

Unification of Electromagnetism and Gravitation

Raymond J. Beach

Lawrence Livermore National Laboratory, L-465, 7000 East Avenue, Livermore,
CA 94551

E-mail: beach2@llnl.gov

Abstract

Using four field equations, a recently proposed classical field theory that covers the phenomenology of classical physics at the level of the Maxwell and Einstein Field Equations (M&EFEs) but then goes further by unifying electromagnetic and gravitational phenomena in a fundamentally new way is reviewed. The four fundamental equations are shown to determine the dynamic fields of the theory in a manner that is logically consistent with the requirements of general covariance. Developing particle-like soliton solutions and then radiative solutions for electromagnetic and gravitational waves, the predictions of the theory are shown to be consistent with those of the M&EFEs. The same solutions are then used to illustrate the unification that the theory brings electromagnetic and gravitational phenomena by, for example, uniting electromagnetic radiation with a specific class of underlying gravitational radiation. A description is given for quantizing the mass, charge and angular momentum of particle-like soliton solutions, a situation that is uniquely possible with the new theory because the mass density and charge density are treated as dynamic fields and not introduced as external parameters. A cosmological solution corresponding to a homogeneous and isotropic universe is investigated. Finally, a description of antimatter is naturally accommodated by the theory and leads to definite predictions regarding the interactions of matter and antimatter with gravitational fields. (*key words: gravitation, general relativity, electromagnetism, unified field theory*)

Introduction

Assuming the geometry of nature is Riemannian with four dimensions, the following four equations have been proposed as an alternative description of classical physics at the level of the M&EFEsⁱ

$$F_{\mu\nu;\kappa} = a^\lambda R_{\lambda\kappa\mu\nu} \quad (1)$$

$$a^\lambda R_\lambda{}^\nu = \rho_c u^\nu \quad (2)$$

$$u^\lambda u_\lambda = -1 \quad (3)$$

$$\left(\rho_m u^\mu u^\nu + F^\mu{}_\lambda F^{\nu\lambda} - \frac{1}{4} g^{\mu\nu} F^{\rho\sigma} F_{\rho\sigma} \right)_{;\nu} = 0 \quad (4)$$

Equation (1) couples the Maxwell tensor $F_{\mu\nu}$ to the Riemann-Christoffel curvature tensor $R_{\lambda\kappa\mu\nu}$. Equations (2) couples the Ricci Tensor R_{λ}^{ν} to the coulombic current density $\rho_c u^{\nu}$. Both equations (1) and (2) are new, as is the vector field a^{λ} that appears in them and serves to couple gravitational to electromagnetic phenomena. Equation (3) normalizes the four-velocity vector field u^{λ} that describes the motion of both the charge density field, ρ_c , and the mass density field, ρ_m , which are assumed to be comoving. Equation (4) describes the conservation of energy and momentum for a particular choice of stress-energy tensor. Much of the discussion that follows will be focused on describing solutions to these equations and demonstrating that such solutions are consistent with those of the classical M&EFEs but then go further by unifying electromagnetic and gravitational phenomena. Taken together, the four field equations are used to axiomatically build up a description of nature in terms of the 6 dynamic fields described in Table I.

Table I. Dynamic fields

Field	Description	Number of components
$g_{\mu\nu}$	Metric tensor	10
$F_{\mu\nu}$	Maxwell tensor	6
u^{λ}	Four-velocity	4
a^{λ}	Four-vector with no counterpart in the classical M&EFEs	4
ρ_c	Charge density	1
ρ_m	Mass density	1
Total number of independent field components		26

An outline of the paper is as follows: After reviewing the development of the theory, a description of classical physics based on the theory is presented which agrees with the conventionally accepted descriptions given by the M&EFEs. Importantly, it is shown that the 26 independent field components listed in Table I are underdetermined by 22 independent equations as required by general covariance, the four remaining degrees of freedom representing the freedom of choice in coordinate system. Using a previously published particle-like soliton solution of the theory, it will be established that the gravitational and electric fields of a spherically symmetric charged particle agree with the fields predicted by the M&EFEs. Unique to the new theory is the view that the source terms of both the electromagnetic and gravitational fields, ρ_c and ρ_m , respectively, are themselves dynamic fields. This departs from the M&EFEs in which both mass and charge are viewed as externally defined parameters, and provides a pathway in the new theory for quantizing the mass, charge and angular momentum of particle-

like solutions. Next, correcting an error in the original publication describing the new theory,ⁱ a detailed description of electromagnetic and gravitational radiation is developed that emphasizes that while the new theory's descriptions are consistent with those of the M&EFEs, the new theory goes further by explaining the undulations in the electromagnetic field amplitudes as due to undulations of the space-time metric itself, intimately connecting electromagnetic radiation to an underlying gravitational radiation. Next, a cosmological solution corresponding to a homogeneous and isotropic universe is investigated, and the analogue of the Friedmann equations describing constraints placed on the scale parameter of the metric are derived. Finally, a discussion of symmetries of the field equations (1) through (4) is used to motivate a description of antimatter and its behavior in a gravitational field.

In this paper geometric units are used throughout along with a metric tensor having signature $[+,+,+,-]$. Commas before tensor indices denote ordinary derivatives while semicolons denote covariant derivatives. Spatial indices run from 1 to 3, with 4 the time index. For the definitions of the Riemann-Christoffel curvature tensor and the Ricci tensor, the conventions used by Weinberg are followed.ⁱⁱ

Maxwell's equations from $F_{\mu\nu;\kappa} = a^\lambda R_{\lambda\kappa\mu\nu}$ and $\rho_c u^\nu = a^\lambda R_\lambda^\nu$

Equations (1) and (2) which relate the Maxwell tensor derivatives and charge current density, respectively, to the Riemann-Christoffel curvature tensor, taken together with the properties of the Riemann-Christoffel curvature tensor are the fundamental relationships from which all of Maxwell's equations follow. The derivation of Maxwell's homogenous equation begins with the first Bianchi identity

$$R_{\lambda\kappa\mu\nu} + R_{\lambda\mu\nu\kappa} + R_{\lambda\nu\kappa\mu} = 0 . \quad (5)$$

Contracting (5) with a^λ gives,

$$a^\lambda R_{\lambda\kappa\mu\nu} + a^\lambda R_{\lambda\mu\nu\kappa} + a^\lambda R_{\lambda\nu\kappa\mu} = 0 . \quad (6)$$

Using (1) to substitute $F_{\mu\nu;\kappa}$ for $a^\lambda R_{\lambda\kappa\mu\nu}$ in (6) leads to

$$F_{\mu\nu;\kappa} + F_{\nu\kappa;\mu} + F_{\kappa\mu;\nu} = 0 , \quad (7)$$

Maxwell's homogenous equation. Maxwell's inhomogeneous equation follows from (1),

$F_{\mu\nu;\kappa} = a^\lambda R_{\lambda\kappa\mu\nu}$, by contracting its μ and κ indices

$$F^{\mu\nu}{}_{;\mu} = -a^\lambda R_\lambda^\nu \quad (8)$$

and making the connection between the RHS of (8) and the coulombic current density $\rho_c u^\nu$ using (2), $a^\lambda R_\lambda^\nu = \rho_c u^\nu$. Combining (8) and (2) then gives the conventional Maxwell-Einstein version of Maxwell's inhomogeneous equation

$$F^{\mu\nu}{}_{;\mu} = -\rho_c u^\nu . \quad (9)$$

Because $F^{\mu\nu}$ is antisymmetric, $F^{\mu\nu}{}_{;\mu;\nu} = 0$ is forced, which in turn forces the coulombic charge density to be a conserved quantity

$$(\rho_c u^\nu)_{;\nu} = (a^\lambda R_\lambda^\nu)_{;\nu} = 0 . \quad (10)$$

Using equations (2) and (3), the coulombic charge density can be solved for directly in terms of the Ricci tensor, a^λ and u^ν

$$\rho_c = \begin{cases} \pm \sqrt{a^\lambda R_\lambda^\nu a^\sigma R_{\sigma\nu}} \\ or \\ -a^\lambda R_\lambda^\nu u_\nu \end{cases} . \quad (11)$$

In the forgoing development, only equations (1), (2) and (3) are fundamental to the new theory. Maxwell's equations (7) and (8), the conservation of the coulombic charge density (10) and the solution for the coulombic charge density (11) are all consequences of (1), (2) and (3), and the properties of the Riemann-Christoffel curvature tensor.

