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Why Does Earth Rotate?

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Abstract: The equation of time (*EoT*) tracks daily deviations in length between the solar day and the mean day. Since the length of the mean day remains constant throughout the year, the *EoT* must mirror daily fluctuations in the length of the solar day. Furthermore, if the *Sun meridian declination* (*SMD*) is dynamically linked to Earth's rotational speed (ERS) the *EoT* must obey to oscillations in ERS. This document examines the position, velocity, acceleration, and *net drive* of the *mean-time Sun* within a *solar sundial noon analemma* considering both its vertical and horizontal dimensions: the *SMD* and the *EoT*. Evidence supports that ERS decreases monotonically along two trans-equinoctial analemmatic *phases* in which the *net drives* of the *EoT* and *SMD* become coordinated (either both accelerating or both decelerating) within the *SMD* interval of −16 to +19 arcdeg, centered at +3. Conversely, ERS increases monotonically along two trans-solstitial analemmatic *phases* in which the *net drives* of the *EoT* and *SMD* become opposed, outside the specified interval of *SMD*. The *ERS* reaches its minima and maxima at the *troughs* and *crests* of the *EoT*, respectively.

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1. Introduction

Despite the fluctuations in Earth's rotational speed being recognized factual for timescales from daily to centennial, the prevailing theory attributes these variations primarily to the conservation of the angular momentum [1] overlooking the complexity of Sun-Earth physics. It is widely accepted that the dynamics of the *Equation of Time* (*EoT*) arise from the shape of Earth's orbit and the tilt of Earth's rotational axis [2][3]. However, the analemmatic dynamics of the *Sun meridian declination* (*SMD*) and the *EoT* remains underexplored. The analysis on these pages builds on a previous study that assessed the dynamics of the *SMD* to estimate the latitudinal budget of natural beam irradiance on Earth [4]. The current work evaluates the combined dynamics (velocity, acceleration, net drive, etc.) and synchrony between the *SMD* and the *EoT* within a *solar noon analemma,* aiming to establish a connection to Earth's rotational speed ($\boldsymbol{\rho}$, km h^{-1}) across daily, monthly, and seasonal timescales.

When the *SMD* is plotted against the *EoT* it yields the *solar noon analemma*, a lemniscate describing the combined *horizontal-vertical* path of the *mean-time noon Sun* along the Gregorian year, as perceived from a specific location on Earth's surface [5]. Following the *SMD* within a *solar noon analemma,* the *mean-time Sun* travels from north to south and backwards, depicting the *analemma*'s y-coordinate or the *Sun's vertical path.* Following the *EoT* within the *solar noon analemma* the *mean-time Sun* travels from east to west and backwards, depicting the *analemma*'s x-coordinate or the *Sun's horizontal path.*

The *EoT* establishes a rhythm for the daily divergence between mean time and solar time, specifically between the lengths of the mean day and the solar day [6]. Consequently, if *SMD* is linked to the variation in Earth's rotation along the planet's orbit, then the *EoT* reflects not only fluctuations in the length of successive solar days but also variations in Earth's rotational speed. This hypothesis

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might seem bold, but the *Sun's vertical path* (*SMD*) and the *Sun's horizontal path* (EoT or $\boldsymbol{\rho}$) are synchronized in such a perfect harmony that evidence supports ca**us**al in lieu of ca**su**al connection.

If natural beam irradiance (*NBI*) is expected to align with the local meridian at mean time noon, but the *EoT* occurs to the left or right of the meridian, this deviation evidences a daily fluctuation in the length of the solar day and Earth's rotational speed. This proposal assumes the Sun-Earth physics is inherently linked to Earth's rotation, even challenging the role of angular moment as the primary driver of Earth's rotational dynamics. Because the *length of the solar day* is tied to the Earth's rotational speed [7], the *light cone* supplying *NBI* to the subsolar point [4] comprises the *clock hand* defining the only time frame Earth adheres to—the solar time. To comprehend the cause-effect relation between the *EoT* and the *SMD*, their interaction must be considered in isolation from any third factor. Earth does not revolve around distant stars nor adhere to civil time, the length of the day should neither be defined by distant stars, nor by a timeframe limited to 24 hours.

The inverse relationship between the *length of the solar day* and Earth's rotational speed requires reversing the scale of the *EoT* for an intuitive analysis, as suggested in a previous study on fluctuations in the length of the planet's sideral day [8]. The given inverse association motivated the analysis of the *Sun's horizontal path* (*EoT*) within a *solar sundial noon analemma*. In this context, negative *EoT* values correspond to periods when the rotational speed falls below the annual average and the *mean-time Sun* lags behind its mean velocity. Conversely, positive *EoT* values correspond to periods when the rotational speed exceeds the annual average and the *mean-time Sun* runs ahead of its mean *velocity*. Following a *sundial noon analemma*, Earth's rotational speed must increase as the *EoT* advances from left to right and decrease as it progresses from right to left. Within a *solar sundial noon analemma*, the *mean-time Sun* travels from left to right during each of two trans-solstitial analemmatic segments, and from right to left during each of two trans-equinoctial analemmatic segments.

2. Materials and Methods

To analyze the joint *vertical-horizontal path* of the *mean-time Sun*, let $\boldsymbol{\delta}$ denote the *SMD* and $\boldsymbol{\delta}'$ the *EoT* (both in arcdeg). Therefore, the joint function $\boldsymbol{\delta}(\boldsymbol{\delta}')$ displays the *solar sundial noon analemma*, which illustrates the bidimensional path of the *mean-time noon Sun* throughout a Gregorian year, because the *sundial analemma* tracks the shifts of the *mean-time Sun* concerning both the *SMD* and the *EoT*. The records of $\boldsymbol{\delta}$ and $\boldsymbol{\delta}'$ correspond to the vertical and horizontal angles from the *mean-time noon Sun* to the Equator ($\boldsymbol{\delta}$) or the local meridian ($\boldsymbol{\delta}'$), respectively. The difference in lengths between the mean day and the solar day (*EoT*) can be expressed in a variety of units including time (min), distance (km), and spheric-coordinates (arcdeg). For the present analysis arcdeg was chosen, to match the units in which *solar declination* is usually stated. The *EoT* and the *Sun's horizontal-path* ($\boldsymbol{\delta}'$) are referred to as synonyms in this document, while the same holds true for the *SMD* (true declination) and the *Sun's vertical-path* ($\boldsymbol{\delta}$).

2.1. Parameters of the Sun's Vertical Path

The *SMD* ($\boldsymbol{\delta}$, Equation (2)) was derived from the fractional year (Equation (1)) modified for *solar noon*, following the analemma's geometric model [9]. Let $\boldsymbol{\delta}$ denote the position of the *Sun's vertical path* (*SMD*), which tracks the angular distance between the *mean time Sun* and the Equator throughout the year. To explore the dynamics of the *Sun's vertical path* in greater depth, five additional parameters were analyzed: angular velocity ($\boldsymbol{\omega}$, arcmin day^{-1}), angular acceleration ($\boldsymbol{\alpha}$, arcsec day^{-2}), angular jerk ($\boldsymbol{j}$, arcjerk day^{-3}), angular snap ($\boldsymbol{\varsigma}$, arcsnap day^{-4}), and *net drive* ($\boldsymbol{\zeta}$) of the *SMD*. The functions $\boldsymbol{\omega}$, $\boldsymbol{\alpha}$, $\boldsymbol{j}$, and $\boldsymbol{\varsigma}$ correspond to the first to fourth derivatives (Equations (3) to (6)) of *SMD* ($\boldsymbol{\delta}$) with respect to time (**t**), same order. The *net drive* ($\boldsymbol{\zeta}$, Equation (7)) is defined by the sign of the product $\boldsymbol{\omega\alpha}$, where $\boldsymbol{\zeta}$ is stated as accelerative (speeding up) when the sign is positive or decelerative (slowing down) when negative.

$$\boldsymbol{x} = [2\pi/365](\mathbf{t} - 1) \qquad (1)$$

$$\boldsymbol{\delta} = [0.006918 - 0.399912\, cos(\boldsymbol{x}) + 0.070257\, sin(\boldsymbol{x}) - 0.006758\, cos(2\boldsymbol{x}) + 0.000907\, sin(2\boldsymbol{x}) - 0.002697\, cos(3\boldsymbol{x}) + 0.00148\, sin(3\boldsymbol{x})]\,(180/\pi) \qquad (2)$$

$\boldsymbol{\omega} = (d\boldsymbol{\delta}/d\mathbf{t}) = [2\pi/365]\,[0.399912\, sin(\boldsymbol{x}) + 0.070257\, cos(\boldsymbol{x}) + 0.013516\, sin(2\boldsymbol{x}) + 0.001814\, cos(2\boldsymbol{x}) + 0.00809\, sin(3\boldsymbol{x}) + 0.00444\, cos(3\boldsymbol{x})]\, (\boldsymbol{\pi/3})(180/\pi)^2$ (3)

$\boldsymbol{\alpha} = (d\boldsymbol{\omega}/d\mathbf{t}) = [2\pi/365]^2\,[0.399912\cos(\boldsymbol{x}) - 0.070257\sin(\boldsymbol{x}) + 0.027032\, cos(2\boldsymbol{x}) - 0.003628\, sin(2\boldsymbol{x}) + 0.024273\cos(3\boldsymbol{x}) - 0.01332\sin(3\boldsymbol{x})]\, [(\boldsymbol{\pi/3})]^{\mathbf{2}}\, (180/\pi)^3$ (4)

$\boldsymbol{j} = (d\boldsymbol{\alpha}/d\mathbf{t}) = [2\pi/365]^3[-0.399912\, sin(\boldsymbol{x}) - 0.070257\, cos(\boldsymbol{x}) - 0.054064\, sin(2\boldsymbol{x}) - 0.007256\, cos(2\boldsymbol{x}) - 0.072819\, sin(3\boldsymbol{x}) - 0.03996\, cos(3\boldsymbol{x})]\, [(\boldsymbol{\pi/3})]^{\mathbf{3}}\, (180/\pi)^4$ (5)

$\boldsymbol{\varsigma} = (d\boldsymbol{j}/d\mathbf{t}) = [2\pi/365]^4[-0.399912\, cos(\boldsymbol{x}) + 0.070257\, sin(\boldsymbol{x}) - 0.108128\, cos(2\boldsymbol{x}) + 0.014512\, sin(2\boldsymbol{x}) - 0.218457\, cos(3\boldsymbol{x}) + 0.11988\, sin(3\boldsymbol{x})]\, [\boldsymbol{\pi/3}]^{\mathbf{4}}(180/\pi)^5$ (6)

$\boldsymbol{\zeta} = sign\,(\boldsymbol{\omega\alpha})$ (7)

where $\boldsymbol{x}$ is the fractional year (*radians*), $\mathbf{t}$ is time within a year (days, 1 to 365).

