Article

Not peer-reviewed version

Scale Invariant Parity Doublet Model: Interplay between the Vector Mesons and the Dilatons /Parity Doubling in Dense Baryonic Matter as an Emergent Symmetry and Pseudo-Conformal Model

Hyun Kyu Lee *

Posted Date: 29 March 2024

doi: 10.20944/preprints202403.1812.v1

Keywords: scale invariance; parity doubling; pseudo-conformal phase; vector meson-dilaton interplay; compact star matter; speed of sound

Article

Scale Invariant Parity Doublet Model: Interplay between the Vector Mesons and the Dilatons/Parity Doubling in Dense Baryonic Matter as an Emergent Symmetry and Pseudo-Conformal Model

Hyun Kyu Lee

Department of Physics, Hanyang University, Seoul, 04763, Korea ; hyunkyu@hanyang.ac.kr

Abstract: The star matter composed of nucleons deep inside the compact star like a neutron star is believed to be very dense and various new types of concepts and physical phenomena due to the nontrivial strong correlations are naturally expected. Peudo-conformality in dense star matter has been discussed recently assuming the hidden underlying scale symmetry, which is partly supported by the recent discussions of infrared conformal window of QCD. It is found that the appearance of dilaton and the dilatonic mean field calculation of nucleonic matter lead to the density independent nucleon mass at higher density regime than normal nuclear matter density. Non-vanishing constant nucleon mass can be considered as a chiral invariant mass of the parity doublet model, in which the chiral symmetry is manifested even in the massive nucleon sector provided with a parity doubling. In this paper we will discuss how the chiral and scale symmetry are correlated to lead the parity doublet symmetry inside the core of the compact star. A simple schematic model is adopted. The star matter at $T = 0$ is basically composed of the nucleons. As a first approximation, they can be treated as free fermions with the effective mass filled up to fermi surface. There is a nontrivial feature due to the interplay between Ω vector meson and nucleon(or equivalently dilaton, χ). The trace of energy momentum tensor approaches density independent, the pseudo-conformal phase. Local fluctuations inside the star matter will be travelling with the speed of sound which depends on the equation of state. In the pseudo-conformal phase, the speed of sound approaches to the conformal velocity for the scale symmetric matter, $v_c = \frac{1}{3}c$. The excitations of the parity doublets in the compact star matter as an emerging phenomena in pseudo-conformal phase will be discussed in connection to the chiral symmetry. The possible implication on the corespondence of hadrons to quark degrees of freedom will be briefly discussed.

Keywords: scale invariance, parity doubling, pseudo-conformal phase, vector meson-dilaton interplay, compact star matter, speed of sound

---

1. Introduction

The scale invariance would be exact if all elementary particle masses(more generally, all dimensionful couplings) vanished [1]. In the real world the scale transformation is not a symmetric transformation. But in the situation where the effects of the masses are not important, such as extremely high energy phenomena, the role of the scale svmmetry when properly formulated can be studied systematically even together with small symmetry breaking effects . The nucleon matter at higher desity much higher than nomal nuclear density n_0 has been of great interest because the recent observations [2] imply the possibiity of high density regime $n \gtrsim 6n_0$ at the core of neutron stars. These density regime has not been fully explored theoretically or experimentally. Kinematically the fermi momentum of nuclear matter at the density of $6n_0$, $k_F \sim 0.65 m_N$. Although it is not high enough for the nucleon mass to be ignored, nontrivial strong correlations of nuclear matter invokes the quest for the scale symmetry at such a high density regime. If the effect of the strong correlation should be such that perturbative QCD works in terms of quark dgree of freedom, then the scale symmetry would be apparent modulo non-vanishing current quark masses. There is some doubt whether the corresponding density is sufficiently reached at the core of neutron star. The relevant degrees of

 doi:10.20944/preprints202403.1812.v1

freedom of dense star matter deep inside the compact star like neutron star are yet supposed to be hadrons, nucleons and mesons , in the frame work of appropriate effective theories.

Various new types of concepts and physical phenomena due to the non-trivial strong correlations are naturally expected. Let us suppose that at higher density the strong correlation of nucleons lead the system to reveal the hidden underlying scale symmetry. There would be excitations in scalar channel, dilaton fields, such that scale invariance can be realized formally in an effective lagrangian with dilatons. One of the interesting results of the dilatonic mean field calculation of nucleonic matter [3] is that the effective nucleon mass gets nonvanishing density-independent constant value at higher density regime relevant to the star matter. It leads to the concept of pseudo-conformality [4,5] in dense star matter, which has been discussed recently assuming the hidden underlying scale symmetry and partly supported by the recent discussions on the infrared conformal window of QCD [6–10].

