigma}_1|\psi(\varphi)\rangle = \langle\psi(\varphi)|\hat{\sigma}_1\hat{\sigma}_3|\psi(\varphi)\rangle = 0$ as $\hat{\sigma}_1\hat{\sigma}_3|\,\psi_k\,\rangle = |\psi_k(\varphi + \pi/2)\rangle$, each term of the resulting commutative relation vanishes and we obtain

$$\langle\psi_k|\,[\hat{\sigma}_1\,,\hat{\sigma}_3]\,|\psi_k\rangle = 0 \tag{18}$$

for the lower limit of the Heisenberg uncertainty relation which needs to be evaluated in the context of a set of wave functions. Thus, the output value is indicative of the input one, and each term of the commutator vanishes for the wave functions $|\,\psi_k(\varphi)\rangle$ of Eq. (14). Consequently, the simultaneous measurement of these two operators in the context

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of the single and independent qubit wave functions is capable of identifying the incoming state as well as the measured one. Thus, a physically meaningful identification of wave-functions will enable simultaneous measurements of well-defined values.

This analysis supports reference [15] in its statement that "There is no mystery. There is no quantum nonlocality". It is the physical process that gives rise to a wave function. The opposite approach of relying on mathematical complexities to conjure up physical processes is bound to generate "quantum mysteries".

5. PHYSICAL ASPECTS OF MEASUREMENTS OF INDEPENDENT PHOTONS FOR INTEGRATED QUANTUM PHOTONICS

The possibility of implementing quantum-strong correlations between polarization states with independent photons opens up new options for replacing complicated single-photon sources with one common multi-photon source for the operation of photonic integrated circuits for quantum data processing.

Polarization states of independent photons can be easily manipulated, controlled, and processed by means of Pauli spin operators that rely on optically integrated phase-controlling devices. A suitable choice of linearly polarized states will enable simultaneous measurements of two Pauli spin operators.

An operational resource has been sought with the ability to predict or determine, remotely, the output state of a physical system of photons by measuring the state of a related pair system. Correlations arise from similarity between the two separate processing circuits, and do *not* require single-photon sources. This is also consistent with the concept of simultaneous operations on all of the possible quantum states because all of these states are present at the same time, being represented by a time-independent mixed state density matrix. The equivalence between the polarization Stokes vectors in the classical and quantum regimes is presented in Appendix B below.

By using a large number of photons per beam front, the correlation function of Eq. (17) can be controlled with adjustable input polarization angles $\varphi_{A\ or\ B}$.

As a matter of fact, the parametric amplification of spontaneously emitted photons is unavoidable, otherwise, the stream of photons in the directions of phase-matching condition will not be able to overcome the quantum Rayleigh scattering. The cases of intensity correlation and quantum dot sources is discussed in reference [23].

It may be instructive to dispel a few misinterpretations about "entangled photons": 1) "While analogies might be seen in the mathematical formulation, the possibility of spatial separation, which is the key aspect of entanglement, does not hold for the classical counterpart "[3]. It is the quantum Rayleigh scatterings that break up any alleged entanglement of two photons; and 2) "The two photons are clearly entangled with each other after passing through the beam splitter. … One crucial point to be made here is that the entangled state is created by the physical action of the beam splitter on both of these photons "[3]. As no physical interaction has ever been identified for the beam splitter entangling two photons, this speculation can be ruled out by quantum Rayleigh scattering taking place in a dielectric medium.

It is claimed in reference [3] that "All contradictions to classical concepts and mind-boggling questions arose upon considering the particle nature of light, i.e. when using single photons. Hence, it is misleading to challenge fundamental concepts using states of light that are fully described by the electromagnetic wave picture and Maxwell's equations" [3]. The challenge to entangled photons comes from the existence in a dielectric medium of the quantum Rayleigh scattering and coupling of photons associated with the photon-dipole interactions. These physically meaningful mechanisms are missing from the conventional interpretations in quantum optics [13]. Furthermore, the Pauli spin operators act on the state of polarization of the beam front, regardless of the number of photons that are carried, instantaneously, by the mode [21].

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This article, by taking into consideration the physically meaningful processes of quantum Rayleigh spontaneous and stimulated emissions, opens up new ways of delivering quantum-strong correlations between the output polarization states of separate measurement setups. Equally, correlations of detected intensity-interference patterns with unity visibility can be designed as explained in reference [23] which will facilitate the design, fabrication, and operation of quantum data processing circuits.

