
Empirical Study on the Influence of Different Mathematical Methods (Algebraic Formula Method and Newton Sum Method) on ChatGPT (AI) Competence in Solving Quadratic Root Functions

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Article

Empirical Study on the Influence of Different Mathematical Methods (Algebraic Formula Method and Newton Sum Method) on ChatGPT (AI) Competence in Solving Quadratic Root Functions

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Abstract: Introduction: This empirical study investigates the impact of two distinct mathematical problem-solving methods – the Algebraic Formula Method and the Newton Sum Method – on enhancing ChatGPT's competence in effectively solving quadratic root functions. The integration of Artificial Intelligence (AI) into mathematical problem-solving has paved the way for innovative approaches. In this study, we delve into the Algebraic Formula Method and the Newton Sum Method, essential techniques for solving quadratic root functions. We aim to showcase the profound influence of these methods on ChatGPT's capacity to excel in solving quadratic equations. **Evidence:** Through concrete evidence, we demonstrate ChatGPT's adept utilization of the Newton Sum Method for quadratic root function calculations. While ChatGPT can compute quadratic root functions of the form $\alpha^{15} + \beta^{15}$ using this method, its proficiency in using algebraic formula methods typically extends only up to $\alpha^4 + \beta^4$. This marked discrepancy underscores the pivotal role that different methods play in amplifying the AI system's mathematical capabilities. **Result:** The results of this study provide concrete evidence of ChatGPT's superior utilization of the Newton Sum Method for calculating quadratic root functions. The model adeptly computes expressions of the form $\alpha^{15} + \beta^{15}$ using this method, while its proficiency using algebraic formula methods is generally limited to $\alpha^4 + \beta^4$. This striking discrepancy underscores the transformative impact that different methods can have on elevating the AI system's mathematical prowess. **Conclusion:** Pushing Boundaries: Pioneering Novel Maths Approaches for Overcoming Limitations in AI. This study serves as an illuminating testament to the significance of pioneering innovative methodologies, rules, theorems, or formulas to surmount the current limitations in AI systems like ChatGPT. These innovative pursuits hold the key to unlocking the untapped potential that lies within, propelling AI systems to greater heights of proficiency. In essence, they offer a strategic pathway towards expanding the capabilities of AI and pushing the boundaries of what can be achieved. **Discussion:** The outcomes derived from this study underscore the significant influence wielded by the method selection in augmenting the mathematical competencies of ChatGPT. Particularly noteworthy is the application of the Newton Sum Method, which surfaces as a compelling exemplar. This method serves as a pivotal conduit through which the model surpasses its prior constraints, allowing it to venture into the realm of calculations entailing higher exponents. **Implications and Future Research:** These findings not only contribute to AI's mathematical competencies but also emphasize the need for pioneering new methods, rules, theorems, or formulas to further enhance AI systems like ChatGPT. Future research could explore the development of novel mathematical techniques tailored to AI systems, thus expanding their capabilities across diverse problem-solving domains.

Keywords: Algebraic Formula Method; Newton Sum; AI system; ChatGPT ; Vieta Theorem; Quadratic Root Functions

1. Introduction

This empirical study investigates the impact of two distinct mathematical problem-solving methods – the Algebraic Formula Method and the Newton Sum Method – on enhancing ChatGPT's competence in effectively solving quadratic root functions.

1.1. ChatGPT¹

Introducing ChatGPT

We've trained a model called ChatGPT which interacts in a conversational way. The dialogue format makes it possible for ChatGPT to answer follow up questions, admit its mistakes, challenge incorrect premises, and reject inappropriate requests.

Introducing ChatGPT Plus

We're launching a pilot subscription plan for ChatGPT, a conversational AI that can chat with you, answer follow-up questions, and challenge incorrect assumptions. **Chat GPT²** knowledge is still limited to 2021 data, which means it can't answer current questions.

1.2. Knowledge Is Power: Why the Future Is not just about the tech³

If we are to rely on machine intelligence, we need to understand the two types of knowledge.

Understanding knowledge means we can distinguish where we want machines to do the mundane work and where we want humans to perform intuitive tasks. Such an approach will be as beneficial for business as for education. As virtual and physical worlds become increasingly interdependent, knowledge – and how we manage it – will become the secret ingredient to manage the situation. And thrive.

Virtual technologies are swiftly becoming intertwined with our physical world, and companies need to adapt. But that doesn't simply mean replacing humans with robots or relying on artificial intelligence (AI) to make all of our decisions. This is because technology, though powerful, is just part of the equation. In fact, human intelligence will be one of the most valuable assets in today's Fourth Industrial Revolution (FIR), and companies may flounder if they fail to strike the right balance of automated technology and human insights.

1.3. Algebra Formula

1.3.1. The following functions are some symmetric function of a quadratic's roots

- a) $\frac{1}{\alpha} + \frac{1}{\beta} = \frac{\alpha + \beta}{\alpha\beta}$,
- b) $\alpha^2 + \beta^2 = (\alpha + \beta)^2 - 2\alpha\beta$
- c) $(\alpha - \beta)^2 = (\alpha + \beta)^2 - 4\alpha\beta$,
- d) $\alpha^3 + \beta^3 = (\alpha + \beta) [(\alpha + \beta)^2 - 3\alpha\beta]$
- e) $\alpha^4 + \beta^4 = (\alpha^2 + \beta^2)^2 - 2\alpha^2\beta^2$
 $= [(\alpha + \beta)^2 - 2\alpha\beta]^2 - 2(\alpha\beta)^2$

1.3.2. The following functions are asymmetric function of a quadratic's roots

- a) $\alpha - \beta = \pm\sqrt{(\alpha + \beta)^2 - 4\alpha\beta}$,
- b) $\alpha^2 - \beta^2 = \pm(\alpha + \beta)\sqrt{(\alpha + \beta)^2 - 4\alpha\beta}$,
- c) $\alpha^3 - \beta^3 = \pm\sqrt{(\alpha + \beta)^2 - 4\alpha\beta} [(\alpha + \beta)^2 - \alpha\beta]$.
- d) $\alpha^4 - \beta^4 = (\alpha^2 - \beta^2)(\alpha^2 + \beta^2)$
 $= \pm[(\alpha + \beta)\sqrt{(\alpha + \beta)^2 - 4\alpha\beta}] [(\alpha + \beta)^2 - 2\alpha\beta]^2 - 2(\alpha\beta)^2]$

Note: Student need to remember above Algebra Formula if they use Veda's theorem method for Construct New Quadratic Equation.

1.4. Newton Sum (Newton's Identities)⁴

Newton's identities, also known as Newton-Girard formulae, is an efficient way to find the power sum of roots of polynomials without actually finding the roots. If x_1, x_2, \dots, x_n are the roots of a polynomial equation, then Newton's identities are used to find the summations like

$$\sum_{i=1}^n x_i^k = x_1^k + x_2^k + \dots + x_n^k.$$

It is mainly used in conjunction with Vieta's formula while working with the (complex) roots (say $\alpha_1, \alpha_2, \dots, \alpha_k$) of a k^{th} degree polynomial. The main idea is that the elementary symmetric polynomials form an algebraic basis to produce all symmetric polynomials. Newton's identity gives us the calculation via a recurrence relation with known coefficients.

Newton's Identities for a Quadratic Polynomial

Suppose that you have a quadratic polynomial $P(x)$ with (complex) roots α_1 and α_2 . Now, you are asked to find the value of $\alpha_1^2 + \alpha_2^2$. This seems very easy since you can use **Vieta's formula** along with the identity $(a + b)^2 = a^2 + b^2 + 2ab$ to find the required result. But what if you need to find $(\alpha_1^{10} + \alpha_2^{10})$? This would take a while if you were to simply use algebraic manipulations. But there's a clever way, using Newton's sums.

Let $P(x) = ax^2 + bx + c$. Then using Vieta's formula, we can get $\alpha_1 + \alpha_2 = -\frac{b}{a}$ and $\alpha_1\alpha_2 = \frac{c}{a}$. Now, denote P_i as the i^{th} power sum of the roots, namely $P_i = \alpha_1^i + \alpha_2^i$. Then we can obtain P_i recursively as follows:

$$\begin{aligned} P_0 &= \alpha_1^0 + \alpha_2^0 &&= 2 \\ P_1 &= \alpha_1^1 + \alpha_2^1 &&= -\frac{b}{a} \\ P_2 &= \alpha_1^2 + \alpha_2^2 = (\alpha_1 + \alpha_2)(\alpha_1 + \alpha_2) - 2\alpha_1\alpha_2 &&= -\frac{b}{a}P_1 - \frac{c}{a}P_0 \\ P_3 &= \alpha_1^3 + \alpha_2^3 = (\alpha_1 + \alpha_2)(\alpha_1^2 + \alpha_2^2) - \alpha_1\alpha_2(\alpha_1 + \alpha_2) &&= -\frac{b}{a}P_2 - \frac{c}{a}P_1 \\ &\vdots &&\vdots \\ P_i &= \alpha_1^i + \alpha_2^i = (\alpha_1 + \alpha_2)(\alpha_1^{i-1} + \alpha_2^{i-1}) - \alpha_1\alpha_2(\alpha_1^{i-2} + \alpha_2^{i-2}) &&= -\frac{b}{a}P_{i-1} - \frac{c}{a}P_{i-2}. \end{aligned}$$

This is a linear recurrence relation that gives us the i^{th} power sum. Note that solving this recurrence to get a closed-form solution is equivalent to finding the roots of the quadratic polynomial.

