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Computing the Total H-irregularity Strength of Edge Comb Product of Graphs

Mohamad Fahruli Wahyujati*, Yeni Susanti**

*Independent researcher, Yogyakarta Indonesia
**Mathematics Department Universitas Gadjah Mada, Yogyakarta Indonesia
Email
*fahruliwahyujati@gmail.com
**yeni_@ugm.ac.id

Abstract.
A simple undirected graph $\Gamma = (V_\Gamma, E_\Gamma)$ admits an H-covering if every edge in E_Γ belongs to at least one subgraph of Γ that isomorphic to a graph H. For any graph Γ admitting H-covering, a total Labelling $\beta : V_\Gamma \cup E_\Gamma \longrightarrow \{1, 2, \ldots, p\}$ is called an H-irregular total p-labelling of Γ if every two different subgraphs H_1 and H_2 of Γ isomorphic to H have distinct weights where the weight $w_\beta(K)$ of subgraph K of Γ is defined as $w_f(K) := \sum_{v \in V_K} f(v) + \sum_{e \in E_K} f(e)$. The smallest number p for which a graph Γ admits an H-irregular total p-labelling is called the total H-irregularity strength of Γ and is denoted by $ths(\Gamma)$. In this paper, we determine the total H-irregularity strength of edge comb product of two graphs.
Keywords : Total H-Irregular Strength; *Edge Comb Product Graph*; *Cycle*; *Path*

1 Introduction

Each graph throughout this paper is simple, finite and undirected. Notation $\Gamma = (V_\Gamma, E_\Gamma)$ denotes a graph Γ with vertex set and edge set V_Γ and E_Γ, respectively. A *labelling* means any mapping that maps a set of graphs elements (vertices or edges, or both) to a set of numbers, called *labels*. If the domain of the mapping is the set of vertices (or edges) then the labelling is called a *vertex labelling* (or an *edge labelling*). If the domain is both vertices and edges then the labelling is called a *total labelling*.

According to Chartrand et al. [1] for any positive integer p, an *edge irregular p-labelling* of a graph $\Gamma = (V_\Gamma, E_\Gamma)$ is a map $\beta : E_\Gamma \longrightarrow \{1, 2, \ldots, p\}$ such that $w_\beta(x) \neq w_\beta(y)$ for every two distinct vertices $x, y \in V_\Gamma$ where $w_\beta(x)$ is defined as $w_\beta(x) = \sum_{xy \in E_\Gamma} \beta(xy)$. In [2] Bača et al. introduced a total edge irregularity strength of graphs. For a graph Γ they defined a labelling $\gamma : V_\Gamma \cup E_\Gamma \longrightarrow \{1, 2, \ldots, p\}$ to be an *edge irregular total p-labelling* of Γ if for every two different edges xy and $x'y'$ in E_Γ it follows that $w_\gamma(xy) \neq w_\gamma(x'y')$, where $w_\gamma(xy) = \gamma(x) + \gamma(xy) + f(y)$. The minimum p for which the graph Γ has an edge irregular total p-labelling is called *the total edge irregularity strength* of G, denoted by $tes(G)$.

A family of subgraphs $H_1, H_2, \ldots, H_t$ of Γ such that each of edge in E_Γ belongs to at least one of the subgraph H_i, $i \in \{1, 2, \ldots, t\}$, is called an edge-covering of Γ and Γ is said to admit an $(H_1, H_2, \ldots, H_t)$-edge covering. If each subgraph H_i is isomorphic to a given graph H, then Γ admits an H-covering.

In 2017, Ashraf F. et al. [3] developed the irregularity strength concept for graph admitting H-covering. Let Γ be a graph admitting H-covering and α be

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a total p-labelling on Γ. The weight of subgraph K of Γ respect to α, denoted by $wt_\alpha(K)$ is defined as

$$wt_\alpha(K) = \sum_{v \in V_K} \alpha(v) + \sum_{e \in E_K} \alpha(e).$$

Any weight of subgraph K that isomorphic to H is called *H-weight. A total p-labelling* α is called an H-irregular total p-labelling of Γ if each two distinct subgraphs K_1 and K_2 isomorphic to H have distinct weights, that is $wt_\alpha(K_1) \neq wt_\alpha(K_2)$. The minimum positive integer p such that Γ has an H-irregular total p-labelling is called total H-irregularity strength of Γ and denoted by $ths(\Gamma, H)$. Furthermore, the authors of [3] also found the lower bound of $ths(\Gamma, H)$ for any graph Γ admitting H-covering, as follows:

$$ths(\Gamma, H) \geq \left\lceil 1 + \frac{t-1}{V_H + E_H} \right\rceil$$

where t is the number of subgraphs of Γ that is isomorphic to H.