The nucleon mass is largely due to the spontaneous symmetry breakings of the chiral and scale symmetry. The nucleon mass at higher density is supposed to be mainly from the vaccum expectation value of the dilaton developed for the spontaneously broken phase of the scale symmetry. Although the finite nucleon mass, equivalantly non-zero value of trace of energy momentum tensor , seems to indicate the apparent violation of scale invariance of the system of nuclear mater, the nature of scale symmetry in the spontaneous broken phase can be emerging in a quite different way. One of the useful dynamical properties of the system is the speed of sound with which local fluctuations of a star matter will be travelling depending on the equation of state. The speed of sound can provide informations from which we can infer the state of matter. The ratio of the pressure , representing the restoring force of star matter, to the energy density , the inertia of the star matter, determine the speed of sound, v_s,

$$v_s^2 = c^2 \frac{dP}{d\epsilon} \tag{1}$$

where c is the speed of light and P and ϵ are the pressure and energy density including the particle's rest mass respectively. The square of sound velocity for scale symmetric matter is $1/3$ times velocity of light.

It is obseerved in the mean field calculation [3] that the nucleon mass is found to be approaching constant, which interestingly leads the speed of veocity to that of scale symmetric matter, conformal velocity $v_c^2 = 1/3$. It is basically due to the density-independent vaccum expectation value of the dilaton. It is interpreted as an indication that the hidden scale symmetry is emerging at high density star matter disguised in the form of conformal speed of sound. It is referred to the pseudo-conformal phase [4].

However the presence of the finite nucleon mass in the nucleon sector seems not to be consistent with the chiral invariance of the system, which is believed to be the symmetry(spontaneously broken or restored) modulo current quark masses of QCD. These observations indicate that the proper way to describe the chiral symmetric nucleonic sector is to implement the parity doubling structure [11–14]. It is popssible because there can be various particle excitations when nucleons get closer with strong correlations. The effect of excitations would be such that hidden symmtries of the system can be manifested. The star matter at high density can provide us the possibility of uncovering the hidden scale symmetry as well as the parity doubled structure of the dense nucleonic matter, which have not been so transparent to be observed or neccessary to implement in describing hadronic matter at low density regime. It is a sort of interplay between chiral symmetry and scale symmetry though not yet accurately formulated. In section 2 , the effective lagrangian which is constructed with the chiral and scale symmetry, adopted for the compact star matter, is described and its mean field calculation results are discussed. Most of the detailed calculations and conventions are from the works by Paeng et al. [3] and later developments [15–17]. One of the interesting observation is the trace of energy momentum tensor becomes density independent at higher density due to the interplay between omega meson and dilaton. Its implications on the emergence of pseudo conformal phase and parity doublings will be

Preprints.org (www.preprints.org) | NOT PEER-REVIEWED | Posted: 29 March 2024 doi:10.20944/preprints202403.1812.v1

discussed in section 3 and 4. Summary and remarks on the possible implication on the correspondence of hadrons to quark degrees of freedom are given in section 5.

2. Effective Lagrangian of Dense Star Matter with Hidden Local Symmetry and Scale Symmetry and Mean Field Calculation

In this discussion we consider the effective theory which would be relevant at highy density regime by adopting the form of effective lagrangian in [3] in which the scale ivariance is implemented by introducing a conformal compensator field, a dilaton field χ, in the lagrangian for massive matter fields. Vector mesons are introduced by the hidden local symmetry [18–20]. We suppose that in the mean field calculation the most relevant degrees of freedom at highly dense nucear matter are ω meson, dilaton χ and nucleon N. Here pion and rho mesons are supposed to be inactive, $<\pi>=0$ and $<\rho>=0$ for a symmetric nuclear matter. We take a simple lagrangian, a truncated form of the lagrangian developed in [3], relevant for the mean field calculation for the symmetric nuclear matter. The lagrangian used in this schematic discussion is given by

$$\mathcal{L}' = \mathcal{L}'_M + \mathcal{L}'_N \tag{2}$$

where

$$\begin{aligned}\mathcal{L}'_M &= -\frac{1}{2}\mathrm{tr}\left[\omega_{\mu\nu}\omega^{\mu\nu}\right] + \frac{f_{\sigma\omega}^2}{2f_\chi^2}\chi^{*2}\left(\frac{\partial_\mu\sigma_\omega}{f_{\sigma\omega}} - g_\omega^*\omega_\mu\right)^2 \\ &\quad + \frac{1}{2}\partial_\mu\chi\cdot\partial^\mu\chi - V(\chi)\end{aligned} \tag{3}$$

and

$$\mathcal{L}'_N = \bar{N}i\left(\partial_\mu - ig_\omega^*\frac{\omega_\mu}{2}\right)N - \bar{N}m_N^*N + g_{V\omega}^*\bar{N}\gamma^\mu\left(\frac{\partial_\mu\sigma_\omega}{2f_{\sigma\omega}} - g_\omega^*\frac{\omega_\mu}{2}\right)N \tag{4}$$

where σ_ω is a would-be Nambu-Goldstone boson of hidden local symmetry that will be Higgsed away and $f_{\sigma\omega}$ is a corresponding decay constant. Now the mean field calculation is performed in the background of degenerated nucleons at $T=0$. The nucleon number density n is given by