2.2. Parameters of the Sun's Horizontal Path

The *EoT* ($\boldsymbol{\delta'}$, Equation (8)) was derived from the fractional year (Equation (1)) modified for *solar noon*, following the geometric model [9] of the analemma. Let $\boldsymbol{\delta'}$ denote the position of the *Sun's horizontal path* (*EoT*) which tracks the angular distance between the *mean time Sun* and the local meridian throughout the year. To explore the dynamics of the *Sun's horizontal path* in greater depth, five additional parameters were analyzed: angular velocity ($\boldsymbol{\omega'}$, arcmin day^{-1}), angular acceleration ($\boldsymbol{\alpha'}$, arcsec day^{-2}), angular jerk ($\boldsymbol{j'}$, arcjerk day^{-3}), angular snap ($\boldsymbol{\varsigma'}$, arcsnap day^{-4}), and *net drive* ($\boldsymbol{\zeta'}$) of the *EoT* ($\boldsymbol{\delta'}$). The functions $\boldsymbol{\omega'}$, $\boldsymbol{\alpha'}$, $\boldsymbol{j'}$, and $\boldsymbol{\varsigma'}$ correspond to the first to fourth derivatives (Equations (9) to (12)) of the *EoT* with respect to time ($\mathbf{t}$), same order. The *net drive* ($\boldsymbol{\zeta'}$, Equation (13)) is defined by the sign of the product $\boldsymbol{\omega'\alpha'}$, where $\boldsymbol{\zeta'}$ is stated as accelerative (speeding up) when the sign of the given product is positive or as decelerative (slowing down) when negative.

$\delta' = EoT = -[0.0000075 + 0.001868\, cos(x) - 0.032077\, sin(x) - 0.014615\, cos(2x) - 0.040849\, sin\,(2x)](180/\pi)$ (8)

$\omega' = \left(\frac{d\delta'}{dt}\right) = -[2\pi/365][-0.001868\sin(x) - 0.032077\cos(x) + 0.02923\, sin(2x) - 0.081698\, cos(2x)]\, [\pi/3]\, (180/\pi)^2$ (9)

$\alpha' = (d\omega'/dt) = -[2\pi/365]^2[-0.001868\cos(x) + 0.032077\, sin(x) + 0.05846\, cos(2x) + 0.163396\, sin(2x)]\, [\pi/3]^2\, (180/\pi)^3$ (10)

$j' = (d\alpha'/dt) = -[2\pi/365]^3[0.001868\, sin(x) + 0.032077\, cos(x) - 0.11692\, sin(2x) + 0.326792\, cos(2x)]\, [\pi/3]^3\, (180/\pi)^4$ (11)

$\varsigma' = (dj'/dt) = -[2\pi/365]^4[0.001868\cos(x) - 0.032077\sin(x) - 0.233840\, cos(2x) - 0.653584\, sin(2x)]\, [\pi/3]^4\, (180/\pi)^5$ (12)

$\zeta' = sign\,(\omega'\alpha')$ (13)

where $\boldsymbol{x}$ is the fractional year (*radians,* Equation (1), $\mathbf{t}$ is the time within a year (days, 1 to 365). The minus sign of Equations (8) to (12) switch the *EoT* from an *on-sky* to a *sundial noon analemma*.

All signs of the *horizontal path* were arbitrarily reversed (Equations (8) to (12)) from those arising from the geometric model of solar declination, to produce the *solar sundial noon analemma*. Inverting the signs of the *EoT* allowed to intuitively associate negative records of $\boldsymbol{\delta'}$ to moments where the *Sun* travels slower on its daily path, yielding longer solar days. Likewise, positive records of $\boldsymbol{\delta'}$ correspond to moments where the *Sun* travels faster on its daily path, yielding shorter solar days. Despite the ordinate axis of the *sundial analemma* being also reversable, such inversion was inappropriate for the planned discussion.

2.3. Adequate Units for the Path of the Mean-Time Noon Sun

When defining appropriate units to express first the parameters of the *Sun's vertical path* ($\boldsymbol{\delta}$, $\boldsymbol{\omega}$, $\boldsymbol{\alpha}$, $\boldsymbol{j}$, and $\boldsymbol{\varsigma}$; Equations (2) to (6), and later those of the *Sun's horizontal path* ($\boldsymbol{\delta'}$, $\boldsymbol{\omega'}$, $\boldsymbol{\alpha'}$, $\boldsymbol{j'}$, and $\boldsymbol{\varsigma'}$; Equations (8) to (12), the guiding fact was that every successive derivative shortened its range of occurrence by a factor of approximately $1/(\pi/3)$ radians (radians being the original unit), equivalent to $1/((\pi/3)(180/\pi))$ arcdeg. Accordingly, the kth derivative of the *Sun´s horizontal position* shortened its range by $1/((\pi/3)^k(180/\pi)^k)$. In order to avoid working with too small numbers, every parameter was scaled up by the factor $(\pi/3)^k\,(180/\pi)^{k+1}$, which is already included in Equations (2) to (6) and

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Equations (8) to (12). The k superscript corresponds to the derivative order, whether for $\boldsymbol{\delta}$ or $\boldsymbol{\delta}'$, which becomes zero for Equation (2) and Equation (8), whereas the *+1* converts the units from radians to arcdeg.

After applying the factor, the numerical value of every successive derivative became scaled up by 60^k with respect to its integral; for instance, the fourth derivative was scaled by 60^4. Consequently, it was only natural to down scale the units for all the parameters [4]. The original unit was rad day^{-k}, whereas the units proposed here for the position, velocity, acceleration, jerk, and snap are: arcdeg, arcmin day^{-1}, arcsec day^{-2}, arcjerk day^{-3}, and arcsnap day^{-4}, same order. After the factors were applied and the units assigned, the records of all the parameters of the *SMD* and the *EoT* fell within a similar range, which allows to display various parameters in a unique ordinate axis despite their differing units. Nonetheless, any comparison proposed between the parameters must refer their scales. In order to establish the similarities between the dynamics of the *SMD* ($\boldsymbol{\delta}$) and that of the *EoT* ($\boldsymbol{\delta}'$) analogous symbols and identical units were proposed to refer equivalent parameter whether for the *Sun's vertical* or *horizontal* path ($\boldsymbol{\omega}$ and $\boldsymbol{\omega}'$, $\boldsymbol{\alpha}$ and $\boldsymbol{\alpha}'$, etc.).

When the positional parameters ($\boldsymbol{\delta}$ and $\boldsymbol{\delta}'$) and their derivatives are plotted against time, they display pseudo sinusoidal functions. Consequently, the classic terminology of sinusoidal curves is adopted from this point onward. In this context, the time interval in which a full *cycle* is completed is known as the *period*. A *cycle* can be further divided into two *half-cycles*, each framed between two *zero-crossing points* (*ZCPs*) two *crests*, or two *troughs*. When the *ZCPs* of the position ($\boldsymbol{\delta}$ and $\boldsymbol{\delta}'$) are chosen to divide a *cycle* into *half-cycles*, a *crest* occurs midway within a *positive half-cycle*, whereas a *trough* occurs midway within a *negative half-cycle*. Unlike the *SMD* and the *EoT* ($\boldsymbol{\delta}$ and $\boldsymbol{\delta}'$), the *crests* and *troughs* of actual sinusoidal are equidistant from their *ZCPs* (the abscissa), and such distance is referred to as the cycle's *amplitude*. For practical purposes, the annual cycle of the *Sun's horizontal path* was considered as a dual cycle —which is framed within a unique cycle of the *Sun's vertical path*—, despite initial and final conditions of the *EoT* only converging by the end of the Gregorian year.

2.4. Sections of the EoT

The *Sun's vertical-path* ($\boldsymbol{\delta}$) was divided into two *half-cycles* by the *ZCPs* of $\boldsymbol{\delta}$, which converge at the equinoxes to either the *crest* or *through* of $\boldsymbol{\omega}$ —where a *trough* corresponds to a *crest* when $\boldsymbol{\delta}$ points downward and $\boldsymbol{\omega}$ is negative—. Every *half-cycle* tracks the *SMD* within one hemisphere. Furthermore, dividing each *half-cycle* of the *SMD* by the solstitial *ZCPs* of $\boldsymbol{\omega}$, yielded *quarter-cycles* which frame an entire season. Furthermore, the *Sun's horizontal-path* ($\boldsymbol{\delta}'$) was divided into two *cycles* by the two *troughs* of $\boldsymbol{\omega}'$ and every *cycle* divided in two *half-cycles* by the *crests* of $\boldsymbol{\omega}'$. Finally, eight *sections* were produced by dividing every *half-cycle* into two sections by the *ZCPs* of $\boldsymbol{\omega}'$. Every resulting section comprised whether the first or the last of the two *quarter-cycles* conforming a season. Because the division was conducted by the key instances of $\boldsymbol{\omega}'$ rather than by time ranges, the resulting sections differ in duration. T

The *Sun's vertical path* ($\boldsymbol{\delta}$) takes *ZCPs* at the Equator, whereas the *Sun's horizontal-path* ($\boldsymbol{\delta}'$) takes *ZCPs* at the local meridian. To analyze the annual dual cycle of the *Sun's horizontal path*, the *EoT* was divided in eight sections by the four *maxima* and four *ZCPs* of $\boldsymbol{\omega}'$. The section boundaries can be either a *crest* and a *ZCP* of $\boldsymbol{\omega}'$, or a *ZCP* and a *trough* of $\boldsymbol{\omega}'$. Let the section names be *early spring*, *late spring*, *early summer*, *late summer*, early autumn, late autumn, early winter, and late winter. Let the section boundaries be denoted *spring equinox*, *midspring*, summer solstice, *midsummer*, autumn equinox, midautumn, winter solstice and midwinter.