THEOREM

For a quadratic polynomial $f(x) = ax^2 + bx + c$ with (complex) roots α_1, α_2 , we denote the i^{th} power sum of the roots as $P_i = \sum_{k=1}^2 \alpha_k^i$ and the sum and product of the roots as A and B , respectively. Then we have

$$\begin{aligned} P_1 &= A \\ P_2 &= AP_1 - 2B \\ &\vdots \\ P_i &= AP_{i-1} - BP_{i-2} \quad \forall i \geq 2. \end{aligned}$$

EXAMPLE

Let a polynomial $P(x)$ be defined as $P(x) = x^2 - 2x + 6$ with its (complex) roots a and b . Then what is the value of $a^{10} + b^{10}$?

By Vieta's formula, the sum of the roots is $2 = A$ (say) and the product of the roots is $6 = B$ (say). Now, using the recurrence relation found above and Newton's identities, we have

$$P_i = AP_{i-1} - BP_{i-2} = 2P_{i-1} - 6P_{i-2} \quad \forall i \geq 2, \quad P_0 = 2, \quad P_1 = A = 2.$$

Now, we can simply use this recurrence relation obtained repeatedly to find P_2, P_3, \dots, P_9 and then finally P_{10} which is the required answer. The answer comes out to be $2(-3808) - 6(-2528)$.

Hence, our final answer is 7552. \square

1.5. The French Mathematician, Vieta⁵.



François Viète (Latin: Franciscus Vieta; 1540 – 23 February 1603)

The French mathematician Viète established the relationship between the equation root and the coefficient in 1615. Viète's theorem states that if α and β are two roots of the quadratic equation $ax^2 + bx + c = 0$ and $a \neq 0$.

Then, The sum of the two roots, $\alpha + \beta = -\frac{b}{a}$. The product of the two roots, $\alpha\beta = \frac{c}{a}$.

$$\text{Prove } \alpha + \beta = -\frac{b}{a} \text{ and } \alpha\beta = \frac{c}{a}.$$

If α and β are two roots of the quadratic equation $ax^2 + bx + c = 0$ and $a \neq 0$.

$$\text{Then } \alpha = \frac{-b + \sqrt{b^2 - 4ac}}{2a}, \quad \beta = \frac{-b - \sqrt{b^2 - 4ac}}{2a}$$

$$\text{So, } \alpha + \beta = \frac{-b + \sqrt{b^2 - 4ac}}{2a} + \frac{-b - \sqrt{b^2 - 4ac}}{2a},$$

$$= \frac{-2b}{2a}$$

$$= -\frac{b}{a}$$

$$\alpha\beta = \frac{-b + \sqrt{b^2 - 4ac}}{2a} \times \frac{-b - \sqrt{b^2 - 4ac}}{2a}$$

$$= \frac{(-b)^2 - (\sqrt{b^2 - 4ac})^2}{(2a)^2} = \frac{b^2 - (b^2 - 4ac)}{4a^2} = \frac{4ac}{4a^2} = \frac{c}{a}$$



2. Evidence

Through concrete evidence, we demonstrate ChatGPT's adept utilization of the Newton Sum Method for quadratic root function calculations. While ChatGPT can compute quadratic root functions of the form $\alpha^{15} + \beta^{15}$ using this method, its proficiency in using algebraic formula methods typically extends only up to $\alpha^4 + \beta^4$. This marked discrepancy underscores the pivotal role that different methods play in amplifying the AI system's mathematical capabilities

2.1. Algebraic Formula Methods. Using Chat GPT⁶ for Calculate

2.1.1. If the roots of the equation $x^2 - 3x + 2 = 0$ are α and β , find $\alpha^2 + \beta^2$

PE If the roots of the equation $x^2 - 3x + 2 = 0$ are α and β , find the value of $\alpha^2 + \beta^2$

 The sum of the squares of the roots of a quadratic equation $ax^2 + bx + c = 0$ is given by: 

$$\alpha^2 + \beta^2 = (\alpha + \beta)^2 - 2\alpha\beta.$$

In your equation $x^2 - 3x + 2 = 0$, the coefficients are $a = 1$, $b = -3$, and $c = 2$.

The sum of the roots $\alpha + \beta$ is equal to the negation of the coefficient of x term divided by the coefficient of the x^2 term:

$$\alpha + \beta = -b/a = -(-3)/1 = 3.$$

The product of the roots $\alpha\beta$ is equal to the constant term divided by the coefficient of the x^2 term:

$$\alpha\beta = c/a = 2/1 = 2.$$

Now you can plug these values into the formula:



$$\alpha^2 + \beta^2 = (\alpha + \beta)^2 - 2\alpha\beta = 3^2 - 2 \cdot 2 = 9 - 4 = 5.$$

So, the value of $\alpha^2 + \beta^2$ is 5.

2.1.2. Algebra Formula, Using Chat GPT for Calculate

If the roots of the equation $x^2 - 3x + 2 = 0$ are α and β , find $\alpha^3 + \beta^3$

PE If the roots of the equation $x^2 - 3x + 2 = 0$ are α and β , find the value of $\alpha^3 + \beta^3$

 The sum of the cubes of the roots of a quadratic equation $ax^2 + bx + c = 0$ is related to the coefficients of the equation through the following formula: 

$$\alpha^3 + \beta^3 = (\alpha + \beta)(\alpha^2 - \alpha\beta + \beta^2).$$

In your equation $x^2 - 3x + 2 = 0$, we've already found that the sum of the roots $\alpha + \beta$ is 3.

We also calculated the value of $\alpha^2 + \beta^2$ as 5 in the previous response.

Now we can use these values to calculate $\alpha^3 + \beta^3$:

$$\alpha^3 + \beta^3 = (\alpha + \beta)(\alpha^2 - \alpha\beta + \beta^2) = 3 \cdot (5 - 2) = 3 \cdot 3 = 9.$$

So, the value of $\alpha^3 + \beta^3$ is 9.

2.1.3. Algebra Formula, Using Chat GPT for Calculate

If the roots of the equation $x^2 - 3x + 2 = 0$ are α and β , find $\alpha^4 + \beta^4$.

PE If the roots of the equation $x^2 - 3x + 2 = 0$ are α and β , find the value of $\alpha^4 + \beta^4$.

The value of $\alpha^4 + \beta^4$ can be found using the following relation:

$$\alpha^4 + \beta^4 = (\alpha^2 + \beta^2)^2 - 2\alpha^2\beta^2.$$

In your equation $x^2 - 3x + 2 = 0$, we've already calculated $\alpha^2 + \beta^2$ as 5.

We can also calculate the product of the squares of the roots $\alpha^2\beta^2$, which is equal to $(\alpha\beta)^2$ (product of the roots) squared:

$$\alpha^2\beta^2 = (\alpha\beta)^2 = (2)^2 = 4.$$

Now, we can plug these values into the formula:

$$\alpha^4 + \beta^4 = (\alpha^2 + \beta^2)^2 - 2\alpha^2\beta^2 = 5^2 - 2 \cdot 4 = 25 - 8 = 17.$$

So, the value of $\alpha^4 + \beta^4$ is 17.

2.1.4. Algebra Formula, Using Chat GPT for Calculate

If the roots of the equation $x^2 - 3x + 2 = 0$ are α and β , find $\alpha^5 + \beta^5$.

PE If the roots of the equation $x^2 - 3x + 2 = 0$ are α and β , find the value of $\alpha^5 + \beta^5$.

The value of $\alpha^5 + \beta^5$ can be found using the following relation:

$$\alpha^5 + \beta^5 = (\alpha + \beta)(\alpha^4 - \alpha^3\beta + \alpha^2\beta^2 - \alpha\beta^3 + \beta^4).$$

In your equation $x^2 - 3x + 2 = 0$, we've already calculated $\alpha + \beta$ as 3.

We can also calculate the following values:

- $\alpha\beta = c/a = 2/1 = 2$
- $\alpha^2\beta^2 = (\alpha\beta)^2 = 2^2 = 4$

Now we need to calculate $\alpha^4 - \alpha^3\beta + \alpha^2\beta^2 - \alpha\beta^3 + \beta^4$:

$$\alpha^4 - \alpha^3\beta + \alpha^2\beta^2 - \alpha\beta^3 + \beta^4 = \alpha^4 - \alpha^3\beta + 4 - 2\alpha\beta^3 + \beta^4.$$

Since we already know that $\alpha^2 + \beta^2 = 5$ from a previous response, we can also find $\alpha^3\beta + \alpha\beta^3$ as follows:

$$\alpha^3\beta + \alpha\beta^3 = \alpha\beta(\alpha^2 + \beta^2) = 2 \cdot 5 = 10.$$

Now we can plug all these values into the formula:

$$\alpha^5 + \beta^5 = (\alpha + \beta)(\alpha^4 - \alpha^3\beta + \alpha^2\beta^2 - \alpha\beta^3 + \beta^4) = 3(5 - 10 + 4 - 2\alpha\beta^3 + \beta^4).$$

Simplifying further:

$$\alpha^5 + \beta^5 = 3(-1 - 2\alpha\beta^3 + \beta^4) = -3(1 + 2\alpha\beta^3 - \beta^4).$$

Now, substituting the values of $\alpha\beta = 2$ and $\alpha^2\beta^2 = 4$, we get:

$$\alpha^5 + \beta^5 = -3(1 + 2 \cdot 2\beta^3 - \beta^4) = -3(1 + 4\beta^3 - \beta^4).$$

The value of β^3 can be calculated from $\alpha\beta = 2$ and the equation $\alpha + \beta = 3$:

$$\beta^3 = (\alpha + \beta)\beta^2 - \alpha\beta^2 = 3\beta^2 - 2\beta^2 = \beta^2.$$

So, we have $\beta^2 = \beta^3$ in this case.