6. CONCLUSIONS

Following the experimental results of quantum-strong correlations obtained with independent photons [5], the analysis presented in this article was motivated by two elements: 1) the vanishing of local expectation values of the Pauli spin operators for maximally entangled single photons, which contradicts the physically measured state of polarization; and 2) the quantum Rayleigh conversion of photons which scatters entangled photons. Quantum-strong correlations of polarized photons can be obtained with independent inputs to identical measuring devices and configurations. The correlation function is reminiscent of the overlap between two polarization Stokes vectors on the Poincaré sphere which can be derived from the Jones vectors. Correlation functions derived from independent mixed quantum states are equivalent to correlations of Stokes vectors on the Poincaré sphere. The correlation function can be controlled through the input angles of polarization, and a large number of photons carried simultaneously by a beam front will deliver a faster result than a sequential counting of single photons.

As the parametric amplification of spontaneously emitted is unavoidable, functional operations commonly attributed to single-photon, entangled states are, in reality, implemented practically with independent, multiple photons per radiation modes that are capable of overcoming the quantum Rayleigh spontaneous emission through stimulated emission. Additionally, the Pauli spin operators act on the state of polarization regardless of the number of photons carried by the mode.

The parallel operations on multiple quantum states, described by a mixed density matrix, correspond to the simultaneous presence of all the modes involved in the operation. One optical multi-photon source will replace multiple one-photon sources, thereby streamlining the design, fabrication, and operations of quantum photonic integrated circuits.

APPENDIX A—A SUMMARY OF QUANTUM RAYLEIGH SCATERRING

The quantum Rayleigh photon conversion (QRPC) involves spontaneous and stimulated emissions of photons associated with absorption and emission of one photon per interaction and corresponds to the optically linear parametric conversion [9-11]. This process underpins the propagation of photons in a dielectric medium by coupling photons from one quadrature of the optical wave into the next quadrature [10]. Equally, the QRPC would bring about various time-delays causing a photon to change direction, back and forth, inside an optical fiber or change its polarization state in any dielectric device such as beam splitters, crystal polarizers, optical fibers, etc., The quantum Rayleigh conversion of photons has been identified as a practical way of implementing phase-sensitive amplification in the linear regime [11-12].

The Hamiltonian of interaction $\hat{H}$ between the electric dipoles and the optical field corresponding to the quantum Rayleigh absorption and emission of one photon has the following form [8]:

$$\hat{H} = \kappa \left(\hat{d}^{\dagger} \cdot \hat{a} + \hat{d} \cdot \hat{a}^{\dagger} \right) \tag{A1}$$

where $\hat{d}$ is the electric dipole operator raising the atomic electron from one level to another, and $\hat{a}$ is the photon annihilation operator, with $\hat{a}^{\dagger}$ its conjugate operator, the photon creation operator. The optically linear susceptibility $\chi^{(1)}$ is included in the coupling coefficient κ.

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An entangled polarization state of a pair of signal (*s*) and idler (*i*) photons polarized in the *x* or *y* directions is described by

$$|\Psi\rangle = |1_x\rangle_s\,|1_x\rangle_i + |1_y\rangle_s\,|1_y\rangle_i$$

When acted upon with the absorption operator of the quantum Rayleigh Hamiltonian of Eq. (*A*1), the state $|\Psi\rangle$ becomes a product state because the empty, zero-photon state does not possess any property, i.e., $\hat{a}_s\,|\Psi\rangle = |0\rangle_s\,|1_x\rangle_i + |0\rangle_s\,|1_y\rangle_i = |0\rangle\,(|1_x\rangle_i + |1_y\rangle_i)$. The spontaneously emitted photon [10], [12] will have an arbitrary state of polarization $|\Psi\rangle_s = cos\,\theta_{sp}\,|1_x\rangle_s + sin\,\theta_{sp}\,|1_y\rangle_s$ where θ_{sp} indicates the angle of polarization in the plane perpendicular to the direction of propagation, and which is unrelated to the other photon of the initially entangled state.

In a nonlinear crystal pumped, e.g., with a pump wave (*p*) and for frequency down-converted photons of $\omega_s + \omega_i = \omega_p$, the gain-providing medium which generates the spontaneous emission, will also amplify the initially single photons, particularly so in the direction of wavevector matching conditions, even for a limited space-time overlap. A phase-pulling effect leading to $\varphi_s + \varphi_i = \varphi_p + \pi/2$ also occurs [12] which facilitates the parametric amplification. Thus, the commonly assumed one single photon output does not physically happen. At least several photons may be associated with each individual and discrete electronic "click". A group of photons of the same frequency propagating inside a dielectric medium will follow a straight-line because a photon locally absorbed by a dipole, will be recaptured by the other photons in the group through stimulated emission. Nevertheless, only one photon may survive the propagation to reach the photodetector.