Ashraf F. et al. [3] found the total P_m-irregularity strength of P_n with m, n both integer, $2 \leq m \leq n$, the total C_m-irregularity strength of ladder graph L_n with $n \geq 3$ and $m = 4$ or $m = 6$, and the total C_3-irregularity strength of fan graph F_n. Ashraf F. et al. [4] determined the exact value of total H-irregularity strengths of G-amalgamation of graphs. Agustin et al. [5] found the total H-irregularity strength of shackle and vertex amalgamation of G, namely $ths(Shack(H, v, n))$, $ths(c(Shack(H, v, n)))$, $ths(Amal(H, v, n))$ and $ths(c(Amal(H, v, n)))$.

In this paper, we investigate the lower bound of the total H-irregularity strength of edge comb product of two graphs and the exact values of the total H-irregularity strength of edge comb product of two classes of graph, including edge comb product of two cycles, edge comb product of path and any graph H, and edge comb product of star and any graph H.

2 Main Results

Let Γ be two connected graphs. Let e be an edge of H. The edge comb product between Γ and H, denoted by $\Gamma \trianglerighteq H$, is graph obtained by taking one copy of Γ and $|E_\Gamma|$ copies of H and then identifying the i-th copy of H at the edge e to the i-th edge of Γ.

By the definition of edge comb product above, it is clear that the edge comb product $H \trianglerighteq H$ between H and its self admits an H-covering. The following theorem provides a sharp lower bound for the total H-irregularity strength of edge comb product of two connected graphs.

Theorem 2.1 *Let Γ and H be two connected graphs. Then*
(i)

$$ths(\Gamma \trianglerighteq H, H) \geq \left\lceil 1 + \frac{|E_\Gamma| - 1}{|V_H| + |E_H|} \right\rceil$$

if Γ does not admit H-covering.
(ii)

$$ths(\Gamma \trianglerighteq H, H) \geq \left\lceil 1 + \frac{t + |E_\Gamma| - 1}{|V_H| + |E_H|} \right\rceil$$

if Γ admits H-covering given by t subgraphs that are isomorphic to H.

Proof.
(i.) Let Γ be a graph that does not admit H-covering. It is obvious that $\Gamma \trianglerighteq H$ admits H-covering given by $|E_\Gamma|$ subgraphs isomorphic to H. Assume that α is an H-irregular total p-labelling of $\Gamma \trianglerighteq H$ with $ths(\Gamma \trianglerighteq H, H) = p$.

The minimum possible weights for all subgraphs of $\Gamma \trianglerighteq H$ that are isomorphic to H are $|V_H|+|E_H|$, $|V_H|+|E_H|+1$, $|V_H|+|E_H|+2, \ldots, |V_H|+|E_H|+|E_\Gamma|-1$. On the other hand, the maximum possible weight for any subgraph of $\Gamma \trianglerighteq H$ isomorphic to H is at most $p(|V_H| + |E_H|)$. Hence

$$\begin{aligned} p(|V_H| + |E_H|) &\geq |V_H| + |E_H| + |E_\Gamma| - 1 \\ p &\geq \frac{|V_H| + |E_H| + |E_\Gamma| - 1}{|V_H| + |E_H|}. \end{aligned}$$

Since $ths(\Gamma \trianglerighteq H, H)$ should be an integer and we need a sharpest lower bound, it implies

$$ths(\Gamma \trianglerighteq H, H) \geq \left\lceil 1 + \frac{|E_\Gamma| - 1}{|V_H| + |E_H|} \right\rceil.$$

(ii.) Let Γ be a graph that admits an H-covering given by t subgraphs isomorphic to H. We have that $\Gamma \trianglerighteq H$ is H-covering given by at least $t + |E_\Gamma|$ subgraphs isomorphic to H. Similar to above, we get

$$ths(\Gamma \trianglerighteq H, H) \geq \left\lceil 1 + \frac{t + |E_\Gamma| - 1}{|V_H| + |E_H|} \right\rceil.$$

□

By considering $\Gamma \cong H$, from Theorem 2.1 (ii) we obtain Corollary 2.2 below.