Finally, substituting β^2 for β^3 :

$$\alpha^5 + \beta^5 = -3(1 + 4\beta^2 - \beta^4) = -3(1 + 4\beta^2 - (\beta^2)^2) = -3(1 + 4\beta^2 - \beta^4).$$

At this point, finding the exact value of β^2 and subsequently $\alpha^5 + \beta^5$ becomes more complex and likely involves solving a higher-degree equation for β . The process becomes intricate and might not lead to a simple numerical result.

In general, the expressions involving higher powers of the roots can become quite involved, and finding specific numerical values can be complex.

When employing the Algebraic Formula, ChatGPT faces limitations in delivering conclusive solutions for problems like $\alpha^5 + \beta^5$, owing to the intricate nature of the solutions.. As we see Chat GPT Reply: At this point, finding the exact value of β^2 and subsequently $\alpha^5 + \beta^5$ becomes more complex and likely involves solving a higher-degree equation for β . The process becomes intricate and might not lead to a simple numerical result. In general, the expressions involving higher powers of the roots can become quite involved, and finding specific numerical values can be complex.

2.2. Newton sum Methods. Using Chat GPT for Calculate

2.2.1. If the roots of the equation $x^2 - 3x + 2 = 0$ are α and β , find the value of $\alpha^2 + \beta^2$ using Newton sum . $S_k = \alpha^k + \beta^k = -a_{(k-1)} \cdot S_{(k-1)} - a_{(k-2)} \cdot S_{(k-2)}$



If the roots of the equation $x^2 - 3x + 2 = 0$ are α and β , find the value of $\alpha^2 + \beta^2$ using Newton sum. $S_k = \alpha^k + \beta^k = -a_{(k-1)} \cdot S_{(k-1)} - a_{(k-2)} \cdot S_{(k-2)}$



Given the quadratic equation $x^2 - 3x + 2 = 0$, we have α and β as its roots. The sum of the roots is $\alpha + \beta = 3$ and the product of the roots is $\alpha\beta = 2$.

We want to find the value of $\alpha^2 + \beta^2$, which can be expressed using Newton's sums as:

$$S_2 = \alpha^2 + \beta^2 = -a_1 \cdot S_1 - a_0 \cdot S_0,$$

where a_1 is the coefficient of the linear term (which is -3 in this case), a_0 is the constant term (which is 2), S_1 is the sum of the roots (which is 3), and S_0 is the number of roots (which is 2).

Plugging in the values:

$$S_2 = -(-3) \cdot 3 - 2 \cdot 2 = 9 - 4 = 5.$$

So, the value of $\alpha^2 + \beta^2$ is 5 .

2.2.2. Newton Sum, Using Chat GPT for Calculate

If the roots of the equation $x^2 - 3x + 2 = 0$ are α and β , find the value of $\alpha^3 + \beta^3$ using Newton sum. $S_k = \alpha^k + \beta^k = -a_1(bS_{k-1} + cS_{k-2})$

If the roots of the equation $x^2 - 3x + 2 = 0$ are α and β , find the value of $\alpha^3 + \beta^3$ using Newton sum. $S_k = \alpha^k + \beta^k = -a_1 S_{k-1} + a_0 S_{k-2}$

Certainly, to find the value of $\alpha^3 + \beta^3$ using Newton's sum, we can use the formula you've provided:

$$S_k = \alpha^k + \beta^k = -a_1 S_{k-1} + a_0 S_{k-2}$$

Given the equation $x^2 - 3x + 2 = 0$, the coefficients are:

$$a_2 = 1$$

$$a_1 = -3$$

$$a_0 = 2$$

Let's use Newton's sum to find $\alpha^3 + \beta^3$:

- $S_0 = \alpha^0 + \beta^0 = 2$
- $S_1 = \alpha^1 + \beta^1 = -(-3) = 3$

Now we can use the formula to find S_2 :

$$S_2 = \alpha^2 + \beta^2 = -a_1 S_1 - a_0 S_0$$

$$= -(-3) * 3 - 2 * 2$$

$$= 9 - 4$$

$$= 5$$

Now let's use the formula again to find S_3 :

$$S_3 = \alpha^3 + \beta^3 = -a_1 S_2 - a_0 S_1$$

$$= -(-3) * 5 - 2 * 3$$

$$= 15 - 6$$

$$= 9$$

There was an error generating a response

2.2.3. Newton Sum, Using Chat GPT for Calculate

If the roots of the equation $x^2 - 3x + 2 = 0$ are α and β , find the value of $\alpha^4 + \beta^4$ using Newton sum. $S_k = \alpha^k + \beta^k = -a_1 S_{k-1} + a_0 S_{k-2}$

If the roots of the equation $x^2 - 3x + 2 = 0$ are α and β , find the value of $\alpha^4 + \beta^4$ using Newton sum. $S_k = \alpha^k + \beta^k = -a_1(bS_{k-1} + cS_{k-2})$

To find the value of $\alpha^4 + \beta^4$ using Newton's sums, we'll use the following formula:

$$S_k = \alpha^k + \beta^k = -a_1 S_{k-1} - a_2 S_{k-2}$$

Given the equation $x^2 - 3x + 2 = 0$, the coefficients are:

$$a_2 = 1$$

$$a_1 = -3$$

$$a_0 = 2$$

Let's use Newton's sum to find $\alpha^4 + \beta^4$:

- $S_0 = \alpha^0 + \beta^0 = 2$
- $S_1 = \alpha^1 + \beta^1 = -(-3) = 3$

Now we can use the formula to find S_2 :

$$S_2 = \alpha^2 + \beta^2 = -a_1 S_1 - a_0 S_0$$

$$= -(-3) * 3 - 2 * 2$$

$$= 9 - 4$$

$$= 5$$

Now we use the formula again to find S_3 :

$$S_3 = \alpha^3 + \beta^3 = -a_1 S_2 - a_0 S_1$$

$$= -(-3) * 5 - 2 * 3$$

$$= 15 - 6$$

$$= 9$$

There was an error generating a response

Now we use the formula again to find S_4 :

$$S_4 = \alpha^4 + \beta^4 = -a_1 S_3 - a_0 S_2$$

$$= -(-3) * 9 - 2 * 5$$


$$= 27 - 10$$

$$= 17$$

So, the value of $\alpha^4 + \beta^4$ is 17.

2.2.4. Newton Sum, Using Chat GPT for Calculate

If the roots of the equation $x^2 - 3x + 2 = 0$ are α and β , find the value of $\alpha^5 + \beta^5$ using Newton sum. $S_k = \alpha^k + \beta^k = -a_1(bS_{k-1} + cS_{k-2})$

 If the roots of the equation $x^2 - 3x + 2 = 0$ are α and β , find the value of $\alpha^5 + \beta^5$ using Newton sum. $S_k = \alpha^k + \beta^k = -a_1 S_{k-1} + a_0 S_{k-2}$

 To find the value of $\alpha^5 + \beta^5$ using Newton's sums, we'll use the following formula:

$$S_k = \alpha^k + \beta^k = -a_1 S_{k-1} + a_0 S_{k-2}$$

Given the equation $x^2 - 3x + 2 = 0$, the coefficients are:

$$a_2 = 1$$

$$a_1 = -3$$

$$a_0 = 2$$

Let's use Newton's sum to find $\alpha^5 + \beta^5$:

1. $S_0 = \alpha^0 + \beta^0 = 2$
2. $S_1 = \alpha^1 + \beta^1 = -(-3) = 3$

Now we can use the formula to find S_2 :

$$\begin{aligned} S_2 &= \alpha^2 + \beta^2 = -a_1 S_1 + a_0 S_0 \\ &= -(-3) * 3 + 2 * 2 \\ &= 9 + 4 \\ &= 13 \end{aligned}$$

Now we use the formula again to find S_3 :

$$\begin{aligned} S_3 &= \alpha^3 + \beta^3 = -a_1 S_2 + a_0 S_1 \\ &= -(-3) * 13 + 2 * 3 \\ &= 39 + 6 \\ &= 45 \end{aligned}$$

Now we use the formula again to find S_4 :

$$\begin{aligned} S_4 &= \alpha^4 + \beta^4 = -a_1 S_3 + a_0 S_2 \\ &= -(-3) * 45 + 2 * 13 \\ &= 135 + 26 \\ &= 161 \end{aligned}$$

Now we use the formula again to find S_5 :

$$\begin{aligned} S_5 &= \alpha^5 + \beta^5 = -a_1 S_4 + a_0 S_3 \\ &= -(-3) * 161 + 2 * 45 \\ &= 483 + 90 \\ &= 573 \end{aligned}$$

So, the value of $\alpha^5 + \beta^5$ is 573.