One of the new pieces of physics in the foregoing development is the introduction of the vector field a^λ , a vector field that has no counterpart in the conventionally accepted development of classical physics but here serves to both couple the Maxwell tensor to the metric tensor through (1), and the charge density to the metric tensor through (2). Much of the analysis and discussion that follows will be focused on the impact of a^λ and how it drives the development of a classical field theory that encompasses the physics covered by the M&EFEs.

A classical field theory that unifies electromagnetism and gravitation with their sources

Equations (1), (2) and (3) provide a basis for deriving Maxwell's homogenous and inhomogeneous equations in curved space-time. Taking the source terms of the gravitational and electromagnetic fields, ρ_m and ρ_c , respectively, as dynamic fields to be solved for, a classical field theory of gravitation and electromagnetism that is logically consistent with the requirements of general covariance is possible. For a theory to be logically consistent with the requirements of general covariance, the N dynamical field components of the theory must be underdetermined by N-4 independent equations, the remaining 4 degrees of freedom representing the freedom in the

choice of coordinate system. Table I lists the six dynamic fields of the theory along with the number of independent components that comprise each field, yielding a total of 26 independent field components across the six fields. I now consider the last of the theory's fundamental equations (4), the energy and momentum conservation equation

$$\left(\rho_m u^\mu u^\nu + F^\mu{}_\lambda F^{\nu\lambda} - \frac{1}{4} g^{\mu\nu} F^{\rho\sigma} F_{\rho\sigma} \right)_{; \nu} = 0 . \quad (4)$$

The specific form of the stress-energy tensor in (4) ensures both that ρ_m is conserved and that there is a Lorentz force law. These two dependent equations are derived by first contracting (4) with u_μ which leads to the conservation of mass

$$\left(\rho_m u^\nu \right)_{; \nu} = 0 , \quad (12)$$

and then combining (4) and (12), which leads to the Lorentz force law

$$\rho_m \frac{Du^\mu}{D\tau} = \rho_c u^\lambda F^\mu{}_\lambda \quad (13)$$

where $\frac{Du^\lambda}{D\tau} \equiv u^\lambda{}_{; \sigma} u^\sigma$. A complete derivation of (12) and (13) is given in the Appendix. Table II collects and summarizes the four fundamental equations of the new theory along with the number of components of each equation.

Table II. Fundamental equations

Equation	Equation number in text	Number of components
$F_{\mu\nu;\kappa} = a^\lambda R_{\lambda\kappa\mu\nu}$	(1)	24
$a^\lambda R_\lambda{}^\nu = \rho_c u^\nu$	(2)	4
$u^\lambda u_\lambda = -1$	(3)	1
$\left(\rho_m u^\mu u^\nu + F^\mu{}_\lambda F^{\nu\lambda} - \frac{1}{4} g^{\mu\nu} F^{\rho\sigma} F_{\rho\sigma} \right)_{; \nu} = 0$	(4)	4
Total number of equations		33

At this point, the total number of fundamental equations at 33 is greater than the total number of field components at 26. This situation over determines the 26 independent field components in Table I and so is not compatible with the requirements of general covariance. The remedy to this situation is to note that not all the 33 equations listed in Table II are independent. As already noted, dependent constraint equations can be derived from the equations listed in Table II and the

properties of the Riemann-Christoffel curvature tensor. Table III collects these dependent constraint equations along with a brief description of their derivation.

Table III. Dependent equations

Equation	Equation number in text	Derivation	Number of components
$F_{\mu\nu;\kappa} + F_{\nu\kappa;\mu} + F_{\kappa\mu;\nu} = 0$	(7)	(1) and $R_{\lambda\kappa\mu\nu} + R_{\lambda\mu\nu\kappa} + R_{\lambda\nu\kappa\mu} = 0$	4
$(\rho_c u^\nu)_{;\nu} = (a^\lambda R_\lambda^\nu)_{;\nu} = 0$	(10)	(1) and (2)	1
$\rho_c = \begin{cases} \pm\sqrt{a^\lambda R_\lambda^\nu a^\sigma R_{\sigma\nu}} \\ or \\ -a^\lambda R_\lambda^\nu u_\nu \end{cases}$	(11)	(2) and (3)	1
$(\rho_m u^\nu)_{;\nu} = 0$	(12)	(4), (3) and (7)	1
$\rho_m \frac{Du^\mu}{D\tau} = \rho_c u^\lambda F^\mu{}_\lambda$	(13)	(4), (3) and (12)	4
Total number of equations			11

The 11 dependent constraint equations listed in Table III reduces the number of independent equations of the theory from the 33 to 22. These 22 remaining independent equations then do satisfy the requirements for a logically consistent and generally covariant theory for the 26 independent field components listed in Table I.

Three important symmetries of the fundamental equations listed in Table II are noted here. The first of these symmetries corresponds to charge-conjugation

$$\begin{pmatrix} u^\lambda \\ a^\lambda \\ F^{\mu\nu} \\ g_{\mu\nu} \\ \rho_c \\ \rho_m \end{pmatrix} \rightarrow \begin{pmatrix} u^\lambda \\ -a^\lambda \\ -F^{\mu\nu} \\ g_{\mu\nu} \\ -\rho_c \\ \rho_m \end{pmatrix}, \quad (14)$$

the second symmetry corresponds to matter-antimatter transformation

$$\begin{pmatrix} u^\lambda \\ a^\lambda \\ F^{\mu\nu} \\ g_{\mu\nu} \\ \rho_c \\ \rho_m \end{pmatrix} \rightarrow \begin{pmatrix} -u^\lambda \\ -a^\lambda \\ -F^{\mu\nu} \\ g_{\mu\nu} \\ \rho_c \\ \rho_m \end{pmatrix}, \quad (15)$$

and the third symmetry is the product of the first two

$$\begin{pmatrix} u^\lambda \\ a^\lambda \\ F^{\mu\nu} \\ g_{\mu\nu} \\ \rho_c \\ \rho_m \end{pmatrix} \rightarrow \begin{pmatrix} -u^\lambda \\ a^\lambda \\ F^{\mu\nu} \\ g_{\mu\nu} \\ -\rho_c \\ \rho_m \end{pmatrix}. \quad (16)$$

In fact, the product of any two of these symmetries gives the remaining symmetry. These symmetries will be useful for defining boundary conditions that lead to quantized mass, charge and angular momentum of particle-like soliton solutions, as well as for the treatment of antimatter.