Corollary 2.2 *Let H be a connected graph. Then*

$$ths(H \trianglerighteq H, H) \geq 2.$$

The following theorem shows the sharpness of bound in Theorem 2.1. We provide Theorem 2.3 for the existence of edge comb product of graphs whose H-irregularity strength satisfies the lower bound in Theorem 2.1.

Theorem 2.3 *Let $m, n \geq 3$ be positive integers. Then:*

$$ths(C_m \trianglerighteq C_n, C_n) = \begin{cases} \left\lceil 1 + \frac{m-1}{2n} \right\rceil & \text{for } m > 2n+1, \\ 2 & \text{for others } m. \end{cases}$$

Proof. Let C_m and C_n be two cycles of m and n vertices, respectively. We define the vertex set and edge set of $C_m \trianglerighteq C_n$ as follows.

$$V_{C_m \trianglerighteq C_n} = \{v_i : 1 \leq i \leq m\} \cup \{u_{i,j} : 1 \leq i \leq m, 1 \leq j \leq n-2\}$$

and

$$\begin{aligned} E_{C_m \trianglerighteq C_n} = &\{v_i v_{i+1} : 1 \leq i \leq m-1\} \cup \{v_1 v_m\} \cup \{v_i u_{i,1} : 1 \leq i \leq m\} \cup \\ &\{v_{i+1} u_{i,n-2} : 1 \leq i \leq m-1\} \cup \{v_1 u_{m,n-2}\} \cup \{u_{i,j} u_{i,j+1} : \end{aligned}$$

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$1 \leq i \leq m, 1 \leq j \leq n-3\}$.

For $m \neq n$, we have $C_m \trianglerighteq C_n$ admits C_n-covering given by m subgraphs of $C_m \trianglerighteq C_n$ isomorphic to C_n. By applying Theorem 2.1 (i), we get

$$ths(C_m \trianglerighteq C_n, C_n) \geq \left\lceil 1 + \frac{m-1}{2n} \right\rceil. \tag{1}$$

Furthermore, if $m \neq n$ and $m \leq 2n+1$, we obtain

$$ths(C_m \trianglerighteq C_n, C_n) \geq 2. \tag{2}$$

For $m = n$, we have $C_n \trianglerighteq C_n$ admits C_n-covering given by $n+1$ subgraphs of $C_n \trianglerighteq C_n$ isomorphic to C_n. According to Corrolary 2.2, we have

$$ths(C_n \trianglerighteq C_n, C_n) \geq 2. \tag{3}$$

Therefore the Equation (1) is the lower bound of the H-irregularity strength of $C_m \trianglerighteq C_n$ for $m \geq 2n+1$, meanwhile Equation (2) and (3) give the lower bound of the H-irregularity strength of $C_m \trianglerighteq C_n$ for $m \leq 2n+1$.

Let $p = \left\lceil 1 + \frac{m-1}{2n} \right\rceil$. In order to show the converse of inequality, we define a C_n-irregular total p-labelling $\alpha : V(C_m \trianglerighteq C_n) \cup E(C_m \trianglerighteq C_n) \to \{1, 2, \ldots, p\}$ as follows

$$\alpha(v_i) = \begin{cases} 1 & \text{for } i = 1, 2 \\ \left\lfloor \frac{i-3}{2n-1} \right\rfloor + 2 & \text{for } 3 \leq i \leq m-p+1 \\ p & \text{for } i = m-p+2 \text{ and } m \not\equiv 2 \pmod{2n} \\ p-1 & \text{for } i = m-p+2 \text{ and } m \equiv 2 \pmod{2n} \\ m-i+2 & \text{for } m-p+3 \leq i \leq m \end{cases}$$

$$\alpha(u_{i,j}) = \begin{cases} 1 & \text{for } i = 1, 2, 3 \\ \left\lfloor \frac{i-4}{2n-1} \right\rfloor + 1 & \text{for } 4 \leq i \leq m-p+1 \text{ and} \\ & \quad j > i - \lfloor \frac{i-4}{2n-1} \rfloor (2n-1) - (n-3) \\ \left\lfloor \frac{i-4}{2n-1} \right\rfloor + 2 & \text{for } 4 \leq i \leq m-p+1 \text{ and} \\ & \quad j \leq i - \lfloor \frac{i-4}{2n-1} \rfloor (2n-1) - (n-3) \\ m-i+1 & \text{for } m-p+1 < i \leq m \end{cases}$$