$$n = \frac{2}{3\pi^2}k_F^3. \tag{5}$$

where k_F is a fermi momentum. The relevant mean field variables are $\chi^*(=<\chi>)$ and $\omega^*(=<\omega_0>)$. Their density dependences are denoted by *. The coupling constants, $g_{V\omega}^*$ and g_ω^*, are also density dependent quantities . The nucleon mass dressed with the compensator field is density dependent linearly in χ^*,

$$m_N^* = m_N\frac{\chi^*}{F_\chi}. \tag{6}$$

From the stationarity conditions of the thermodynamical potential the gap equations for χ^* and ω^* is obtained

$$\frac{m_N^2\chi^*}{\pi^2 f_\chi^2}\left[k_F E_F - m_N^{*2}\ln\left(\frac{k_F+E_F}{m_N^*}\right)\right] - \frac{f_{\sigma\omega}^2}{f_\chi^2}g_\omega^{*2}\omega_0^*\chi^* + \left.\frac{\partial V(\chi)}{\partial\chi}\right|_{\chi=\chi^*} = 0 \tag{7}$$

$$g_\omega^*(g_{V\omega}^*-1)n - f_{\sigma\omega}^2 g_\omega^{*2}\frac{\chi^{*2}}{f_\chi^2}\omega_0^* = 0\,. \tag{8}$$

Preprints.org (www.preprints.org) | NOT PEER-REVIEWED | Posted: 29 March 2024 doi:10.20944/preprints202403.1812.v1

where

$$E_F = \sqrt{m_N^{*2} + k_F^2} \tag{9}$$

Using (7) and (8), the vacuum expectation value of the trace of energy-momentum tensor θ^μ_μ can be obtained

$$\begin{aligned} \langle \theta^\mu_\mu \rangle &= \langle \theta^{00} \rangle - \sum_i \langle \theta^{ii} \rangle \\ &= 4V(\chi^*) - \chi \frac{\partial V(\chi)}{\partial \chi}\bigg|_{\chi=\chi^*}. \end{aligned} \tag{10}$$

There is no contribution from matter fields in eq.(10), because the lagrangian is scale invariant for the matter fields by construction. The fermi surface does not spoil scale symmetry. The nonvanishing trace of energy momentum tensor would be soley from the symmetry breaking nature of the dilaton potential, $V(\chi)$. This shows that the scale invariant formalism provides a simple description that the symmetry breaking due to the finite nucleon mass can be transfered onto the spontaneous symmtry breaking of the scale symmetry, represented by the dilaton potential.

Hence howthe scale symmtry breaking evolves with the density is determined by the density dependence of the dilaton field, χ^*, which can be obtained from the gap equations. Using eq.(8), eq.(7) becomes

$$\frac{m_N^2 \chi^*}{\pi^2 f_\chi^2}\left[k_F E_F - m_N^{*2} \ln\left(\frac{k_F + E_F}{m_N^*}\right)\right] - \frac{(g^*_{V\omega} - 1)^2 n^2}{F^2_{\sigma\omega}/F^2_\chi \chi^{*3}} + \frac{\partial V(\chi)}{\partial \chi}\bigg|_{\chi=\chi^*} = 0\,, \tag{11}$$

The first part is representing the dilaton coupling to fermi gas and the third term is from the dilaton potential. The second term is due to ω-nucleon(dilaton) coupling which is also density dependent. The density dependence of ω-nucleon coupling, $g^*_{V\omega}$, is taken as a parameteric form to find solutions. Hence the density depence of dilaton is a result of the interplay between omega meson and dilaton. The details are given in [3].

Interestingly the interplay [21] is such that we can get a solution of χ^* which becomes density independent at higher density $n > n_A(\sim 1/2n_0)$[1] . The exact numerics of n_A is dependent specifically on the model considered. Then we can notice the trace of enrgy momentum in eq.(10), which is a function of χ, is density independent.

This feature obtained in the schematic approach is consistent with the numerical results of the energy density ϵ and pressure P with full use of the effective lagrangian in [3] , which can be captured very well by the simple formula,

$$\epsilon = Bn^{4/3} + D \tag{12}$$

$$P = \frac{1}{3}Bn^{4/3} - D \tag{13}$$

where B and D are the density independent parameters determined numerically. Then the trace of energy momentum tensor is given by

$$\theta^\mu_\mu = \epsilon - 3P = 4D \tag{14}$$

which is density independent. It is interesting to note that density independent part D in the energy density is well explained in terms of the density independent dilaton condensate χ^* in eq.(10).

1 Without the presence of the second term χ^* is determined only by the nucleon number density. One can see that without ω-nucleon coupling the density independent solution of $\chi^*(\neq 0)$ would be impossible.

Preprints.org (www.preprints.org) | NOT PEER-REVIEWED | Posted: 29 March 2024 doi:10.20944/preprints202403.1812.v1

In this effective theory of star matter formulated with guidance of the scale invariance, which is supposed to be broken spontaneousley, $\chi^* \neq 0$, the trace of energy momentum tensor is non-vanishing. This is equivalent to the absence of the exact scale symmetry. However the density independency of χ^* might be one of the ways how the hidden scale symmetry in spontaneously broken phase is manifested itself and emerging in dense matter.