Particle-like solution: Electric field, gravitational field, and quantization

Here I investigate an exact solution of the new theory representing a charged, spherically symmetric particle-like soliton. This example is useful because an exact solution to the field equations facilitates a clear comparison between the gravitational and electric fields predicted by the new theory and those predicted by the M&EFEs. To proceed I draw on a solution for a spherically symmetric charged particle that was previously derived in reference [i].ⁱⁱⁱ Working in spherical coordinates (r, θ, ϕ, t) , it was shown there that the following expressions for the dynamic fields given in Table I are an exact solution to the field equations given in Table II

$$\begin{aligned}
g_{\mu\nu} &= \begin{pmatrix} 1 & 0 & 0 & 0 \\ 1 - \frac{2m}{r} + \frac{q^2}{r^2} & 0 & 0 & 0 \\ 0 & r^2 & 0 & 0 \\ 0 & 0 & r^2 \sin^2[\theta] & 0 \\ 0 & 0 & 0 & -\left(1 - \frac{2m}{r} + \frac{q^2}{r^2}\right) \end{pmatrix} \\
F_{\mu\nu} &= \begin{pmatrix} 0 & B_\phi & -B_\theta & E_r \\ -B_\phi & 0 & B_r & E_\theta \\ B_\theta & -B_r & 0 & E_\phi \\ -E_r & -E_\theta & -E_\phi & 0 \end{pmatrix} = s \begin{pmatrix} 0 & 0 & 0 & \frac{q}{r^2} - \frac{q^3/m}{r^3} \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ -\frac{q}{r^2} + \frac{q^3/m}{r^3} & 0 & 0 & 0 \end{pmatrix} \\
u^\lambda &= s \begin{pmatrix} 0, 0, 0, \frac{1}{\sqrt{1 - \frac{2m}{r} + \frac{q^2}{r^2}}} \end{pmatrix} \\
a^\lambda &= s \left(0, 0, 0, \frac{q}{m} \right) \\
\rho_c &= \frac{q^3}{m} \frac{\sqrt{1 - \frac{2m}{r} + \frac{q^2}{r^2}}}{r^4} \\
\rho_m &= \frac{q^4}{m^2} \frac{\left(1 - \frac{2m}{r} + \frac{q^2}{r^2}\right)}{r^4}
\end{aligned} \tag{17}$$

where $s = \pm 1$ as will be explained later. Solution (17) is straightforward to verify by direct substitution into the equations of Table II.^{iv} The physical interpretation of this solution is that of a particle having charge $\pm q$ and mass m . Of note is the metric tensor which is identical to the Reissner-Nordstrom metric, establishing that the new theory predicts gravitational fields in agreement with the Einstein field equations. Furthermore, the electric field is radial and agrees with the coulomb field of the conventional Maxwell equations to leading order in $1/r$.

Regarding solution (17), several points are worth emphasizing. First, the fundamental equations in Table II, which look very different than the M&EFEs, give the same solutions for the gravitational and electric fields as the M&EFEs, at least for the case of the spherically symmetric charged particle investigated here. This lends credence to the claim that the new theory's predictions are consistent with those of the M&EFEs. Second, the new theory's predictions go further than the M&EFEs by giving the spatial distribution of the mass and charge density as

part of their solution, *i.e.*, the mass and charge density are dynamic fields in the new theory. As discussed below, having the mass density and charge density as dynamic fields, when combined with boundary conditions that impose self-consistency on the field solutions, leads to quantization conditions on the particle's mass and charge.

As previously proposed in reference [i], a methodology for quantizing the charge of particle-like solutions such as (17) proceeds by imposing a boundary condition requiring the asymptotic value of the electric field be consistent with the spatially integrated charge density

$$q = \int \rho_c u^4 \sqrt{\gamma_{sp}} d^3x = \lim_{r \rightarrow \infty} r^2 F_{14} \quad (18)$$

where q is the total charge of the particle, F_{14} is the radial electric field component of the Maxwell tensor, and γ_{sp} is the determinant of the spatial metric defined by^v

$$\gamma_{sp\ ij} = g_{ij} - \frac{g_{i4} g_{j4}}{g_{44}} \quad (19)$$

where i and j run over the spatial dimensions 1, 2 and 3. An analogous quantizing boundary condition for the mass of the particle is arrived at by requiring the asymptotic value of its gravitational field be consistent with the spatially integrated mass density of the solution

$$m = \int \rho_m |u^4| \sqrt{\gamma_{sp}} d^3x = \lim_{r \rightarrow \infty} r \frac{1 + g_{44}}{2} \quad (20)$$

where m is the total mass of the particle. The reason for the absolute value of u^4 in the mass boundary condition (20) but not in the charge boundary condition (18) is the symmetry (15), and the requirement that the boundary conditions have the same symmetry as the theory's fundamental equations. The boundary conditions (18) and (20) represent self-consistency constraints on the charge parameter q and the mass parameter m that appear in the metric (17). The proposal here is that these boundary or self-consistency conditions represent additional constraints on *physically allowable solutions* beyond the fundamental equations presented in Table II.

For the spherically symmetric solution investigated in (17), the LHS of both (18) and (20) diverge leaving no hope for satisfying those quantization boundary conditions. The upshot of this observation is that while (17) represents a solution that describes the gravitational and electrical fields of a point charge that formally satisfy the equations of the theory in Table II, (17) cannot represent a physically allowed solution. The possibility of finding solutions that satisfy both the equations of the theory in Table II and the quantized charge and mass boundary conditions remains an open question at this point. However, interesting possibilities exist beyond the specific solution investigated here. For example, the modified Reissner-Nordstrom and modified Kerr-Newman metrics developed by S.M. Blinder,^{vi} give finite values for the LHS

of both (18) and (20).^{vii} Finally, when considering metrics that include nonzero angular momentum, as for example would be required for particles having an intrinsic magnetic field, the same methodology used here to quantize the particle's mass and charge can be used to quantize its angular momentum.

The particle-like solution (17) illustrates one restriction that the charge-conjugation symmetry (14) places on metrics that contain a charge parameter q . By (14), the charge-conjugation transformation takes $g_{\mu\nu} \rightarrow g_{\mu\nu}$, and $\rho_c \rightarrow -\rho_c$ or equivalently $q \rightarrow -q$ by (18). This forces the conclusion that the sign of q has no impact on the metric, *i.e.*, the metric can only depend on the absolute value of q since the metric is unchanged by the transformation $q \rightarrow -q$. This result is in line with the known charge containing solutions of the Einstein field equations such as the Reissner-Nordstrom and Kerr-Newman metrics which depend on q^2 .

One of the unique features of the classical field theory being proposed here is that it allows for the inclusion of antimatter in a very natural way. The multiplicative factor s in the expressions for $F_{\mu\nu}$, a^λ and u^λ in solution (17) is defined by

$$s = \begin{cases} +1 & \text{for matter} \\ -1 & \text{for antimatter} \end{cases} \quad (21)$$

and accounts for the matter-antimatter symmetry expressed in (15). The physical interpretation is the $s = -1$ solution represents a particle having the same mass but opposite charge and four-velocity as the $s = +1$ solution. This is equivalent to the view today that a particle's antiparticle is the particle moving backwards through time.^{viii} Said another way, the time-like component of the four-velocity is positive for matter and negative for antimatter

$$u^4 \begin{cases} > 0 & \text{for matter} \\ < 0 & \text{for antimatter} \end{cases} \quad (22)$$

With these definitions for the four-velocity of matter and antimatter, charged mass density can annihilate similarly charged antimass density and satisfy both local conservation of charge (10) and local conservation of mass (12). Additionally, because total energy is conserved by (4), the annihilation of matter by antimatter must be accompanied by the generation of electromagnetic energy, the only other available energy channel in the theory.

Because I am endeavoring to develop a theory that flows from the four fundamental equations in Table II axiomatically, an interesting observation is that there appears to be nothing at this point in the development that precludes the existence of negative mass density, $\rho_m < 0$, and negative mass parameter, $m < 0$. Indeed, the existence of negative mass in the context of classical General Relativity has been studied^{ix, x} and invoked when trying to find stable particle-like solutions using the conventional Einstein field equations.^{xi, xii, xiii} However, in the context of the

present theory, the existence of negative mass density leads to a logical contradiction that can only be resolved by requiring the mass density be non-negative, *i.e.*, $\rho_m \geq 0$ always. I'll come back to this point and develop this logical inconsistency when investigating the behavior of matter and antimatter in electric and gravitational fields.