$$\alpha(v_i v_{i+1}) = \begin{cases} 1 & \text{for } i = 1 \\ \left\lfloor \frac{i-2}{2n-1} \right\rfloor + 2 & \text{for } 2 \leq i \leq m-p+1 \\ p-1 & \text{for } i = m-p+2 \text{ and } m \not\equiv 2 \pmod{2n} \\ p & \text{for } i = m-p+2 \text{ and } m \equiv 2 \pmod{2n} \\ m-i+1 & \text{for } m-p+3 \leq i \leq m-1 \end{cases}$$

$$\alpha(v_m v_1) = 1$$

$$\alpha(v_i u_{i,1}) = \begin{cases} 1 & \text{for } 1 \leq i \leq n+1 \text{ or } i = m \\ \left\lfloor \frac{i-(n+2)}{2n-1} \right\rfloor + 2 & \text{for } n+2 \leq i \leq m-p+1 \\ m-i+1 & \text{for } m-p+2 \leq i \leq m-1 \end{cases}$$

$$\alpha(v_{i+1} u_{i,n-2}) = \begin{cases} \left\lfloor \frac{i-1}{2n-1} \right\rfloor + 1 & \text{for } 1 \leq i \leq m-p+1 \\ m-i+1 & \text{for } m-p+2 \leq i \leq m-1 \end{cases}$$

$$\alpha(v_1 u_{m,n-2}) = 1$$

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$$\alpha(u_{i,j}u_{i,j+1}) = \begin{cases} 1 & \text{for } 1 \le i \le n+2 \\ \left\lfloor \frac{i-(n+3)}{2n-1} \right\rfloor + 1 & \text{for } n+3 \le i \le m-p+1 \text{ and} \\ & \quad j > i - \lfloor \frac{i-(n+3)}{2n-1} \rfloor (2n-1) - (n+2) \\ \left\lfloor \frac{i-(n+3)}{2n-1} \right\rfloor + 2 & \text{for } n+3 \le i \le m-p+1 \text{ and} \\ & \quad j \le i - \lfloor \frac{i-(n+3)}{2n-1} \rfloor (2n-1) - (n+2) \\ m-i+1 & \text{for } m-p+1 < i \le m. \end{cases}$$