To investigate the connection between the *EoT* and Earth's rotation, the analemma was further divided into four *phases*: trans-equinoctial *phases* I and III and trans-solstitial *phases* II and IV. Each *phase* includes two consecutive sections of the *EoT*, while the numbers obey to the order in which they occur. Unlike the seasons of the *SMD*, which consist of two *sections* with opposing *horizontal* directions—the direction of the *EoT*—, each analemmatic *phase* consists of two consecutive sections whose horizontal direction is consistent both within and between. The direction of the *EoT* is identified by the sign of $\boldsymbol{\omega}'$, where $\boldsymbol{\delta}'$ points right when the sign is positive or left when negative.

Each analemmatic *phase* extends between two *ZCPs* of $\boldsymbol{\omega}'$. The trans-equinoctial *phase* I occurs between the *midwinter* and *midspring*, and the trans-equinoctial *phase* III occurs between *midsummer* and *midautumn*. Therefore, the trans-equinoctial *phase* I encompasses late winter and *early spring* while the trans-equinoctial *phase* III includes *late summer* and early autumn. The trans-solstitial *phase* II occurs from *midspring* to *midsummer* and the trans-solstitial *phase* IV spans from *midautumn* to *midwinter*. Therefore, the trans-solstitial *phase* II encompasses *late spring* and *early summer*, while the trans-solstitial *phase* IV includes *late autumn* and *early winter*.

Each *phase* shows homogeneity regarding four aspects: (1) the sign of $\boldsymbol{\omega}'$, and consequently (2) the direction of $\boldsymbol{\delta}'$; which inherits a consisting direction to both (3) Earth's rotational speed and (4) to the length of the solar day. When only numerical data are available, the sign of $\boldsymbol{\omega}'$ is the sole indicator for the direction in which the *Sun's horizontal path* occurs. The concept of *phase* within the *EoT*, was key for studying the dependance between the *EoT*, the length of the solar day, and Earth's rotational speed.

2.5. Earth's Rotational Speed and the EoT

Because the Earth orbits the Sun rather than the galaxy or distant stars, studies on Earth dynamics must prioritize Sun-Earth interactions. Given that Earth's rotational speed ($\boldsymbol{\rho}$) is discussed alongside the *vertical* ($\boldsymbol{\omega}$) and *horizontal* ($\boldsymbol{\omega}'$) velocities of the *mean-time noon Sun*, the concept "speed" is reserved for the planet's rotation. This is appropriate because the rotational speed is always positive.

This document defines solar day as the unique true day and assumes the *Sun's horizontal path* is inherently tied to Earth's rotation. On a hypothetical day when the *EoT* runs 16 min ahead of the meridian, this offset corresponds to a *horizontal* angle of +4 arcdeg beyond the local meridian, equivalent to 240 arcmin per day or 10 arcmin per hour compared to the *mean day*. This deviation indicates the *Sun's horizontal path* extends 445 km above the mean day and the equatorial rotational speed is 18.5 km hour^{-1} faster than the annual average speed. The *EoT* ($\boldsymbol{\delta}'$), must be either added to or subtracted from the average speed of rotation (1669.78 km h^{-1}). In fact, Earth's rotational speed varies from 1653 to 1688 km h^{-1} at the Equator (or 27.5 to 28.1 km min^{-1}). A fluctuation involving ±1 % throughout the year , equivalent to 852 km day^{-1}, 35.5 km h^{-1}, or 0.6 km min^{-1} between the maximum and minimum speeds.

Because Earth's linear rotational speed varies with latitude, it is convenient to derive an expression which accounts for this factor. The linear regressions in Equations (15) and (16) establish the association between Earth's rotational speed and the *EoT,* for the Equator and the Tropic of Cancer, respectively, both derived from Equation (14). A general formula for Earth's speed of rotation is also given in Equation (17), which considered the following steps: (1) assesses Earth's circumference (km) at the desired latitude $\boldsymbol{l}$, which is the distance the *Sun's horizontal path* travels within a day, given by $2\pi r\ Cos\ \boldsymbol{l}$, (2) derive an equivalence between angular and linear distances by dividing Earth's circumference by 360 arcdeg, (3) add the *EoT* to Earth's circumference, both in km [10], and (4) dividing the outcome by the 24 hours of a mean day. The radius of Earth's circumference and the Earth's average speed were $r = 40075\ km/2\pi$ and $\boldsymbol{\rho} = 1669.78$.

$$\rho_i = (1669.78\,)\,(1 + \delta'/360\,) \qquad (14)$$

$$\rho_i = 1669.78 + 4.63827\,\delta' \qquad (15)$$

$$\rho_i = 1531.13 + 4.25770\,\delta' \qquad (16)$$

$$\rho_i = (1/24)\,(2\pi r\ Cos\ \boldsymbol{l})(1 + \delta'/360) \qquad (17)$$

where $\boldsymbol{\rho_i}$ is the Earth's rotational speed on day i, $\boldsymbol{l}$ is the site's latitude and $\boldsymbol{\delta}'$ is the *EoT* on day i.

3. Results and Discussion

3.1. The Signs Paradox

The signs of the parameters describing the *Sun's vertical path* ($\boldsymbol{\delta}$, $\boldsymbol{\omega}$ and $\boldsymbol{\alpha}$) arise because the Northern Hemisphere is the customary reference across science. Nonetheless, the signs can be

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reversed without semantic consequence, which would mean the Southern Hemisphere is the reference. Actually, the signs of *SMD* reverse naturally between hemispheres without interfering with the *net drive* of declination (speeding up or slowing down) of any season. Otherwise stated, the dynamics of the *SMD* would remain unaffected whether analyzed through austral or boreal seasons. Accordingly, whether a *ZCP* of $\boldsymbol{\omega}$ occurs downward or upward is irrelevant from the standpoint of physics, which holds true for the *ZCPs* of $\boldsymbol{\alpha}$. Hence, absolute values can be taken from $\boldsymbol{\delta}$, $\boldsymbol{\omega}$ and $\boldsymbol{\alpha}$ when their association is assessed within hemisphere.

Unlike the signs of the parameters describing the *Sun's vertical path* ($\boldsymbol{\delta}$, $\boldsymbol{\omega}$ and $\boldsymbol{\alpha}$…), those of the *horizontal path* ($\boldsymbol{\delta'}$, $\boldsymbol{\omega'}$ and $\boldsymbol{\alpha'}$…) cannot be reversed without consequence. For the *Sun's horizontal path* ($\boldsymbol{\delta'}$), the meaning of a positive *half-cycle* opposes that of a negative *half-cycle*. The analyses undertaken in this work are based on a *sundial meridional analemma*, on which the direction of $\boldsymbol{\delta'}$ reverses from west to east and vice versa compared to a *sky solar noon analemma*. Reversing the signs is beneficial when relating the *EoT* to the Earth's rotational speed or the length of the solar day. For instance, at the *ZCPs* of the *EoT*, the length of the solar day and Earth's rotational speed converge to their annual averages (24 hours and 1669.78 km h^{-1}). A *ZCP* of $\boldsymbol{\delta'}$ can occur both within a trans-equinoctial *phase* —where the rotational speed decreases progressively— and within a trans-solstitial *phase* —where the rotational speed is progressively increasing. Therefore, the former conforms a downward *ZCP* and the latter conforms an upward *ZCP*, which switch from positive to negative records or from negative to positive records, respectively.

Every time $\boldsymbol{\omega'}$ switches signs at a *phase*'s boundary (midseason *ZCPs* of $\boldsymbol{\omega'}$), Earth's rotational speed switches between from a growing to a decreasing streak, or vice versa, while the dynamics of the length of the solar day reverses direction simultaneously, varying inversely with Earth's rotational speed.

3.2. Dynamics of the Sun Meridian Declination

The within season averages for position ($\boldsymbol{\delta}$), angular velocity ($\boldsymbol{\omega}$) acceleration ($\boldsymbol{\alpha}$), jerk ($\boldsymbol{j}$), snap ($\boldsymbol{\varsigma}$) and net drive ($\boldsymbol{\zeta}$) of the *SMD* are shown in Table 1. Given that the *Sun meridian declination* ($\boldsymbol{\delta}$) defines the *y-coordinate* of the *solar noon analemma* as perceived from a given site on Earth's surface, the annual cycle of *SMD* is referred through this document as the *vertical path of the Sun* ($\boldsymbol{\delta}$). The *period* of the *Sun's vertical-path* ($\boldsymbol{\delta}$) extents for one year, which can be split into a positive *half-cycle* that spans the Northern Hemisphere and a negative *half-cycle* that spans the Southern Hemisphere. The *crest* and *trough* of $\boldsymbol{\omega}$ converge with the equinoxes, whereas the *ZCPs* of $\boldsymbol{\omega}$ concur with the solstices.

The functions $\boldsymbol{\delta}(\boldsymbol{\omega})$ and $\boldsymbol{\alpha}(\boldsymbol{\omega})$ approach circumference-like shapes. Accordingly, $|\boldsymbol{\omega}|$ and $|\boldsymbol{\delta}|$ vary inversely, so the highest $|\boldsymbol{\omega}|$ occurs at the lowest $|\boldsymbol{\delta}|$ and vice versa. An analogous association occurs between $|\boldsymbol{\omega}|$ and $|\boldsymbol{\alpha}|$. Furthermore, $|\boldsymbol{\alpha}|$ and $|\boldsymbol{\delta}|$ vary directly and $\boldsymbol{\alpha}(\boldsymbol{\delta})$ approaches a straight line. The factor $[\pi/3]^k\,[180/\pi]^k$, included in Equations (3) to (6), was applied because each successive derivative of $\boldsymbol{\delta}$ occurred within 1/60th of the range spanned by its integral when all the parameters were expressed in radians. For instance, $\boldsymbol{\alpha}$ and $\boldsymbol{\varsigma}$ span 1/60^2 or 1/60^4 compared to $\boldsymbol{\delta}$'s range, respectively. The proposed units clarify the actual associations and indicate their differing scales. For instance, after switching units, the circle $\boldsymbol{\delta}(\boldsymbol{\omega})$ associates $\boldsymbol{\delta}$ in arcdeg with $\boldsymbol{\omega}$ in arcmin day^{-1}.

Table 1. Parameters of the *Sun's vertical path* for every season of the year: solar declination ($\boldsymbol{\delta}$), velocity ($\boldsymbol{\omega}$), acceleration ($\boldsymbol{\alpha}$), jerk ($\boldsymbol{j}$), snap($\boldsymbol{\varsigma}$) and net drive ($\boldsymbol{\zeta}$).