3. Speed of Sound and Pseudo-Confornality

Among the useful dynamical properties of the system, we consider the speed of sound inside the compact star matter. The general expression for the speed of sound, v_s, is

$$v_s^2 = c^2 \frac{dP}{d\epsilon} = c^2 \frac{dP}{dn} / \frac{d\epsilon}{dn} \tag{15}$$

where c is the speed of light. $\frac{dP}{dn}$ representing the compressibility of the fermion system determines the restoring force while the energy density ϵ including the particle's rest mass determines the inertia. This is the speed with which small local fluctuations of a star matter are travelling. It depends on the equation of state and therefore on the underlying hidden scale symmetry we are exploring.

It is an old idea that at high energies masses of particles become unimportant and scale invariance of the system can be inferred at this kinetic regime [1]. Alternatively it is also interesting to see whether the high density of nucleonic matter can be a possible window for investigating the effect of the scale invarince.

The divergence of dilatation current, s^μ, induced by scale transformations, is given by the trace of energy momentum tensor, θ^μ_μ

$$\partial_\mu s^\mu = \theta^\mu_\mu. \tag{16}$$

For the conformal invariance window it becomes zero. In terms of energy density ϵ and pressure P the trace of energy momentum tensor eq.(10) is given by

$$\theta^\mu_\mu \quad = \quad \epsilon - 3P = 0. \tag{17}$$

Then, using eq. (15), the speed of sound bocomes conformal velocity v_c,

$$v_c = 1/\sqrt{3} \tag{18}$$

in the conformal window.

As an example let us cosider a nonineracting degenerate fermion matter at zero temperature . The energy density and pressure are given by

$$\begin{aligned} \epsilon &= \frac{1}{4\pi^2}\left[2E_F^3 k_F - m_N^2 E_F k_F - m_N^4 \ln\left(\frac{E_F + k_F}{m_N}\right)\right], \\ P &= \frac{1}{4\pi^2}\left[\frac{2}{3}E_F k_F^3 - m_N^2 E_F k_F + m_N^4 \ln\left(\frac{E_F + k_F}{m_N}\right)\right]. \end{aligned} \tag{19}$$

At high density with the fermi momentum, k_F , much larger than the mass

$$k_F \gg m \tag{20}$$

energy density and pressure becomes

$$\epsilon \rightarrow \frac{1}{2\pi^2} k_F^4, \tag{21}$$

$$P \rightarrow \frac{1}{6\pi^2} k_F^4 = \frac{1}{3}\epsilon \tag{22}$$

Preprints.org (www.preprints.org) | NOT PEER-REVIEWED | Posted: 29 March 2024 doi:10.20944/preprints202403.1812.v1

Therefore one can see a high enough density regime is a possible conformal window where $\theta^\mu_\mu = \epsilon - 3P \to 0$ and the speed of sound approaches conformal velocity v_c. One can see however even in the high energy limit the trace of energy momentum tensor does not vanish exactly. If we keep the beyond leading order terms in $k_F \gg m$ limit,

$$\theta^\mu_\mu \quad \to \quad \frac{m^2}{\pi^2}\left[E_F k_F - m^2 \ln\left(\frac{k_F + E_F}{m}\right)\right] \tag{23}$$

It is nonvanishing and of oder $\mathcal{O}(m^2 k_F^2)$. It is interesting to note that the speed of sound can have a conformal velocity limit even the trace of energy momentum tensor does not vanish exactly. This tells us that the effect of the finite mass would not affect the speed of sound reaching the conformal velocity in the conformal window ($k_F \gg m$ in this case). Therefore it suggests that a useful quantity to discuss properly the conformal limt as far as a conformal velocity is concerned at high energy is not the energy momentum tensor itself but might be the ratio of the trace of energy mementum tensor to the energy density , Δ, as proposed in [22]

$$\Delta = \frac{\theta^\mu_\mu}{3\epsilon}, \tag{24}$$

which has a proper limit in the conformal window , $k_F \gg m_N$,

$$\Delta \quad \to \quad \mathcal{O}(\frac{m^2}{k_F^2}) \to 0 \tag{25}$$

It can be compared with the limiting behavior of the trace of energy momentum tensor

$$\theta^\mu_\mu \to \mathcal{O}(m^2 k_F^2). \tag{26}$$

The speed of sound approaches v_c in the conformal limit , $\Delta \to 0$

$$v_s^2 \to \frac{1}{3} \tag{27}$$

For the compact-star matter, the core densities are supposed to be 5 - 10 n_0 depending on the models. The corresponding fermi momentum for $n_{core} = 6n_0$, as an example, is given by

$$k_F = \left[\frac{3\pi^2}{2} n_{core}\right]^{1/3} \sim 0.65 m_N \tag{28}$$

which doesn't seem to be high enough to be a conformal window in the form of $k_F \gg m_N$. However , since it is the criterior applicable to the noninteracting massive fermions , the same criterior might not be simply imposed on strongly interacting fermionic matter at the core of a compact-star, in which a conformal window may appear in a different context.

