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Friedmann Type Equations in Thermodynamical Form Lead to Much Tighter Constraints on the Energy Density of the Universe

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Abstract: Based on recent progress in cosmological thermodynamics, we are presenting a new thermodynamic formulation of the Friedmann equations, as well as for other similar equations put forward in the literature from different solutions to Einstein's field equation. Since the CMB temperature has been measured much more accurately than the Hubble constant, we demonstrate that the thermodynamic formulation of the Friedmann equation dramatically narrows down the acceptable range for the mass-energy density in the universe.

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1. The Friedmann Equation on Thermodynamical Form

From the Friedemann–Lamítre-Robertson-Walker (FLRW) metric is given by:

$$ds^2 = c^2dt^2 - a^2(t)\left(\frac{dr^2}{1-kr^2} + r^2\Omega^2\right) \tag{1}$$

where $\Omega^2 = d\theta^2 + \sin^2\theta 2\phi^2$.The Friedmann equation, cited as Friedmann [1], forms the backbone of multiple cosmological models, including the Λ-CDM model. It can be derived from the FLWE metric and is typically expressed as:

$$H_0^2 = \frac{8\pi\rho + \Lambda c^2}{3} - \frac{k^2c^2}{3} \tag{2}$$

where k is the curvature parameter and Λ is the cosmological constant. In 2015, Tatum et al. [2] heuristically suggested the following equation for the CMB temperature:

$$T_{cmb} = \frac{\hbar c}{k_b 4\pi\sqrt{R_h 2l_p}} \tag{3}$$

where R_h represents the Hubble radius of the universe and l_p denotes the Planck length [3,4]. However, few have taken notice of this formula, and there could be several reasons for this. It was published in a low-ranked journal, and additionally, no derivation of the formula based on known laws in physics was presented. Haug and Wojnow [5,6] recently demonstrated that the equation can be derived from the Stefan-Boltzmann law. Furthermore, Haug and Tatum demonstrated that it can also be derived from more general geometric principles, where the observed CMB temperature can then be seen as the geometric mean of the lowest and highest possible temperatures in the Hubble sphere. Tatum et al. [7] has also reformulated this formula to (see also [8,9]) :

$$H_0 = \mho T_0^2. \tag{4}$$

Where $\mho$ is a composite constant of the form

$$\mho = \frac{k_b^2 32\pi^2 G^{1/2}}{c^{5/2}\hbar^{3/2}} = 2.91845601\times10^{-19} \pm 0.00003279\times10^{-19}\ s^{-1}\cdot K^{-2}. \tag{5}$$

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The value is based on NIST CODATA 2018. The only uncertainty in the Upsilon composite constant comes from the uncertainty in G, as the other constants, k_b, c, and $\hbar$, are exactly defined according to the NIST CODATA standard at the time of writing.

This means that in the Friedmann equation, we can rewrite it as:

$$\begin{aligned} H_0^2 &= \frac{8\pi G\rho + \Lambda c^2}{3} - \frac{k^2c^2}{3} \\ T_0^4\mho^2 &= \frac{8\pi G\rho + \Lambda c^2}{3} - \frac{k^2c^2}{3} \\ T_0^4 &= \frac{8\pi G\rho + \Lambda c^2}{3\mho^2} - \frac{k^2c^2}{3\mho^2} \\ T_0 &= \left(\frac{8\pi G\rho + \Lambda c^2}{3\mho^2} - \frac{k^2c^2}{3\mho^2}\right)^{\frac{1}{4}} \end{aligned} \tag{6}$$

or alternatively solved for the density:

$$\rho = \frac{3T_0^4\mho^2 - \Lambda c^2 + kc^2}{8\pi G}. \tag{7}$$

We will from now only focus on flat-space cosmology and set $k = 0$, which involves for example several forms of $R_h = ct$ cosmological models. In addition we will first focus on the critical universe density, so we will also set $\Lambda = 0$, this gives

$$T_0 = \left(\frac{8\pi G\rho}{3\mho^2}\right)^{\frac{1}{4}} \tag{8}$$

and alternatively from the density:

$$\rho_c = \frac{3T_0^4\mho^2}{8\pi G} = \frac{3T_0^4\left(\frac{k_b^2 32\pi^2 G^{1/2}}{c^{5/2}\hbar^{3/2}}\right)^2}{8\pi G} = \frac{3T_0^4 k_b^4 128\pi^3}{c^5\hbar^3} \tag{9}$$

This thermodynamical formula for the critical density was recently provided by Haug and Tatum [9], but here it is derived even further so that we can see that G has canceled out, and the only uncertainty in the estimate for the critical density is therefore the uncertainty in the observed CMB temperature, T_0.

Dhal et al. [10] conducted one of the most recent and accurate measurements of the CMB temperature, obtaining $T_0 = 2.725007 \pm 0.000024$ K (see also [11,12]). Therefore, we will use the CMB temperature as input to find the estimate of the critical density, resulting in:

$$\rho_c = \frac{3T_0^4\mho^2}{8\pi G} = \frac{3T_0^4\left(\frac{k_b^2 32\pi^2 G^{1/2}}{c^{5/2}\hbar^{3/2}}\right)^2}{8\pi G} = \frac{3T_0^4 k_b^4 128\pi^3}{c^5\hbar^3} = 8.399481^{+0.000296}_{-0.000296} \times 10^{-27}\ kg \cdot m^{-3} \tag{10}$$

This gives a much more narrow estimates of the critical density of the universe than one can get from going through the traditional formula $\rho_c = \frac{3H_0^2}{8\pi G}$.