	Length (days)	$\boldsymbol{\delta}$ (arcdeg)	$\boldsymbol{\omega}$ (arcmin day^{-1})	$\boldsymbol{\alpha}$ (arcsec day^{-2})	$\boldsymbol{j}$ (arcjerk day^{-3})	$\boldsymbol{\varsigma}$ (arcsnap day^{-4})	$\boldsymbol{\zeta}$
Spring	93	+14.969	+15.048	−15.416	−15.355	14.206	−
Summer	93	+14.815	−15.097	−14.934	15.416	12.377	+
Autumn	90	−14.946	−15.391	+15.956	19.198	−13.662	−
Winter	89	−14.545	+16.059	+15.766	−19.925	−14.209	+

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Weighted average	0.356	−0.005	0.002	−0.003	−0.017

A negative net drive is decelerative, while a positive net drive is accelerative. The sign of the net drive is the sign of the product $\boldsymbol{\omega}\,\boldsymbol{\alpha}$.

After scaling the records up and assigning proper units, the resemblance between the within season averages is remarkable for each parameter. For instance, when signs are dismissed, the records occur within the interval 12 to 20. The weighted averages show a yearly equilibrium in the *SMD* dynamic-system, as the summatory of each individual parameter approaches zero. Although a deviation of 0.356 arcdeg in *SMD* is indicative of some strength not being considered. The average records shown in Table 1 must be interpreted with caution, given the that $\boldsymbol{\omega}$, $\boldsymbol{\alpha}$, $\boldsymbol{j}$, and $\boldsymbol{\varsigma}$ were scaled up.

In boreal spring, positive records of $\boldsymbol{\delta}$ and $\boldsymbol{\omega}$ coincide with a negative $\boldsymbol{\alpha}$, which means the Sun *decelerates* as it departs from the Equator and approaches the Tropic of Cancer. In boreal summer, a positive $\boldsymbol{\delta}$ converges to negative records of $\boldsymbol{\omega}$ and $\boldsymbol{\alpha}$, which indicates the apparent Sun *accelerates* as it departs from the Tropic of Cancer and approaches the Equator. In boreal autumn, negative records of $\boldsymbol{\delta}$ and $\boldsymbol{\omega}$ coincide with a positive $\boldsymbol{\alpha}$, which means the Sun *decelerates* as it departs from the Equator and approaches the Tropic of Capricorn. In boreal winter, a negative $\boldsymbol{\delta}$ converges with positive records of $\boldsymbol{\omega}$ and $\boldsymbol{\alpha}$, which indicates the apparent Sun *accelerates* as it departs from the Tropic of Capricorn and approaches the Equator.

Summarizing the *Sun's vertical path*, the sign of $\boldsymbol{\delta}$ can be dismissed and the *net drive* be stated as decelerative when the signs of $\boldsymbol{\omega}$ and $\boldsymbol{\alpha}$ diverge, or accelerative when the signs of the parameters $\boldsymbol{\omega}$ and $\boldsymbol{\alpha}$ coincide. As the sign of both $\boldsymbol{\omega}$ and $\boldsymbol{\alpha}$ are consistent within season, an identical *net drive* endures throughout each of the four seasons. Every particular $\boldsymbol{\delta}$ holds characteristic records of $\boldsymbol{\omega}$ and $\boldsymbol{\alpha}$ within an hemisphere, regardless of whether such association occurs within a decelerative (spring or autumn) or an accelerative season (summer or winter), although differences do occur between hemispheres.

3.3. Dynamics of the Equation of Time

The position ($\boldsymbol{\delta}'$), angular velocity ($\boldsymbol{\omega}'$) acceleration ($\boldsymbol{\alpha}'$), jerk ($\boldsymbol{j}'$), snap ($\boldsymbol{\varsigma}'$) and net drive ($\boldsymbol{\zeta}'$) of the *EoT*, as well as the equatorial rotational speed of the Earth ($\boldsymbol{\rho}$) are shown in Table 2. Given that the *EoT* ($\boldsymbol{\delta}'$) comprises the abscissa of the *solar noon analemma* as perceived from a given site on Earth's surface, the annual cycle of *EoT* is referred through this document as the *Sun's horizontal path* ($\boldsymbol{\delta}'$). Unlike the *SMD*, the *EoT* reaches *ZCPs*—through the local meridian—four times a year, therefore, the annual dynamics of the *EoT* was divided into two cycles whose *periods* last 173 and 192 days, for the Northern and Southern Hemisphere, respectively. Because the *EoT* was split by the key instances of $\boldsymbol{\omega}'$ rather than by those of $\boldsymbol{\delta}$, the boundaries of the analemmatic *cycles*, *phases* and *sections* correspond to *troughs*, crests, and *ZCPs* of $\boldsymbol{\omega}'$.

Despite *ZCPs* of $\boldsymbol{\delta}'$ being the natural boundaries to divide the *EoT*, a most intuitive analysis arises by splitting $\boldsymbol{\delta}'$ from the key moments of $\boldsymbol{\omega}'$. For instance, every *half-cycle* of $\boldsymbol{\delta}'$ becomes virtually synchronized with one of the four seasons of the year. Furthermore, dividing *half-cycles* (seasons) in *quarter-cycles* (sections) proved useful because the parameters switch sign and/or direction at midseason boundaries.

Dividing the *EoT* by the key instances of $\boldsymbol{\omega}'$ brought a number of advantages. Unlike the *extrema* of $\boldsymbol{\delta}'$, the *crests* and *troughs* of $\boldsymbol{\omega}'$ occur at the solstices and near the equinoxes, respectively. Moreover, this approach allowed for the combined path $\boldsymbol{\delta}(\boldsymbol{\delta}')$ to be analyzed in *sections* whose horizontal *net drives* are consistent within but differing between. Thus, each cycle of the *Sun's horizontal path* includes the path of the *mean time noon Sun* within one hemisphere, although the equinoctial boundaries denoted *spring equinox* and autumn equinox occur at +3 arcdeg of *SMD* rather than at the Equator. Accordingly, every season consists of two sections with opposing *net drives* and varying lengths.

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Table 2. Parameters of the *Sun's horizontal path* by analemmatic section: equation of time ($\boldsymbol{\delta}'$), velocity ($\boldsymbol{\omega}'$), acceleration ($\boldsymbol{\alpha}'$), jerk ($\boldsymbol{j}'$), snap($\boldsymbol{\varsigma}'$), net drive ($\boldsymbol{\zeta}$), and Earth's rotational speed ($\boldsymbol{\rho}$).

Period	Length (days)	δ' arcdeg	ω' (arcmin day^{-1})	α' (arcsec day^{-2})	j' (arcjerk day^{-3})	ς' (arcsnap day^{-4})	ζ'	ρ km h^{-1}
Early spring	47	−0.129	−3.007	+6.083	11.284	−31.860	−	1669.2
Late spring	38	−0.488	+2.118	+5.227	−14.057	−25.659	+	1667.5
Early summer	36	+1.172	+2.146	−5.366	−15.134	23.282	−	1675.2
Late summer	52	+0.569	−3.512	−6.596	10.972	32.590	+	1672.4
Early autumn	45	−3.108	−3.598	+7.533	15.849	−26.036	−	1655.5
Late autumn	52	−2.700	+4.435	+8.064	−14.248	−30.721	+	1657.3
Early winter	52	+2.186	+4.403	−7.957	−13.075	32.492	−	1679.9
Late winter	43	+2.773	−3.057	−6.810	15.946	23.700	+	1682.6
Weighted average	365	−0.157	0.000	0.000	0.000	0.000		1669.8

The last column can be derived from $\boldsymbol{\delta}'$ in accordance with Equations (14), (15), and (17).

The dynamics of the *Sun's horizontal path* is analogous to that of the *Sun's vertical path*. Thus, (1) the maxima and *ZCPs* of $|\boldsymbol{\delta}'|$ and $|\boldsymbol{\alpha}'|$ occur nearby, therefore they vary directly, (2) whereas $\boldsymbol{\delta}'$ and $\boldsymbol{\omega}'$ vary inversely within every *quarter-cycle* of the *EoT*, (3) $\boldsymbol{\omega}'$ and $\boldsymbol{\alpha}'$ vary inversely, where the *crests* of $|\boldsymbol{\alpha}'|$ nearly coincide with *ZCPs* of $\boldsymbol{\omega}'$ and the *crests* of $|\boldsymbol{\omega}'|$ nearly meet the *ZCPs* of $\boldsymbol{\alpha}'$—so that $|\boldsymbol{\omega}'|$ and $|\boldsymbol{\alpha}'|$ cannot maximize together, and (4) initial and final conditions converge by the end of the Gregorian year, therefore the $\boldsymbol{\delta}'$ dual cycle repeats annually. Despite the synchrony between *crests* of $\boldsymbol{\omega}'$ and *ZCPs* of $\boldsymbol{\alpha}'$ or that between *ZCP* of $\boldsymbol{\omega}'$ and *crest/trough* of $\boldsymbol{\delta}'$ —where the *EoT* shifts directions, $\boldsymbol{\alpha}'$ and $\boldsymbol{\delta}'$ do not maximize nor they reach *ZCPs* together.

Regarding the *net dive*, a section of the *EoT* is always *accelerative* when the *mean-time Sun* approaches the local meridian or *decelerative* when the *mean-time Sun* departs from the local meridian, disregarding whether the direction of the actual motion goes to the right or left. As every section tagged early tracks a departure from the local meridian and every season tagged late tracks an approach to the local meridian, the four sections of the *EoT* labeled *late* are *accelerative*, whereas the four sections labeled *early* are *decelerative*. Thus, every season contains a *decelerative* and an *accelerative* section, in that order.

As the direction of $\boldsymbol{\delta}'$ is not given by the values or signs of the *EoT*, the direction of the motion can be retrieved from the sign of $\boldsymbol{\omega}'$. A positive $\boldsymbol{\omega}'$ characterizes the trans-solstitial *phases* II and IV, where the *mean-time noon Sun* travels right. A negative $\boldsymbol{\omega}'$ characterizes the trans-equinoctial *phases* I and III, where the *mean-time noon Sun* travels left.

