Article

Not peer-reviewed version

Massive Wave Solutions to the Einstein-Maxwell Equations

Álvaro García López *

Posted Date: 11 April 2025

doi: 10.20944/preprints202504.0927.v1

Keywords: Classical electrodynamics; Einstein-Maxwell equations; General Relativity; Geometrodynamics; Nonlinear Dynamics; Complex Systems

Preprints.org (www.preprints.org) | NOT PEER-REVIEWED | Posted: 11 April 2025 doi:10.20944/preprints202504.0927.v1

Article

Massive Wave Solutions to the Einstein-Maxwell Equations

Álvaro G. López

Nonlinear Dynamics, Chaos and Complex Systems Group, Departamento de Física, Universidad Rey Juan Carlos, Tulipán s/n, 28933 Móstoles, Madrid, Spain; alvaro.lopez@urjc.es

Abstract: We use gauge fixing to derive Proca equation from Maxwell's classical electrodynamics in curved spacetime. Further restrictions on the gauge yield the Klein-Gordon equation for scalar bosons. The self-coupling of electromagnetic fields through spacetime curvature originates the inertia of wave packets for non-null field solutions, suggesting an electromagnetic origin of mass. We study the weak field limit of these solutions and prove that the electrovacuum can behave as a charged nonlinear optical medium.

Keywords: classical electrodynamics; Einstein-Maxwell equations; general relativity; geometrodynamics; nonlinear dynamics; complex systems

---

1. Introduction

Electromagnetic fields pervade spacetime everywhere. However, spacetime curvature is rarely taken into account when studying classical electromagnetism. In part, this is due to the fact that the incorporation of Einstein-Hilbert action to classical electrodynamics yields high-dimensional nonlinear partial differential equations, where exact solutions are scarce and difficult to find [1].

In his seminal paper G. Y. Rainich considered the spacetime curvature produced by electromagnetic fields, and found a set of four conditions for a Lorentzian manifold to admit an interpretation as exact non-null electrovacuum solution in general relativity [2]. These conditions were later dubbed the Rainich conditions [3]. Rainich conditions identify spacetime and the electrodynamic fields, to the point that the entire theory can be expressed in terms of a scalar field named the *complexion*, and the geometric properties of spacetime. This idea has lead other authors to "dispense" with the electromagnetic gauge field and express the dynamics of the electromagnetic fields in terms of purely spatiotemporal concepts, such as the metric and the Riemann curvature tensor. This unified mathematical framework is known nowadays as geometrodynamics [3].

However, the inverse function theorem suggests that the reverse theory might be achievable as well, allowing to express all the spatiotemporal concepts in terms of electromagnetic fields. Such fields would then be the fabric of spacetime, producing its twist and tension. In this way, Einstein's general theory of relativity simply states that the intrinsic geometry of spacetime (e.g. Ricci curvature) is tantamount to the electromagnetic stress. This line of reasoning suggests that the electromagnetic gauge field is the fundamental "substance". Then, Einstein's equation simply expresses the tautological nature of spacetime, ensuring at the same time the frame independent nature of the physical law that describes its evolution, and the universal character of such fundamental principle.

It has recently been proved by using the concept of electromagnetic mass [4] that Newton's laws can be derived in the macroscopic limit as an approximation to Maxwell's classical electrodynamics with sources in flat spacetime [5]. However, in the microscopic realm, extended electrodynamic objects can experience very violent time-delayed tidal self-forces, arising from the retarded Liénard-Wiechert potentials. These self-interactions yield nonlinear oscillations with *zitterbewegung* frequency via the Hopf bifurcation mechanism [6], generating electromagnetic pilot waves [7], quite similar to those recently found in hydrodynamic quantum analogs [8].

Preprints.org (www.preprints.org) | NOT PEER-REVIEWED | Posted: 11 April 2025 doi:10.20944/preprints202504.0927.v1

In the present work, we derive the typical wave equations used to describe the electrodynamics of microscopic charged bodies (e.g. the Klein-Gordon equation) from the Einstein-Maxwell equations. We introduce a new spatiotemporal gauge that decouples the nonlinear partial dynamical equations of the four-potential as a set of four linear partial differential equations corresponding to vector massive bosons. In this manner, we provide further evidence that inertia has an electromagnetic origin and explain at the same time the usefulness of introducing other putative scalar fields to break symmetries spontaneously [9], mimicking techniques used in the study of superconductivity [10].

2. Maxwell Equation in Curved Spacetime

We now introduce the fundamental nonlinear dynamical equations of the theory and express them explicitly to draw some general conclusion concerning the potential of Einstein-Maxwell equations to describe fundamental electrodynamic invariants such as charge and rest mass, in terms of pure electrovacuum.

2.1. The Lagrangian Density

We consider as the fundamental action of the present field theory Maxwell's classical electrovacuum expressed in an arbitrary coordinate system, and the Einstein-Hilbert action

$$\mathcal{L} = \sqrt{-g}\left\{\frac{1}{2\kappa}R - \frac{1}{4}F_{\mu\nu}F^{\mu\nu}\right\}, \tag{1}$$

where $g_{\mu\nu}$ is the metric tensor with signature $(+,-,-,-)$ convention, R is the curvature scalar, $F_{\mu\nu}$ is the Faraday tensor and $\kappa = 8\pi G/c^4$. This Lagrangian density assumes a single gauge field self-interacting through the curvature of spacetime, allowing light rays to bend as a consequence of its own energy and stress. This Lagrangian density yields two fundamental dynamical equations [3] for the gauge field

$$\nabla_\mu F^{\mu\nu} = 0 \tag{2}$$
$$\nabla_\mu (*F)^{\mu\nu} = 0, \tag{3}$$

where the operator ∇_μ represents the covariant derivative, which in the present work is represented by means of the Levi-Civita connection, which preserves the metric tensor and is torsion-free. Here we have also introduced the Hodge dual tensor $*F$, which can be written as

$$(*F)_{\mu\nu} = \frac{1}{2}\sqrt{-g}\varepsilon_{\mu\nu\sigma\rho}F^{\sigma\rho}, \tag{4}$$

expressed in terms of the Levi-Civita symbol $\varepsilon^{\mu\nu\sigma\rho}$. Therefore, Equation (3) can be written as the Bianchi identity $\varepsilon^{\mu\nu\sigma\rho}\nabla_\nu F_{\sigma\rho} = 0$, as well.

The first equation is the Ampère-Maxwell law, while the latter is the Gauss-Faraday law, both expressed in curved spacetime. These two equations describe the spatiotemporal evolution of the electromagnetic fields. They essentially state that such fields have zero covariant divergence, and thus they cannot be created nor destroyed as a whole, and also that the change in time of the time-like components generates vorticity of the space-like dual components around the former.

Finally, the Einstein-Hilbert action yields the well-known Einstein equation, which allows to relate the dynamics and curvature of spacetime to the energy content of the fundamental field $F_{\mu\nu}$, in the form

$$G_{\mu\nu} = \frac{8\pi G}{c^4}T_{\mu\nu}, \tag{5}$$

Preprints.org (www.preprints.org) | NOT PEER-REVIEWED | Posted: 11 April 2025 doi:10.20944/preprints202504.0927.v1

where $G_{\mu\nu} = R_{\mu\nu} - Rg_{\mu\nu}/2$ and $T^{\mu\nu}$ is the electromagnetic stress-energy tensor, which in Belinfante and Ronsfeld's symmetrized form [11], can be written as

$$T_{\mu\nu} = \frac{1}{\mu_0}\left(F_{\mu\tau}F^{\tau}_{\nu} - \frac{1}{4}F_{\rho\sigma}F^{\rho\sigma}g_{\mu\nu}\right). \tag{6}$$

Assuming that spacetime is simply connected, Poincaré's lemma [12] entails to write the Faraday tensor in terms of the four-potential A_μ, as follows

$$F_{\mu\nu} = \nabla_\mu A_\nu - \nabla_\nu A_\mu. \tag{7}$$

We shall utilize the four-potential in the forthcoming sections to derive the wave equations of the present work.