Electromagnetic and gravitational radiative solutions

Working in the weak field limit, here I derive expressions for a propagating electromagnetic plane wave in terms of the vector field a^λ and the metric tensor $g_{\mu\nu}$.^{xiv} This example is useful as it makes clear the relationship between electromagnetic and gravitational radiation imposed by the fundamental equations in Table II, and predicts that an electromagnetic wave cannot exist without an underlying gravitational wave. To begin, consider an electromagnetic plane wave having frequency ω , propagating in the +z-direction and polarized in the x-direction. The Maxwell tensor for this field is given by

$$F_{\mu\nu} = \begin{pmatrix} 0 & 0 & -B_y & E_x \\ 0 & 0 & 0 & 0 \\ B_y & 0 & 0 & 0 \\ -E_x & 0 & 0 & 0 \end{pmatrix} e^{i\omega(t-z)} \quad (23)$$

where E_x and B_y are the constant field amplitudes. Assuming a near Minkowski weak field metric

$$g_{\mu\nu} = \eta_{\mu\nu} + h_{\mu\nu} e^{i\omega(t-z)} \quad (24)$$

$$|h_{\mu\nu}| \ll 1$$

where the $h_{\mu\nu}$ are complex constants, and a constant vector field a^λ ,

$$a^\lambda = (a^1, a^2, a^3, a^4), \quad (25)$$

I proceed by substituting for $F_{\mu\nu}$, $g_{\mu\nu}$ and a^λ into (1) and only retain terms to first order in the fields $h_{\mu\nu}$ and $F_{\mu\nu}$, which I assume are of the same order of magnitude. Doing this leads to a set of 8 independent linear equations for the 16 unknown constants: $h_{\mu\nu}$, a^λ , E_x and B_y .

Imposing these constraining equations, I then solve for the field components E_x , B_y , $g_{\mu\nu}$ and a^λ in terms of 8 free constants

$$E_x = i \omega \frac{(h_{11}^2 + h_{12}^2)}{2 h_{11}} a^1, \quad (26)$$

$$B_y = E_x$$

$$g_{\mu\nu} = \eta_{\mu\nu} + \begin{pmatrix} h_{11} & h_{12} & -h_{14} & h_{14} \\ h_{12} & -h_{11} & -h_{24} & h_{24} \\ -h_{14} & -h_{24} & h_{33} & -\frac{1}{2}(h_{33} + h_{44}) \\ h_{14} & h_{24} & -\frac{1}{2}(h_{33} + h_{44}) & h_{44} \end{pmatrix} e^{i\omega(t-z)}, \quad (27)$$

and

$$a^\lambda = \left(a^1, a^1 \frac{h_{12}}{h_{11}}, a^4, a^4 \right). \quad (28)$$

This solution illustrates several ways in which the new theory departs from the traditional view of electromagnetic radiation. In the approach being put forth here the undulations in the electromagnetic field are due to undulations in the metrical field (27) via the coupling defined in (1). This result also underlines that the existence of electromagnetic radiation is forbidden in strictly flat space-time. An interesting aspect of this solution is that while electromagnetic radiation necessitates the presence of an underlying gravitational radiation field, the gravitational radiation is not completely defined by the electromagnetic radiation. The supporting gravitational radiation has 6 undetermined constants ($h_{11}, h_{12}, h_{14}, h_{24}, h_{33}, h_{44}$), with the only restriction being that they satisfy $|h_{\mu\nu}| \ll 1$ and $h_{11} \neq 0$ as required by (26). Further insight into the content of the metric (27) is evident after making the coordinate transformation from $x^\mu \rightarrow x'^\mu$ given by

$$\begin{pmatrix} x \\ y \\ z \\ t \end{pmatrix} \rightarrow \begin{pmatrix} x' \\ y' \\ z' \\ t' \end{pmatrix} = \begin{pmatrix} x + \frac{i}{\omega} h_{14} \\ y + \frac{i}{\omega} h_{24} \\ z - \frac{i}{2\omega} h_{33} \\ t - \frac{i}{2\omega} h_{44} \end{pmatrix} \quad (29)$$

and only retaining terms to first order in the h 's. Doing this, the metric (27) is transformed to

$$g'_{\mu\nu} = \eta_{\mu\nu} + \begin{pmatrix} h_{11} & h_{12} & 0 & 0 \\ h_{12} & -h_{11} & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix} e^{i\alpha(t-z)}, \quad (30)$$

while E'_x and B'_y , the transformed electric and magnetic field amplitudes, respectively, are identical to E_x and B_y given in (26). Note that only the h_{11} and h_{12} components of the metric (30) have an absolute physical significance and $h_{22} = -h_{11}$, which makes the plane wave solution (30) identical to the plane wave solution of the classical Einstein field equations.^{xv, xvi} Because the underlying gravitational wave couples to both charged and uncharged matter, one consequence of solution here is that there will be an uncertainty when describing the interaction of electromagnetic radiation with matter if the gravitational wave component of the problem is ignored. However, for a nonrelativistic matter, this gravitational interaction (30) vanishes to first order in the h 's. To see this, consider the following expansion of the Lorentz force law

$$\begin{aligned} \rho_m \frac{Du^\mu}{D\tau} &= \rho_c u^\lambda F^\mu{}_\lambda \\ &\downarrow \\ \rho_m \frac{du^\mu}{d\tau} &= -\rho_m u^\nu u^\lambda \Gamma^\mu{}_{\nu\lambda} + \rho_c u^\lambda F^\mu{}_\lambda \end{aligned} \quad (31)$$

The first term on the RHS in the line above represents the gravitational interaction. This gravitational interaction term vanishes for nonrelativistic matter, $u^\lambda \approx (0, 0, 0, 1)$ because for the metric (30) all the $\Gamma^\mu{}_{44}$ vanish to first order in the h 's.

The forgoing analysis demonstrates the necessity of having an underlying gravitational wave to support the presence of an electromagnetic wave, but the converse is not true, and gravitational radiation can exist independent of any electromagnetic radiation. The following analysis demonstrates this by solving for the structure of gravitational radiation in the absence of electromagnetic radiation. Following the same weak field formalism for the unknown fields $h_{\mu\nu}$ given in (24), but this time zeroing out E_x and B_y in (23), leads to the following solutions for $g_{\mu\nu}$ and a^λ

$$g_{\mu\nu} = \eta_{\mu\nu} + \begin{pmatrix} h_{11} & h_{12} & -h_{14} & h_{14} \\ h_{12} & \frac{h_{12}^2}{h_{11}} & -h_{24} & h_{24} \\ -h_{14} & -h_{24} & h_{33} & -\frac{1}{2}(h_{33} + h_{44}) \\ h_{14} & h_{24} & -\frac{1}{2}(h_{33} + h_{44}) & h_{44} \end{pmatrix} e^{i\omega(t-z)} \quad (32)$$

and

$$a^\lambda = \left(a^1, -a^1 \frac{h_{11}}{h_{12}}, a^4, a^4 \right). \quad (33)$$

Both $g_{\mu\nu}$ given by (32) and a^λ given by (33) are modified from their solutions in the presence of an electromagnetic wave as given by (27) and (28), respectively. Performing a transformation to the same primed coordinate system as given in (29), here gives the metric field

$$g'_{\mu\nu} = \eta_{\mu\nu} + \begin{pmatrix} h_{11} & h_{12} & 0 & 0 \\ h_{12} & \frac{h_{12}^2}{h_{11}} & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix} e^{i\omega(t-z)} \quad (34)$$

again, illustrating that only the h_{11} and h_{12} components have an absolute physical significance.