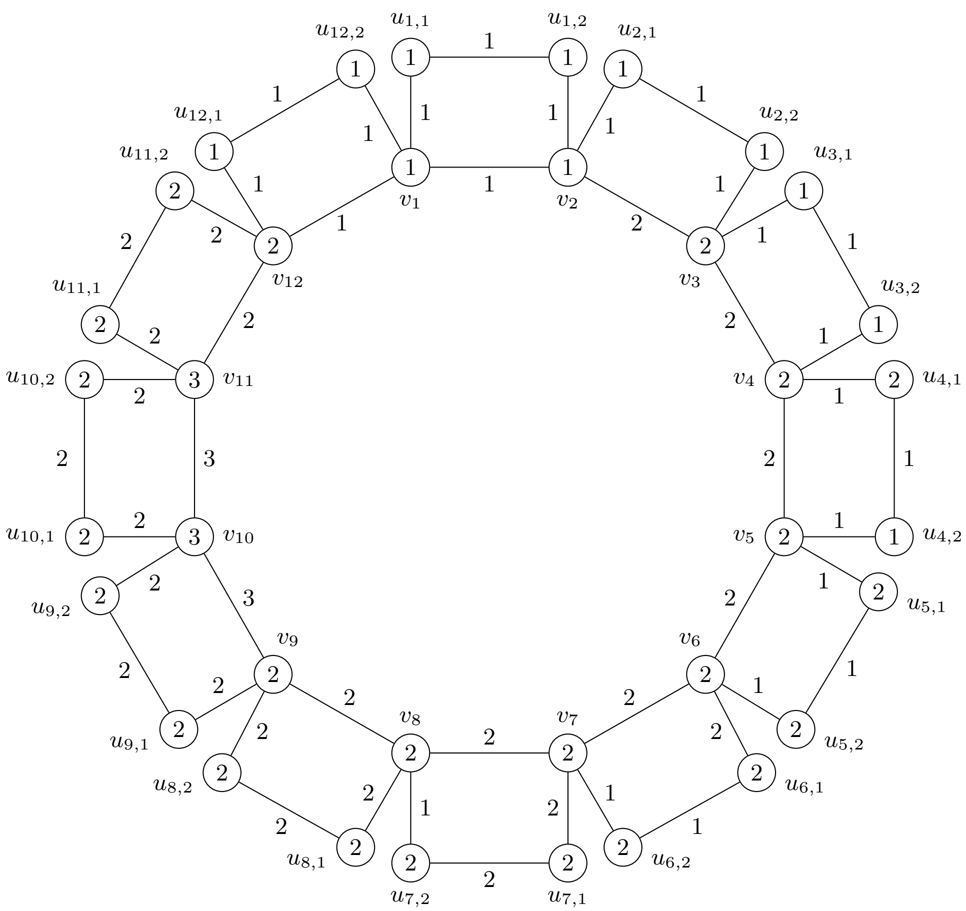

Fig. 1. A C_4-irregular total 3-labelling of $C_{12} \trianglerighteq C_4$

It is readily seen that all labels of vertices and edges of $C_m \trianglerighteq C_n$ under α labelling above are at most p. Let C_n^i be a subgraph of $C_m \trianglerighteq C_n$ isomorphic to C_n with $V_{C_n^i} = \{v_i, v_{i+1 \pmod m}\} \cup \{u_{i,j} : 1 \le j \le n-2\}$ for any $1 \le i \le m$. For case $m = n$, there is an addition subgraph isomorphic to C_n, namely $C_n^{'}$ with $V_{C_n^{'}} = \{v_i : 1 \le i \le m\}$.

For the weight of subgraphs $C_n^{'}$ and C_n^i, $i = 1, 2, \ldots, n$, we get the following

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$$wt_\alpha(C_n^{'}) = \sum_{v\in V_{C_n^{'}}} \alpha(v) + \sum_{e\in E_{C_n^{'}}} \alpha(e) = 4n-4$$

and

$$\begin{aligned} wt_\alpha(C_n^{i}) &= \sum_{v\in V_{C_n^{i}}} \alpha(v) + \sum_{e\in E_{C_n^{i}}} \alpha(e) \\ &= \begin{cases} 2n & \text{for } i=1 \\ i - \left\lfloor \frac{i-2}{2n-1} \right\rfloor (2n-1) + \left\lceil \frac{i-1}{2n-1} \right\rceil 2n & \text{for } 2\le i\le m-p \\ (m-i+1)2n+1 & \text{for } m-p+1\le i\le m. \end{cases} \end{aligned}$$

Under the labelling α above, it is easily to check that $wt_\alpha(C_n^i) \neq wt_\alpha(C_n^p)$ for any $1\le i,k\le m$, $i\neq k$. Thus, $ths(C_m \trianglerighteq C_n) \le \left\lceil 1+\frac{m-1}{2n} \right\rceil$. For $m\le 2n+1$, we get $ths(C_m \trianglerighteq C_n)\le 2$. This completes the proof. □

In the next theorem, we determine the exact value of the total H-irregularity strenth of path graph and an arbitrary 2-connected graph H.

Theorem 2.4 *Let P_m be a path graph of order $m\ge 2$ and H be any 2-connected graph. Then*

$$ths(P_m \trianglerighteq H, H) = 1 + \left\lceil \frac{m-2}{|V_H|+|E_H|} \right\rceil.$$

Proof. As H is 2-connected graph, we have $P_m \trianglerighteq H$ admits H-covering and it contains exactly $m-1$ subgraphs of $P_m \trianglerighteq H$ isomorphic to H. Let us denote the vertex set and edge set of subgraph P_m of $P_m \trianglerighteq H$ by $V_{P_m} = \{v_i : 1\le i\le n\}$ and $E_{P_m} = \{v_iv_{i+1} : 1\le i\le m-1\}$. Let us also denote the elements (both vertices and edges) of the graph $P_m \trianglerighteq H$ from the i-th copy of H that are not vertices of the P_m by the symbol a_i^j, for any $1\le i\le m-1$ and $1\le j\le s-2$, where $s=|V_H|+|E_H|$.