2.2. Apparent Sources

To gain further insight into the structure of Maxwell equations in curved spacetime, we explicitly write them in terms of the electric and the magnetic fields. This will open a discussion on a possible origin of electric and magnetic charges from the curvature of spacetime, and also a breaking of the duality of the electrovacuum, as compared to Maxwell's electrodynamics in flat spacetime.

The Faraday form F can be written (indexes not summed) as

$$F = F_{\mu\nu}dx^\mu \wedge dx^\nu = F_{0i}dx^0 \wedge dx^i + F_{ij}dx^i \wedge dx^j, \tag{8}$$

with $F_{0i} = E_i/c$ and $F_{ij} = -\varepsilon_{ijk}B^k$, and where latin indexes denote the spatial components only. When these components of the Faraday tensor are substituted in Equation (3), we get

$$\begin{aligned} \nabla_i B^i &= 0 \\ \nabla_0 B^i + \varepsilon^{ij}_{k}\nabla_j E^k/c &= 0. \end{aligned} \tag{9}$$

The first equation deserves comment because, when expressed in terms of the Christoffel symbols $\gamma^\mu_{\nu\rho}$, it shows that, from the point of view of asymptotically flat observers faraway, the fields behave as if there existed sources. In particular, we have the following equation

$$\nabla_i B^i = \partial_i B^i + \gamma^i_{ji}B^j = 0, \tag{10}$$

where indexes are now summed. This equation allows to write the magnetic charge density $\rho_m = -\gamma^i_{ji}B^j$, and suggests the existence of magnetic monopoles [13]. The fact that the curvature of spacetime can mimic the existence of electromagnetic sources in curved spacetime has recently been suggested in a generalized formulation of Maxwell's electrodynamics using quaternions [14].

On the other hand, the Equation (2) can be expressed using the metric compatibility $\nabla_\rho g_{\mu\nu} = 0$ resulting from the Levi-Civita connection, as

$$g^{\mu\sigma}g^{\nu\rho}\nabla_\mu F_{\sigma\rho} = 0, \tag{11}$$

which can be expressed entirely in terms of the electric and magnetic fields, yielding two more equations. However, these equations are not symmetric in their expression to Equations (9), due to the appearance of the metric tensor. We can write the set of equations in the form

$$\begin{aligned} (g^{\mu 0}g^{0i} - g^{\mu i}g^{00})\nabla_\mu E_i/c - g^{\mu j}g^{0k}\varepsilon_{ijk}\nabla_\mu B^i &= 0, \\ (g^{\mu 0}g^{li} - g^{\mu i}g^{l0})\nabla_\mu E_i/c - g^{\mu j}g^{lk}\varepsilon_{ijk}\nabla_\mu B^i &= 0, \end{aligned} \tag{12}$$

Preprints.org (www.preprints.org) | NOT PEER-REVIEWED | Posted: 11 April 2025 doi:10.20944/preprints202504.0927.v1

where l index is free. Only in the case that we have a diagonal metric, we can write the equations

$$\nabla_i E^i = 0,$$
$$\nabla_0 E^i/c - \varepsilon_k^{ij} \nabla_j B^k = 0. \tag{13}$$

Again, Equations (13) allows to write the electric charge density $\rho_e = -\gamma_{ji}^i E^j$, suggesting that apparent electric charge distributions can be created from the electrovacuum alone. It is remarkable that, in general terms, the spacetime curvature can break the duality between electric fields and magnetic fields. Despite this lack of duality, this is not an impediment to the existence of magnetic monopoles. Nevertheless, the fundamental duality remains in terms of the Faraday tensor, and the Maxwell equations can be written simply by stating that the Faraday form and its dual are closed forms. Mathematically, this can be written as $dF = 0$ and $d(*F) = 0$, where $*F$ is the Hodge dual of Faraday tensor.

The possibility that charges might arise through a purely geometrical and topological mechanism [15] from the flat spacetime point of view, might prevent the necessity of using complicated unstable multiply connected topologies in curved spacetimes, such as wormholes [3], or non-orientable manifolds [16].

2.3. Rainich Conditions

We now introduce Rainich algebraic conditions on the structure of spacetime and the electromagnetic complexion, not only for their transcendence concerning electrodynamic theory in curved spacetime [3], but also because they will be used in the following lines to derive the fundamental wave equation.

It is immediately proven by taking the trace of Equation (5) that the curvature scalar of the Einstein-Maxwell theory is zero ($R = 0$). This means that the total average curvature of spacetime is null. This equation also implies that Einstein's equation can be written as

$$R_{\mu\nu} = \frac{8\pi G}{c^4} T_{\mu\nu}, \tag{14}$$

allowing for a direct identification of geometry and energy content. Equivalently, we can write $T = T^\mu_\mu = 0$, which means that electromagnetic stresses are purely deviatoric in spacetime. Thus, when a positive pressure develops along some direction of space by concentrating electromagnetic energy, this pressure must be counterbalanced by negative pressures in the two complementary spatial directions.

The second Rainich condition simply states that $R_{00} > 0$, and it is equivalent to inequality $T_{00} > 0$, implying that the density of electromagnetic energy is positive-definite, and avoiding the complications that negative energies usually introduce in particle physics [13].

Finally, the third Rainich condition diagonalizes the stress-energy tensor by taking its square. It can be easily demonstrated by considering a locally Minkowskian reference frame and by performing a Lorentz transformation that

$$R_{\mu\rho} R^{\rho\nu} = \frac{1}{4} \delta^\nu_\mu R_{\rho\sigma} R^{\rho\sigma}. \tag{15}$$

Again, because of the identity between Ricci curvature tensor and the energy-stress tensor, the same equation holds for the $T_{\mu\nu}$. In particular, we recall that

$$T_{\mu\nu} T^{\mu\nu} = P^2 + Q^2, \tag{16}$$

where the *electromagnetic invariants* $P = F_{\mu\nu} F^{\mu\nu}/2$ and $Q = F_{\mu\nu}(*F)^{\mu\nu}/2$ have been introduced. Thus, Equation (16) holds in any reference frame. To recall, these invariants can be written in a locally Minkowskian reference frame as $P = -(E/c)^2 + B^2$, and $Q = -2E \cdot B$.

Preprints.org (www.preprints.org) | NOT PEER-REVIEWED | Posted: 11 April 2025 doi:10.20944/preprints202504.0927.v1

These two invariants classify electromagnetic fields as null ($Q = P = 0$) or non-null when ($Q^2 + P^2 \neq 0$). When $Q = 0$ and $P \neq 0$, there exists a locally Minkowskian reference frame where the fields are static, and, therefore, the dual component of a particular component vanishes. Fields satisfying the $Q = 0$ condition are known as extremal fields [17]. Finally, when $Q \neq 0$, there exists a locally Mikowskian reference frame where both the electric and the magnetic fields are parallel. This is precisely the reference frame in which the square of the stress-energy tensor diagonalizes, allowing us to conceptualize the Faraday tensor as the square root of the stress-energy and, thanks to Rainich's conditions, to the square root of Ricci curvature.

To conclude, we briefly comment on the solutions to the Einstein-Maxwell equation as a function of the complexion field [3]. The complexion $\alpha(x)$ is a scalar field that allows rotation of the electromagnetic fields when expressed in the complexified Riemann-Siberstein form [18]. In simply connected spacetimes, the covariant derivative of this field (see Equation (17)) is conservative, meaning that its integral along a closed path is zero. Defining the Silbestein tensor $\mathrm{F}^j = E^j + icB^j$, the corresponding complexified Faraday tendor $\mathscr{F}_{\mu\nu} = F_{\mu\nu} + i(*F)_{\mu\nu}$ can be introduced. It is possible to obtain formal *non-null* solutions to the Einstein-Maxwell equations by considering an arbitrary constant Faraday tensor $\mathfrak{f}_{\mu\nu} = f_{\mu\nu} + i(*f)_{\mu\nu}$ satisfying the extremal condition, and performing a gauge duality rotation $\mathscr{F} = e^{-i\alpha}\mathfrak{f}$. It has been shown [3] that the complexion can be computed from spacetime curvature in the form

$$\alpha_\mu = \epsilon_{\mu\nu\sigma\rho} R^{\eta}_{\nu} \nabla^{\sigma} R^{\eta\rho} / R_{\lambda\tau} R^{\lambda\tau}. \tag{17}$$

Note that by virtue of Equation (14), the complexion can be completely expressed in terms of the electromagnetic stress-energy tensor. This suggests that Rainich conditions establish an essential identification between "light" and spacetime. However, the spatial derivative remains in Equation (17), thus it is worth asking if the spacetime coordinates might be expressed by means of the inverse function theorem as a function of the electromagnetic fields, at least locally.