The interaction of nonrelativistic matter with the gravitational wave (34) vanishes for the same reason that it vanished for the gravitational wave (30) that accompanies electromagnetic radiation. Of particular note is the change in the value of the h_{22} component depending on whether the gravitational wave supports and electromagnetic wave as in (30) or is standalone as in (34).

It seems remarkable that the fundamental equations in Table II that lead to Maxwell's equations and electromagnetic radiation can also lead to gravitational waves, unifying both phenomena as undulations of the metric field. On the other hand, equation (1) with $F_{\mu\nu} = 0$ is a system of second order partial differential equations, $a^\lambda R_{\lambda\mu\nu} = 0$, in the metric field components $g_{\mu\nu}$ just as the Einstein field equations are, so the fact that both sets of field equations give similar solutions for gravitational waves is not to be completely unexpected. Combining the solutions in this section for gravitational and electromagnetic radiation with those of the last section for the static electric and gravitational fields of a particle lends additional credence to the claim that the new theory's solutions are consistent with those of the M&EFs.

Cosmological solution

As shown in a previous section, the M&EFEs and the new theory's field equations in Table II share particle-like solutions having similar character. However, when considering non-static metrics, differences between the predictions of the two theories start to emerge. To illustrate where these differences are, here I investigate the Friedmann–Lemaître–Robertson–Walker (FLRW) metric

$$g_{\mu\nu} = \begin{pmatrix} \frac{R^2(t)}{1-kr^2} & 0 & 0 & 0 \\ 0 & R^2(t)r^2 & 0 & 0 \\ 0 & 0 & R^2(t)r^2 \sin^2(\theta) & 0 \\ 0 & 0 & 0 & -1 \end{pmatrix} \quad (35)$$

where k equals +1, 0 or -1 depending on whether the spatial curvature is positive, zero or negative, respectively, and $R(t)$ is a spatial scale factor. Just as in the case of classical General Relativity where the FLRW metric is a cosmological solution representing a homogenous and isotropic universe, it is the same for the field equations in Table II with an appropriate choice for $R(t)$. To see this, I first demonstrate that (35) is indeed a solution of the Table II equations, and then derive a constraints on $R(t)$, the analogue of the Friedmann equations of classical General Relativity. In an isotropic universe, a^μ must be time-like

$$a^\mu = (0, 0, 0, a^4), \quad (36)$$

and $F_{\mu\nu;\kappa}$ must be identically zero, which when substituted into (1) gives

$$a^\lambda R_{\lambda\kappa\mu\nu} = 0. \quad (37)$$

Equation (37) immediately forces,

$$a^\lambda R_\lambda{}^\nu = 0, \quad (38)$$

which is just equation (2) with $\rho_c = 0$, and which will be automatically satisfied if (37) is satisfied. Substituting a^μ given by (36) and the FLRW metric given by (35) into (37) leads to the following constraint on the cosmic scale factor $R(t)$

$$\frac{d^2 R(t)}{dt^2} = 0 \quad (39)$$

with solution

$$R(t) = R_0 + v_R t, \quad (40)$$

where R_0 is the scale factor at $t=0$ and v_R is the rate of change of the scale factor. The solution for $R(t)$ given in (40) ensures that the metric (35) satisfies both (37) and (38) for all values of k . Based on this solution, the predictions of the new theory for a homogenous and isotropic universe are:

1. It must be charge neutral, *i.e.*, $\rho_c = 0$.
2. The cosmic scale factor changes linearly with cosmic time.

The second prediction above runs counter to the prevailing view today that the growth of the cosmic scale factor is divided into three regimes: the radiation dominated regime in which the scale factor grows as $t^{1/2}$, the matter dominated regime in which the scale factor grows as $t^{2/3}$, and the dark energy dominated regime in which the scale factor grows exponentially with time. That equation (40) for $R(t)$ gives a time dependence different than do the Friedmann equations of classical General Relativity is not surprising because in the new theory the Riemann-Christoffel curvature tensor is not directly tied to the stress-energy tensor as it is in the classical Einstein field equations. Assuming a commoving coordinate system

$$u^\mu = (0, 0, 0, 1), \quad (41)$$

and a homogenous and isotropic stress-energy tensor in (4); a stress-energy tensor that takes the form of a perfect fluid,

$$T^{\mu\nu} = pg^{\mu\nu} + (p + \rho_m)u^\mu u^\nu \quad (42)$$

where p is pressure, the conservation of energy and momentum,

$$(pg^{\mu\nu} + (p + \rho_m)u^\mu u^\nu)_{;\nu} = 0, \quad (43)$$

leads to the standard constraint between p , ρ_m , and R ,^{xvii}

$$\frac{d}{dR}(\rho_m R^3) = -3pR^2. \quad (44)$$

Substituting the scale factor (40) into (44) gives

$$\frac{d\rho_m}{dt} = -\frac{3v_R}{R_0 + v_R t}(\rho_m(t) + p(t)) \quad (45)$$

which when combined with an equation of state can give the temporal development of ρ_m and p in terms of v_R for a homogeneous and isotropic universe that satisfies the fundamental equations of Table II.

Antimatter and its behavior in electric and gravitational fields

The distinction between matter and antimatter is naturally accommodated in the new theory, with antimatter solutions generated from their corresponding matter solutions using transformation (15). As already mentioned, antimatter can be viewed as matter moving backwards through time. To see this more rigorously, consider the four-velocity associated with a fixed quantity of charge and mass density,

$$u^\lambda = \frac{dx^\lambda}{d\tau} . \quad (46)$$

Under the matter-antimatter transformation (15), $u^\lambda \rightarrow -u^\lambda$, or equivalently $d\tau \rightarrow -d\tau$. This motivates the following expression for the four-velocity in terms of the coordinate time

$$u^\lambda = \frac{dx^\lambda}{d\tau} = s\gamma \frac{dx^\lambda}{dt} = s\gamma \begin{pmatrix} \mathbf{v} \\ 1 \end{pmatrix} \quad (47)$$

where s is the matter-antimatter parameter defined in (21), $\mathbf{v} = (v_x, v_y, v_z)$ is the ordinary 3-space velocity of the charge or mass density, and $\gamma = 1 / \sqrt{1 - \mathbf{v}^2}$. Equation (47) establishes that the corresponding matter-antimatter solutions travel in opposite time directions relative to each other. One of the unusual aspects of the matter-antimatter transformation (15) is that ρ_c does not change sign under the transformation. To see that this is consistent with the usual view in which antiparticles have the opposite charge of their corresponding particles, I'll use (47) to illustrate the behavior of a charged matter and antimatter density in an electromagnetic field. Consider a region with an externally defined electromagnetic field

$$F_{\mu\nu} = \begin{pmatrix} 0 & B_z & -B_y & E_x \\ -B_z & 0 & B_x & E_y \\ B_y & -B_x & 0 & E_z \\ -E_x & -E_y & -E_z & 0 \end{pmatrix} \quad (48)$$

and with no, or at least a very weak gravitational field so that $g_{\mu\nu} \approx \eta_{\mu\nu}$ and $\Gamma^\lambda_{\mu\nu} \approx 0$. Starting with the Lorentz force law (13) we have