3.4. The Sun's Combined Path at the Section Boundaries

During solstices or equinoxes $\boldsymbol{\omega}'$ reaches a *maxima* in perfect synchrony with a *ZCP* of $\boldsymbol{\alpha}'$, but such coincidence holds an opposite meaning. At solstices, *crests* of $\boldsymbol{\omega}'$ (3.288 or 6,99 arcmin day^{-1}) converge to *downward ZCPs* of $\boldsymbol{\alpha}'$, which occurs five days after or four days before an *upward ZCP* of $\boldsymbol{\delta}'$, for the summer or winter solstice, respectively. On the other hand, near the equinoctial *troughs* of $\boldsymbol{\omega}'$ (−7.786 or −5.65 arcmin day^{-1} for the vernal or autumnal equinox, same order) meets *upward ZCPs* of $\boldsymbol{\alpha}'$, which occurs 18 days before (spring equinox) or 15 days after (autumn equinox) a *downward ZCP* of $\boldsymbol{\delta}'$, at characteristic $\boldsymbol{\delta}$ records of 9.832 and 7.907 arcdeg, respectively.

At each of the four midseason boundaries, $\boldsymbol{\omega}'$ reaches a *ZCP* in perfect synchrony with an *extrema* of $\boldsymbol{\delta}'$. In seasons whose *net drive* of *SMD* is *accelerative*, a *downward ZCP* of $\boldsymbol{\omega}'$ meets a *crest* of $\boldsymbol{\delta}'$, which takes place eight days before (midsummer) or six days after (midwinter) a *trough* of $\boldsymbol{\alpha}'$. In seasons whose *net drive* of *SMD* is *decelerative*, an *upward ZCP* of $\boldsymbol{\omega}'$ meets a *trough* of $\boldsymbol{\delta}'$, which

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takes place five days after (midspring) or four days before a *crest* of $\boldsymbol{\alpha}'$. The midseason boundaries, defined by *ZCPs* of $\boldsymbol{\omega}'$, converge to a similar analemmatic $\boldsymbol{\delta}$ whether for the Northern (−18.59 or −19.19 for midspring and midsummer) or Southern Hemisphere (−14.35 or −13.29 for midautumn and midwinter), but such $\boldsymbol{\delta}$ differs between them. The given fact reinforces the strong association between the *SMD* and the *EoT*.

3.5. Combined Horizontal-Vertical Path of the Sun

In this document, the *true solar declination* ($\boldsymbol{\delta}$) is referred to as the *Sun's vertical path*, whose parameters are denoted $\boldsymbol{\delta}$, $\boldsymbol{\omega}$, $\boldsymbol{\alpha}$ and $\boldsymbol{\zeta}$. Likewise, the EoT ($\boldsymbol{\delta}'$) is referred to as the *Sun's horizontal path*, whose parameters are denoted $\boldsymbol{\delta}'$, $\boldsymbol{\omega}'$, $\boldsymbol{\alpha}'$ and $\boldsymbol{\zeta}'$. To explore the connection between the *Sun's vertical path* (*SMD*) and the Sun's *horizontal path* (*EoT*) along the year, their parameters were explored in the same timeline. Two cycles of the *Sun's horizontal path* ($\boldsymbol{\delta}'$) occur in synchrony with a unique cycle of the *Sun's vertical path* (*SMD*) along the year. A *cycle* of $\boldsymbol{\delta}'$ concurs with a *half-cycle* of the *SMD* in the Northern Hemisphere, which encompasses two seasons of the Gregorian year. In the *horizontal path of the Sun*, $\boldsymbol{\delta}'$ switches left at midsummer or midwinter and switches right at midspring or midautumn, corresponding to the *crests* or *troughs* of the *EoT*, in the same order.

The direction of the *mean time noon Sun* regarding whether the *EoT* or the *SMD* is identified by the sign of the corresponding velocity. A positive record of $\boldsymbol{\omega}$ indicates *the mean-time Sun* is moving toward the Tropic of Cancer (boreal direction), whereas a positive record of $\boldsymbol{\omega}'$ indicates the *mean-time Sun* is heading right. The association between the *SMD* and the *EoT* is summarized in Fig 1, where every parameter of *Sun's vertical path* is plotted against its corresponding parameter of the *Sun's horizontal path*.

In **Figure 1**, every section denoted *late* is tagged grey to avoid overmarking. The ranges on which the velocity and acceleration of the *SMD* take place double those of the *EoT*, whereas jerk and snap vary within the same range for both the *vertical* and *horizontal paths* of the *mean time noon Sun*. **Figure 1** indicates that the *SMD* and the *EoT* show a cause- effect association when each and every dynamic parameter is considered alone.

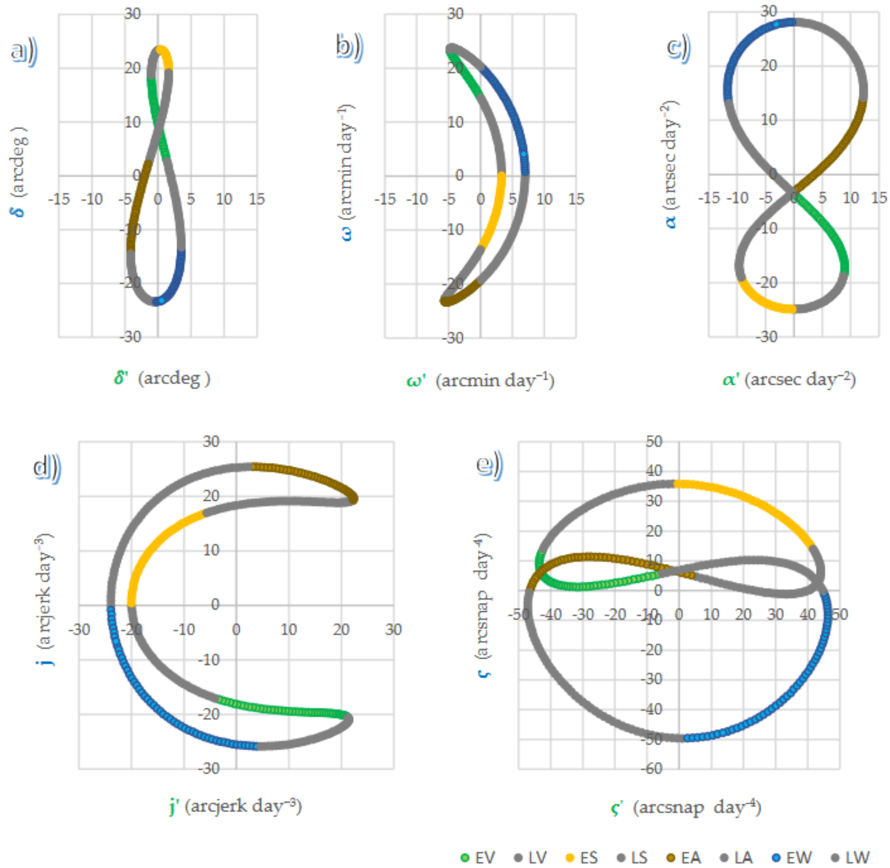

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Figure 1. Association between the parameters of the *Sun's vertical path* (y-axis) and those of the *Sun's horizontal path* (x-axis), including position (a),velocity (b), acceleration (c), jerk (d), and snap (e). EV: Early spring, LV: Late spring (V means vernal), ES: Early summer, LS: Late summer, EA: Early autumn, LA: Late autumn, EW: Early winter and LW: Late winter.

Regarding the *net drive* of the *SMD*, a season is *accelerative* when the mean time noon Sun approaches the Equator, or *decelerative* when the mean time noon Sun departs from the Equator. The *net drive* switches to decelerative when the *SMD* goes through a *ZCP*, where such *ZCP* signifies the *equilibrium point* and corresponds to the Equator. Every analemmatic section which departs de local meridian as well as every season (*SMD)* which departs from the Equator are *decelerative*. The signs of the parameters of the *Sun's horizontal path* (*EoT*) occur in the very same order in either hemisphere. Although every parameter of the *Sun's vertical path* (*SMD*) reverses signs between hemispheres, the *net drives* of the *SMD* occur in the exact same order for the sequence of analemmatic sections of either hemispheres. Therefore, the same outcome would have arisen whether a positive declination was assigned to the Southern Hemisphere, as there is no up or down in the universe.

Following the *solar sundial noon analemma* and disregarding some equinoctial deviance, one *cycle* of the *EoT* occur while the meridional Sun travels on one hemisphere, while a *half-cycle* of the *EoT* concurs to a season of the Gregorian year. Every *half-cycle* of the *EoT* consists of two sections whose *resultant drives* oppose. According to the reversed $\boldsymbol{\delta}'$, each *half-cycle* of $\boldsymbol{\delta}'$ that spans the left side of the *solar sundial noon analemma* (spring and autumn) has a *decelerative* section followed by an *accelerative* section, such contrasting *drives* switching at a midseason *trough* of $\boldsymbol{\delta}'$. Each *half-cycles* of $\boldsymbol{\delta}'$ that spans the right side of the *solar noon sundial analemma* (summer and winter) has an *accelerative* section followed by a *decelerative* section, such contrasting *drives* switching at a midseason *crest* of $\boldsymbol{\delta}'$. The second section of a season reverses the deviation from the local meridian caused by the first section; therefore, the Sun occurs near the local meridian around the end of the second section. The sign of the main three parameters (position, velocity and acceleration), whether for the *Sun's horizontal* or *vertical path*, remains consistent within every section of the *EoT*.

The association between the *SMD* and the *EoT* becomes clear with the *sundial noon analemma* alone—even dismissing the vertical and horizontal velocities and accelerations. For instance, the Pearson correlations for the association between $\boldsymbol{\delta}'$ and $\boldsymbol{\delta}$ yield -0.98 (P<0.001) for the trans-equinoctial *phases* and 0.90 (P<0.001) for the trans-solstitial *phases*. The association between the *Sun's horizontal* and *Sun's vertical path* is also noticed in the coordination of their *resultant drives* in *phases* I and III, or by their opposition in *phases* II and IV . The analysis of the independent and combined *net drives* is presented in Table 3.

Table 3. Signs of the main parameters and *resultant drive* of the *Sun's horizontal* and *vertical paths*, according to a *solar sundial noon analemma*: position, velocity, acceleration and *net drive*.