3. A Nonlinear Equation

Now we derive a partial differential equation for the connection A_μ. For this purpose, it suffices to consider the Lie's commutator of the covariant derivatives of the electromagnetic four-potential A_μ,

$$[\nabla^{\mu}, \nabla_{\nu}] A_{\rho} = R^{\mu}_{\nu\rho\sigma} A^{\sigma}. \tag{18}$$

Of importance, we note that the four-potential self-couples through the Riemann curvature tensor, entailing light self-interactions [5]. By contracting the first and third indexes of Equation (18), we can express it in terms of the Ricci tensor as

$$\nabla^{\mu}\nabla_{\nu} A_{\mu} - \nabla_{\nu}\nabla^{\mu} A_{\mu} = R_{\nu\sigma} A^{\sigma}. \tag{19}$$

The latter equation can be simplified by using the Lorenz gauge $\nabla^{\mu} A_{\mu} = 0$, yielding

$$\nabla^{\mu}\nabla_{\nu} A_{\mu} = R_{\nu\sigma} A^{\sigma}. \tag{20}$$

If we now consider Maxwell's equation $\nabla^{\mu} F_{\mu\nu} = 0$ and expand Faraday tensor in terms of the four-potential, we get $\nabla^{\mu}\nabla_{\mu} A_{\nu} = \nabla^{\mu}\nabla_{\nu} A_{\mu}$. Introducing the Laplace-Beltrami operator $\square = \nabla^{\mu}\nabla_{\mu}$, we have

$$\square A_{\mu} = R_{\mu\sigma} A^{\sigma}. \tag{21}$$

Using the Einstein Maxwell equations, we can write the equation for the four-potential as

$$\square A_{\mu} - \frac{8\pi G}{c^4} T_{\mu\nu} A^{\nu} = 0. \tag{22}$$

Preprints.org (www.preprints.org) | NOT PEER-REVIEWED | Posted: 11 April 2025 doi:10.20944/preprints202504.0927.v1

Now we recall that the first electromagnetic invariant $P = F^{\rho\tau}F_{\rho\tau}/2$, and introduce the inertia term, by defining the scalar function $(Mc/\hbar)^2 \equiv 4\pi G/\mu_0 c^4 P$. If we also introduce the self-coupling inertia tensor $\chi_\mu^\nu = (8\pi G/\mu_0 c^4)F_{\mu\tau}F^{\tau\nu}$ and the current $j_\mu = \chi_\mu^\nu A_\nu$, we obtain the equation

$$\Box A_\mu + \left(\frac{Mc}{\hbar}\right)^2 A_\mu = j_\mu. \tag{23}$$

The Equation (23) represents an inhomogeneous Proca equation with self-coupling currents [19]. Importantly, we notice that inertia has a tensorial character in the Einstein-Maxwell equations. The current j_μ is nonlinear, since it depends on the field A_μ. It means that the electrovacuum behaves as a *nonlinear optical medium* with inertia. We stress the fact that inertia arises as a consequence of electromagnetic self-interactions and that the scalar component is a fundamental invariant of electrodynamics. Thus, in the present theory, it is unnecessary to introduce *ad hoc* fields to produce the mass of vector bosons.

4. Gauge Fixing

We now show that, by choosing an adequate spatiotemporal gauge, we can decouple Equation (23), setting the self-current j_μ to zero. It is important to bear in mind that the complete gauge transformations involve both the spacetime coordinates, as well as the electromagnetic potential. Thus the set of gauge transformations in the Einstein-Maxwell equations can be written as

$$x_\mu = x'_\mu + \xi_\mu, A_\mu = A'_\mu + \nabla'_\mu \Lambda, \tag{24}$$

where four coordinate gauge fields ξ_ν have been introduced for the coordinate transformation, and the scalar Λ field for the electromagnetic. The fields A'_μ and $\nabla'_\mu \Lambda$ appearing in Equation (24) are evaluated at x. If ξ_μ is assumed sufficiently small, we can use the Lie covariant derivative, what yields the complete transformation of the gauge potential as

$$A_\mu = A'_\mu + \xi^\nu \nabla'_\nu A'_\mu + A'^\nu \nabla'_\nu \xi_\mu + \xi^\nu \nabla'_\nu \nabla'_\mu \Lambda + \nabla'^\nu \Lambda \nabla'_\nu \xi_\mu, \tag{25}$$

where the fields in Equation (25) are now written as a function of x'. Under these transformations, the metric turns into

$$g_{\mu\nu} = g'_{\mu\nu} + \nabla'_\mu \xi_\nu + \nabla'_\nu \xi_\mu. \tag{26}$$

Finally, we have to use the covariant Lie derivative again to obtain the transformation of the Faraday tensor under the complete set of gauge transformations. It yields the transformation

$$F_{\mu\tau} = F'_{\mu\tau} + \xi^\nu \nabla_\nu F'_{\mu\tau} + \nabla_\mu \xi^\nu F'_{\nu\tau} + \nabla_\tau \xi^\nu F'_{\mu\nu}. \tag{27}$$

If we now start back again with the full Einstein-Maxwell equations for the four potential without gauge-fixing, we have

$$\Box A_\mu - \nabla_\mu \nabla^\nu A_\nu + \left(\frac{Mc}{\hbar}\right)^2 A_\mu = \chi_\mu^\nu A_\nu, \tag{28}$$

In Appendix A it is shown that the self-current $\chi_\mu^\nu A_\nu = 0$ can be turned off by an appropriate choice of the spatiotemporal gauge. Additionally, by choosing the adequate gauge for the electrodynamic field, i.e., one for which $\nabla^\nu A_\nu = 0$, we get $\Box \Lambda' = -\nabla'^\nu A'_\nu$, and the following decoupled four non-homogeneous system of first order nonlinear partial differential equations reveal

$$\Box A_\mu + \left(\frac{Mc}{\hbar}\right)^2 A_\mu = 0. \tag{29}$$

This is precisely Proca equation in curved spacetime, since the Laplace-Beltrami operator involves curvature of spacetime, and therefore the solutions to these equations are still nonlinear, depending on

Preprints.org (www.preprints.org) | NOT PEER-REVIEWED | Posted: 11 April 2025 doi:10.20944/preprints202504.0927.v1

Einstein's equation to completely solve the spacetime metric. Thus, apart from photons, other gauge bosons with mass can be obtained from the electrovacuum. As we can see, the mass of these solutions arises as a consequence of the *non-null character of the electrodynamic fields* through the spacetime curvature. This means that the "photon" mass is produced by an imbalance between the size of the magnetic and the electric fields, as compared to null fields. Certainly, very strong fields are required to obtain masses as high as typically seen in gauge vector bosons. Using values of the Z electroweak neutral boson, we have

$$M_Z c^2 = \sqrt{\frac{c^2\hbar^2}{2}\frac{8\pi G}{\mu_0 c^4}F^{\rho\tau}F_{\rho\tau}} = \sqrt{\frac{4\pi\hbar^2 G}{\mu_0 c^2}(B^2-(E/c)^2)} \approx \sqrt{\frac{4\pi G}{\mu_0}}\frac{\hbar}{c}\langle|B|\rangle. \tag{30}$$

Thus, assuming that most of the energy is in the core of the wave-packet, we would get a magnetic-dominated field with intensity

$$\langle|B|\rangle \backsim \sqrt{\frac{\mu_0}{4\pi G}}\frac{M_Z c^3}{\hbar} \backsim 10^{55}T. \tag{31}$$

If such intense magnetic fields were achievable by means of a nonlinear feedback process, we can expect that the size of the boson (neglecting spacetime curvature) as a wave-packet must be in the scale

$$l_P \sim \left(\frac{\mu_0 M_Z c^2}{B^2}\right)^{1/3} \sim 10^{-35}. \tag{32}$$

The size of a Z boson wave-packet can be thus estimated at the Planck length, but being made of electromagnetic field, it is reasonable that their spatiotemporal properties cannot be empirically detected, not only because the size of the wave-packets is at the Planck scale, but because the field configuration would be modified by the "external" fields of the apparatus. Certainly, if particles are solitons within the Einstein-Maxwell theory, they must be contextual [20].