$$\begin{aligned}
\rho_p \frac{Du^\mu}{D\tau} &= \rho_c u^\lambda F^\mu{}_\lambda \\
&\downarrow \\
\rho_p s \gamma \frac{du_\mu}{dt} &= \rho_c F_{\mu\lambda} u^\lambda \\
&\downarrow \\
\rho_p s \gamma \frac{d}{dt} \begin{pmatrix} s \gamma \vec{v} \\ -s \gamma \end{pmatrix} &= \rho_c \begin{pmatrix} 0 & B_z & -B_y & E_x \\ -B_z & 0 & B_x & E_y \\ B_y & -B_x & 0 & E_z \\ -E_x & -E_y & -E_z & 0 \end{pmatrix} \begin{pmatrix} s \gamma v_x \\ s \gamma v_y \\ s \gamma v_z \\ s \gamma \end{pmatrix} \\
&\downarrow \\
\rho_p \frac{d}{dt} \begin{pmatrix} \vec{\gamma v} \\ \gamma \end{pmatrix} &= s \rho_c \begin{pmatrix} \vec{E} + \vec{v} \times \vec{B} \\ \vec{v} \cdot \vec{E} \end{pmatrix} \tag{49}
\end{aligned}$$

which on the last line above ends up at the conventional form of the Lorentz force law except for the extra factor of s on the RHS. This factor of s in (49) gives the product $s\rho_c$ the appearance that antimatter charge density has the opposite sign to that of matter charge density. The definition of q given in (18) is also equivalent to this point of view because making the matter-antimatter transformation (15) changes the sign of u^λ but not ρ_c in (18), thus changing the sign of q .

Next I investigate antimatter in a gravitational field. There is no question about the gravitational fields generated by matter and antimatter, they are identical under the matter-antimatter symmetry (15), as $g_{\mu\nu}$ is unchanged by that transformation. To understand whether antimatter is attracted or repelled by a gravitational field, I go again to the Lorentz force law (13), but this time assume there is no electromagnetic field present, just a gravitational field corresponding to a Schwarzschild metric generated by a central mass m corresponding to either matter or antimatter. Placing a test particle a distance r from the center of the gravitational field and assuming it to be initially at rest, the equation of motion for the test particle, a geodesic trajectory, is given by the following development

$$\begin{aligned}
& \frac{Du^\mu}{D\tau} = 0 \\
& \downarrow \\
& s\gamma \frac{du^\mu}{dt} = -\Gamma^\mu_{\nu\rho} u^\nu u^\rho \quad (50) \\
& \downarrow \\
& s\gamma \frac{d}{dt} \left(s\gamma \frac{d}{dt} \begin{pmatrix} r \\ \theta \\ \phi \\ t \end{pmatrix} \right) = -\Gamma^\mu_{\nu\rho} u^\nu u^\rho \approx -\Gamma^\mu_{44} s^2 = \begin{pmatrix} -\frac{m}{r^2} \left(1 - \frac{2m}{r} \right) \\ 0 \\ 0 \\ 0 \end{pmatrix} s^2
\end{aligned}$$

where in the last line above I have approximated the RHS using the initial at rest value of u^μ , $u^\mu = (0, 0, 0, s)$ and additionally used the fact that the only nonzero Γ^μ_{44} in a Schwarzschild metric is $\Gamma^1_{44} = \frac{m}{r^2} \left(1 - \frac{2m}{r} \right)$. Simplifying the LHS of the last line in (50) by noting that initially $\gamma=1$, gives

$$\frac{d^2 r}{dt^2} \approx -\frac{m}{r^2} \quad (51)$$

independent of s , and so demonstrating that the proposed theory predicts both matter and antimatter will be attracted by a gravitational field because they follow the same geodesic trajectory, and this regardless of whether matter or antimatter generated the gravitational field.

As already noted, there appears to be nothing in the fundamental equations of Table II that preclude the possibility of negative mass density, $\rho_m < 0$. However, there are inconsistencies that are introduced if negative mass density were to exist. Consider equation (51) with $m > 0$. As just shown, a test particle at some distance from the origin will feel an attractive gravitational force regardless of whether it is comprised of matter or antimatter. But this attraction is also independent of whether the test particle is comprised of positive or negative mass because the test particle's mass does not enter the calculation; all test particles, regardless of their composition, follow the same geodesic trajectory. Now consider equation (51) with $m < 0$. In this case a test particle at some distance from the origin will feel a repulsive gravitational force regardless of whether the test particle is matter or antimatter and regardless of whether the test particle has positive or negative mass. These two situations directly contradict each other, making the new theory logically inconsistent if negative mass density exists. Thus, the only way to avoid this logical contradiction is to require mass density be non-negative always.

Discussion and summary

Equations (1) and (2) are the most important concepts being proposed in this manuscript, fundamentally tying the Maxwell tensor and charge density to the Riemann-Christoffel curvature tensor and thereby unifying electromagnetism and gravitation. The cost of this unification is the introduction of a new field a^λ , a field that has no counterpart in classical physics but in the new theory couples electromagnetic phenomena to gravitational phenomena. The unification that ensues puts electromagnetic phenomena on par with gravitational phenomena, with both intrinsically tied to nonzero curvatures. On the surface, this central role for curvature in all electromagnetic phenomena might be construed as problematic due to the prevalent view today that electromagnetic phenomena can exist in flat space-time. However, looking a little deeper it is not hard to see the connection between the new theory that requires a nonzero curvature for all electromagnetic phenomena and the classical Maxwell theory that works in flat space-time. If one were not aware of the vector field a^λ , but recognized the existence of the coulombic current density as is the view today, that part of the new theory not directly connected to a^λ is exactly the classical Maxwell theory. To see this, consider the equations in Tables II and III that result from replacing $a^\lambda R_{\lambda}{}^\nu$ with the classical coulombic current density. The resulting equations, none of which reference a^λ or have a connection to the Riemann-Christoffel curvature tensor are exactly the classical Maxwell field equations and their consequences. Considering the new theory being put forth here, one might view classical physics at the level of the Maxwell field equations as incomplete, there being a hidden field a^λ that has gone unrecognized. The value of the new theory with its introduction of the vector field a^λ is the reductionism it brings to classical physics at the level of the M&EFEs. In addition to the electromagnetic coverage of the new theory, it also contains solutions replicating those of the Einstein field equations. In fact, the Reissner-Nordstrom metric and the Schwarzschild metric as a limiting case of the Reissner-Nordstrom metric are exact solutions demonstrating that the new theory replicates gravitational physics at the level of the Einstein field equations, at least in these spherically symmetric cases. Additionally, the FLRW metric representing a homogenous and isotropic universe in classical General Relativity represents the same in the new theory although the analogue to the Friedmann equations of classical General Relativity lead to different constraints on the cosmic scale factor in the new theory than those derived using classical General Relativity. Specifically, in the new theory the scale factor varies linearly with cosmic time. This difference between classical General Relativity and the new theory may be a motivation for further study of the new theory as the inability of General Relativity and the Standard Model of particle physics to correctly describe astronomical observations beyond the scale of our own solar system has motivated modifications to General Relativity and specifically the cosmological field equations which continue to be debated in the scientific literature.^{xviii, xix, xx}

The weak field solutions for gravitational and electromagnetic radiation require that electromagnetic radiation be supported by an underlying gravitational radiation, but not the converse; gravitational radiation can exist standalone, being independent of any electromagnetic radiation. A consequence of this is that a test particle in the path of an electromagnetic wave would in addition to feeling the effects of the undulating electric and magnetic fields, also feel the effects of the gravitational wave. However, for the special case of nonrelativistic matter, the gravitational wave exerts no influence on the test mass to first order in the h 's. This prediction of the new theory, requiring electromagnetic radiation to be supported by an underlying gravitational radiation does not agree with the classical Maxwell field equations and their prediction that electromagnetic radiation can exist in a rigorously flat space-time.