2.2.5. Newton Sum, Using Chat GPT for Calculate

If the roots of the equation $x^2 - 3x + 2 = 0$ are α and β , find the value of $\alpha^6 + \beta^6$ using Newton sum. $S_k = \alpha^k + \beta^k = -a_1 S_{k-1} + a_0 S_{k-2}$.

If the roots of the equation $x^2 - 3x + 2 = 0$ are α and β , find the value of $\alpha^6 + \beta^6$ using Newton sum. $S_k = \alpha^k + \beta^k = -a_1(bS_{k-1} + cS_{k-2})$

To find the value of $\alpha^6 + \beta^6$ using Newton's sums, we'll use the following formula:

$$S_k = \alpha^k + \beta^k = -a_1 S_{k-1} - a_2 S_{k-2}$$

Given the equation $x^2 - 3x + 2 = 0$, the coefficients are:

$$a_2 = 1$$

$$a_1 = -3$$

$$a_0 = 2$$

Let's use Newton's sum to find $\alpha^6 + \beta^6$:

- $S_0 = \alpha^0 + \beta^0 = 2$
- $S_1 = \alpha^1 + \beta^1 = -(-3) = 3$

Now we can use the formula to find S_2 :

$$\begin{aligned} S_2 &= \alpha^2 + \beta^2 = -a_1 S_1 - a_0 S_0 \\ &= -(-3) \cdot 3 - 2 \cdot 2 \\ &= 9 - 4 \\ &= 5 \end{aligned}$$

Now we use the formula again to find S_3 :

$$\begin{aligned} S_3 &= \alpha^3 + \beta^3 = -a_1 S_2 - a_0 S_1 \\ &= -(-3) \cdot 5 - 2 \cdot 3 \\ &= 15 - 6 \\ &= 9 \end{aligned}$$

There was an error generating a response

Now we use the formula again to find S_4 :

$$\begin{aligned} S_4 &= \alpha^4 + \beta^4 = -a_1 S_3 - a_0 S_2 \\ &= -(-3) \cdot 9 - 2 \cdot 5 \\ &= 27 - 10 \\ &= 17 \end{aligned}$$

Now we use the formula again to find S_5 :

$$\begin{aligned} S_5 &= \alpha^5 + \beta^5 = -a_1 S_4 - a_0 S_3 \\ &= -(-3) \cdot 17 - 2 \cdot 9 \\ &= 51 - 18 \\ &= 33 \end{aligned}$$

Now we use the formula again to find S_6 :

$$\begin{aligned} S_6 &= \alpha^6 + \beta^6 = -a_1 S_5 - a_0 S_4 \\ &= -(-3) \cdot 33 - 2 \cdot 17 \\ &= 99 - 34 \\ &= 65 \end{aligned}$$

So, the value of $\alpha^6 + \beta^6$ is 65.

2.2.6. Newton Sum, Using Chat GPT for Calculate

If the roots of the equation $x^2 - 3x + 2 = 0$ are α and β , find the value of $\alpha^7 + \beta^7$ using Newton sum. $S_k = \alpha^k + \beta^k = -a_1(bS_{k-1} + cS_{k-2})$.

If the roots of the equation $x^2 - 3x + 2 = 0$ are α and β , find the value of $\alpha^7 + \beta^7$ using Newton sum. $S_k = \alpha^k + \beta^k = -a_1(bS_{k-1} + cS_{k-2})$

To find the value of $\alpha^7 + \beta^7$ using Newton's sums, we'll use the following formula:

$$S_k = \alpha^k + \beta^k = -a_1 S_{k-1} - a_2 S_{k-2}$$

Given the equation $x^2 - 3x + 2 = 0$, the coefficients are:

$$a_2 = 1$$

$$a_1 = -3$$

$$a_0 = 2$$

Let's use Newton's sum to find $\alpha^7 + \beta^7$:

- $S_0 = \alpha^0 + \beta^0 = 2$
- $S_1 = \alpha^1 + \beta^1 = -(-3) = 3$

Now we can use the formula to find S_2 :

$$S_2 = \alpha^2 + \beta^2 = -a_1 S_1 - a_0 S_0$$

$$= -(-3) * 3 - 2 * 2$$

$$= 9 - 4$$

$$= 5$$

Now we use the formula again to find S_3 :

$$S_3 = \alpha^3 + \beta^3 = -a_1 S_2 - a_0 S_1$$

$$= -(-3) * 5 - 2 * 3$$

$$= 15 - 6$$

$$= 9$$

Now we use the formula again to find S_4 :

$$S_4 = \alpha^4 + \beta^4 = -a_1 S_3 - a_0 S_2$$

$$= -(-3) * 9 - 2 * 5$$

$$= 27 - 10$$

$$= 17$$

Now we use the formula again to find S_5 :

$$S_5 = \alpha^5 + \beta^5 = -a_1 S_4 - a_0 S_3$$

$$= -(-3) * 17 - 2 * 9$$

$$= 51 - 18$$

$$= 33$$

Now we use the formula again to find S_6 :

$$S_6 = \alpha^6 + \beta^6 = -a_1 S_5 - a_0 S_4$$

$$= -(-3) * 33 - 2 * 17$$

$$= 99 - 34$$

$$= 65$$

Now we use the formula again to find S_7 :

$$S_7 = \alpha^7 + \beta^7 = -a_1 S_6 - a_0 S_5$$

$$= -(-3) * 65 - 2 * 33$$


$$= 195 - 66$$


$$= 129$$

So, the value of $\alpha^7 + \beta^7$ is 129.

2.2.7. Newton Sum, Using Chat GPT for Calculate

If the roots of the equation $x^2 - 3x + 2 = 0$ are α and β , find the value of $\alpha^8 + \beta^8$ using Newton sum. $S_k = \alpha^k + \beta^k = -a_1(bS_{k-1} + cS_{k-2})$.

 If the roots of the equation $x^2 - 3x + 2 = 0$ are α and β , find the value of $\alpha^8 + \beta^8$ using Newton sum. $S_k = \alpha^k + \beta^k = -a_1(bS_{k-1} + cS_{k-2})$

 To find the value of $\alpha^8 + \beta^8$ using Newton's sums, we'll use the following formula:

$$S_k = \alpha^k + \beta^k = -a_1 * S_{(k-1)} - a_2 * S_{(k-2)}$$

Given the equation $x^2 - 3x + 2 = 0$, the coefficients are:

$$a_2 = 1$$

$$a_1 = -3$$

$$a_0 = 2$$

Let's use Newton's sum to find $\alpha^8 + \beta^8$:

$$1. S_0 = \alpha^0 + \beta^0 = 2$$

$$2. S_1 = \alpha^1 + \beta^1 = -(-3) = 3$$

Now we can use the formula to find S_2 :

$$\begin{aligned} S_2 &= \alpha^2 + \beta^2 = -a_1 * S_1 - a_0 * S_0 \\ &= -(-3) * 3 - 2 * 2 \\ &= 9 - 4 \\ &= 5 \end{aligned}$$

Now we use the formula again to find S_3 :

$$\begin{aligned} S_3 &= \alpha^3 + \beta^3 = -a_1 * S_2 - a_0 * S_1 \\ &= -(-3) * 5 - 2 * 3 \\ &= 15 - 6 \\ &= 9 \end{aligned}$$

Now we use the formula again to find S_4 :

$$\begin{aligned} S_4 &= \alpha^4 + \beta^4 = -a_1 * S_3 - a_0 * S_2 \\ &= -(-3) * 9 - 2 * 5 \\ &= 27 - 10 \\ &= 17 \end{aligned}$$

Now we use the formula again to find S_5 :

$$\begin{aligned} S_5 &= \alpha^5 + \beta^5 = -a_1 * S_4 - a_0 * S_3 \\ &= -(-3) * 17 - 2 * 9 \\ &= 51 - 18 \\ &= 33 \end{aligned}$$

Now we use the formula again to find S_6 :

$$\begin{aligned} S_6 &= \alpha^6 + \beta^6 = -a_1 * S_5 - a_0 * S_4 \\ &= -(-3) * 33 - 2 * 17 \\ &= 99 - 34 \\ &= 65 \end{aligned}$$

Now we use the formula again to find S_7 :

$$\begin{aligned} S_7 &= \alpha^7 + \beta^7 = -a_1 * S_6 - a_0 * S_5 \\ &= -(-3) * 65 - 2 * 33 \\ &= 195 - 66 \\ &= 129 \end{aligned}$$

Now we use the formula again to find S_8 :

$$\begin{aligned} S_8 &= \alpha^8 + \beta^8 = -a_1 * S_7 - a_0 * S_6 \\ &= -(-3) * 129 - 2 * 65 \\ &= 387 - 130 \\ &= 257 \end{aligned}$$

So, the value of $\alpha^8 + \beta^8$ is 257.

2.2.8. Newton Sum, Using Chat GPT for Calculate

If the roots of the equation $x^2 - 3x + 2 = 0$ are α and β , find the value of $\alpha^9 + \beta^9$ using Newton sum. $S_k = \alpha^k + \beta^k = -a_1(bS_{k-1} + cS_{k-2})$.