Comimg back to the compact star matter, strongly interacting nuclear matter at high density , formulated scale invariant way in the previous section. The energy density and the pressure are given by

$$\begin{aligned} \epsilon \quad = \quad & \frac{1}{4\pi^2}\left[2E_F^3 k_F - m_N^{*2} E_F k_F - m_N^{*4} \ln\left(\frac{E_F + k_F}{m_N^*}\right)\right] \\ & + g_\omega^*(g_{V\omega}^* - 1)\omega_0^* n - \frac{1}{2} f_{\sigma\omega}^2 g_\omega^{*2} \frac{\chi^{*2}}{f_\chi^2} \omega_0^{*2} + V(\chi^*) \end{aligned} \tag{29}$$

Preprints.org (www.preprints.org) | NOT PEER-REVIEWED | Posted: 29 March 2024 doi:10.20944/preprints202403.1812.v1

and

$$P = \frac{1}{4\pi^2}\left[\frac{2}{3}E_F k_F^3 - m_N^{*2} E_F k_F + m_N^{*4}\ln\left(\frac{E_F + k_F}{m_N^*}\right)\right] + \frac{1}{2} f_{\sigma\omega}^2 g_\omega^{*2} \frac{\chi^{*2}}{f_\chi^2}\omega_0^{*2} - V(\chi^*) . \tag{30}$$

which have nontrivial contributions from the hadronic interactions as compared with eq.(19). The trace of energy momentum tensor is given by

$$\theta_\mu^\mu = \frac{m^2}{\pi^2}\left[E_F k_F - m^2\ln\left(\frac{k_F + E_F}{m}\right)\right] - f_{\sigma\omega}^2 g_\omega^2 \frac{\chi^{*2}}{f_\chi^2}\omega_0^{*2} + 4V(\chi^*) \tag{31}$$

$$= 4V(\chi^*) - \chi \frac{\partial V(\chi)}{\partial \chi}\bigg|_{\chi=\chi^*} \tag{32}$$

where eq.(7) has been used in the last step. As in the previous section there is no explicit nucleon mass contribution. This shows that the Fermi surface does not spoil the nature of the scale symmetry. It is because the matter part is constructed scale invariantly from the begining.

As mentioned above the criterior of a conformality $k_F \gg m$ derived for the noninteracting massive fermion turns out to be not so useful since the trace of energy-momentum tensor is depend only on the dilaton potential, no explicit dependence on the nucleon mass. Moreover the density at the core of compact star inferred from the recent observations is not sufficiently high as shown in eq.(28) to be an extreme relativistic limit.

We may consider eq.(25) as a possible criterior for conformality in dense hadronic matter

$$\Delta \to 0 \tag{33}$$

Suppose the vacuum expectation value of the dilaton potential does not increase rapidly with the density so that $\Delta \to 0$ at high density. Then we may expect the conformal sound velocity $v_c = 1/\sqrt{3}$. This implies that the conformality can be revealed at the high density limit. It is not easy to figure out the relevant density scale to pin down the density for conformal window.

Alternatively we can take the speed of sound as as a physical quantity that can quantify the conformal window as $v_s \to v_c$ rather than $\theta_\mu^\mu \to 0$ or $\Delta \to 0$. In fact it is observed in the previous section that the vacuum expectation value of the dilaton field approaches constant, equivalently the trace of energy momentum tensor appears to be constant as shown in eq.(14). It is a result of many body effects in nuclear matter, which happens curiously at moderately high density , $n > n_A \sim 2n_0$. In this density regime, the variation of the trace of energy momentum tensor with respect to density is zero,

$$\frac{\partial}{\partial n}\theta_\mu^\mu = 0. \tag{34}$$

Then we get

$$\frac{\partial \epsilon}{\partial n}\left(1 - 3\frac{v^2}{c^2}\right) = 0, \tag{35}$$

and the speed of sound becomes the conformal velocity,

$$\frac{v^2}{c^2} = \frac{1}{3} \tag{36}$$

provided that there is no extremum in the energy density, $\frac{\partial \epsilon}{\partial n} \neq 0$. Interestingly this density independent feature can be taken as a signatuture for the conformal window even though $\Delta \neq 0$. It is the many

Preprints.org (www.preprints.org) | NOT PEER-REVIEWED | Posted: 29 March 2024 doi:10.20944/preprints202403.1812.v1

body effect, specifically interplay between the omega meson excitation and nucleons in the dense medium, that reveals the conformal velocity precociousely for $n > n_{1/2} \sim 2n_0$, which is one of the characteristics of underlying scale symmetry in dense matter. This feature accounts for the emergent pseudo conformal symmetry in the compact-star matter and suggests that the core of the compact star provides a new window for investigating the effect of the scale symmetry(spontaneousely broken) hidden in the dense hadronic matter.