The particle-like solution presented within is important because it demonstrates that the fundamental equations in Table II replicate the results of the Einstein field equations, at least for the case of spherical symmetry. While it is evident from preceding discussions that the Maxwell field equations and the classical physics that springs from them are contained in the four fundamental equations in Table II, the same cannot be said of the Einstein field equations. It is only through the demonstrated particle-like solution (17) that the claim is made that the new theory covers the physics of the classical Einstein field equations. The coverage of the physics of both the Maxwell field equations and the Einstein field equations along with the unification it brings to them puts the new theory in a very strong position. Another reason that the particle-like solution (17) is so important is that it establishes that exact solutions to the theory do exist. This is not at all evident from equation (1), which represents a mixed system of first order partial differential equations for $F_{\mu\nu}$ and so carries with it specific integrability conditions that must be satisfied for solutions to exist.^{xxi, xxii} The existence of the particle-like solution (17) allays that concern, proving by direct demonstration that solutions do indeed exist.

A detraction of the new theory, at least at its present level of development, is the somewhat *ad hoc* inclusion of the conserved stress-energy tensor (4) as one of its four fundamental equations; this rather than a more elegant and integrated, variational derivation of the field equations from an appropriate action. The strongest argument for the inclusion of the conserved stress-energy tensor (4) is the logical completeness it brings to the theory, *i.e.*, the number of independent field equations is four less than the number of independent field variables. This leaves four degrees of freedom in the determination of the independent field variables, corresponding to the four degrees of freedom in the choice of coordinate system, and is consistent with the requirements of general covariance. This internal consistency with the requirements of general covariance is a very strong argument for the theory when compared to the situation with the M&EFEs, where attempts to find particle-like soliton solutions have noted that there is generally one more unknown than there are equations for determining the unknowns.^{xi}

As proposed here, the new theory is a theory of everything at the level of classical physics. This claim rests on the fact the both the charge density and mass density are treated as dynamic fields in the theory, leaving no external parameters that need be introduced. This of course highlights

one shortcoming of the particle-like solution (17). As already discussed that solution cannot satisfy the charge and mass boundary conditions (18) and (20), respectively, because the spatial integrals in both of those equations diverge due to singularities at the origin. This is a technical problem due to the metric solution in (17), the Reissner-Nordstrom metric with its singularity at the origin. One way to get around this difficulty is to investigate other choices of metric such as, for example, the Blinder-Reissner-Nordstrom metric [vi] which is well behaved at the origin. Still other possibilities include cylindrically symmetric metrics to account for angular momentum about an axis; such metrics being capable of modeling particles having a magnetic field, but this goes well beyond the level of analysis presented within.

An attractive feature of the new theory is the way in which antimatter is naturally accommodated in it. Using the matter-antimatter transformation (15), a solution corresponding to antimatter can be generated from its symmetric matter solution by changing the sign of a^λ , u^λ and $F_{\mu\nu}$, and leaving ρ_c , ρ_m and $g_{\mu\nu}$ unchanged. This leads to the same picture as is accepted today in which antimatter density is viewed as matter density moving backwards through time. The change in sign of u^λ between matter and antimatter ensures that matter density and similarly charged antimatter density can annihilate each other and conserve charge (10), mass (12) and overall energy (4). By direct calculation, the behavior of matter and antimatter in a gravitational field is demonstrated to be attractive always. Also by direct calculation, it is shown that the mass density field, ρ_m , must be non-negative always to eliminate the possibility of a logical contradiction within the theory.

Conclusion

The proposed classical field theory of electromagnetism and gravitation developed here encompasses classical physics at the level of the M&EFEs using four fundamental field equations as detailed in Table II. The coverage of electromagnetism at the level of the classical Maxwell equations is clear as the classical Maxwell equations are derivable from the equations of Table II. The coverage of the Einstein field equations is not as evident, being demonstrated by solutions of the equations in Table II that are consistent with analogous solutions of the classical Einstein field equations. Specifically, the Reissner-Nordstrom metric and the Schwarzschild metric as a limiting case of the Reissner-Nordstrom metric are exact solutions of the new theory, as is the LFRW metric with a suitable choice for the scale parameter's time development. Departures from the predictions of the M&EFEs by the new theory occur when one considers fundamental equations (1) and (2) with their introduction of the vector field a^λ that physically couples gravitational and electromagnetic phenomena. In the case of electromagnetic phenomena, this coupling, for example, requires that electromagnetic waves be supported by an underlying gravitational wave. In the case of the FLRW metric solution for a homogenous and isotropic universe, a restriction is placed on the time development of the scale

parameter that is different than the prediction of the Friedmann equations of classical General Relativity. One of the strengths of the new theory is the unification it brings to electromagnetism and gravitation through a set of field equations that are logically consistent from the standpoint of general covariance. Another strength is the reductionism the new theory brings to electromagnetic and gravitational phenomena by treating the sources of these fields as dynamic variables rather than parameters that are externally put into the theory; a development which explains the quantization of the mass, the charge and the angular momentum of particles in the context of a classical field theory.

One of the very fortunate circumstances with the development of the new theory here is the existence of exact solutions to the fundamental field equations of Table II. These exact solutions lend credence to the proposed theory being a viable physical theory because such solutions can be compared to well-known physical situations, for example, the electric and gravitational fields of a spherically symmetric source, and the electromagnetic wave solution. To push deeper will require the development of approximate solution methods that can not only solve the field equations in Table II but also impose the boundary conditions on mass, charge and angular momentum. Particularly for the case of more complicated metrics having angular momentum; metrics that could represent known particles with intrinsic magnetic fields, this pursuit of quantized particle-like solutions from a classical field theory is an interesting prospect.

The genesis of the work presented within was reported in reference [i]. The same fundamental equations and quantizing boundary conditions reviewed here were first reported there. Based on the symmetry properties of the fundamental equations in Table II, the interpretation of the particle-like solution has been advanced here. Specifically, the sign of the charge density field is unrestricted in the present work, and the mass density field is restricted to be non-negative always to avoid internal logical contradictions. Additionally, the present work considers a cosmological solution based on the FLRW metric. The present work also corrects an error in the weak field analysis of reference [i], leading to the expanded discussion of electromagnetic radiation and its underlying gravitational radiation.

Acknowledgement

This work was performed under the auspices of the U.S. Department of Energy by Lawrence Livermore National Laboratory under Contract DE-AC52-07NA27344.

Appendix – Derivation of the Lorentz force law and the conservation of mass

The Lorentz force law (13) and the conservation of mass (12) follow from the fundamental equations of the theory. To derive the conservation of mass equation (12) I begin with equation (4) contracted with u_μ

$$u_\mu \left(\rho_m u^\mu u^\nu + F^\mu{}_\lambda F^{\nu\lambda} - \frac{1}{4} g^{\mu\nu} F^{\rho\sigma} F_{\rho\sigma} \right)_{;\nu} = 0 . \quad (52)$$