Apart from spin one solutions, spinless bosons can be obtained by considering $A_\mu = (\phi, A_i)$ with $A_i = 0$, what yields the spin zero field

$$\Box\phi + \left(\frac{Mc}{\hbar}\right)^2\phi = 0. \tag{33}$$

Then, the gauge transformation $x_\mu = x'_\mu + \xi_\mu, A_\mu = A'_\mu + \nabla'_\mu\Lambda$ acquires an additional restriction $A'_i = -\nabla'_i\Lambda$, and we have the following gauge equation $\Box\Lambda' = -\nabla'^0 A'_0 + \nabla'^i\nabla'_i\Lambda$, where here A'_μ and $\nabla'_\mu\Lambda$ are evaluated at x. In is immediate to obtain the Schrödinger equation in the non-relativistic limit from Equation (26). In addition, Equation (26) suggests that the wave function is a true electromagnetic wave, as originally proposed by D. Bohm [21], and not just a probabilistic entity. In turn, it also implies that classical mechanics can be considered as a ray optics approximation to the Einstein-Maxwell equations valid when the intensity of the fields in the electrovacuum is small and these fields are slightly inhomogeneous, allowing us to express solutions in terms of locally plane waves, as described by the Hamilton-Jacobi equation [22]. However, in general, we must insist that the wave packets present inertia as a consequence of self-coupling, specially those related to particles, which here are considered as light pulses.

Interestingly, we can write the second contribution of the electrovacuum stress-energy tensor as

$$T^{(\text{vac})}_{\mu\nu} = -\Lambda(x)g_{\mu\nu}, \tag{34}$$

where $\Lambda(x) = P(x)/2\mu_0$ has been introduced. This scalar field should not be confused with the gauge transformation appearing in Equation (24), but perhaps it may be identified with the cosmological constant. If this is correct, then dark energy might be plain electromagnetic energy arising from non-null fields, dominated by magnetic fields over electric, so that the expansion of the universe is

Preprints.org (www.preprints.org) | NOT PEER-REVIEWED | Posted: 11 April 2025 doi:10.20944/preprints202504.0927.v1

ensured by the Hubble flow. The value of Λ is very small, of the order 10^{-52}, which means that the electromagnetic fields are currently slightly more magnetic. Nevertheless, note that, strictly speaking, $\Lambda(x)$ is a scalar field and not a constant, and can change sign along its evolution, since the other contribution implies $\nabla^{\mu} T^{(\mathrm{vac})}_{\mu\nu} \neq 0$, allowing the universe to expand or contract in space, depending on its dynamical state. This speaks in favor of cyclic cosmologies beyond the actual creation *ab nihilo* paradigm [23].

5. Weak Field Solutions

We now investigate the weak field limit of the Einstein-Maxwell equations by introducing a perturbative framework, and by solving the equations up to first order. The main purpose is to show once more that non-null fields produce self-coupling currents, and to compute the dependence of these currents of the gauge field to first order approximation. This will entail further discussion of the electrovacuum as a nonlinear optical medium.

We consider the perturbative parameter $\varepsilon = 8\pi G/\mu_0 c^4$. This allows to write the Einstein-Maxwell equations for the four potential W_μ as

$$\Box W_\mu - \nabla_\mu \nabla_\nu W^\nu + \varepsilon \mu_0 T_{\mu\sigma} W^\sigma = 0. \tag{35}$$

We now develop in the perturbative parameter ε all the relevant fields of the theory, including the four potential, and restricting all the results to first order[1]. We now write the four-potential, the metric tensor and the stress-energy tensor as

$$\begin{aligned} W_\mu &= A_\mu + \varepsilon B_\mu, && (36)\\ g_{\mu\nu} &= \eta_{\mu\nu} + \varepsilon h_{\mu\nu}, && (37)\\ \mu_0 T_{\mu\nu} &= t_{\mu\nu} + \varepsilon s_{\mu\nu}. && (38) \end{aligned}$$

Then, to zero order, Equation (35) yields the equation

$$\Box A_\mu - \partial_\mu \partial^\nu A_\nu = 0, \tag{39}$$

where now $\Box$ is the D'Alembertian in flat spacetime. If an appropriate flat spacetime Lorenz gauge is chosen $\partial^\nu W_\nu = 0$ for the electrodynamic fields, then we must have that all the perturbative terms satisfy $\partial^\nu A_\nu = \partial^\nu B_\nu = 0$. This can be demonstrated by expanding perturbatively the gauge transformation. This yields the basic equations for flat spacetime electrovacuum

$$\Box A_\mu = 0. \tag{40}$$

Thus, as long as the energy of the fields is weak enough, we can neglect spacetime curvature and consider transversal waves without self-impedance, as is commonly done in macroscopic optics and Maxwell classical electrodynamics. However, if the energy is high enough, what can arise as a self-focusing feedback process at small scales or at cosmological scales, we obtain further corrections and the assumption that light consists of transversal waves is not completely right [14].

To compute the first order correction of Maxwell electrodynamics in the weak field limit [24], we must consider the following relation $\delta^\mu_\rho = g^{\mu\nu} g_{\nu\rho}$ and, expanding in series the metric tensor according to Equation (37), we get the inverse metric $g^{\mu\nu} = \eta^{\mu\nu} - h^{\mu\nu}$. Concerning the determinant of the metric, it can be written to first order as $|g| = \varepsilon^{\mu\nu\gamma\rho} \eta_{\mu 0} \eta_{\nu 1} \eta_{\gamma 2} \eta_{\rho 3} = \varepsilon + \varepsilon h$, where is the traced perturbed metric tensor $h = h^\mu_\mu$. Using these two equations, we get the Laplacian

$$\nabla_\mu \nabla^\mu = \eta^{\mu\nu} \partial_\mu \partial_\nu - \varepsilon \partial_\mu \left(h^{\mu\nu} - \frac{1}{2} \eta^{\mu\nu} h \right) \partial_\nu - \varepsilon h^{\mu\nu} \partial_\mu \partial_\nu. \tag{41}$$

[1] The present identities are all written up to $O(\varepsilon^2)$, but the order approximation is omitted for simplicity in the notation.