We define an H-irregular total labelling β in the following way:

$$\beta(v_i) = \begin{cases} 1 & \text{for } i=1,2 \\ \left\lfloor \frac{i-3}{s} \right\rfloor + 2 & \text{for } 3\le i\le m \end{cases}$$

$$\beta(a_i^j) = \begin{cases} \left\lfloor \frac{i-1}{s} \right\rfloor + 1 & \text{for } i\equiv 1 \pmod s \text{ or } i\equiv 2 \pmod s \text{ or } i\equiv 3 \pmod s, \\ & 1\le i\le m-1 \text{ and } 1\le j\le s-2 \\ \left\lfloor \frac{i-1}{s} \right\rfloor + 2 & \text{for } i\not\equiv 1 \pmod s \text{ or } i\not\equiv 2 \pmod s \text{ or } i\not\equiv 3 \pmod s, \\ & 1\le i\le m-1 \text{ and } 1\le j\le i-\lfloor\frac{i-4}{s}\rfloor s-3 \\ \left\lfloor \frac{i-1}{s} \right\rfloor + 1 & \text{for } i\not\equiv 1 \pmod s \text{ or } i\not\equiv 2 \pmod s \text{ or } i\not\equiv 3 \pmod s, \\ & 1\le i\le m-1 \text{ and } i-\lfloor\frac{i-4}{s}\rfloor s-2\le j\le s-2. \end{cases}$$

If $m-1\equiv 1 \pmod s$, then the maximal used label is

$$\left\lfloor \frac{(m-1)-1}{s} \right\rfloor + 1 = \frac{(m-1)-1}{s} + 1 = \frac{m-2}{s} + 1.$$

If $m-1\equiv 2 \pmod s$, then the maximal used label is

$$\left\lfloor \frac{m-3}{s} \right\rfloor + 2 = \left(\frac{m-3}{s}+1\right) + 1 = \left\lceil \frac{m-2}{s} \right\rceil + 1.$$

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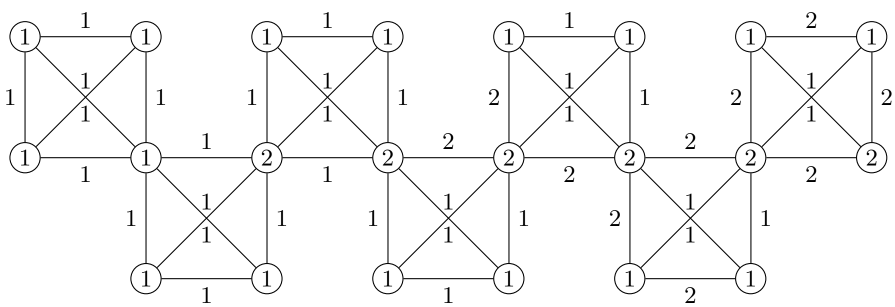

Fig. 2. A K_4-irregular total 2-Labelling of $P_8 \trianglerighteq K_4$

If $m-1 \equiv 3 \pmod{s}$, then the maximal used label is

$$\left\lfloor \frac{m-3}{s} \right\rfloor + 2 = \left(\left\lfloor \frac{m-3}{s} \right\rfloor + 1 \right) + 1 = \left\lceil \frac{m-2}{s} \right\rceil + 1.$$

If $m-1 \not\equiv 1 \pmod{s}$ or $m-1 \not\equiv 2 \pmod{s}$ or $m-1 \equiv 3 \pmod{s}$, then the maximal used label is

$$\left\lfloor \frac{(m-1)-1}{s} \right\rfloor + 2 = \left(\left\lfloor \frac{m-2}{s} \right\rfloor + 1 \right) + 1 = \left\lceil \frac{m-2}{s} \right\rceil + 1.$$

Thus β is $\left(\left\lceil \frac{m-2}{s} \right\rceil + 1 \right)$-labelling.

Let H_i be subgraph of $P_m \trianglerighteq H$ that isomorphic to H, $i = 1, 2, \ldots, m-1$, with element set as follows $V_{H_i} \cup E_{H_i} = \{v_i, v_{i+1}\} \cup \{a_i^j : 1 \leq j \leq s-2\}$. For the weight of subgraphs H_i, $i = 1, 2, \ldots m-1$, we obtain

$$wt_\beta(H_i) = |V_H| + |E_H| - 1 + i.$$

Hence, under the labelling β, all H-weights form a consecutive sequence $(|V_H| + |E_H|, |V_H| + |E_H| + 1, \ldots, |V_H| + |E_H| + m - 2)$. It implies that all H-weights are distinct.