4. Parity Doubling

In the pseudo-conformal phase discuused in section 2, the mass of nucleon becomes density independent and nonvanishing. It might be expected that at such a high density the chriral smmetry is getting restored with the nucleon mass decreasing or becoming zero. In the scheme of linear sigma model like Gell-Mann Levy type [23], the finite mass of nucleon is not contradictory with the chiral symmetry, since the most part of nucleon mass is due to the spontaneous symmetry breaking of the chiral symmetry. However the nonvanishing constant nucleon mass in the pseudo-conformal phase seems not to be consistent with the chiral symmetry since the chiral codensate contribution is expected to be unimportant. The mass term in eq.(5) violates the chiral symmetry explicitly . The chiral invariance in the nucleon sector can be restored if there are excitations of the parity odd nucleons at high density. This possibility has been explored in the frame work of the parity doubling model, in which the finite mass of the nucleon can be introduced in a chiral invariant way [11]. There can be various excitations when nucleons get closer with strong correlations. The excitations would be such the hidden symmtries of the system can be manifested. In dense hadronic matter, we consider excitations of the dilaton and the odd parity nucleons(parity partner of nucleon) for the scale symmetry and the chiral symmetry respectively. Parity doubling in dense mater is emerging as a result of the interplay betwen the scale and chiral symmetry. The relevant fields can be π, ρ, ω and dilatons $\chi's$ and the parity doublet nucleon B. The nucleon field N in eq.(37) is replaced by the parity doublet field B and the mass term, $\bar{N} m_N^* N$, by the parity doublet mass mass term [24,25] ,

$$\begin{aligned} \mathcal{L}_N^{pdm} &= \bar{B} i \left(\partial_\mu - i g_\omega^* \frac{\omega_\mu}{2} \right) B + g_{V\omega}^* \bar{B} \gamma^\mu \left(\frac{\partial_\mu \sigma_\omega}{2 f_{\sigma\omega}} - g_\omega^* \frac{\omega_\mu}{2} \right) B \\ &\quad + \mathcal{L}_{nucleon\, mass} \end{aligned} \tag{37}$$

$$\mathcal{L}_{nucleon\, mass} = \ldots - g_1 \sqrt{\kappa} \chi_s \bar{B} B + g_2 \sqrt{\kappa} \chi_s \bar{B} \rho_3 B - i \bar{m}_0 \bar{B} \rho_2 \gamma_5 B \,, \tag{38}$$

where $B = (B1, B2)$ denotes the nucleon in parity doublet in the chiral eigenstate, the ρ_i are the Pauli matrices in the parity pair space. g_1, g_2 are dimensionless parametrs for the contribution from the spontaneousely broken chiral symmetry signified by the pion decay constant F_π in $\kappa = (F_\pi / F_{\chi_s})^2$. $\bar{m}_0 = (\chi^* / F_\chi) m_0$ is supposed to be from the scale symmetry breaking, where m_0 is a mass parameter. To make eq.(38) scale invariant we introduce two dilatons, χ_s and χ, soft and hard dilatons respectively. The soft dilaton (χ_s) is introduced to make the spntaneous symmetry breaking part with the pion decay contant F_π to be scale invariant and the hard dilaton(χ) for the invariant mass($\bar{m}_0$) part, the last term, to be scale invariant. Baryons B are not mass eigenstate because of the last term. After diagonalizing the mass matrix, two mass eigen states are identified as positive parity nucleon, N_+, and its chiral partner, N_- . Their masses are given by

$$m_{N_\pm} = \mp g_2 \kappa \chi_s + \sqrt{(g_1 \kappa \chi_s)^2 + \bar{m}_0^2} \,. \tag{39}$$

It is to be noted that it is a functional of dialtons such that it transforms as scale dimension 1. In a linearized scheme suitable for the study of dynamical aspects of chiral symmetry, the vacuum

Preprints.org (www.preprints.org) | NOT PEER-REVIEWED | Posted: 29 March 2024 doi:10.20944/preprints202403.1812.v1

expectation value of scalar field , σ in Gell-Mann Levy type model, becomes zero at chiral symmetry restoration,

$$< \sigma > \to 0 \text{ equivalently } < \chi_s > \to 0 \tag{40}$$

which corresponds to the dilaton limit [26] in which ρ-nucleon coupling is supposed to be no longer active. The nucleon mass becomes $\bar{m}_0$.

$$m_{N_+} \to \bar{m}_0 = \frac{\chi^*}{F_\chi} m_0 \tag{41}$$

Identifying m_{N_+}, $< \chi_h >$ and m_0 with m_N^*, χ^* and m_N respectively, one can get the same mass as in eq.(6). Therefore the parity doubling provides the nonvanishing nucleon mass in the pseudo-conformal phase in chiral symmetric way.[2] It is consistent with that the emergence of chiral invariant mass is essentialy due to the interplay between the hard dilaton and omega meson.

A part of nucleon mass is generated by dynamically via spontaneous symmetry breaking of the chiral symmetry. The rest of the nucleon mass up to 70 % is unconnected with the chiral symmetry breaking. It is supposed to be the contribution from the spontaneous symmetry breaking of the scale symmetry. The excitation of parity doublets in the compact star matter makes these features in its mass formula compatible with the chiral symmetry. The parity doubling is one of the emergent phenomena in the pseudo-conformal phase of strongly correlated dense compact star matter.