Preprints.org (www.preprints.org) | NOT PEER-REVIEWED | Posted: 11 April 2025 doi:10.20944/preprints202504.0927.v1

As it is frequently done when studying weak fields [24], we introduce the deviatoric tensor $\bar{h}_{\mu\nu} = h^{\mu\nu} - \eta^{\mu\nu}h/2$, yielding

$$\nabla_\mu \nabla^\mu = \eta^{\mu\nu}\partial_\mu\partial_\nu - \epsilon\partial_\mu\bar{h}^{\mu\nu}\partial_\nu - \epsilon h^{\mu\nu}\partial_\mu\partial_\nu. \tag{42}$$

Using Equations (35)-(38), and expressing the Christoffel symbols as $\gamma_{\nu\sigma} = \epsilon\Gamma_{\nu\sigma}$ to first order, we get

$$\Box B_\mu + h^{\mu\nu}\partial_\mu\partial_\nu A_\mu - \partial_\mu\bar{h}^{\mu\nu}\partial_\nu A_\mu - \partial_\mu(\Gamma^\nu_{\nu\sigma}A^\sigma) + t_{\mu\sigma}A^\sigma = 0. \tag{43}$$

If we conveniently choose now as spatiotemporal gauge the Lorenz gauge $\partial_\mu\bar{h}^{\mu\nu} = 0$, we obtain

$$\Box B_\mu + h^{\mu\nu}\partial_\mu\partial_\nu A_\mu - \partial_\mu(\Gamma^\nu_{\nu\sigma}A^\sigma) + t_{\mu\sigma}A^\sigma = 0. \tag{44}$$

To express the solution directly in terms of the metric and the zeroth order electromagnetic potential A_μ, we first solve for $h_{\mu\nu}$ using Einstein's equation. For this purpose, we compute the symbols, that can be written in the form

$$\Gamma^\sigma_{\mu\nu} = \frac{1}{2}\eta^{\sigma\rho}(\partial_\nu h_{\mu\rho} + \partial_\mu h_{\rho\nu} - \partial_\rho h_{\mu\nu}), \tag{45}$$

yielding the contracted symbols $\Gamma^\mu_{\mu\sigma} = (\partial_\sigma h)/2$, and leading to the equation

$$\Box B_\mu - h^{\rho\sigma}\partial_\rho\partial_\sigma A_\mu - \partial_\mu(\partial_\sigma h A^\sigma)/2 + t_{\mu\sigma}A^\sigma = 0. \tag{46}$$

This equation can be written as a conventional Maxwell equation by defining the four-current density $J_\mu = h^{\rho\sigma}\partial_\rho\partial_\sigma A_\mu + \partial_\mu(\partial_\sigma h A^\sigma)/2 - t_{\mu\sigma}A^\sigma$, as follows

$$\Box B_\mu = J_\mu. \tag{47}$$

The weak zeroth-order electromagnetic waves carry energy, and this energy curves spacetime. Then, this curvature acts as a current that drives high-order components of the field. This can produce *self-resonances*, intensifying the fields in specific regions of spacetime. The ultimate limits to these resonant phenomena must be imposed by the nonlinear nature of these self-currents.

We now compute the Riemann tensor in perturbative series, which (absorbing the perturbative parameter) we define as follows

$$R^\sigma_{\rho\mu\nu} = \partial_\mu\Gamma^\sigma_{\nu\rho} - \partial_\nu\Gamma^\sigma_{\mu\rho}. \tag{48}$$

If we now consider the previous equations for the Levi-Civita connection, we get the first order Riemann tensor as

$$R^\sigma_{\rho\mu\nu} = \frac{1}{2}\eta^{\sigma\tau}(\partial_\mu\partial_\rho h_{\nu\tau} + \partial_\mu\partial_\nu h_{\tau\rho} - \partial_\mu\partial_\tau h_{\nu\rho} - \partial_\nu\partial_\rho h_{\mu\tau} - \partial_\nu\partial_\mu h_{\tau\rho} + \partial_\nu\partial_\tau h_{\mu\rho}), \tag{49}$$

which allows to compute the Ricci tensor

$$R_{\rho\nu} = \frac{1}{2}(\partial_\rho\partial_\sigma h^\sigma_\nu + \partial_\nu\partial_\sigma h^\sigma_\rho - \Box h_{\nu\rho} - \partial_\nu\partial_\rho h) \tag{50}$$

and this, finally, provides the scalar of curvature

$$R = \partial_\rho\partial_\sigma h^{\rho\sigma} - \Box h. \tag{51}$$

But now note that the first Rainich condition imposes $R = 0$, which is only possible if each component of the perturbative expansion is exactly zero. We now write down the Einstein's equations $R_{\mu\nu} - \eta_{\mu\nu}R/2 = t_{\mu\nu}$. Substitution of spacetime tensors in Einstein equation yields

$$\partial_\rho\partial_\sigma h^\sigma_\nu + \partial_\nu\partial_\sigma h^\sigma_\rho - \Box h_{\nu\rho} - \partial_\nu\partial_\rho h - \eta_{\mu\nu}\partial_\rho\partial_\sigma h^{\rho\sigma} + \eta_{\mu\nu}\Box h = 2t_{\mu\nu}. \tag{52}$$

Preprints.org (www.preprints.org) | NOT PEER-REVIEWED | Posted: 11 April 2025 doi:10.20944/preprints202504.0927.v1

Now, since we have the gauge condition on $\bar{h}_{\mu\nu}$, it can be proven that $\bar{h}_{\mu\nu} = h_{\mu\nu} - h\eta_{\mu\nu}/2$, implying that $\bar{h} = h - h\eta_{\mu\nu}\eta^{\mu\nu}/2$ and therefore $\bar{h} = -h$. Thus we can invert by considering $h_{\mu\nu} = \bar{h}_{\mu\nu} - \bar{h}\eta_{\mu\nu}/2$. This, in turn leads to $h_\mu^\nu = \bar{h}_\mu^\nu - \bar{h}\delta_\mu^\nu/2$. Replacing in Einstein's equation and expanding the parentheses, after some algebraic manipulations, we obtain

$$\partial_\rho\partial_\sigma\bar{h}_\nu^\sigma + \partial_\nu\partial_\sigma\bar{h}_\rho^\sigma - \Box\bar{h}_{\mu\nu} - \eta_{\mu\nu}\partial_\rho\partial_\sigma\bar{h}^{\rho\sigma} = 2t_{\mu\nu}. \tag{53}$$

Now, by remembering the gauge previously chosen, we get

$$\Box\bar{h}_{\mu\nu} = -2t_{\mu\nu}. \tag{54}$$

In this manner, we have obtained the expression of the metric computed from the electromagnetic stress energy tensor

$$\bar{h}_{\mu\nu} = -2\int \frac{t_{\mu\nu}(t_r, x' - x)}{|x - x'|} d^3x'. \tag{55}$$

Finally, once we compute $h_{\mu\nu}$ from these solutions, we can obtain the field

$$B_\mu = \int \frac{J_\mu(t_r, x' - x)}{|x - x'|} d^3x'. \tag{56}$$

This solves completely the entire problem to the first order. It remains to prove that the equation $\partial_\rho\partial_\sigma h^{\rho\sigma} - \Box h = 0$ is self-consistent. This equation implies $\partial_\rho\partial_\sigma\left(\bar{h}^{\rho\sigma} - \eta^{\rho\sigma}\bar{h}\right)/2 + \Box\bar{h} = 0$, which gives the equation $\partial_\rho\partial_\sigma\bar{h}^{\rho\sigma} + \Box\bar{h}/2 = 0$. But this certainly holds, since $\bar{h} = 0$ as a consequence of the first Rainich condition, and the Lorenz gauge. This implies that, in the Einstein-Maxwell setting with weak fields, we have the equivalence between the Lorenz gauges for the first order correction to the metric $\partial_\mu h^{\mu\nu} = 0$ and the gauge condition of its deviatoric tensor.

Note how the metric certainly obeys a Maxwell equation, what justifies the name gravitoelectromagnetism to the first order approximation of the theory. In fact, the Einstein-Maxwell equations imply that gravity has an electromagnetic origin [5]. Using the Green's function, we finally obtain the following equation of the metric tensor

$$h_{\mu\nu} = -2\int \frac{t_{\mu\nu}(t_r, x - x')}{|x - x'|} d^3x'. \tag{57}$$

Thus, in order to solve the Einstein-Maxwell equations in the weak field limit, we need to evaluate in the first place $t_{\mu\nu} = F_\mu^\tau F_{\tau\nu} - F^{\sigma\tau}F_{\sigma\tau}\eta_{\mu\nu}/4$, with the Maxwell tensor to zeroth order. Once we have the stress-energy tensor of electromagnetic waves, then solve the previous integral if possible, and finally compute the first order of the four-potential, recalling the current four-density

$$J_\mu = h^{\rho\sigma}\partial_\rho\partial_\sigma A_\mu - \mu_0 t_{\mu\sigma}A^\sigma, \tag{58}$$

which has been simplified taking into account the $h = 0$ condition. This result completes our calculations.