By applying Theorem 2.1 (i), we have $ths(P_m \trianglerighteq H, H) \geq 1 + \left\lceil \frac{m-2}{|V_H|+|E_H|} \right\rceil$ as the lower bound. It proves that the irregular total labelling β has the required properties. □

As an example of Theorem 2.4, we give Figure 2 with H is the complete graph K_4.

Theorem 2.5 *Let S_m be a star graph of order $m+1$ and let H be 2-connected graph. Then*

$$ths(S_m \trianglerighteq H) = 1 + \left\lceil \frac{m-1}{|V_H| + |E_H| - 1} \right\rceil.$$

Proof. The edge comb product of star graph S_m and any connected graph H is equivalent to the vertex-amalgamation $Amal(H, K_1, m)$. Thus,

$$ths(S_m \trianglerighteq H) = 1 + \left\lceil \frac{m-1}{|V_H| + |E_H| - |V_{K_1}| - |E_{K_1}|} \right\rceil \qquad (see[4])$$

$$= 1 + \left\lceil \frac{m-1}{|V_H|+|E_H|-1} \right\rceil. \qquad □$$

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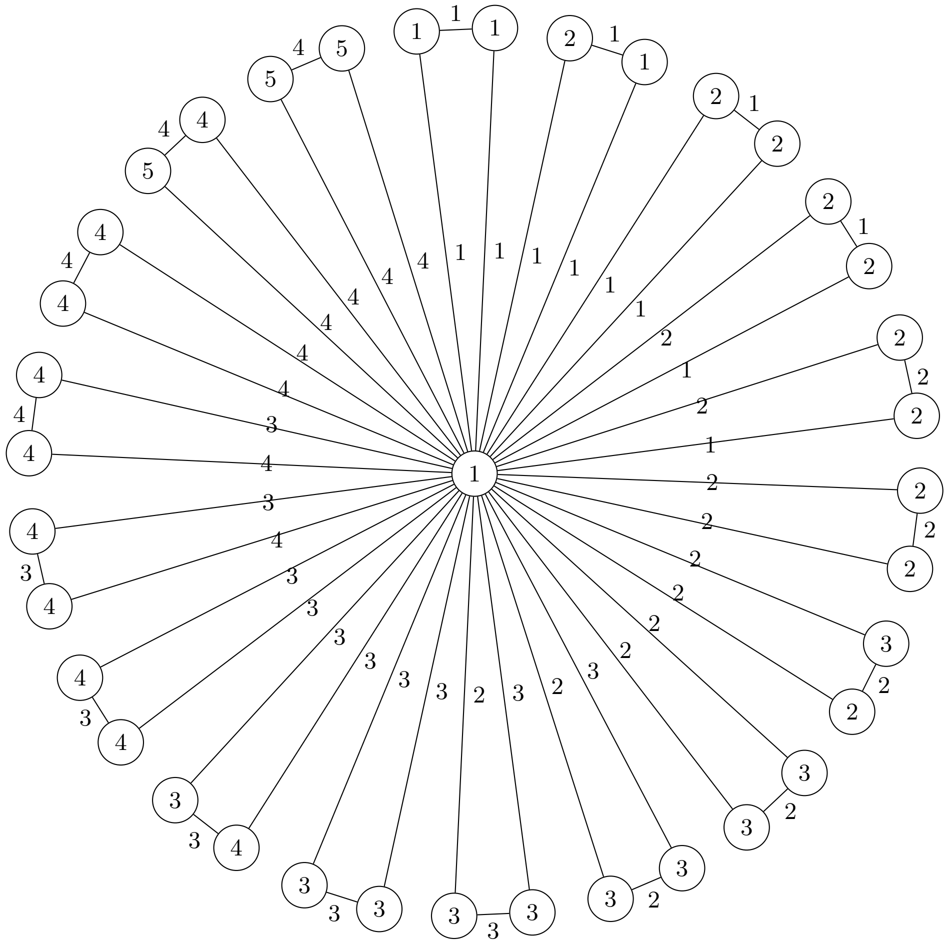

Fig. 3. A C_3-irregular total 5-Labelling of $S_{18} \trianglerighteq C_3$

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