If the roots of the equation $x^2 - 3x + 2 = 0$ are α and β , find the value of $\alpha^9 + \beta^9$ using Newton sum. $S_k = \alpha^k + \beta^k = -a_1(bS_{k-1} + cS_{k-2})$

To find the value of $\alpha^9 + \beta^9$ using Newton's sums, we'll use the following formula:

$$S_k = \alpha^k + \beta^k = -a_1 S_{k-1} - a_2 S_{k-2}$$

Given the equation $x^2 - 3x + 2 = 0$, the coefficients are:

$$a_2 = 1$$

$$a_1 = -3$$

$$a_0 = 2$$

Let's use Newton's sum to find $\alpha^9 + \beta^9$:

$$1. S_0 = \alpha^0 + \beta^0 = 2$$

$$2. S_1 = \alpha^1 + \beta^1 = -(-3) = 3$$

Now we can use the formula to find S_2 :

$$\begin{aligned} S_2 &= \alpha^2 + \beta^2 = -a_1 S_1 - a_0 S_0 \\ &= -(-3) \cdot 3 - 2 \cdot 2 \\ &= 9 - 4 \\ &= 5 \end{aligned}$$

Now we use the formula again to find S_3 :

$$\begin{aligned} S_3 &= \alpha^3 + \beta^3 = -a_1 S_2 - a_0 S_1 \\ &= -(-3) \cdot 5 - 2 \cdot 3 \\ &= 15 - 6 \\ &= 9 \end{aligned}$$

There was an error generating a response

Now we use the formula again to find S_4 :

$$\begin{aligned} S_4 &= \alpha^4 + \beta^4 = -a_1 S_3 - a_0 S_2 \\ &= -(-3) \cdot 9 - 2 \cdot 5 \\ &= 27 - 10 \\ &= 17 \end{aligned}$$

Now we use the formula again to find S_5 :

$$\begin{aligned} S_5 &= \alpha^5 + \beta^5 = -a_1 S_4 - a_0 S_3 \\ &= -(-3) \cdot 17 - 2 \cdot 9 \\ &= 51 - 18 \\ &= 33 \end{aligned}$$

Now we use the formula again to find S_6 :

$$\begin{aligned} S_6 &= \alpha^6 + \beta^6 = -a_1 S_5 - a_0 S_4 \\ &= -(-3) \cdot 33 - 2 \cdot 17 \\ &= 99 - 34 \\ &= 65 \end{aligned}$$

Now we use the formula again to find S_7 :

$$\begin{aligned} S_7 &= \alpha^7 + \beta^7 = -a_1 S_6 - a_0 S_5 \\ &= -(-3) \cdot 65 - 2 \cdot 33 \\ &= 195 - 66 \\ &= 129 \end{aligned}$$

Now we use the formula again to find S_8 :

$$\begin{aligned} S_8 &= \alpha^8 + \beta^8 = -a_1 * S_7 - a_0 * S_6 \\ &= -(-3) * 129 - 2 * 65 \\ &= 387 - 130 \\ &= 257 \end{aligned}$$

Now we use the formula again to find S_9 :

$$\begin{aligned} S_9 &= \alpha^9 + \beta^9 = -a_1 * S_8 - a_0 * S_7 \\ &= -(-3) * 257 - 2 * 129 \\ &= 771 - 258 \\ &= 513 \end{aligned}$$

So, the value of $\alpha^9 + \beta^9$ is 513.

2.2.9. Newton Sum, Using Chat GPT for Calculate

If the roots of the equation $x^2 - 3x + 2 = 0$ are α and β , find the value of $\alpha^{10} + \beta^{10}$ using Newton sum. $S_k = \alpha^k + \beta^k = -a_1(bS_{k-1} + cS_{k-2})$.



If the roots of the equation $x^2 - 3x + 2 = 0$ are α and β , find the value of $\alpha^{10} + \beta^{10}$ using Newton sum. $S_k = \alpha^k + \beta^k = -a_1(bS_{k-1} + cS_{k-2})$



To find the value of $\alpha^{10} + \beta^{10}$ using Newton's sums, we'll use the following formula:

$$S_k = \alpha^k + \beta^k = -a_1 * S_{(k-1)} - a_2 * S_{(k-2)}$$

Given the equation $x^2 - 3x + 2 = 0$, the coefficients are:

$$a_2 = 1$$

$$a_1 = -3$$

$$a_0 = 2$$

Let's use Newton's sum to find $\alpha^{10} + \beta^{10}$:

$$1. S_0 = \alpha^0 + \beta^0 = 2$$

$$2. S_1 = \alpha^1 + \beta^1 = -(-3) = 3$$

Now we can use the formula to find S_2 :

$$\begin{aligned} S_2 &= \alpha^2 + \beta^2 = -a_1 * S_1 - a_0 * S_0 \\ &= -(-3) * 3 - 2 * 2 \\ &= 9 - 4 \\ &= 5 \end{aligned}$$

Now we use the formula again to find S_3 :

$$\begin{aligned} S_3 &= \alpha^3 + \beta^3 = -a_1 * S_2 - a_0 * S_1 \\ &= -(-3) * 5 - 2 * 3 \\ &= 15 - 6 \\ &= 9 \end{aligned}$$

This was an approximation response

Now we use the formula again to find S_4 :

$$\begin{aligned} S_4 &= \alpha^4 + \beta^4 = -a_1 * S_3 - a_0 * S_2 \\ &= -(-3) * 9 - 2 * 5 \\ &= 27 - 10 \\ &= 17 \end{aligned}$$

Now we use the formula again to find S_5 :

$$\begin{aligned} S_5 &= \alpha^5 + \beta^5 = -a_1 * S_4 - a_0 * S_3 \\ &= -(-3) * 17 - 2 * 9 \\ &= 51 - 18 \\ &= 33 \end{aligned}$$

Now we use the formula again to find S_6 :

$$\begin{aligned} S_6 &= \alpha^6 + \beta^6 = -a_1 * S_5 - a_0 * S_4 \\ &= -(-3) * 33 - 2 * 17 \\ &= 99 - 34 \\ &= 65 \end{aligned}$$

Now we use the formula again to find S_7 :

$$\begin{aligned} S_7 &= \alpha^7 + \beta^7 = -a_1 * S_6 - a_0 * S_5 \\ &= -(-3) * 65 - 2 * 33 \\ &= 195 - 66 \\ &= 129 \end{aligned}$$

Now we use the formula again to find S_8 :

$$\begin{aligned} S_8 &= \alpha^8 + \beta^8 = -a_1 * S_7 - a_0 * S_6 \\ &= -(-3) * 129 - 2 * 65 \\ &= 387 - 130 \\ &= 257 \end{aligned}$$

Now we use the formula again to find S_9 :

$$\begin{aligned} S_9 &= \alpha^9 + \beta^9 = -a_1 * S_8 - a_0 * S_7 \\ &= -(-3) * 257 - 2 * 129 \\ &= 771 - 258 \\ &= 513 \end{aligned}$$

Now we use the formula again to find S_{10} :

$$\begin{aligned} S_{10} &= \alpha^{10} + \beta^{10} = -a_1 * S_9 - a_0 * S_8 \\ &= -(-3) * 513 - 2 * 257 \\ &= 1539 - 514 \\ &= 1025 \end{aligned}$$

So, the value of $\alpha^{10} + \beta^{10}$ is 1025.

2.2.10. Newton Sum, Using Chat GPT for Calculate

If the roots of the equation $x^2 - 3x + 2 = 0$ are α and β , find the value of $\alpha^{11} + \beta^{11}$ using Newton sum. $S_k = \alpha^k + \beta^k = -a_1(bS_{k-1} + cS_{k-2})$.

If the roots of the equation $x^2 - 3x + 2 = 0$ are α and β , find the value of $\alpha^{11} + \beta^{11}$ using Newton sum. $S_k = \alpha^k + \beta^k = -a_1(bS_{k-1} + cS_{k-2})$

To find the value of $\alpha^{11} + \beta^{11}$ using Newton's sums, we'll use the following formula:

$$S_k = \alpha^k + \beta^k = -a_1 S_{(k-1)} - a_2 S_{(k-2)}$$

Given the equation $x^2 - 3x + 2 = 0$, the coefficients are:

$$a_2 = 1$$

$$a_1 = -3$$

$$a_0 = 2$$

Let's use Newton's sum to find $\alpha^{11} + \beta^{11}$:

- $S_0 = \alpha^0 + \beta^0 = 2$
- $S_1 = \alpha^1 + \beta^1 = -(-3) = 3$

Now we can use the formula to find S_2 :

$$S_2 = \alpha^2 + \beta^2 = -a_1 S_1 - a_0 S_0$$

$$= -(-3) * 3 - 2 * 2$$

$$= 9 - 4$$

$$= 5$$

Now we use the formula again to find S_3 :

$$S_3 = \alpha^3 + \beta^3 = -a_1 S_2 - a_0 S_1$$

$$= -(-3) * 5 - 2 * 3$$

$$= 15 - 6$$

$$= 9$$

Now we use the formula again to find S_4 :

$$S_4 = \alpha^4 + \beta^4 = -a_1 S_3 - a_0 S_2$$

$$= -(-3) * 9 - 2 * 5$$

$$= 27 - 10$$

$$= 17$$

Now we use the formula again to find S_5 :

$$S_5 = \alpha^5 + \beta^5 = -a_1 S_4 - a_0 S_3$$

$$= -(-3) * 17 - 2 * 9$$

$$= 51 - 18$$

$$= 33$$

Now we use the formula again to find S_6 :

$$S_6 = \alpha^6 + \beta^6 = -a_1 S_5 - a_0 S_4$$

$$= -(-3) * 33 - 2 * 17$$

$$= 99 - 34$$

$$= 65$$

Now we use the formula again to find S_7 :

$$S_7 = \alpha^7 + \beta^7 = -a_1 S_6 - a_0 S_5$$

$$= -(-3) * 65 - 2 * 33$$

$$= 195 - 66$$

$$= 129$$

```

Now we use the formula again to find S_8:

S_8 = α^8 + β^8 = -a_1 * S_7 - a_0 * S_6
     = -(-3) * 129 - 2 * 65
     = 387 - 130
     = 257

Now we use the formula again to find S_9:

S_9 = α^9 + β^9 = -a_1 * S_8 - a_0 * S_7
     = -(-3) * 257 - 2 * 129
     = 771 - 258
     = 513

Now we use the formula again to find S_10:

S_10 = α^10 + β^10 = -a_1 * S_9 - a_0 * S_8
      = -(-3) * 513 - 2 * 257
      = 1539 - 514
      = 1025

Now we use the formula again to find S_11:

S_11 = α^11 + β^11 = -a_1 * S_10 - a_0 * S_9
      = -(-3) * 1025 - 2 * 513
      = 3075 - 1026
      = 2049

So, the value of α^11 + β^11 is 2049.