Section	Sun's horizontal path				Sun's vertical path				Combined drive
	$\boldsymbol{\delta}'$	$\boldsymbol{\omega}'$	$\boldsymbol{\alpha}'$	$\boldsymbol{\zeta}'$	$\boldsymbol{\delta}$	$\boldsymbol{\omega}$	$\boldsymbol{\alpha}$	$\boldsymbol{\zeta}$	$(\boldsymbol{\zeta}', \boldsymbol{\zeta})$
Early spring	−	−	+	−	+	+	−	−	Coordinated
Late spring	−	+	+	+	+	+	−	−	Opposed
Early summer	+	+	−	−	+	−	−	+	Opposed
Late summer	+	−	−	+	+	−	−	+	Coordinated
Early autumn	−	−	+	−	−	−	+	−	Coordinated
Late autumn	−	+	+	+	−	−	+	−	Opposed
Early winter	+	+	−	−	−	+	+	+	Opposed
Late winter	+	−	−	+	−	+	+	+	Coordinated

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A negative *net drive* is named decelerative and a positive *net drive* is named accelerative. The *net drives* of the *EoT* and *SMD* are coordinated when they align (trans-equinoctial *phases*) or opposed when they differ (trans-solstitial *phases*).

The sign of $\boldsymbol{\omega}$ was dismissed to assess the connection between $\boldsymbol{\omega}'$ and $|\boldsymbol{\omega}|$. The Pearson correlation indicates $\boldsymbol{\omega}'$ and $|\boldsymbol{\omega}|$ are inversely correlated throughout the year. Because the association fluctuates with *SMD* the analemma was sliced in four segments of *SMD* in order to compare the strength of the association between $\boldsymbol{\omega}'$ and $|\boldsymbol{\omega}|$ for the two sections occurring in the same interval of *SMD* —but opposing directions of declination—. For instance, the correlation coefficients relating $\boldsymbol{\omega}'$ with $|\boldsymbol{\omega}|$ were similar between: late spring and early autumn (**−0.96**8, **−0.96**9), early spring and late summer (**−0.99**9, **−0.99**9), late winter and early autumn (**−0.97**2, **−0.97**8), or between early winter and late autumn (**−0.96**3, **−0.96**1). Moreover, a *crest* of $\boldsymbol{\omega}'$ meets a *ZCP* of $\boldsymbol{\omega}$ at each solstice; whereas a *trough* of $\boldsymbol{\omega}'$ takes place near either equinoctial *crests* of $\boldsymbol{\omega}$.

The sections within a trans-solstitial *phase* hold differing dynamics, the first section is characterized by an increasing $\boldsymbol{\omega}'$ and a decreasing $|\boldsymbol{\omega}|$, while the second section is characterized by a decreasing $\boldsymbol{\omega}'$ and an increasing $|\boldsymbol{\omega}|$. In the trans-equinoctial *phases*, $\boldsymbol{\omega}'$ and $|\boldsymbol{\omega}|$ increase together as the combined direction of the *mean-time Sun* approaches simultaneously the Equator and the local meridian, but decrease together as the direction of the *mean-time Sun* departs at once from the Equator and the local meridian.

Despite the consistent increasing or decreasing pattern of $\boldsymbol{\alpha}$ within season, $\boldsymbol{\alpha}'$ increases monotonically along the first section and decreases monotonically along the second section of every season, whereas the signs of both remain unchanged within season. The relationship between the horizontal and vertical accelerations, $\boldsymbol{\alpha}'$ and $\boldsymbol{\alpha}$, vary directly within the first section of every season but inversely within the second season. The Pearson correlation coefficients between $\boldsymbol{\alpha}'$ and $\boldsymbol{\alpha}$ are virtually identical between the two sections of the same season, but their signs oppose. This fact remains true when comparing early spring to late spring (r = 0.96 or −0.93), early summer to late summer (r = 0.94 or −0.94), early autumn to late autumn (r =0.90 and −0.90), or early winter to late winter (r= 0.89 or −0.88). As the sign of $\boldsymbol{\alpha}$ was dismissed for this analysis, the sign of the correlation coefficient obeys to changes in $\boldsymbol{\alpha}'$ between the two sections of the same season, because the direction of $\boldsymbol{\alpha}'$shifts at midseason.

A *ZCP* of $\boldsymbol{\alpha}'$ converges to the maxima of $\boldsymbol{\alpha}$ at the solstices, while the extrema of $\boldsymbol{\alpha}'$ occur close to every midseason boundary at characteristic records of $\boldsymbol{\delta}$, whereas a *ZCP* of $\boldsymbol{\alpha}'$ occurs near the vernal and autumnal equinoxes, at $\boldsymbol{\delta}$ records of 2.928 and 2.714, respectively. Whether for the *Sun's vertical or horizontal path*, an extrema of acceleration converges with a change in direction, a behavior characteristic of pendular motion.

Given that the local meridian and the Equator conform the *equilibrium points* for the *EoT* and *SMD*, respectively, every shift departing from the *equilibrium point* is *decelerative* and every shift approaching the *equilibrium point* is *accelerative*, whether for the *Sun's horizontal* or *vertical path*. The *net drive* of a section becomes accelerative when the velocity is monotonically increasing or decelerative when the velocity is monotonically decreasing. An accelerative *net drive* can occur on either a positive or a negative direction, as long as the motion directions occurs towards the *equilibrium point*. Analogously, a decelerative *net drive* can occur on either a positive or a negative direction, as long as such direction departs from the *equilibrium point*. The direction of the actual motion can be retrieved from the sign of the velocity. This facts apply separately for the *SMD* or the *EoT*, disregarding of whether their *net drives* become coordinated or opposed.

3.6. Earth's Speed of Rotation

Dividing the *EoT* in sections allowed for a close examination of the within season and interseason *net drives*. Nonetheless, to analyze the dynamics behind the *length of the solar day* and the Earth's rotational speed, a more effective analysis arises by dividing the analemma in four *phases* according to their horizontal direction, where each *phase* encompasses two successive sections of the *EoT*. The key moments of Earth's rotational speed within a *sundial noon analemma* are inherited from the *EoT*. The *crests* and *troughs* of $\boldsymbol{\rho}$ correspond to the *crests* and *troughs* of $\boldsymbol{\delta}$ —at *ZCPs* of $\boldsymbol{\omega}'$. For instance, the midspring and midautumn *troughs* of the *EoT* correspond to *troughs* of $\boldsymbol{\rho}$ (1665.3 and 1650.8 km h^{-1}), whereas the midsummer and midwinter *crests* of the *EoT* correspond to *crests* of $\boldsymbol{\rho}$ (1677.4 and 1686.3 km h^{-1}).

Earth's rotational speed accomplishes two *phases* of progressive <u>decreases</u> throughout the Gregorian year, denoted trans-equinoctial *phases* I and III. The *phase* I encompasses late winter and early spring, whereas the *phase* III encompasses late summer and early autumn. In the *phases* I and III , $\boldsymbol{\rho}$ behaves monotonically decreasing from a *crest* to a *trough* of $\boldsymbol{\delta}'$ (both including a *ZCPs* of $\boldsymbol{\omega}'$). In the trans-equinoctial *phases*, $\boldsymbol{\rho}$ decreases despite the *net drives* of the $\boldsymbol{\delta}'$ sections being *accelerative* before the near equinoctial *trough* of $\boldsymbol{\omega}'$ and *decelerative* after the near equinoctial *trough* of $\boldsymbol{\omega}'$, yielding first <u>growing drops</u> and then <u>decreasing drops</u> in $\boldsymbol{\rho}$, respectively.

The *net drives* of the *EoT* and *SMD* become opposed along the trans-solstitial *phases* II and IV. In the trans-solstitial *phase* II, an accelerative $\boldsymbol{\delta}'$ and a decelerative $\boldsymbol{\delta}$ characterize late spring, but both *net drives* reverse for early summer. In the trans-solstitial *phase* IV an accelerative $\boldsymbol{\delta}'$ and a decelerative $\boldsymbol{\delta}$ characterize late autumn, but the *net drives* of the *EoT* and *SMD* reverse for early winter. Earth-Sun dynamics causes Earth's rotational speed to increase during the trans-solstitial *phases* of the *EoT*, for *SMD* records exceeding the *SMD* ranges parenthetically specified above.

The Earth rotates below its average speed during most of spring and autumn, but above its average speed during most of summer and winter. The average rotational speed is reached only four times a year, either at the downward or upward *ZCPs* of the *EoT* ($\boldsymbol{\delta}'$=0), on 16 Apr, 15 Jun, 2 Sept, and 26 Dec (days 106, 166, 245 and 360 of the year); whereas the $\boldsymbol{\rho}$ *crests* occur on 14 Feb and 28 Jul and the $\boldsymbol{\rho}$ *troughs* fall on 15 May and 1 Nov. Accordingly, the trans-equinoctial phases I and II, where the Earth's rotational speed decreases monotonically, span from 14 Feb to 15 May and from 28 Jul to 1 Nov, respectively; each lasting three months. Hence, the trans-solstitial phases II and IV, where $\boldsymbol{\rho}$ increases monotonically, span from 1 Nov to 14 Feb, and from 15 May to 28 July.

Earth's rotational speed accomplishes two *phases* of progressive <u>increases</u> throughout the Gregorian year, denoted trans-solstitial *phases* II and IV. The *phase* II encompasses late spring and early summer, and the *phase* IV encompasses late autumn and early winter. In the *phases* II and IV , $\boldsymbol{\rho}$ behaves monotonically increasing from a *trough* to a *crest* of $\boldsymbol{\delta}'$ (both tagged by *ZCPs* of $\boldsymbol{\omega}'$). In the trans-solstitial *phases*, $\boldsymbol{\rho}$ increases despite the *net drives* of the $\boldsymbol{\delta}'$ sections being *accelerative* before the <u>solstice</u> and *decelerative* after the solstice (going midway through an $\boldsymbol{\alpha}'$ *ZCPs* and a $\boldsymbol{\omega}'$ *crest*), yielding first <u>growing increments</u> and then <u>decreasing increments</u> in $\boldsymbol{\rho}$, respectively.

The *net drives* of *EoT* and *SMD* are *coordinated* throughout either trans-equinoctial *phase*. In the trans-equinoctial *phase* I, both exhibit a *coordinated* accelerative *net drive* in late winter ($\boldsymbol{\delta}$: −13.29, 3.08, range 16.3) but a *coordinated* decelerative *net drive* in early spring ($\boldsymbol{\delta}$: 3.08 to 18.59, range 15.5). In the trans-equinoctial *phase* II, both exhibit a *coordinated* accelerative *net drive* in late summer ($\boldsymbol{\delta}$: 19.19 to 2.57, range 16.2) but a *coordinated* decelerative *net drive* in early autumn ($\boldsymbol{\delta}$: 2.57 to −14.35, range 16.9).