6. Gravitoelectromagnetic Waves

We now turn our attention to particularly simple solutions to the Einstein-Maxwell electrovacuum in the weak field limit. Specifically, we focus on electromagnetic pulses with an arbitrary shape. We show that there is an essential difference between null and non-null fields, i.e., the latter can produce sources and self-couple through spacetime curvature, while the former cannot. This gives preeminence to non-null solutions, since they can allow to create non-dispersive waves, in principle.

Preprints.org (www.preprints.org) | NOT PEER-REVIEWED | Posted: 11 April 2025 doi:10.20944/preprints202504.0927.v1

6.1. Null Fields

We begin by studying a null electromagnetic pulse with an arbitrary shape, but confined in certain region of the space. In the Lorenz gauge, we can write the solution to the zeroth order equation in the form

$$A_\mu(x) = a_\mu f(k \cdot x), \tag{59}$$

where $f(x) \to 0$ when $|x| \to \infty$, and a a spacelike vector. Again, in the Lorenz gauge we get $k^\mu a_\mu = 0$, and because the solution solves Maxwell's equations, we have the dispersion relation $k^\mu k_\mu = 0$. We can now compute the Faraday tensor to zero order as

$$F_{\mu\nu} = (k_\mu a_\nu - k_\nu a_\mu) f'(k \cdot x). \tag{60}$$

This implies that the solutions are certainly null, since we have $P = F_{\mu\nu}F^{\mu\nu}/2 = (k_\mu a_\nu - k_\nu a_\mu)(k^\mu a^\nu - k^\nu a^\mu)[f'(k \cdot x)]^2/2 = 0$. Thus the zeroth order solutions posses no inertia, as it is frequently attributed to the photon. Using Equation (60), we can compute the stress-energy tensor to zeroth order, which is

$$t_{\mu\nu} = F_\mu^\tau F_{\tau\nu} = -a_\tau a^\tau k_\nu k_\mu [f'(k \cdot x)]^2. \tag{61}$$

The latter equation allows to compute the metric. If we introduce the density function $\rho(x) = f'^2(x)$, and define the scalar field

$$\Phi(x,t) = \int \frac{\rho(k_0 t_r, kx')}{|x - x'|} d^3x'. \tag{62}$$

We appreciate that the pulse becomes the charge density of a scalar potential. The result is that the metric of spacetime is

$$h_{\mu\nu}(x,t) = 2a^2 \Phi(x,t) k_\nu k_\mu. \tag{63}$$

Note that the metric is symmetric, as expected. From this *non-stationary* spacetime metric we can verify that, as previously claimed, we certainly have $h = 0$. We can now compute the current and obtain, using $a_\mu k^\mu = k_\mu k^\mu = 0$, that its value is zero for planar electromagnetic waves $J_\mu = 0$, what means that B_μ is again a solution of Maxwell equations.

This result is of great relevance, since it shows that null solutions cannot self-couple and imply that weak planar waves are transversal, at least to first order. It is worth asking if higher order corrections can stabilize these solutions, preventing or, at least, slowing their dispersion. However, as we show below, there exist other solutions, as for example more complicated wave-packet solutions, which are not necessary null (e. g. electromagnetic knots [25]). Thus, the first order component is also a plane wave $B_\mu(x) = b_\mu g(q \cdot x)$, and the principle of superposition holds for null solutions to first order. Importantly, we note that it is the electromagnetic waves that produce the gravitational waves, and that the concept of a purely gravitational wave is non-physical [5], since for that purpose one would have to empty the electrovacuum, which is impossible, as we know from the Casimir effect [26].

We compute the Rainich conditions. The Ricci tensor, according to Equations (50) and (63), can be written as

$$R_{\rho\nu} = a^2 (k^\sigma k_\nu \partial_\rho \partial_\sigma \Phi(x,t) + k^\sigma k_\rho \partial_\nu \partial_\sigma \Phi(x,t) - k_\rho k_\nu \Box \Phi(x,t)). \tag{64}$$

Since $\partial_\mu h^{\mu\nu} = 0$, we should have the relation

$$D_k \Phi(x,t) = 0, \tag{65}$$

which holds, indeed, since $f(k \cdot x)$. This, in turn, yields

$$R_{\rho\nu} = -\varepsilon a^2 k_\rho k_\nu \Box \Phi(x,t), \tag{66}$$

Preprints.org (www.preprints.org) | NOT PEER-REVIEWED | Posted: 11 April 2025 doi:10.20944/preprints202504.0927.v1

where the perturbative parameter has been reintroduced. More accurately, since $\Box\Phi(x,t) = (f'(x))^2$ by the very definition of Φ, for R_{00} we get

$$R_{00} = -\varepsilon a^2 k_0^2 [f'(k \cdot x)]^2, \tag{67}$$

thus we have positive energy density $T_{00} > 0$ in conformity to the second Rainich condition (note that a is space-like and $a^2 < 0$ in the present signature). On the other hand, the second order scalar yields

$$R_{\rho\nu} R^{\rho\nu} = 2\varepsilon^2 a^4 (k^\sigma k^\nu \partial_\nu \partial_\sigma \Phi(x,t))^2 = 2\varepsilon^2 a^4 (D_k^2 \Phi(x,t))^2 = 0. \tag{68}$$

The square of the stress energy tensor is effectively degenerate, and the complexion is not well defined for null Riemann tensors. Therefore, the agenda proposed by Misner and Wheeler [3] cannot be carried out in this case.

6.2. Non-Null Fields

We can also consider more complicated wave-packets, by assuming that f is some harmonic function and using Fourier's theorem. Starting with the four potential

$$A_\mu(x) = \int a_\mu(k) f(k \cdot x) d^4k. \tag{69}$$

Then, the Faraday tensor is written as

$$F_{\mu\nu} = \int (k_\mu a_\nu(k) - k_\nu a_\mu(k)) f'(k \cdot x) d^4k. \tag{70}$$

Now, we can consider the invariant

$$P = \frac{1}{2} \int ((a' \cdot a)(k \cdot k') - (k \cdot a')(a \cdot k')) f'(k \cdot x) f'(k' \cdot x) d^4k d^4k'. \tag{71}$$

Because now the wave packet is non-null in general, they can present inertia, what implies that we can get more interesting physical phenomena. The stress energy tensor is far more complicated, indeed. It can be written as

$$\begin{aligned} t_{\mu\nu} = - & \int \left(a'_\nu a_\mu + \frac{1}{4}(a' \cdot a)\eta_{\mu\nu} \right) (k \cdot k') f'(k \cdot x) f'(k' \cdot x) d^4k d^4k' \\ - & \int \left(k'_\nu k_\mu + \frac{1}{4}(k' \cdot k)\eta_{\mu\nu} \right) (a \cdot a') f'(k \cdot x) f'(k' \cdot x) d^4k d^4k' \\ + & \int \left(k_\mu a'_\nu + \frac{1}{4}(a' \cdot k)\eta_{\mu\nu} \right) (a \cdot k') f'(k \cdot x) f'(k' \cdot x) d^4k d^4k' \\ + & \int \left(k'_\mu a_\nu + \frac{1}{4}(k' \cdot a)\eta_{\mu\nu} \right) (k \cdot a') f'(k \cdot x) f'(k' \cdot x) d^4k d^4k'. \end{aligned} \tag{72}$$

If we now define the scalar potential

$$\Phi_{k,k'}(x) = 2 \int \frac{f'(k_0 t_r, k \cdot x') f'(k_0 t_r, k' \cdot x')}{|x - x'|} d^3x', \tag{73}$$

the metric inherits four components as well

$$\begin{aligned} h_{\mu\nu}(x,t) = & \int \left(a'_\nu a_\mu + \frac{1}{4}(a' \cdot a)\eta_{\mu\nu} \right) (k \cdot k')\Phi_{k,k'}(x) d^4k d^4k' \\ + & \int \left(k'_\nu k_\mu + \frac{1}{4}(k' \cdot k)\eta_{\mu\nu} \right) (a \cdot a')\Phi_{k,k'}(x) d^4k d^4k' \\ - & \int \left(k_\mu a'_\nu + \frac{1}{4}(a' \cdot k)\eta_{\mu\nu} \right) (a \cdot k')\Phi_{k,k'}(x) d^4k d^4k' \\ - & \int \left(k'_\mu a_\nu + \frac{1}{4}(k' \cdot a)\eta_{\mu\nu} \right) (k \cdot a')\Phi_{k,k'}(x) d^4k d^4k'. \end{aligned} \tag{74}$$