```

2.2.11. Newton Sum, Using Chat GPT for Calculate

If the roots of the equation $x^2 - 3x + 2 = 0$ are α and β , find the value of $\alpha^{12} + \beta^{12}$ using Newton sum. $S_k = \alpha^k + \beta^k = -a_1(bS_{k-1} + cS_{k-2})$.

```

If the roots of the equation  $x^2 - 3x + 2 = 0$  are  $\alpha$  and  $\beta$ , find the value of  $\alpha^{12} + \beta^{12}$  using Newton sum.  $S_k = \alpha^k + \beta^k = -a_1(bS_{k-1} + cS_{k-2})$ 

To find the value of  $\alpha^{12} + \beta^{12}$  using Newton's sums, we'll use the following formula:

 $S_k = \alpha^k + \beta^k = -a_1 S_{(k-1)} - a_2 S_{(k-2)}$ 

Given the equation  $x^2 - 3x + 2 = 0$ , the coefficients are:
a_2 = 1
a_1 = -3
a_0 = 2

Let's use Newton's sum to find  $\alpha^{12} + \beta^{12}$ :

1.  $S_0 = \alpha^0 + \beta^0 = 2$ 
2.  $S_1 = \alpha^1 + \beta^1 = -(-3) = 3$ 

Now we can use the formula to find S_2:

 $S_2 = \alpha^2 + \beta^2 = -a_1 S_1 - a_0 S_0$ 
     = -(-3) * 3 - 2 * 2
     = 9 - 4
     = 5

Now we use the formula again to find S_3:

 $S_3 = \alpha^3 + \beta^3 = -a_1 S_2 - a_0 S_1$ 
     = -(-3) * 5 - 2 * 3
     = 15 - 6
     = 9

There was an error generating a response

```

Now we use the formula again to find S_4 :

$$\begin{aligned} S_4 &= \alpha^4 + \beta^4 = -a_1 S_3 - a_0 S_2 \\ &= -(-3) * 9 - 2 * 5 \\ &= 27 - 10 \\ &= 17 \end{aligned}$$

Now we use the formula again to find S_5 :

$$\begin{aligned} S_5 &= \alpha^5 + \beta^5 = -a_1 S_4 - a_0 S_3 \\ &= -(-3) * 17 - 2 * 9 \\ &= 51 - 18 \\ &= 33 \end{aligned}$$

Now we use the formula again to find S_6 :

$$\begin{aligned} S_6 &= \alpha^6 + \beta^6 = -a_1 S_5 - a_0 S_4 \\ &= -(-3) * 33 - 2 * 17 \\ &= 99 - 34 \\ &= 65 \end{aligned}$$

Now we use the formula again to find S_7 :

$$\begin{aligned} S_7 &= \alpha^7 + \beta^7 = -a_1 S_6 - a_0 S_5 \\ &= -(-3) * 65 - 2 * 33 \\ &= 195 - 66 \\ &= 129 \end{aligned}$$

Now we use the formula again to find S_8 :

Now we use the formula again to find S_8 :

$$\begin{aligned} S_8 &= \alpha^8 + \beta^8 = -a_1 S_7 - a_0 S_6 \\ &= -(-3) * 129 - 2 * 65 \\ &= 387 - 130 \\ &= 257 \end{aligned}$$

Now we use the formula again to find S_9 :

$$\begin{aligned} S_9 &= \alpha^9 + \beta^9 = -a_1 S_8 - a_0 S_7 \\ &= -(-3) * 257 - 2 * 129 \\ &= 771 - 258 \\ &= 513 \end{aligned}$$

Now we use the formula again to find S_{10} :

$$\begin{aligned} S_{10} &= \alpha^{10} + \beta^{10} = -a_1 S_9 - a_0 S_8 \\ &= -(-3) * 513 - 2 * 257 \\ &= 1539 - 514 \\ &= 1025 \end{aligned}$$

Now we use the formula again to find S_{11} :

$$\begin{aligned} S_{11} &= \alpha^{11} + \beta^{11} = -a_1 S_{10} - a_0 S_9 \\ &= -(-3) * 1025 - 2 * 513 \\ &= 3075 - 1026 \\ &= 2049 \end{aligned}$$

Now we use the formula again to find S_{12} :

$$\begin{aligned} S_{12} &= \alpha^{12} + \beta^{12} = -a_1 S_{11} - a_0 S_{10} \\ &= -(-3) * 2049 - 2 * 1025 \\ &= 6147 - 2050 \\ &= 4097 \end{aligned}$$

So, the value of $\alpha^{12} + \beta^{12}$ is 4097.

2.2.12. Newton Sum, Using Chat GPT for Calculate

If the roots of the equation $x^2 - 3x + 2 = 0$ are α and β , find the value of $\alpha^{13} + \beta^{13}$ using Newton sum. $S_k = \alpha^k + \beta^k = -a_1(bS_{k-1} + cS_{k-2})$.

If the roots of the equation $x^2 - 3x + 2 = 0$ are α and β , find the value of $\alpha^{13} + \beta^{13}$ using Newton sum. $S_k = \alpha^k + \beta^k = -a_1(bS_{k-1} + cS_{k-2})$

To find the value of $\alpha^{13} + \beta^{13}$ using Newton's sums, we'll use the following formula:

$$S_k = \alpha^k + \beta^k = -a_1 S_{k-1} - a_2 S_{k-2}$$

Given the equation $x^2 - 3x + 2 = 0$, the coefficients are:

$$a_2 = 1$$

$$a_1 = -3$$

$$a_0 = 2$$

Let's use Newton's sum to find $\alpha^{13} + \beta^{13}$:

- $S_0 = \alpha^0 + \beta^0 = 2$
- $S_1 = \alpha^1 + \beta^1 = -(-3) = 3$

Now we can use the formula to find S_2 :

$$S_2 = \alpha^2 + \beta^2 = -a_1 S_1 - a_0 S_0$$

$$= -(-3) * 3 - 2 * 2$$

$$= 9 - 4$$

$$= 5$$

Now we use the formula again to find S_3 :

$$S_3 = \alpha^3 + \beta^3 = -a_1 S_2 - a_0 S_1$$

$$= -(-3) * 5 - 2 * 3$$

$$= 15 - 6$$

$$= 9$$

There was an error generating a response

Now we use the formula again to find S_4 :

$$S_4 = \alpha^4 + \beta^4 = -a_1 S_3 - a_0 S_2$$

$$= -(-3) * 9 - 2 * 5$$

$$= 27 - 10$$

$$= 17$$

Now we use the formula again to find S_5 :

$$S_5 = \alpha^5 + \beta^5 = -a_1 S_4 - a_0 S_3$$

$$= -(-3) * 17 - 2 * 9$$

$$= 51 - 18$$

$$= 33$$

Now we use the formula again to find S_6 :

$$S_6 = \alpha^6 + \beta^6 = -a_1 S_5 - a_0 S_4$$

$$= -(-3) * 33 - 2 * 17$$

$$= 99 - 34$$

$$= 65$$

Now we use the formula again to find S_7 :

$$S_7 = \alpha^7 + \beta^7 = -a_1 S_6 - a_0 S_5$$

$$= -(-3) * 65 - 2 * 33$$

$$= 195 - 66$$

$$= 129$$

Now we use the formula again to find S₈:

$$\begin{aligned} S_8 &= \alpha^8 + \beta^8 = -a_1 S_7 - a_0 S_6 \\ &= -(-3) * 129 - 2 * 65 \\ &= 387 - 130 \\ &= 257 \end{aligned}$$

Now we use the formula again to find S₉:

$$\begin{aligned} S_9 &= \alpha^9 + \beta^9 = -a_1 S_8 - a_0 S_7 \\ &= -(-3) * 257 - 2 * 129 \\ &= 771 - 258 \\ &= 513 \end{aligned}$$

Now we use the formula again to find S₁₀:

$$\begin{aligned} S_{10} &= \alpha^{10} + \beta^{10} = -a_1 S_9 - a_0 S_8 \\ &= -(-3) * 513 - 2 * 257 \\ &= 1539 - 514 \\ &= 1025 \end{aligned}$$

Now we use the formula again to find S₁₁:

$$\begin{aligned} S_{11} &= \alpha^{11} + \beta^{11} = -a_1 S_{10} - a_0 S_9 \\ &= -(-3) * 1025 - 2 * 513 \\ &= 3075 - 1026 \\ &= 2049 \end{aligned}$$

Now we use the formula again to find S₁₂:

$$\begin{aligned} S_{12} &= \alpha^{12} + \beta^{12} = -a_1 S_{11} - a_0 S_{10} \\ &= -(-3) * 2049 - 2 * 1025 \\ &= 6147 - 2050 \\ &= 4097 \end{aligned}$$

Now we use the formula again to find S₁₃:

$$\begin{aligned} S_{13} &= \alpha^{13} + \beta^{13} = -a_1 S_{12} - a_0 S_{11} \\ &= -(-3) * 4097 - 2 * 2049 \\ &= 12291 - 4098 \\ &= 8193 \end{aligned}$$

So, the value of $\alpha^{13} + \beta^{13}$ is 8193.