The probability of emitting a photon with momentum $\boldsymbol{k}$ and polarization μ is related to the decay rate γ_s [1/s] of the excited dipole and evaluated as [8]:

$$\gamma_s(\boldsymbol{k},\,\mu,\,\omega) = \frac{9\,\epsilon^{5/2}}{(2\,\epsilon+1)^2}\,\frac{\omega^3}{\hbar\,c}\left(\frac{\boldsymbol{d}\cdot\boldsymbol{e}_{k\mu}}{4\,\pi}\right)^2 \tag{A2}$$

with $\boldsymbol{d}$ denoting the electric dipole moment which is excited by an optical field of the same polarization, and $\boldsymbol{e}_{k\mu}$ is the polarization unit vector of the emitted photon, and which is perpendicular to the direction of propagation $\boldsymbol{k}$. The dielectric constant is ϵ.

A series of sequential events of absorption and spontaneous emissions by electric dipoles will generate a fairly symmetric distribution of polarization states on the *x y* plane perpendicular to the propagation direction.

The generic eigenstates of polarization associated with spontaneous emission through quantum Rayleigh conversion of photons on the two-dimensional Hilbert space $\mathcal{H}$ will take the form of single and independent qubits $\Psi(\varphi_{em})$ identified as:

$$|\Psi(\varphi_{em})\rangle = cos\,\varphi_{em}\,|x\rangle + sin\,\varphi_{em}\,|y\rangle \tag{A3}$$

These state vectors with polarization angles φ_{em} in the range $-\pi \le \varphi_{em} \le \pi$ will describe any possible polarization perpendicular to the direction of propagation of the spontaneous emission. Incoming photons initially polarized in the *x* –direction will reappear with an angle φ_{em} -rotated polarization, thereby enabling them to pass through a φ_{em}-rotated polarization analyzer. This is the physical process of the polarization Malus law.

APPENDIX B—EQUAL QUANTUM AND CLASSICAL CORRELATIONS OF POLARIZED PHOTONS

The correlation function between two remote sets of polarization Stokes vectors is found, classically, as follows.

For two experimentally identical configurations with identical devices, Stokes parameters for Stokes vectors on the Poincaré sphere are obtained from the Jones vector by

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applying the Pauli spin operators, with classical Pauli unit vectors $\overline{\sigma}_{1,3}$ on the Poincaré sphere [21]. The input polarized beam vector is denoted by

$$\vec{v} = (\vec{x} + \vec{y})/\sqrt{2} \tag{B1}$$

and the detection polarization filter $\vec{s}(\theta)$ rotated by an angle θ from the x -axis is

$$\vec{s}(\theta) = cos\theta\vec{x} + sin\theta\vec{y} \tag{B2}$$

By means of dyadic operators and products, the polarization Stokes vectors have the projection operation $\bar{\bar{p}}(\theta)$ with polarization eigenstates $\vec{s}(\theta)$ and $\vec{s}(\theta + \pi/2)$, taking the form:

$$\begin{aligned}
\bar{\bar{p}}(\theta) = \cdot\, \vec{s}(\theta)\, \vec{s}(\theta) \cdot\ - \cdot \vec{s}\left(\theta + \frac{\pi}{2}\right) \vec{s}\left(\theta + \frac{\pi}{2}\right) \cdot = \\
= \cdot\, [cos^2\,\theta\, \vec{x}\vec{x} + sin^2\,\theta\ \vec{y}\,\vec{y} + sin\,\theta\, cos\,\theta\ (\vec{x}\,\vec{y} + \vec{y}\,\vec{x}) - \ sin^2\,\theta\, \vec{x}\vec{x} - \\
cos^2\,\theta\ \vec{y}\,\vec{y} + sin\,\theta\, cos\,\theta\ (\vec{x}\,\vec{y} + \vec{y}\,\vec{x})] \cdot = \\
=\cdot\, [(cos^2\,\theta - sin^2\theta)(\vec{x}\vec{x} - \vec{y}\,\vec{y}) + \\
+ 2\, sin\,\theta\, cos\,\theta\, (\vec{x}\,\vec{y} + \vec{y}\,\vec{x})] \cdot = \\
= sin\,2\theta\ (\cdot\, \overline{\sigma}_1 \cdot) + cos\,2\theta\, (\cdot\, \overline{\sigma}_3 \cdot)
\end{aligned} \tag{B3}$$

This is a Stokes vector on the Poincaré sphere. The correlation function E_c between two data sets of measured Stokes vectors, apart from each other, and with reference to the same frame of coordinates, becomes by means of an overlapping operation:

$$E_c = \vec{v} \cdot \bar{\bar{p}}(\theta_1) \cdot \bar{\bar{p}}(\theta_2) \cdot \vec{v} = cos\ 2\ (\theta_1 - \theta_2) \tag{B4}$$

where the dot products of the dyadic operations have been pointed out.