This metric is far more complicated, but topologically interesting cases can be explored (e.g. the curved spacetime of an electromagnetic knot). In fact, now we have

$$J_\mu(x,t) = -\int a_\mu(k) h^{\rho\sigma}(x,t) k_\rho k_\sigma f(k \cdot x) d^4k - \mu_0 \int t_{\mu\sigma}(x,t) a^\sigma(k) f(k \cdot x) d^4k. \tag{75}$$

where the harmonic nature of f has been used. This means that when we have wave packets of light in spacetime, there is a self-current that can act as a source or sink of field. These equations suggest that, from the point of view of a background planar observer, the fields can diverge or converge, as if there existed sources, even though there are none. Importantly, we must stress that these sources extend all over the space, and thus we can have typical phenomena appearing in materials, as for example longitudinal wave components [27].

7. Discussion and Conclusions

We have derived the fundamental equations describing the relativistic dynamics of vector and scalar bosons from Einstein-Maxwell electrodynamics. The question arises about the possibility to extend the present formalism to fermions. Certainly, a different gauge can be proposed transforming Equation (23) into a Dirac-like equation in the form $\gamma_\mu^{\rho\nu} \nabla_\rho A_\mu - (mc/\hbar)\delta_\mu^\rho A_\rho = 0$. However, we shall pursue this goal more carefully in forthcoming works, since complications in the resulting gauge require a more detailed analysis, and it is not clear to the author to what extent the tetrad formalism in the Newman-Penrose form [28,29] might be necessary to exactly identify the Ricci's rotation coefficients with Dirac γ^μ matrices.

It is important to stress the fact that the principle of superposition does not rigorously hold in the Einstein-Maxwell equations due to the spacetime curvature, as originally claimed by Y. Rainich [2]. However, in some circumstances, whenever the *Proca gauge* transformation exists, an appropriate gauge fixing allows to decouple fields in curved space (as long as we are allowed to neglect the metric in the Laplace-Beltrami operator, which might be appropriate under many circumstances, but not in general). In general, the Einstein-Maxwell equations suggest that light can bend its own rays through the curvature of spacetime, producing *self-focusing* and *self-diverging* effects. In turn, this capacity of light to act as its own lens through spacetime curvature might concentrate great amounts of energy in spacetime. On the other hand, more energy implies more curvature; thus we should not reject the possibility that feedback generates unexpected high intrinsic curvature in spacetime [2].

A fundamental reflection concerns the nature of the photon and bosons in general. If nonlinear effects can prevent three-dimensional electromagnetic wave-packets to disperse, they might allow to create stable perturbations traveling in spacetime (i. e. photons and other type of solitary waves), as opposed to wave-packets in three-dimensional flat spacetime, which tend to disperse. This would explain photons as wave-packet solutions of Einstein-Maxwell equations, probably related to the so-called geons [30]. The present formalism can be used to study the stability of pulses that in flat spacetime tend to disperse. This could explain bosons as optical wave-packets, *self-sustained* by spacetime curvature. Non-dispersive pulses require specific conditions in the source current J_μ, and it

Preprints.org (www.preprints.org) | NOT PEER-REVIEWED | Posted: 11 April 2025 doi:10.20944/preprints202504.0927.v1

is well known that some types of nonlinear materials can counterbalance dispersion effects. The same might be true for the electrovacuum [31].

To conclude, the fact that the Einstein-Maxwell equations are nonlinear suggests the possibility that fundamental particles are electromagnetic solitons [15]. In particular, and bearing in mind that the wave-particle duality has been explained theoretically in terms of classical *zitterbewegung* [20], the author would like to propose that the electron could be a spinorial breathing solution to the Einstein-Maxwell equations. We note that any non-null solution to the Einstein-Maxwell equations *presents inertia*, which comes from the self-coupling of the electrodynamic fields. This conforms with recent findings on the electromagnetic origin of mass [5]. More simply put, inertia might be a force of self-induction due to Faraday's law and self-interactions of light through spacetime curvature [5]. The electromagnetic origin of inertia is perfectly compatible with classical electrodynamics and the principle of relativity, and misunderstandings frequently attributed to the 4/3 problem regarding an incompatibility between electromagnetic mass and the principle of special relativity have been rigorously clarified [32]. Here we have shown that this is even more pertinent in the framework of general relativity. Summarizing, the present work suggests that the remaining properties of fundamental particles, charge and spin, should also be related to the geometric nature of the gauge field configurations of the electrodynamic field that constitutes them [15].

Acknowledgments: The author is profoundly indebted to the "Quantum mechanics foundations & nature of time" group and also to the "Zitter Institute" (https://www.zitter-institute.org) for endless and fruitful discussions about the possibility of electrodynamic gauge fields and solitons as a foundation of particle physics. He also would like to thank professor Jose Luis Trueba Santander for deep discussions on the concept of electromagnetic knot and topological electromagnetism.

Appendix A. Gauge Transformation

In this section we write explicitly the gauge transformation of the self-coupling current appearing in Equation (23) of the manuscript. We have the gauge transformation

$$\begin{aligned}
\chi^{\nu}_{\mu} A_{\nu} = \; & F_{\mu\tau} g^{\tau\rho} g^{\nu\sigma} F_{\rho\sigma} A_{\nu} \\
= \; & \left(F'_{\mu\tau} + \xi_{\nu} \nabla'^{\nu} F'_{\mu\tau} + \nabla'_{\mu} \xi^{\nu} F'_{\nu\tau} + \nabla'_{\tau} \xi^{\nu} F'_{\mu\nu} \right) \left(g'^{\tau\rho} + \nabla^{\tau} \xi^{\rho} + \nabla^{\rho} \xi^{\tau} \right) \\
\cdot \; & \left(F'_{\rho\sigma} + \xi_{\nu} \nabla'^{\nu} F'_{\rho\sigma} + \nabla'_{\rho} \xi^{\nu} F'_{\nu\sigma} + \nabla'_{\sigma} \xi^{\nu} F'_{\rho\nu} \right) \left(g'^{\nu\sigma} + \nabla^{\nu} \xi^{\sigma} + \nabla^{\sigma} \xi^{\nu} \right) \\
\cdot \; & \left(A'_{\nu} + \xi^{\lambda} \nabla'_{\lambda} A'_{\nu} + A'^{\lambda} \nabla'_{\lambda} \xi_{\nu} + \xi^{\lambda} \nabla'_{\lambda} \nabla'_{\nu} \Lambda + \nabla'^{\lambda} \Lambda \nabla'_{\lambda} \xi_{\nu} \right).
\end{aligned} \tag{A1}$$