2.2.13. Newton Sum, Using Chat GPT for Calculate

If the roots of the equation $x^2 - 3x + 2 = 0$ are α and β , find the value of $\alpha^{14} + \beta^{14}$ using Newton sum. $S_k = \alpha^k + \beta^k = -a_1(bS_{k-1} + cS_{k-2})$.

If the roots of the equation $x^2 - 3x + 2 = 0$ are α and β , find the value of $\alpha^{14} + \beta^{14}$ using Newton sum. $S_k = \alpha^k + \beta^k = -a_1(bS_{k-1} + cS_{k-2})$

To find the value of $\alpha^{14} + \beta^{14}$ using Newton's sums, we'll use the following formula:

$$S_k = \alpha^k + \beta^k = -a_1 S_{(k-1)} - a_2 S_{(k-2)}$$

Given the equation $x^2 - 3x + 2 = 0$, the coefficients are:

$$\begin{aligned} a_2 &= 1 \\ a_1 &= -3 \\ a_0 &= 2 \end{aligned}$$

Let's use Newton's sum to find $\alpha^{14} + \beta^{14}$:

- $S_0 = \alpha^0 + \beta^0 = 2$
- $S_1 = \alpha^1 + \beta^1 = -(-3) = 3$

Now we can use the formula to find S₂:

$$\begin{aligned} S_2 &= \alpha^2 + \beta^2 = -a_1 S_1 - a_0 S_0 \\ &= -(-3) * 3 - 2 * 2 \\ &= 9 - 4 \\ &= 5 \end{aligned}$$

Now we use the formula again to find S₃:

$$\begin{aligned} S_3 &= \alpha^3 + \beta^3 = -a_1 S_2 - a_0 S_1 \\ &= -(-3) * 5 - 2 * 3 \\ &= 15 - 6 \\ &= 9 \end{aligned}$$

Now we use the formula again to find S_4 :

$$\begin{aligned} S_4 &= \alpha^4 + \beta^4 = -a_1 * S_3 - a_0 * S_2 \\ &= -(-3) * 9 - 2 * 5 \\ &= 27 - 10 \\ &= 17 \end{aligned}$$

Now we use the formula again to find S_5 :

$$\begin{aligned} S_5 &= \alpha^5 + \beta^5 = -a_1 * S_4 - a_0 * S_3 \\ &= -(-3) * 17 - 2 * 9 \\ &= 51 - 18 \\ &= 33 \end{aligned}$$

Now we use the formula again to find S_6 :

$$\begin{aligned} S_6 &= \alpha^6 + \beta^6 = -a_1 * S_5 - a_0 * S_4 \\ &= -(-3) * 33 - 2 * 17 \\ &= 99 - 34 \\ &= 65 \end{aligned}$$

Now we use the formula again to find S_7 :

$$\begin{aligned} S_7 &= \alpha^7 + \beta^7 = -a_1 * S_6 - a_0 * S_5 \\ &= -(-3) * 65 - 2 * 33 \\ &= 195 - 66 \\ &= 129 \end{aligned}$$

Now we use the formula again to find S_8 :

$$\begin{aligned} S_8 &= \alpha^8 + \beta^8 = -a_1 * S_7 - a_0 * S_6 \\ &= -(-3) * 129 - 2 * 65 \\ &= 387 - 130 \\ &= 257 \end{aligned}$$

Now we use the formula again to find S_9 :

$$\begin{aligned} S_9 &= \alpha^9 + \beta^9 = -a_1 * S_8 - a_0 * S_7 \\ &= -(-3) * 257 - 2 * 129 \\ &= 771 - 258 \\ &= 513 \end{aligned}$$

Now we use the formula again to find S_{10} :

$$\begin{aligned} S_{10} &= \alpha^{10} + \beta^{10} = -a_1 * S_9 - a_0 * S_8 \\ &= -(-3) * 513 - 2 * 257 \\ &= 1539 - 514 \\ &= 1025 \end{aligned}$$

Now we use the formula again to find S_{11} :

$$\begin{aligned} S_{11} &= \alpha^{11} + \beta^{11} = -a_1 * S_{10} - a_0 * S_9 \\ &= -(-3) * 1025 - 2 * 513 \\ &= 3075 - 1026 \\ &= 2049 \end{aligned}$$

Now we use the formula again to find S_{12} :

$$\begin{aligned} S_{12} &= \alpha^{12} + \beta^{12} = -a_1 * S_{11} - a_0 * S_{10} \\ &= -(-3) * 2049 - 2 * 1025 \\ &= 6147 - 2050 \\ &= 4097 \end{aligned}$$

Now we use the formula again to find S_{13} :

$$\begin{aligned} S_{13} &= \alpha^{13} + \beta^{13} = -a_1 * S_{12} - a_0 * S_{11} \\ &= -(-3) * 4097 - 2 * 2049 \\ &= 12291 - 4098 \\ &= 8193 \end{aligned}$$

Now we use the formula again to find S_{14} :

$$\begin{aligned} S_{14} &= \alpha^{14} + \beta^{14} = -a_1 * S_{13} - a_0 * S_{12} \\ &= -(-3) * 8193 - 2 * 4097 \\ &= 24579 - 8194 \\ &= 16385 \end{aligned}$$

So, the value of $\alpha^{14} + \beta^{14}$ is 16385.

2.2.14. Newton Sum, Using Chat GPT for Calculate

If the roots of the equation $x^2 - 3x + 2 = 0$ are α and β , find the value of $\alpha^{15} + \beta^{15}$ using Newton sum . $S_k = \alpha^k + \beta^k = -a_1(bS_{k-1} + cS_{k-2})$.

If the roots of the equation $x^2 - 3x + 2 = 0$ are α and β , find the value of $\alpha^{15} + \beta^{15}$ using Newton sum . $S_k = \alpha^k + \beta^k = -a_1(bS_{k-1} + cS_{k-2})$

To find the value of $\alpha^{15} + \beta^{15}$ using Newton's sums, we'll use the following formula:

$$S_k = \alpha^k + \beta^k = -a_1 * S_{(k-1)} - a_2 * S_{(k-2)}$$

Given the equation $x^2 - 3x + 2 = 0$, the coefficients are:

$$\begin{aligned} a_2 &= 1 \\ a_1 &= -3 \\ a_0 &= 2 \end{aligned}$$

Let's use Newton's sum to find $\alpha^{15} + \beta^{15}$:

- $S_0 = \alpha^0 + \beta^0 = 2$
- $S_1 = \alpha^1 + \beta^1 = -(-3) = 3$

Now we can use the formula to find S_2 :

$$\begin{aligned} S_2 &= \alpha^2 + \beta^2 = -a_1 * S_1 - a_0 * S_0 \\ &= -(-3) * 3 - 2 * 2 \\ &= 9 - 4 \\ &= 5 \end{aligned}$$

Now we use the formula again to find S_3 :

$$\begin{aligned} S_3 &= \alpha^3 + \beta^3 = -a_1 * S_2 - a_0 * S_1 \\ &= -(-3) * 5 - 2 * 3 \\ &= 15 - 6 \\ &= 9 \end{aligned}$$

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Now we use the formula again to find S_4 :

$$\begin{aligned} S_4 &= \alpha^4 + \beta^4 = -a_1 S_3 - a_0 S_2 \\ &= -(-3) * 9 - 2 * 5 \\ &= 27 - 10 \\ &= 17 \end{aligned}$$

Now we use the formula again to find S_5 :

$$\begin{aligned} S_5 &= \alpha^5 + \beta^5 = -a_1 S_4 - a_0 S_3 \\ &= -(-3) * 17 - 2 * 9 \\ &= 51 - 18 \\ &= 33 \end{aligned}$$

Now we use the formula again to find S_6 :

$$\begin{aligned} S_6 &= \alpha^6 + \beta^6 = -a_1 S_5 - a_0 S_4 \\ &= -(-3) * 33 - 2 * 17 \\ &= 99 - 34 \\ &= 65 \end{aligned}$$

Now we use the formula again to find S_7 :

$$\begin{aligned} S_7 &= \alpha^7 + \beta^7 = -a_1 S_6 - a_0 S_5 \\ &= -(-3) * 65 - 2 * 33 \\ &= 195 - 66 \\ &= 129 \end{aligned}$$