5. Summary and Discussion

In this work the results of the schematic approach together with the full mean field calculation on the emerging pseudo-conformal phase and parity doubling in dense hadronic matter inside the compact star core are discussed. it is observed that ω-nucleon couling is crucial to have pseudo-conformal symmetry in dense nuclear matter. It happens on the top of the fermi surface of nucleons with density-dependent effective masses. The scale symmetry is implemented via a dilaton field χ .

One of the interesting results of the dilatonic mean field calculation is that the effective nucleon mass gets nonvanishing density-independent constant value at higher density regime relevant to the star matter. It is due to the nontrivial interplay between ω -nucleon(dilaton χ) coupling encoded in the asymptotic behavior of ω coupling constant $g_{V\omega}$ [4]. The trace of energy momentum tensor which is a function of χ becomes density independent as well. Although it is nonvanishing, the speed of sound approaches the conformal velocity $v_s \to 1/\sqrt{3}$. One of the characteristics of underlying scale symmetry in dense matter, the conformal velocity, appears precociousely for $n > n_A(\sim 2n_0)$. This feature accounts for the emergent pseudo-conformal symmetry in compact-star matter and suggests that the core of the compact star provides a new window for investigating the effect of scale symmetry(spontaneousely broken) hidden in dense medium.

The nucleon mass is largely due to the spontaneous symmetry breakings of the chiral and scale symmetry. The nucleon mass at higher density is supposed to be mainly from the vaccum expectation value of the dilaton developed for the spontaneously broken phase of the scale symmetry. The finite nucleon mass which is unconnected to chiral condensation is apparently chiral symmetry breaking. The excitation of parity doublets in the compact star matter makes the system chiral invariant. This is an interesting observation that in dense matter there is an additional interplay between scale and chiral symmmetry. It can be understood that the constraint of the chiral symmetry induces the parity doublet excitations in the pseudo-conformal matter. The parity doubling is an emergent phenomenon in the pseudo-conformal phase of strongly-correlated compact star matter. The dilaton field might

[2] What is observed in [27], is actually the behavior of χ at higher density not χ_s. We do not have the meanfield result on χ_s which does not couple to omega meson directly in this work, but χ_s is assumed to be sufficiently small enough to be ignored at high density, $n > n_A$.

be considered to be coupled to particle-hole excitation (0^+) on the fermi surface. Then the emergent pseudo-conformal symmetrty is considered basically as a result of additional excitations in the highly dense hadronic matter. The precise mechanism how particle-hole excitations are such that the system becomes pseudo-conformal is not clear yet.

There has been always a quest for excitations of quarks or deconfinent in the extreme conditions, high temerature and/or high density. For the highly dense matter expected at core of compact stars, the analytic parametrization of the numerical result of the energy density in section 2 is

$$\epsilon \quad = \quad Bn^{4/3} + D. \tag{42}$$

If we take it seriously as it is , one can notice that the first term has the same density dependense as that of ideal relativistic quark matter. D looks like a bag constant in the bag model [28,29]. The apperance of bag-like constant in dense hadronic matter [30] can be considered as an incomplete confinement, leakage of bag constant throughout the dense core. What is interesting of this approach is that although no explicit QCD degrees of freedom are involved, the property of the equation of state in this approach is quite similar to that of "deconfined quark matter ". This might be a useful hint for exploring the quark-hadron continuity in dense hadronic matter [31–33].

Acknowledgments: The author would like to thank Mannque Rho for valuable discussions during the long time collaboration on which this article is based.