This correlation function is as strong as any quantum correlation, and has been derived classically. It will facilitate the implementation of coordinated outputs between various subsections of an integrated photonic system.

REFERENCES

[1] G. Moody *et al*, "2022 Roadmap on integrated quantum photonics ", *J. Phys. Photonics*, **4,** 012501, (2022).

[2] P. Kok and B. W. Lovett, *Introduction to Optical Quantum Information Processing*, Cambridge University Press, 2010.

[3] D. Paneru, E. Cohen, R. Fickler, R. W. Boyd and E. Karimi, "Entanglement: quantum or classical? ", *Rep. Prog. Phys.*, **83**, 064001 (2020).

[4] S. M. Barnett1 and S. Croke, "Quantum state discrimination", *Adv. Opt. Phot.* (AOP), **1**, 238–278, (2009).

[5] M. Iannuzzia, R. Francini, R. Messi, and D. Moricciani," Bell-type Polarization Experiment With Pairs Of Uncorrelated Optical Photons", *Phys. Lett. A*, **384** (9), 126200, (2020).

[6] W. H. Louisell, *Quantum Statistical Properties of Radiation*, John Wiley & Sons, 1973.

[7] D. Marcuse, *Principles of Quantum Electronics*, Academic Press, 1980.

[8] R. J. Glauber and M. Lewenstein, "Quantum optics of dielectric media", *Phys. Rev. A*, **43**, 467- 491, (1991).

[9] A. Vatarescu, "The Scattering and Disappearance of Entangled Photons in a Homogeneous Dielectric Medium," Rochester Conference on Coherence and Quantum Optics (CQO-11), (2019). doi.org/10.1364/CQO.2019.M5A.19.

[10] A. Vatarescu, "Photonic coupling between quadrature states of light in a homogeneous and optically linear dielectric medium", *J. Opt. Soc. Am. B*, .**31**, 1741–1745, (2014).

[11] A. Vatarescu, "Phase-Sensitive Amplification with Low Pump Power for Integrated Photonics ", OSA Advanced Photonics Congress, paper ID: IM3A.6., 2016.

[12] A. Vatarescu, "Photonic Quantum Noise Reduction with Low-Pump Parametric Amplifiers for Photonic Integrated Circuits", *Photonics,* **3**, 61, (2016).

[13] J. C. Garrison and R.Y. Chiao, *Quantum Optics*, Oxford University Press, 2008.

[14] R. B. Griffiths, "Nonlocality claims are inconsistent with Hilbert-space quantum mechanics", Phys. Rev. A **101**, 022117 (2020).

[15] F. J. Tipler, "Quantum nonlocality does not exist," PNAS **111** (31), 11281-11286, (2014). doi.org/10.1073/pnas.1324238111

[16] K. Hess, "What Do Bell-Tests Prove? A Detailed Critique of Clauser-Horne-Shimony-Holt Including Counterexamples", *J. Mod. Physics*, **12**, 1219-1236, (2021).

[17] S. Boughn, "Making Sense of Bell's Theorem and Quantum Nonlocality", Found. Phys., **47**, 640-657 (2017).

[18] A. Khrennikov, "Get Rid of Nonlocality from Quantum Physics ", Entropy, **21**, 806-815, (2019).

[19] M. Kupczynski, "Closing the Door on Quantum Nonlocality", Entropy, **20**, 877-890, (2018).

[20] U. Fano, "Description of States in Quantum Mechanics by Density Matrix and Operator Techniques", *Rev. Mod. Phys.*, **29**, 74-93, (1957).

[21] J. P. Gordon and H. Kogelnik, "PMD fundamentals: Polarization mode dispersion in optical fibers", PNAS, **97** (9), 4541-4550, (2000) www.pnas.org/cgi/content/abstract/97/9/4541.

[22] D. J. Griffiths, 2005, *Introduction to Quantum Mechanics*, Publisher: Pearson Prentice Hall.

[23] A. Vatarescu**,** "Instantaneous Quantum Description of Photonic Wavefronts and Applications ", MDPI prepints, DOI: 10.20944/preprints201811.0196.v2.