As a simplified and practical conclusion, the *EoT* and the *SMD* exhibit *coordinated net drives* within the $\boldsymbol{\delta}$ interval of −13 to 19 *arcdeg*, centered in $\boldsymbol{\delta}$ = +3. Consequently, each of the analemmatic trans-equinoctial *phase*—each including two sections—spans approximately 16 *arcdeg* of *SMD*. According to the synchrony between the *SMD* and the *EoT*, the Sun influences significantly Earth's rotation. To begin with, Earth's rotational axis is a perpendicular projection to the Equator. Meanwhile the axis of the *sunlight cone*—extending from the Sun's center to the subsolar point— also conforms a normal projection to the Earth's rotational axis, a relationship that holds true throughout the year. The angular distance between the last vector and Earth's Equator is known as *SMD*. Because the *SMD* is faultlessly synchronized with the four seasons along Earth's orbit, the association here described between the Earth's rotational speed and the *SMD*, may obey to the dynamic interaction between the *SMD* and Earth's revolution. For instance, the Sun-Earth gravity imposes a torque which periodically forces Earth's Equator into the ecliptic [11].

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The increasing rotational speed of Earth characteristic of the trans-solstitial analemmatic *phases* II and IV at high *SMDs*, suggests that the angle at which the Sun reaches Earth modifies the Sun's influence on Earth's rotation. This perspective somehow implies *NBI* tags the axis of the Sun-Earth gravity, because it marks the shortest distance between the Sun and Earth's surface, by landing on Earth's surface at the subsolar point. The coordination in the *net drives* of the *SMD* and the *EoT* along the trans-equinoctial *phases* I and III of the *solar sundial analemma* suggests Earth resists rotation as *NBI* approaches the *SMD* +3 arcdeg. Conversely, the proximity of *SMD* to either the Tropic of Cancer or the Tropic of Capricorn enables a faster rotation. Thus, Earth's rotational speed increases as the length of the parallel hosting the *NBI* is shorter, and decreases as the length of the parallel grows.

Although the center of mass-density controlling *SMD* lie in the Equator, the latitude +3 arcdeg conforms the equilibrium center for the association between the *SMD* and Earth's rotational speed, as records of *SMD* above or below +3 arcdeg promote lower rate of change for $\boldsymbol{\rho}$**,** or likewise, a lower velocity for $\boldsymbol{\omega}'$. Because the $\boldsymbol{\rho}$ dynamics differs between hemispheres, it can be hypothesized that the higher share of continental land of the Northern Hemisphere modifies the effect of *SMD* over Earth's rotational speed. The midseason boundaries (midspring, midsummer, etc.) of the analemma, where $\boldsymbol{\omega}'$=0, define the beginning and ending of the four analemmatic *phases* on which the dynamics of the *EoT* and $\boldsymbol{\rho}$ progress in a consistent direction.

As Earth's linear speed of rotation varies with latitude, rotational speed can be assessed by dividing Earth's circumference of a particular latitude by the 24 hours in a *mean day*. At the Equator, Earth's perimeter is 40,075 km, a distance which the apparent Sun spans in 24 hours every *solar day* (average). Accordingly, Earth's linear rotational speed averages 1669.78 km h^{-1} at the Equator, or 27.8 km min^{-1}, where each degree of latitude encompasses 111.319 km. For the Tropics of Cancer and Capricorn, the same parameters correspond to 1532 km h^{-1}, 25.53 km min^{-1} and 102 km, respectively. The association between Earth's linear rotational speed and the length of the parallel holding NBI is displayed on **Figure 2**, first as a join function, then both parameters are compared on the same time line along the Gregorian year and finally their accelerations are contrasted.

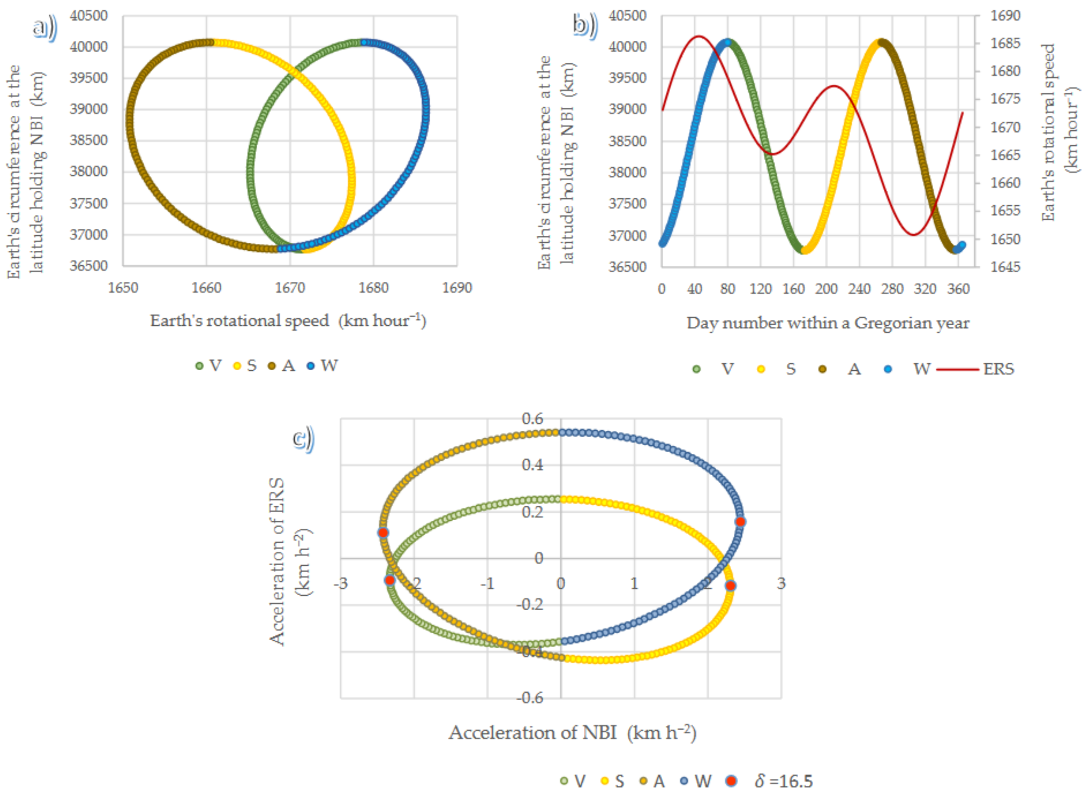

Figure 2. Association between Earth's circumference at the latitude holding *NBI* and Earth's linear rotational speed (ERS) (a), then both given parameters plotted against day number within a Gregorian year (b), and finally,

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the association between the acceleration of the same parameters (c). V: Vernal S: Summer, A: Autumn, W: Winter, ERS: Earth's rotational speed, NBI Natural Beam Irradiance.

According to **Figure (2a)** Earth's rotational speed ($\boldsymbol{\rho}$) varies on par with the length of the parallel holding *NBI* (**L**). However, the association differs between the ascending and descending halves of *SMD*. For instance, the function **L**($\boldsymbol{\rho}$) can be read as the intersection of two elliptical shapes, the one in green-blue corresponds to winter and spring (ascending *SMD*) while the yellow-brown corresponds to summer and autumn (descending *SMD*),where **L** = $2\pi\ r\ Cos\ \delta$.

In **Figure (2b)**, Earth's rotational speed (ERS) conforms a linear transformation of the *EoT*, with *crests* occurring in midsummer and midwinter and *troughs* in midspring and midautumn. Because **L** defines the distance spanned by *NBI* within a day, it can be thought as the *NBI* velocity across longitude (**L** km day^{-1}). Therefore, the daily shift in length between successive latitudes where *SMD* occurs can be read as the *NBI* acceleration ($\boldsymbol{a}$). Furthermore, switching the units of $\boldsymbol{a}$ from km day^{-2} to km hour^{-2} allows for a direct comparison between the accelerations of ERS and *NBI*.

Despite $\boldsymbol{a}$ ranging from 0.01 to 2.5 km hour^{-2}, the two maxima of $\boldsymbol{\rho}$ occur at $\boldsymbol{a}$ of 2.31 and 2.14 (midwinter and midsummer), whereas the two minima of $\boldsymbol{\rho}$ occur at $\boldsymbol{a}$ records of −2.27 and −2.30 km hour^{-2} (midspring and midautumn). During ascending *SMD*, the *crest* and *trough* of $\boldsymbol{\rho}$ occur on the 9th or 7th day (respectively) following a maximum of $\boldsymbol{a}$ (2.443 and−2.328). Likewise, during descending *SMD* the *crest* and *trough* of $\boldsymbol{\rho}$ occur on the 10th or 9th day (respectively) preceding a maximum of $\boldsymbol{a}$ (2.308 and −2.422). The four given extrema of $\boldsymbol{a}$ occur within a narrow range of *SMD* records*:* at 16.47, −16.86, 16.64, and − 16.94 arcdeg, corresponding to days 35, 128, 219, 314 of the Gregorian year. This coincidence highlights the association between the acceleration of ERS and $\boldsymbol{a}$; except the *crest* or *trough* of $\boldsymbol{\rho}$ follows the $\boldsymbol{a}$ extrema when *SMD* ascends**,** whereas during descending *SMD*, the *crest* or *trough* of $\boldsymbol{\rho}$ precede the $\boldsymbol{a}$ extrema.

Figure **(2c)**, summarizes, and provides additional proof for the association between the *SMD* and the *EoT* trough *NBI*. On one hand, **L** represents the distance that *NBI* spans across longitude; on the other, $\boldsymbol{\rho}$ represents the Earth's linear rotational speed at the Equator, both parameters being characteristic for each full rotation of the planet.

A *SMD* of 16.5 (whether positive or negative) represents the midseason threshold for the directional shifts in the joint function comparing the accelerations of **L** and $\boldsymbol{\rho}$. *Crests* in ERS acceleration occur at *ZCP* of $\boldsymbol{a}$ at the solstices, while *troughs* in ERS acceleration occur near *ZCP* of $\boldsymbol{a}$ around the equinoxes. Furthermore, the *crests* and *troughs* of the $\boldsymbol{a}$ closely align with the *SMD* of 16.5 arcdeg, at the points denoted midseason boundaries, on which the ERS acceleration occurs below 0.2 km h^{-1}. Given the circular association between the accelerations of ERS and NBI, the highest values of ERS acceleration converge near ZCP of NBI acceleration, whereas the opposite holds also true.