References

1. Coleman, S. *Aspects of Symmetry: Selected Erice lectures*, Cambridge University Press, Cambridge, U.K., 1985.
2. Lattimer, J.M. Neutron stars and the nuclear matter equation of state. *Ann. Rev. Nucl. Part. Sci.* **2021**, *71*, 433; Krastep, P.G. A deep learning approach to extracting nuclear matter properties from neutron star observations. *Symmetry* **2023**, *15*, 1123 .
3. Paeng, W.G.; Kuo, T.T.S.; Lee, H.K. ; Ma, Y.L.; Rho, M. Scale-invariant hidden local symmetry, topology change and dense baryonic matter II. *Phys. Rev. D* **2017**, *96*, 014031 .
4. Rho, M. Pseudo-conformal sound speed in the core of compact stars. *Symmetry* **2022**, *14*, 2154.
5. Rho, M. Dense baryonic matter predicted in "Pseudo Conformal Model”. *Symmetry* **2023** *15*, 1271 (2023).
6. Crewther R. J. ; Tunstall, L. C. Δ I = 1/2 rule for kaon decays derived from QCD infrared fixed point. *Phys. Rev. D* **2015**, *91*, 034016.
7. Crewther, R. J. Genuine dilatons in gauge theories. *Universe*, **2020**, *6*, 96.
8. Del Debbio, L. ; Zwicky, R. Dilaton and massive hadrons in a conformal phase. *JHEP*, **2022**, *08*, 007 .
9. Zwicky, R. QCD with an infrared fixed point – Pion sector. *arXiv* **2023**, arXiv:2306.06752 ; Zwicky, R. QCD with an infrared fixed point and a dilaton. *arXiv* **2023**, arXiv:2312.13761.
10. Yamawaki, K. On the anomalous dimension in QCD. *Symmetry* **2024** *16*, 2 .
11. Detar, C. E. ; Kunihiro, T. Linear sigma model with parity doubling. *Phys. Rev. D* **1989** , *39*, 2805 .
12. Jido D. ; Hatsuda, T. ; Kunihiro , T. Chiral-Symmetry Realization for Even- and Odd-Parity Baryon Resonances. *Phys. Rev. Lett.* **2000**, *84*, 3252.
13. Jido, D. ; Oka, M. ; Hosaka, A. Chiral Symmetry of Baryons. *Prog. Theor. Phys.* **2001**, *106*, 873 .
14. Motohiro, Y. ; Kim Y. ; Harada, M. Asymmetric nuclear matter in a parity doublet model with hidden local symmetry. *Phys. Rev. C***2015**, *92*, 025201.
15. Rho, M. ; Ma, Y.L. Manifestation of hidden symmetries in baryonic matter: From finite nuclei to neutron stars. *Mod. Phys. Lett. A* **2021**, *36*, 2130012.
16. Ma, Y.L. ; Rho,M. Pseudoconformal structure in dense baryonic matter. *Phys. Rev. D* **2019**, *99*, 014034 ;
17. Lee, H.K. ; Ma, Y.L. ; Paeng, W.G. ; Rho, M. Cusp in the symmetry energy, speed of sound in neutron stars and emergent pseudo-conformal symmetry. *Modern Physics Letters A*, **2022**, *37*, 2230003 .
18. Bando, M.; Kugo, T. ; Uehara, S.; Yamawaki K. ; Yanagida, T. Is rho meson a dynamical gauge boson of hidden local symmetry? *Phys. Rev. Lett.* **1985**, *54*, 1215.
19. Harada, M. ; Yamawaki, K. Hidden local symmetry at loop: A New perspective of composite gauge boson and chiral phase transition. *Phys. Rept.* **2003**, *381*, 1.

 doi:10.20944/preprints202403.1812.v1

20. Yamawaki, K. Proving rho meson be a dynamical gauge boson of hidden local symmetry. *Symmetry* **2023**, *15*, 2209.
21. Paeng, W.G. ; Lee, H.K. ; Ma, Y.L. ; Rho, M. ; Sasaki, C. Interplay between omega-Nucleon Interaction and Nucleon Mass in Dense Baryonic Matter . *Phys. Rev. D* **2013**, *88*, 105019 .
22. Fujimoto,Y. ; Fukushima, K. ; McLerran, L. D. ; Praszalowicz, M. , Trace Anomaly as Signature of Conformality in Neutron Stars. *Phys. Rev. Lett.* **2022**, *129*, 252702.
23. Gell-Mann, M. ; Levy, M. The axial vector current in beta decay. *Nuovo Cim.* **1960**, *16*, 705 (1960).
24. Sasaki, C. ; Lee, H.K. ; Paeng, W.G. ; Rho, M. Conformal anomaly and the vector coupling in dense matter *Phys. Rev.D* **2011**, *84*, 034021.
25. Paeng, W. G. ; Lee, H. K. ; Rho, M. ; Sasaki, C. Dilaton-Limit Fixed Point in Hidden Local Symmetric Parity Doublet Model. *Phys. Rev. D* **2012**,*85*, 054022 .
26. Beane. S.R. ; van Kolck, U. The dilated chiral quark model. *Phys. Lett. B* **1994**, *328*, 137 .
27. Paeng, W.G. ; Lee, H.K. ; Ma, Y.L. ; Rho, M. ; Sasaki, C. Interplay between omega-Nucleon Interaction and Nucleon Mass in Dense Baryonic Matter. em Phys. Rev. D **2013***88*, 105019 .
28. Chodos, A. ; Jaffe, R.L. ; Johnson, K. ; Thorn, C.B. Baryon structure in the bag theory. *Phys. Rev. D* **1974**, *10*, 2599.
29. Brown, G.E. ; Rho, M. The Little Bag. *Phys.Lett. B* **1979**, *82* ,177.
30. Lee, H.K. ; Rho, M. Dilatons in Hidden Local Symmetry for Hadrons in Dense Matter. *Nucl. Phys. A* **2009**, *829*, 76.
31. Annala, E. ; Gorda, T. ; Kurkela, A. ; Nattila, J. ; Vuorinen, A. Evidence for quark-matter cores in massive neutron stars. *Nature Phys.* **2020** , *16*, 907.
32. Rho, M. Dense Baryonic Matter Predicted in "Pseudo-Conformal Model". *arXiv* **2023**, arXiv: 2305.0471.
33. Fujimoto Y. ; Kojo ,T. ; McLerran, L.D. Momentum Shell in Quarkyonic Matter from Explicit Duality: A Dual Model for Cold, Dense QCD. *Phys. Rev. Lett.* **2024** , *132*, 112701.