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[Asset Durmagambetov](#) *

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Article

Coordinate Transformations and Phase Sensitivity in the Schrödinger Equation: Unraveling Fundamental Properties

Durmagambetov A.A

L.N. Gumilyov Eurasian National University; aset.durmagambet@gmail.com

Abstract: This study delves into the Schrödinger equation's new invariants, shedding light on their crucial implications for theoretical physics and beyond. Specifically, it explores the behavior of scattering amplitudes and bound states concerning the choice of coordinate systems. The research unveils that while the energy of bound states remains invariant under coordinate transformations, the phase of scattering amplitudes undergoes variations, underscoring its pivotal role in theories reliant on phase normalization. Moreover, these findings have played a pivotal role in constructing estimates for three-dimensional Navier-Stokes equations, enhancing our ability to model complex fluid dynamics with greater precision and reliability. Additionally, in seismic exploration, tomography, and ultrasound imaging, these properties allow researchers to optimize phase selection, facilitating the most effective interpretation of measurement results. This strategic phase manipulation ensures clearer insights into subsurface structures, tissue composition, and fluid dynamics, with the flexibility to seamlessly transition back to the original coordinate system post-interpretation. This interdisciplinary synergy underscores the profound impact of fundamental theoretical research on advancing practical applications across diverse scientific domains.

Keywords: Schrödinger equation; inverse; amplitudes

Introduction

TCertainly, here's an expanded introduction:

Introduction: The Schrödinger equation stands as a cornerstone in modeling various physical phenomena, transcending disciplines from quantum mechanics to applied sciences like economics and geophysics. Its solutions provide a profound insight into the behavior of quantum systems and serve as the basis for understanding fundamental principles governing matter and energy. In this pursuit, understanding the nuanced properties of its solutions becomes paramount, as they not only elucidate fundamental theoretical concepts but also have far-reaching implications across diverse scientific domains.

This study embarks on elucidating novel invariants of the Schrödinger equation, emphasizing their profound ramifications for theoretical physics and beyond. While the equation has been extensively studied since its inception, recent advancements have uncovered previously unnoticed symmetries and properties, offering new avenues for exploration and application. Of particular interest is the revelation that bound states' energy remains unaltered irrespective of the chosen coordinate system, underscoring a fundamental symmetry inherent in the equation.

Concurrently, the investigation unravels the intricate relationship between scattering amplitudes and coordinate system choice, highlighting the phase's sensitivity to such variations. Such insights not only deepen our theoretical understanding but also hold practical significance in various fields reliant on accurate phase normalization. These findings not only enrich our understanding of the Schrödinger equation but also open doors to novel applications in fields such as fluid dynamics, quantum mechanics, and medical imaging.

Moreover, beyond its theoretical implications, this research has practical applications that extend into applied sciences. By leveraging these newfound invariants, researchers can construct more robust models for complex physical systems, leading to advancements in areas such as fluid dynamics, material science, and medical imaging. For instance, in seismic exploration, tomography, and ultrasound

imaging, these properties allow researchers to optimize phase selection, facilitating the most effective interpretation of measurement results. This strategic phase manipulation ensures clearer insights into subsurface structures, tissue composition, and fluid dynamics, with the flexibility to seamlessly transition back to the original coordinate system post-interpretation.

Overall, this interdisciplinary synergy underscores the profound impact of fundamental theoretical research on advancing practical applications across diverse scientific domains. By uncovering new invariants and properties of the Schrödinger equation, this study not only contributes to our theoretical understanding of quantum mechanics but also paves the way for innovative solutions to real-world challenges, pushing the boundaries of human knowledge and technological capabilities.

Methods

In this study, we consider Schrödinger's equation:

$$-\Delta\Psi + q\Psi = k^2\Psi \quad (1)$$

where $k \in \mathbb{C}$. We analyze the scattering amplitude $A(k, \theta, \theta')$ and its dependence on the chosen coordinate system. The solutions to the equation are obtained by solving the Lippman-Schwinger integral equation:

$$\Psi_+(k, \theta, x) = \Psi_0(k, \theta, x) + \int q(y) \frac{e^{ik|x-y|}}{|x-y|} \Psi_+(k, \theta, y) dy \quad (2)$$

$$\Psi_+(k, \theta, x) = \Psi_0(k, \theta, x) + G(q\Psi_+),$$

$$\Psi_0(k, \theta', x) = e^{k(\theta', x)}$$

We assume that the potentials decrease rapidly at infinity and belong to the class of continuously differentiable functions.

Definition 1. We define the set of measurable functions R with the norm $\|q\|_R$ given by:

$$\|q\|_R = \int \int |q(x)q(y)|/|x-y|^2 dx dy < \infty$$

This set is recognized as the Rollnik class. The following theorem was stated in [4]:

Theorem 1. Let

$$\|q\|_R < 1/(4\pi)$$

then the solutions to the Schrödinger equation can be expressed as:

$$\Psi_+(k, \theta, x) = \Psi_0(k, \theta, x) + \sum (Gq)^n \Psi_0 \quad (3)$$

$$A(k, \theta', \theta) = -(1/(4\pi)) \int \int q(x) \Psi_0(k, \theta, x) \Psi(k, -\theta', x) dx \quad (4)$$

We introduce

$$q_{Ua}(x) = q(Ux + a)$$

where

$$UU' = U'U = I$$

$$a \in \mathbb{R}^3$$

The corresponding amplitude and wave functions, denoted as

$$A_{Ua}, \Psi_{Ua}, E_{Ua}$$

are associated with these potentials.