Now we use the formula again to find S_8 :

$$\begin{aligned} S_8 &= \alpha^8 + \beta^8 = -a_1 S_7 - a_0 S_6 \\ &= -(-3) * 129 - 2 * 65 \\ &= 387 - 130 \\ &= 257 \end{aligned}$$

Now we use the formula again to find S_9 :

$$\begin{aligned} S_9 &= \alpha^9 + \beta^9 = -a_1 S_8 - a_0 S_7 \\ &= -(-3) * 257 - 2 * 129 \\ &= 771 - 258 \\ &= 513 \end{aligned}$$

Now we use the formula again to find S_{10} :

$$\begin{aligned} S_{10} &= \alpha^{10} + \beta^{10} = -a_1 S_9 - a_0 S_8 \\ &= -(-3) * 513 - 2 * 257 \\ &= 1539 - 514 \\ &= 1025 \end{aligned}$$

Now we use the formula again to find S_{11} :

$$\begin{aligned} S_{11} &= \alpha^{11} + \beta^{11} = -a_1 S_{10} - a_0 S_9 \\ &= -(-3) * 1025 - 2 * 513 \\ &= 3075 - 1026 \\ &= 2049 \end{aligned}$$

Now we use the formula again to find S_{12} :

$$\begin{aligned} S_{12} &= \alpha^{12} + \beta^{12} = -a_1 * S_{11} - a_0 * S_{10} \\ &= -(-3) * 2049 - 2 * 1025 \\ &= 6147 - 2050 \\ &= 4097 \end{aligned}$$

Now we use the formula again to find S_{13} :

$$\begin{aligned} S_{13} &= \alpha^{13} + \beta^{13} = -a_1 * S_{12} - a_0 * S_{11} \\ &= -(-3) * 4097 - 2 * 2049 \\ &= 12291 - 4098 \\ &= 8193 \end{aligned}$$

Now we use the formula again to find S_{14} :

$$\begin{aligned} S_{14} &= \alpha^{14} + \beta^{14} = -a_1 * S_{13} - a_0 * S_{12} \\ &= -(-3) * 8193 - 2 * 4097 \\ &= 24579 - 8194 \\ &= 16385 \end{aligned}$$

Now we use the formula again to find S_{15} :

$$\begin{aligned} S_{15} &= \alpha^{15} + \beta^{15} = -a_1 * S_{14} - a_0 * S_{13} \\ &= -(-3) * 16385 - 2 * 8193 \\ &= 49155 - 16386 \\ &= 32769 \end{aligned}$$

So, the value of $\alpha^{15} + \beta^{15}$ is 32769.

2.2.15. Newton Sum, Using Chat GPT for Calculate

If the roots of the equation $x^2 - 3x + 2 = 0$ are α and β , find the value of $\alpha^{28} + \beta^{28}$.

PE If the roots of the equation $x^2 - 3x + 2 = 0$ are α and β , find the value of $\alpha^{28} + \beta^{28}$



To find the value of $\alpha^{28} + \beta^{28}$, we can continue using Vieta's formulas and Newton's Sums.

Given the quadratic equation $x^2 - 3x + 2 = 0$ with roots α and β , we have the following values from Vieta's formulas:

1. Sum of roots: $\alpha + \beta = 3$
2. Product of roots: $\alpha\beta = 2$

We can use Newton's Sums to find the value of $\alpha^{28} + \beta^{28}$. The formula for S_k with $k = 28$ is:

$$S_{28} = \alpha^{28} + \beta^{28} = -\frac{1}{a} (bS_{27} + cS_{26})$$

We can calculate the values of S_{27} and S_{28} using the previously calculated values:

1. $S_{27} = \alpha^{27} + \beta^{27}$ - Unfortunately, we don't have a formula for S_{27} at this point, but we can still find it using the relationship between S_k and S_{k-1} derived from Newton's Sums:

$$S_k = -\frac{1}{a} (bS_{k-1} + cS_{k-2})$$

So, we can start by calculating S_{27} and then use it to find S_{28} .

Let's first calculate S_{27} :

$$S_{27} = -\frac{1}{1} (-3S_{26} + 2S_{25})$$

We'll need to calculate S_{26} and S_{25} using the same recursive formula. Continuing this process, we'll eventually calculate S_{27} , and then we can calculate S_{28} using the formula above.

However, this recursive process can become quite complex and time-consuming for higher values of k . If you're looking for a more efficient method, you might consider using a mathematical software or programming language that can handle symbolic computations.

3. Results

The results of this study provide concrete evidence of ChatGPT's superior utilization of the Newton Sum Method for calculating quadratic root functions. The model adeptly computes expressions of the form $\alpha^{15} + \beta^{15}$ using this method, while its proficiency using algebraic formula methods is generally limited to $\alpha^4 + \beta^4$. This striking discrepancy underscores the transformative impact that different methods can have on elevating the AI system's mathematical prowess.

In the realm of quadratic root function calculations, the Newton Sum Method emerges as a powerful tool compared to the Algebra Formula method, enabling AI systems like ChatGPT to solve expressions like $\alpha^{15} + \beta^{15}$ which cannot be solved by the Algebra formula method. However, a noteworthy facet emerges as ChatGPT encounters challenges in yielding definitive solutions for expressions such as $\alpha^{28} + \beta^{28}$. The intricate nature of these solutions proves to be a stumbling block, leading to limitations in the scope of deliverable conclusions.

The core reason behind these limitations lies in the inherent methodology of the Newton Sum calculation technique. Rooted in historical calculations, this method relies on previous results to determine the current outcome. As a consequence, when confronted with higher-order quadratic root function calculations like $\alpha^{28} + \beta^{28}$, the computational process undergoes a transformation into a labyrinthine endeavour, entailing increased intricacy and time consumption.

Engaging with this limitation, ChatGPT offers a comprehensive strategy: "We'll need to calculate S_{26} and S_{25} using the same recursive formula. Continuing this process, we'll eventually calculate S_{27} , and then we can calculate S_{28} using the formula above. **However, this recursive process can become quite complex and time-consuming for higher values of k .** If you're looking for a more efficient method, you might consider using a mathematical software or programming language that can handle symbolic computations."

4. Conclusion

Pushing Boundaries: Pioneering Novel Maths Approaches for Overcoming Limitations in AI

This study serves as an illuminating testament to the significance of pioneering innovative methodologies, rules, theorems, or formulas to surmount the current limitations in AI systems like ChatGPT. These innovative pursuits hold the key to unlocking the untapped potential that lies within, propelling AI systems to greater heights of proficiency. In essence, they offer a strategic pathway towards expanding the capabilities of AI and pushing the boundaries of what can be achieved.

In the broader panorama of AI's evolutionary trajectory, these findings resonate as a resonant call to action. They invite researchers, developers, and practitioners to seize the opportunity and embark on a transformative journey. The limitations encountered today are not mere roadblocks but stepping stones towards a more capable AI landscape. As the digital realm continues to evolve, the pursuit of novel methodologies becomes an integral part of this evolutionary symphony.

These findings cast a compelling vision for future research endeavours. They beckon researchers to embark on audacious explorations into uncharted territories. This involves delving deep into the unexplored realm of novel mathematical techniques, intricately tailored to accommodate the unique characteristics of AI systems. Such an endeavour, though challenging, promises to be transformative. The resulting synergy between innovative mathematics and advanced AI systems holds the potential to reshape the technological landscape.

In conclusion, this study resonates as a clarion call to innovation, urging stakeholders to embrace the challenge and forge ahead. The limitations encountered today are the building blocks of progress. The pioneering of novel methodologies is not only an aspiration but a necessity to shape the future of AI. As we stand at the precipice of an AI-powered future, the beckoning of uncharted territories invites us to create a new paradigm, where the potential of AI systems is truly unleashed and their impact resonates across the technological landscape.

5. Discussion

The results obtained from this study strongly emphasize the substantial impact that the choice of method holds in bolstering the mathematical proficiencies of ChatGPT. One method that particularly stands out is the application of the Newton Sum Method, which emerges as a compelling and illustrative example.

This method acts as a pivotal gateway, enabling the model to transcend its previous limitations and boldly explore the domain of calculations involving higher exponents. This significant breakthrough not only showcases the Newton Sum Method's efficacy but also highlights its pivotal role in pushing the boundaries of ChatGPT's mathematical capabilities.

6. Implications and Future Research

The discoveries unearthed by this study carry a dual impact, contributing not solely to the advancement of AI's mathematical competencies but also underscoring the imperative of forging ahead with innovation. These findings serve as a clarion call for the exploration and creation of pioneering methodologies, novel rules, inventive theorems, and groundbreaking formulas. The aim is clear: to propel AI systems like ChatGPT to even greater heights of proficiency and utility.

Looking ahead, the vista of future research beckons. A promising avenue lies in the exploration and refinement of novel mathematical techniques that are expressly tailored to the unique demands of AI systems. This deliberate alignment holds the potential to significantly expand the purview of AI's capabilities, allowing them to permeate a diverse spectrum of problem-solving domains.

In essence, this study isn't merely a snapshot of AI's current capabilities; it's a catalyst for future innovation. It reinforces the idea that as AI continues its evolution, it must continue to push boundaries, to delve into uncharted territories, and to chart new courses of advancement. By embracing this ethos, researchers can fuel the ongoing transformation of AI from a tool of the present

into an even more formidable force in the future, reshaping the way we approach complex problem-solving across a multitude of domains.

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