Next let us add the cosmological constant. It is assumed in the Λ-CDM model that

$$\Lambda = 3\left(\frac{H_0}{c}\right)^2 \Omega_\Lambda \tag{11}$$

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where Ω_Λ is the ratio of the energy density due to the cosmological constant and the critical density, according to the Planck collobration 2018 we have $\Omega_\Lambda = 0.6889 \pm 0.0056$. Next we will use a Hubble constant of $H_0 = 73.01 \pm 1.04\ km/s/Mpc$ as given by Riess et. al [13] in our Thermodynamical Friedmann formula density based on flat-space ($k = 0$):

$$\rho = \frac{3T_0^4\mho^2 - \Lambda c^2}{8\pi G} = 1.49 \times 10^{-27} \pm 0.25 \times 10^{-27}\ kg \cdot m^{-3}. \tag{12}$$

This is way below the WMAP value of $9.9 \times 10^{-27}\ kg \cdot m^{-3}$. To get to this density one must raise the current CMB temperature to minimum $T_0 = 3.23K$ which is far outside any confidence interval for the measured CMB temperature. Recently Haug and Tatum [14] has demonstrated that one in the Λ-CDM model likely have a incomplete model for the cosmological red-shifts and that one therefore have got too high value estimates for H_0 from a series of supernova studies, like the well known Riess et. al study.

Alternatively, one could argue that it is not relevant to include Λ when calculating the density of the universe if the prediction will be used to compare with observed energy density. In the Λ-CDM model, Λ is associated with dark energy, which appears to be not directly observable. Therefore, this remains an open question.

2. The Extremal Universe and the Haug-Spavieri Cosmology

Haug [15] has recently derived a similar equation to that of the Friedmann equation from the extremal solution of the Reisner-Nordström [16], Kerr [17] and Kerr-Newman [18,19] metric it is given by :

$$H_0^2 = \frac{8\pi G\rho - \Lambda c^2}{3}. \tag{13}$$

The same universe equation is also derived from the recent exact solution to Einstein's field equation [20], as provided by Haug-Spavieri [21,22]. Firstly, this solution does not yield a curvature constant k, indicating a prediction of flat space. Secondly, the cosmological constant is not introduced manually into Einstein's 1916 field equation, as Einstein did in 1917 in his extended field equation [23]. Thirdly, the cosmological constant is negative rather than positive. For researchers primarily familiar with the Λ-CDM model, this may seem unfamiliar, but the possibility of a negative cosmological constant is actively discussed in the literature (see, for example, [24–30]). In this model, the cosmological constant is precisely given by $\Lambda = 3\left(\frac{H_0^2}{c^2}\right)$, simplifying the equation further to:

$$H_0^2 = \frac{4\pi G\rho}{3}. \tag{14}$$

Next, we can utilize the fact that $H_0 = \mho T_0^2$ and input this into the equation above. This gives us the thermodynamical version of the Haug-Spavieri universe.

$$T_0^4\mho^2 = \frac{4\pi G\rho}{3} \tag{15}$$

that can be rewritten as

$$T_0 = \left(\frac{4\pi G\rho}{3\mho^2}\right)^{\frac{1}{4}}. \tag{16}$$

Solved with respect to ρ this gives

$$\rho = \frac{3T_0^4\mho^2}{4\pi G} = \frac{3T_0^4\left(\frac{k_b^2 32\pi^2 G^{1/2}}{c^{5/2}\hbar^{3/2}}\right)^2}{4\pi G} = \frac{3T_0^4 k_b^4 256\pi^3}{c^5\hbar^3}. \tag{17}$$

The only uncertainty in the mass-energy density comes from the uncertainty in the measured CMB temperature, as k_b, $\hbar$, and c are all defined as exact constants by NIST CODATA 2018. If we again use the Dhal et al. [10] $T_0 = 2.725007 \pm 0.000024$ K, we get:

$$\rho = \frac{3T_0^4 k_b^4 256\pi^3}{c^5\hbar^3} = 1.679896^{+0.000592}_{-0.000592} \times 10^{-27}\ kg \cdot m^{-3} \tag{18}$$

Exactly half of this density comes from gravitational relativistic effects not taken into account for example in the Schwarzschild metric. This relativistic gravitational energy can in some sense be labeled dark energy as it likely not can be detected by any other means than observed gravitational effects. To compare with observed energy levels one should therefore likely compare with only half of this, that is $8.399481^{+0.000296}_{-0.000296} \times 10^{-27}\ kg \cdot m^{-3}$.

This means that both the Haug-Spavieri universe and the extremal universe seem to accurately predict the observed energy density in the universe. Additionally, they provide a much better explanation for the cosmological constant. For an in-depth discussion, refer to [15,22].

3. Conclusion

We have demonstrated that the Friedmann equation, as well as other similar cosmological models derived from general relativity theory, can also be expressed in thermodynamic form as a function of the Cosmic Microwave Background (CMB) temperature instead of the Hubble constant. Since the CMB temperature is measured much more precisely than the Hubble constant, this leads to much tighter constraints on parameters such as the critical energy density. Furthermore, it facilitates the comparison of different cosmological models.

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