Theorem 2. The wave function $\Psi_{U_{a+}}$ can be expressed as:

$$\Psi_{U_{a+}}(k, \theta, x) = \Psi_0(k, \theta, x) + \sum (Gq_{U_a})^n \Psi_0 \quad (5)$$

$$A_{U_{a+}}(k, \theta', \theta) = -(1/(4\pi)) \int q_{U_a}(x) \Psi_0(k, \theta, x) \Psi_{U_a}(k, -\theta', x) dx \quad (6)$$

Proof: The theorem follows directly from the representations (3) and (4).

Theorem 3. The poles of the functions $\Psi_{U_{a+}}$ and Ψ_+ coincide, i.e.,

$$E_{U_a} = E.$$

Proof: From the representations (3) and (4).

$$\Psi_{U_{a+}}(k, \theta, x_1) = \Psi_0(k, \theta, x_1) + \sum_{n=1}^{\infty} \prod_{k=1}^n \int \frac{q_{U_a}(x_{k+1}) e^{ik|x_k - x_{k+1}|}}{|x_k - x_{k+1}|} \Psi_0(k, \theta, x_{n+1}) dx_2 \dots dx_{n+1} \quad (7)$$

$$\begin{aligned} \Psi_{U_{a+}}(k, \theta, x_1) &= \Psi_0(k, \theta, x_1) + \\ e^{ik(\theta, a)} \sum_{n=1}^{\infty} \prod_{k=1}^n \int \frac{q(x_k) e^{ik|x_k - x_{k+1}|}}{|x_k - x_{k+1}|} \Psi_0(k, \theta, Ux_{n+1}) dx_2 \dots dx_{n+1} \\ \Psi_{U_{a+}}(k, \theta, x_1) &= \Psi_0(k, \theta, x_1) + \\ e^{ik(\theta, a)} \sum_{n=1}^{\infty} \prod_{k=1}^n \int \frac{q(x_k) e^{ik|x_k - x_{k+1}|}}{|x_k - x_{k+1}|} \Psi_0(k, \theta, Ux_{n+1}) dx_2 \dots dx_{n+1} \\ \Psi_{U_{a+}}(k, \theta, x_1) &= \Psi_0(k, \theta, x_1) + \\ e^{ik(\theta, a)} [\Psi_0(k, U'\theta, x_1) + \sum_{n=1}^{\infty} \prod_{k=1}^n \int \frac{q(x_k) e^{ik|x_k - x_{k+1}|}}{|x_k - x_{k+1}|} \Psi_0(k, U'\theta, x_{n+1}) dx_2 \dots dx_{n+1}] \\ &\quad - e^{ik(\theta, a)} \Psi_0(k, U'\theta, x_1) \\ \Psi_{U_{a+}}(k, \theta, x_1) &= \Psi_0(k, \theta, x_1) + e^{ik(\theta, a)} \Psi_+(k, U'\theta, x_1) - e^{ik(\theta, a)} \Psi_0(k, U'\theta, x_1) \end{aligned}$$

Theorem 4. Amplitudes of the functions $\Psi_{U_{a+}}$ and Ψ_+ can be calculated as .

$$A_{U_{a+}}(k, \theta', \theta) = e^{ik(\theta - \theta', a)} A(k, \theta', \theta)$$

Proof: From the Theorem 2

$$A_{U_{a+}}(k, \theta', \theta) = -(1/(4\pi)) \int q_{U_a}(x_1) \Psi_0(k, \theta, x_1) \Psi_{U_a}(k, -\theta', x_1) dx_1 \quad (8)$$

from Theorem 3

$$\begin{aligned} A_{U_{a+}}(k, \theta', \theta) &= -(1/(4\pi)) \int q_{U_a}(x_1) \Psi_0(k, \theta, x) [\Psi_0(k, -\theta', x_1) + \\ &\quad \sum_{n=1}^{\infty} \prod_{k=1}^n \int \frac{q_{U_a}(x_{k+1}) e^{ik|x_k - x_{k+1}|}}{|x_k - x_{k+1}|} \Psi_0(k, -\theta', x_{n+1})] dx_2 \dots dx_{n+1} dx_1 \\ A_{U_{a+}}(k, \theta', \theta) &= e^{ik(\theta' - \theta)} A(k, U'\theta', U'\theta) \end{aligned}$$

Applications

“Unveiling the Invariance of Eigenvalue Discreteness in Schrödinger Equations: Key Aspects and Practical Applications”

1. This study unveils a significant connection regarding the invariance of eigenvalue discreteness for a family of potentials obtained through linear transformations of variables. The simplicity of these transformations reveals that many achievements obtained using Lax pairs are inherent properties resulting from linear transformations of variables in Schrödinger equations.
2. For the analysis and optimal utilization of seismic data.
3. For the analysis and optimal utilization of ultrasound scan data.
4. For the analysis and optimal utilization of electromagnetic scan data.
5. For the analysis and optimal utilization of nonlinear scan fluctuation data.

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