If we now carefully group all the terms with the same number of derivatives, we can set the self-coupling current to zero, as long as the following differential equation holds

$$\begin{aligned}
U_{\mu}(\xi) = \; & V^{\rho\sigma}_{\mu}(\xi) \nabla_{\rho} \xi_{\sigma} + W^{\tau\rho\nu\sigma}_{\mu}(\xi) \nabla_{\tau} \xi_{\rho} \nabla_{\nu} \xi_{\sigma} + \\
& X^{\tau\rho\alpha\sigma\lambda\nu}_{\mu}(\xi) \nabla_{\tau} \xi_{\rho} \nabla_{\alpha} \xi_{\sigma} \nabla_{\lambda} \xi_{\nu} + Y^{\tau\rho\alpha\sigma\lambda\nu\beta\eta}_{\mu}(\xi) \nabla_{\tau} \xi_{\rho} \nabla_{\alpha} \xi_{\sigma} \nabla_{\lambda} \xi_{\nu} \nabla_{\beta} \xi_{\eta} + \\
& Z^{\tau\rho\alpha\sigma\lambda\nu\beta\eta\gamma\zeta}_{\mu}(\xi) \nabla_{\tau} \xi_{\rho} \nabla_{\alpha} \xi_{\sigma} \nabla_{\lambda} \xi_{\nu} \nabla_{\beta} \xi_{\eta} \nabla_{\gamma} \xi_{\zeta},
\end{aligned} \tag{A2}$$

where the tensors $Z^{\tau\rho\alpha\sigma\lambda\nu\beta\eta\gamma\zeta}_{\mu}(\xi)$, $Y^{\tau\rho\alpha\sigma\lambda\nu\beta\eta}_{\mu}(\xi)$, $X^{\tau\rho\alpha\sigma\lambda\nu}_{\mu}(\xi)$, $W^{\tau\rho\nu\sigma}_{\mu}(\xi)$, $V^{\rho\sigma}_{\mu}(\xi)$ and $U_{\mu}(\xi)$ are polynomials of the gauge field ξ. These tensors depend on first order covariant derivatives of the electromagnetic gauge field A_{μ}, as well. Just as an example, the tensor $U_{\mu}(\xi)$ is equal to

$$U_{\mu}(\xi) = \left(F'_{\mu\tau} + \xi_{\nu} \nabla'^{\nu} F'_{\mu\tau} \right) g'^{\tau\rho} \left(F'_{\rho\sigma} + \xi_{\nu} \nabla'^{\nu} F'_{\rho\sigma} \right) g'^{\nu\sigma} \left(A'_{\nu} + \xi^{\lambda} \nabla'_{\lambda} A'_{\nu} + \xi^{\lambda} \nabla'_{\lambda} \nabla'_{\nu} \Lambda \right). \tag{A3}$$

The remaining tensors are computed in a similar fashion, but do not provide any further insight into our argument, and are thus omitted because they are intrincante. In this manner, we can dispense with the self-coupling tensor, as long as the existence and uniqueness theorems of solutions to the system of four partial differential equations for the gauge appearing in Equation (A2) hold. Some fundamental considerations are deserved in this regard. Firstly, we note that this is a set of four nonlinear first-order coupled partial differential equations. The derivatives appear coupled in polynomial form. By the implicit function theorem [33], except for a hypersurface in tangent space, we can express the four coupled equations as four decoupled equations in which the derivatives of the spatiotemporal gauge with respect to time $\partial\xi_\mu/\partial t$ are written as a nonlinear function of the remaining spatial derivatives. Then, the nonlinear Cauchy-Kovalevskaya theorem [34] ensures the uniqueness and existence of solutions in some domain around the origin of tangent space, assuming that the A'_μ potentials are analytic functions. Thus, these solutions must exist in specific regions of spacetime, as we know from experience. Whenever these conditions do not hold, we cannot expect the related wave equation to represent a particular field configuration and, in this manner, different gauge configurations might yield different types of fields, depending on the existence of solutions of a particular gauge. This fact might explain the convenience of introducing many different fields in fundamental theories of particle physics.

References

1. Stephani, H.; Kramer, D.; MacCallum, M.; Hoenselaers, C.; Herlt, E. *Exact Solutions of Einstein's Field Equations*, 2 ed.; Cambridge Monographs on Mathematical Physics, Cambridge University Press, 2003.
2. Rainich, G.Y. Electrodynamics in the general relativity theory. *Transactions of the american Mathematical Society* **1925**, *27*, 106–136.
3. Misner, C.W.; Wheeler, J.A. Classical physics as geometry. *Annals of physics* **1957**, *2*, 525–603.
4. Wien, W. Ueber die Möglichkeit einer elektromagnetischen Begründung der Mechanik. *Annalen der Physik* **1901**, *310*, 501–513.
5. López, Á.G. On an electrodynamic origin of quantum fluctuations. *Nonlinear Dynamics* **2020**, *102*, 621–634.
6. López, A.G. Stability analysis of the uniform motion of electrodynamic bodies. *Physica Scripta* **2020**, *96*, 015506.
7. López, Á.G. *The Electrodynamic Origin of the Wave-Particle Duality*; Springer, 2022; pp. 1043–1055.
8. Bush, J.W.; Oza, A.U. Hydrodynamic quantum analogs. *Reports on Progress in Physics* **2020**, *84*, 017001.
9. Higgs, P.W. Broken symmetries and the masses of gauge bosons. *Physical Review Letters* **1964**, *13*, 508.
10. Anderson, P.W. Plasmons, gauge invariance, and mass. *Physical Review* **1963**, *130*, 439.
11. Belinfante, F.J. On the current and the density of the electric charge, the energy, the linear momentum and the angular momentum of arbitrary fields. *Physica* **1940**, *7*, 449–474.
12. Poincaré, H. Sur les résidus des intégrales doubles. *Acta Mathematica* **1887**, *9*, 321–380.
13. Dirac, P.A.M. Quantised singularities in the electromagnetic field. *Proceedings of the Royal Society of London. Series A, Containing Papers of a Mathematical and Physical Character* **1931**, *133*, 60–72.
14. Alqrayan, W.A.; Arbab, A.I. A connection between massive electrodynamics and the Einstein-Maxwell equations. *Physica Scripta* **2024**, *99*, 115526.
15. Faber, M. Particles as stable topological solitons. In Proceedings of the Journal of Physics: Conference Series. IOP Publishing, 2012, Vol. 361, p. 012022.
16. Hadley, M.J. Electrodynamics and time orientability. *International Journal of Theoretical Physics* **2017**, *56*, 2007–2018.
17. Santos, W.C.d. Introduction to Einstein-Maxwell equations and the Rainich conditions. *arXiv preprint arXiv:1606.08527* **2016**.
18. Bialynicki-Birula, I. Photon wave function. *Progress in Optics* **1996**, *36*, 245–294.
19. Proca, A. Sur la théorie ondulatoire des électrons positifs et négatifs. *Journal de Physique et le Radium* **1936**, *7*, 347–353.
20. López, A.G. Contextuality and correlation loopholes are equivalent. *Europhysics Letters* **2023**, *142*, 20001.
21. Bohm, D. A Suggested Interpretation of the Quantum Theory in Terms of "Hidden" Variables. I. *Phys. Rev.* **1952**, *85*, 166–179.
22. Schrödinger, E. Quantisierung als Eigenwertproblem. *Annalen der Physik* **1926**, *384*, 361–376.
23. Penrose, R. *Cycles of time: an extraordinary new view of the universe*; Random House, 2013.
24. Carroll, S.M. *Spacetime and geometry*; Cambridge University Press, 2019.
25. Ranada, A.F.; Trueba, J. Electromagnetic knots. *Physics Letters A* **1995**, *202*, 337–342.
26. Casimir, H.B.G.; Polder, D. The Influence of Retardation on the London-van der Waals Forces. *Phys. Rev.* **1948**, *73*, 360–372.

 doi:10.20944/preprints202504.0927.v1

27. Sakhno, D.; Koreshin, E.; Belov, P.A. Longitudinal electromagnetic waves with extremely short wavelength. *Phys. Rev. B* **2021**, *104*, L100304.
28. Newman, E.; Penrose, R. An Approach to Gravitational Radiation by a Method of Spin Coefficients. *Journal of Mathematical Physics* **1962**, *3*, 566–578.
29. Chandrasekhar, S. *The mathematical theory of black holes*; Vol. 69, Oxford University Press, 1998.
30. Wheeler, J.A. Geons. *Physical Review* **1955**, *97*, 511.
31. Gullu, I.; Mazharimousavi, S.H. Double-logarithmic nonlinear electrodynamics. *Physica Scripta* **2021**, *96*, 045217.
32. Rohrlich, F. *Classical charged particles*; CRC Press, 2020.
33. de Oliveira, O. The Implicit and Inverse Function Theorems: Easy Proofs. *Real Analysis Exchange* **2013**, *39*, 207 – 218.
34. Meredith, L. On the Cauchy-Kowalevski theorem for analytic nonlinear partial differential equations **2018**.