4. Conclusion

Establishing the four *maxima* and four *zero crossing* points of the *EoT* velocity as section boundaries to divide the *solar sundial noon analemma*, unveiled the *Sun meridian declination* (*SMD*) and the *EoT* hold a cause-effect interaction throughout the year. The combined dynamics can be summarized by their *net drives*. The *vertical* and *horizontal* dimensions of the *solar sundial noon analemma* (*EoT* and *SMD*) showed a coordinated *net drive* during two trans-equinoctial *phases*, within the *SMD* interval of −13 to 19 arcdeg —a range 32 arcdeg wide centered in +3 arcdeg of *SMD*—. Conversely, the *EoT* and the *SMD* showed opposing *net drives* during two trans-solstitial *phases*, outside the specified interval of *SMD*.

The dynamics of Earth's rotational speed is closely linked to that of the *EoT* when examined in the context of a *solar sundial noon analemma*. Two trans-equinoctial analemmatic *phases* progress from right to left, characterized by a negative *EoT* velocity and a coordinated *net drive* between *SMD* and *EoT*, conveying successive decreases in Earth's rotational speed and corresponding increases in the length of the solar day. Conversely, two trans-solstitial analemmatic *phases* progress from left to right, characterized by a positive *EoT* velocity and opposing *net drives* between *SMD* and *EoT*, conveying

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successive increases in Earth's rotational speed and corresponding decreases in the length of the solar day.

The dynamics of the *SMD*, *EoT* and Earth's rotational speed are synchronized with Earth's orbit. Increments in the rotational speed occur when the orbit closely approaches or departs from the solstices, while reductions in the rotational speed occur when the orbit closely approaches or departs from the equinoxes. Because Earth's rotational speed fluctuates by only $\pm 1\%$ throughout the year, and from the dynamical association discussed above: (1) *the Sun causes the Earth to rotate* as a consequence of Earth's dynamics of revolution and *declination*. Furthermore, because the sections on which the rotational speed decreases are always near equinoctial, it follows that (2) the Earth becomes heavier to spin when the *SMD* is closer to +3 arcdeg of *SMD* but lighter to spin as the *SMD* approaches the Tropic of Cancer or Capricorn. Summarizing, the closer the *SMD* occurs to the latitude + 3 arcdeg, the slower the Earth rotates, which implies the Earth's rotational speed varies with the latitude at which the *sunlight cone* reaches the planet.

Earth's rotational speed and *SMD* are linked through *NBI*. The acceleration in the daily path of *NBI* —measured by daily shifts in length between successive latitudes where *SMD* occurs— varies inversely with the acceleration in Earth's rotational speed.

5. Definitions

Amplitude. In sinusoidal curves, the function's amplitude is the vertical distance between a crest or a trough and the abscise axis.

Angular velocity of *SMD* ($\boldsymbol{\omega} = d\boldsymbol{\delta}/dt$, arcmin day^{-1}). First order derivative for the vertical position of the Sun (*SMD*) within a solar meridional analemma.

Angular acceleration of the true declination ($\boldsymbol{\alpha} = d\boldsymbol{\omega}/dt$, arcsec day^{-2}). Second order derivative for the vertical position of the Sun (*SMD*) within a meridional analemma.

Angular jolt of the true declination ($\boldsymbol{j} = d\boldsymbol{\alpha}/dt$, arc**jerk** day^{-3}). Third order derivative for the position of the *SMD*.

Angular snap of the true declination ($\boldsymbol{\varsigma} = d\boldsymbol{\alpha}/dt$, arc**snap** day^{-4}). Fourth order derivative for the position of the *SMD*.

Angular velocity of the EoT ($\boldsymbol{\omega}' = d\delta'/dt$, arcmin day^{-1}). First order derivative for the position of the *EoT*.

Angular acceleration of the EoT ($\boldsymbol{\alpha}' = d\boldsymbol{\omega}'/dt$, arcsec day^{-2}). Second order derivative for the position of the *EoT*.

Angular jolt of the EoT ($\boldsymbol{j}' = d\boldsymbol{\alpha}'/dt$, arcjerk day^{-3}). Third order derivative for the position of the *EoT*.

Angular snap of the EoT ($\boldsymbol{\varsigma}' = d\boldsymbol{j}'/dt$, arcsnap day^{-4}). Fourth order derivative for the position of the EoT.

arc**jerk**: A unit required to undertake this analysis. 1 arcjerk is equivalent to $1/60^3$ arcdeg, $1/60^2$ arcmin, 1/60 arcsec or 60 arcsnap. The unit is proposed after using the factor $[\boldsymbol{\pi/3}]^{\boldsymbol{k}}\,[180/\pi]^{k+1} = [180/\pi]60^3$ for the third order derivative of position, whether for the *EoT* or for the *SMD*.

arc**snap**: A unit required to undertake this analysis. 1 arcsnap is equivalent to $1/60^4$ arcdeg, $1/60^3$ arcmin, $1/60^2$ arcsec and 1/60 arcjerk. The unit is proposed after using the factor $[\pi/3]^{k}\,[180/\pi]^{k+1} = [180/\pi]60^4$ for the fourth order derivative of position, whether for the *EoT* or for the *SMD*.

Cycle. Repetitive shape of a sinusoidal curve. The cycle is completed when final conditions converge to the initial conditions of the next cycle. A *cycle* can be split into *half-cycles* and/or *quarter-cycles*, following fixed instances of the position or following a parameter derived from the position.

Equation of Time, or EoT (δ'). Also referred here as the *Sun's horizontal path*, tracks the daily discrepancies between the true solar time and the mean-time, throughout a Gregorian year. The *EoT* also locates the longitude at which the *lumbra* occurs at the mean-time noon when the *SMD* converges the site's latitude.

Crest. A maximum of a function where the direction of the motion switches from upward to downward, or from right to left. The slope of a function becomes zero at the *crest*.

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Light cone: A figurative conic section interrupted by two circular planes whose wider plane corresponds to the solar disk, along its full two-dimensional shape on the sky, and whose smaller plane is the *lumbra*.

Lumbra. A circular shape drawn by the *light-cone* on the Earth surface which receives natural beam irradiance simultaneously.

Net drive. The *net drive*, or *resultant drive*, explains the association between the direction of the strength causing the motion and the direction of the actual motion. Only two kinds of resultant drive can take place within a one-dimensional displacement: *accelerative* drive (speeding-up) and *decelerative* drive (slowing-down : braking). An *accelerative net drive* occurs when $\boldsymbol{\alpha}$ and $\boldsymbol{\omega}$ hold the same sign, whereas a *decelerative net drive* occurs when those sings oppose.

Natural beam irradiance (*NBI*). The amount of solar irradiance (Nm^{-2}) delivered as a normal projection from the solar ring to the *lumbra*. *NBI* lands on a site when the *Sun meridian declination* converges the site's latitude and the Sun aligns to the local meridian.

Period. The time it takes for the entire cycle of a sinusoidal function to be completed.

Phase of *EoT*. Each of 4 segments defined for the *EoT* within a solar sundial noon analemma. A phase always starts and finishes at a *zero-crossing point* of the velocity of the *EoT*.

Pseudo-sinusoidal function. A graph which resembles a sinusoidal function (derived from a sine or cosine function), but whose amplitudes, periods and zero crossing points fail to occur on a perfect rhythm. The pseudo-sinusoidal may be comprised by the combination of both sine and cosine functions together.

Solar noon analemma (δ', δ). Function built by plotting the *SMD* against the *EoT*. For any given site on Earth's surface, the solar noon analemma displays the combined vertical-horizontal path of the mean Sun on the sky for the 365 days of a Gregorian year.

Solar noon sundial analemma: When the sky analemma is inverted with regards to the abscissa (*EoT*), the horizontal axis becomes inverted from its center; the name comes from the way in which this analemma can be recorded on land by a gnomon. Despite a sundial analemma also reverses vertically, the present document does not reverse the *SDMN*, but only the *EoT*, because such approach suffices the proposed analysis.

Sun meridian declination (δ). Also referred here as the *Sun's vertical path*, accounts for the *true declination*. The true declination is the angle the Sun and Earth's Equator and locates the latitude at which the *lumbra* occurs on Earth's surface, for any given day of a Gregorian year.

Trough. A minimum (negative extrema) of a function where the direction of the motion switches from downward to upward, or from left to right. The slope of a function becomes zero at the *trough*.

Zero crossing point (ZCP). When a sinusoidal or pseudo-sinusoidal curve oscillates around zero, a *ZCP* is the point where the curve crosses the abscise axis. An upward ZCP occurs when the curve crosses the abscissa's axis from negative to positive records, while a downward *ZCP* occurs when the function crosses the abscissa's axis from positive to negative records.

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References

1. Gross, R.S. Theory of Earth Rotation Variations. In Proceedings of the International Association of Geodesy Symposia; Springer Verlag, 2016; Vol. 142, pp. 41–46.
2. Nurick, A. A Closed Form Equation of the Time Component for the Tilt of the Earth's Axis. *Solar Energy* **2011**, *85*, 295–298, doi:10.1016/j.solener.2010.11.022.
3. Raisz, E. The Analemma. *Journal of Geography*, **1941**, *40*, 90–97.
4. Rueda, J.A.; Ramírez, S.; Sánchez, M.A.; Guerrero, J. de D. Sun Declination and Distribution of Natural Beam Irradiance on Earth. *Atmosphere (Basel)* **2024**, *15*, doi:10.3390/atmos15081003.
5. Shaw, S.G. Resurrecting the Analemma. *Navigation, Journal of the Institute of Navigation* **2002**, *49*, 1–5, doi:10.1002/j.2161-4296.2002.tb00251.x.
6. Athayde Jr, L.S. Paradoxical Variation of the Solar Day Related to Kepler/Newton System. *Journal of Aeronautics & Aerospace Engineering* **2015**, *04*, doi:10.4172/2168-9792.1000145.
7. Muller, M. Equation of Time - a Problem in Astronomy. *Acta Phys. Pol. A* **1995**, *88*, 49.
8. Georgieva, K. Solar Dynamics and Solar Terrestrial Influences. In *Space Science*; Nova Science Inc: New York, 2006; pp. 35–81.
9. Spencer, J.W. Fourier Series Representation of the Position of the Sun. *Search (Syd)* **1971**, *2*, 172.
10. Williams, D.R. Earth Fact Sheet. NASA Goddard Space Flight Center.
11. Torge, W.; Muller, J.; Pail, R. *Geodesy*; 5th ed.; De Gruyter Oldenbourg Verlag: Berlin, 2023;