Expanding (52) and then simplifying as per the following development

$$\begin{aligned} & u_\mu \left((\rho_m u^\nu)_{;\nu} u^\mu + \rho_m u^\nu u^\mu_{;\nu} + F^\mu{}_{\lambda;\nu} F^{\nu\lambda} + F^\mu{}_\lambda F^{\nu\lambda}_{;\nu} - \frac{1}{2} g^{\mu\nu} F^{\rho\sigma} F_{\rho\sigma;\nu} \right) = 0 \\ & \downarrow \\ & (\rho_m u^\nu)_{;\nu} (u^\mu u_\mu) + \rho_m u^\nu (u_\mu u^\mu_{;\nu}) + u_\mu F^\mu{}_{\lambda;\nu} F^{\nu\lambda} + u_\mu F^\mu{}_\lambda (F^{\nu\lambda}_{;\nu}) - \frac{1}{2} (u_\mu g^{\mu\nu}) F^{\rho\sigma} F_{\rho\sigma;\nu} = 0 \\ & \downarrow \\ & (\rho_m u^\nu)_{;\nu} (-1) + \rho_m u^\nu (0) + u_\mu F^\mu{}_{\lambda;\nu} F^{\nu\lambda} + u_\mu F^\mu{}_\lambda (-\rho_c u^\lambda) - \frac{1}{2} (u^\nu) F^{\rho\sigma} F_{\rho\sigma;\nu} = 0 \\ & \downarrow \\ & -(\rho_m u^\nu)_{;\nu} + (u_\mu F^\mu{}_{\lambda;\nu} F^{\nu\lambda}) - \rho_c (u_\mu u^\lambda F^\mu{}_\lambda) - \frac{1}{2} (u^\nu F^{\rho\sigma} F_{\rho\sigma;\nu}) = 0 \\ & \downarrow \\ & -(\rho_m u^\nu)_{;\nu} + u^\mu F_{\mu\lambda;\nu} F^{\nu\lambda} - \rho_c (0) - \frac{1}{2} u^\mu F^{\nu\lambda} F_{\nu\lambda;\mu} = 0 \\ & \downarrow \\ & -(\rho_m u^\nu)_{;\nu} + u^\mu F^{\nu\lambda} \left(F_{\mu\lambda;\nu} - \frac{1}{2} F_{\nu\lambda;\mu} \right) = 0 \\ & \downarrow \\ & -(\rho_m u^\nu)_{;\nu} + u^\mu F^{\nu\lambda} \left(F_{\mu\lambda;\nu} + \frac{1}{2} F_{\lambda\mu;\nu} + \frac{1}{2} F_{\mu\nu;\lambda} \right) = 0 \\ & \downarrow \\ & -(\rho_m u^\nu)_{;\nu} + u^\mu F^{\nu\lambda} \left(\frac{1}{2} F_{\mu\lambda;\nu} + \frac{1}{2} F_{\mu\nu;\lambda} \right) = 0 \\ & \downarrow \\ & (\rho_m u^\nu)_{;\nu} = 0 \end{aligned} \quad (53)$$

leads to the conservation of mass equation (12) on the last line above. The Lorentz force law (13) is now derived using the conservation of mass result just derived and equation (4).

Expanding and then simplifying per the following development

$$\begin{aligned}
& \left(\rho_m u^\mu u^\nu + F^\mu{}_\lambda F^{\nu\lambda} - \frac{1}{4} g^{\mu\nu} F^{\rho\sigma} F_{\rho\sigma} \right)_{;\nu} = 0 \\
& \downarrow \\
& (\rho_m u^\nu)_{;\nu} u^\mu + \rho_m u^\nu u^\mu{}_{;\nu} + F^\mu{}_{\lambda;\nu} F^{\nu\lambda} + F^\mu{}_\lambda (F^{\nu\lambda}{}_{;\nu}) - \frac{1}{2} g^{\mu\nu} F^{\rho\sigma} F_{\rho\sigma;\nu} = 0 \\
& \downarrow \\
& (0) u^\mu + \rho_m \frac{Du^\mu}{D\tau} + F^\mu{}_{\lambda;\nu} F^{\nu\lambda} - F^\mu{}_\lambda (\rho_c u^\lambda) - \frac{1}{2} F^{\nu\lambda} F_{\nu\lambda}{}^{;\mu} = 0 \\
& \downarrow \\
& \rho_m \frac{Du^\mu}{D\tau} - \rho_c u^\lambda F^\mu{}_\lambda + F^{\nu\lambda} \left(F^\mu{}_{\lambda;\nu} - \frac{1}{2} F_{\nu\lambda}{}^{;\mu} \right) = 0 \\
& \downarrow \\
& \rho_m \frac{Du^\mu}{D\tau} - \rho_c u^\lambda F^\mu{}_\lambda + F^{\nu\lambda} g^{\mu\sigma} \left(F_{\sigma\lambda;\nu} - \frac{1}{2} F_{\nu\lambda;\sigma} \right) = 0 \\
& \downarrow \\
& \rho_m \frac{Du^\mu}{D\tau} - \rho_c u^\lambda F^\mu{}_\lambda + F^{\nu\lambda} g^{\mu\sigma} \left(F_{\sigma\lambda;\nu} + \frac{1}{2} F_{\lambda\sigma;\nu} + \frac{1}{2} F_{\sigma\nu;\lambda} \right) = 0 \\
& \downarrow \\
& \rho_m \frac{Du^\mu}{D\tau} - \rho_c u^\lambda F^\mu{}_\lambda + F^{\nu\lambda} g^{\mu\sigma} \left(\frac{1}{2} F_{\sigma\lambda;\nu} + \frac{1}{2} F_{\sigma\nu;\lambda} \right) = 0 \tag{54} \\
& \downarrow \\
& \rho_m \frac{Du^\mu}{D\tau} - \rho_c u^\lambda F^\mu{}_\lambda = 0
\end{aligned}$$

leads to the Lorentz force law (13) on the last line above.

References and endnotes

ⁱ Beach, Raymond J. "A Classical Field Theory of Gravity and Electromagnetism." *Journal of Modern Physics* 2014 (2014).

ⁱⁱ S. Weinberg, *Gravitation and Cosmology*, John Wiley & Sons, New York, NY 1972. The definition of the

Riemann-Christoffel curvature tensor is $R^\lambda{}_{\mu\nu\kappa} \equiv \partial_\kappa \Gamma^\lambda{}_{\mu\nu} - \partial_\nu \Gamma^\lambda{}_{\mu\kappa} + \Gamma^\eta{}_{\mu\nu} \Gamma^\lambda{}_{\kappa\eta} - \Gamma^\eta{}_{\mu\kappa} \Gamma^\lambda{}_{\nu\eta}$, and the

definition of the Ricci tensor is $R_{\mu\kappa} \equiv R^\lambda{}_{\mu\lambda\kappa}$.

ⁱⁱⁱ The interpretation of the solution here is different than in reference [i]. Specifically, in reference [i] the charge density was restricted to be positive, a restriction that is lifted here.

-
- ^{iv} Calculations within were performed in Mathematica: Wolfram Research, Inc., Mathematica, Version 11.0, Champaign, IL (2016).
- ^v L.D. Landau and E.M. Lifshitz, *The Classical Theory of Fields*, Fourth Edition, p. 235, Pergamon Press, New York, NY 1975.
- ^{vi} Blinder, S. M. "General relativistic models for the electron." *Reports on Mathematical Physics* 47.2 (2001): 279-285.
- ^{vii} These metric solutions and others are being pursued as a subject of a future publication.
- ^{viii} R.P. Feynman, "The Theory of Positrons," *Phys Rev*, 1949, **76**(6): p. 749-759.
- ^{ix} Bondi, Hermann. "Negative mass in general relativity." *Reviews of Modern Physics* 29, no. 3 (1957): 423.
- ^x Bonnor, W. B. "Negative mass in general relativity." *General relativity and gravitation* 21, no. 11 (1989): 1143-1157.
- ^{xi} Bailyn, M., and D. Eimerl. "General-Relativistic Interior Metric for a Stable Static Charged Matter Fluid with Large $e m$." *Physical Review D* 5, no. 8 (1972): 1897.
- ^{xii} Bonnor, W.B. and Cooperstock, F.I., 1989. Does the electron contain negative mass? *Physics Letters A*, 139(9), pp.442-444.
- ^{xiii} Herrera, L., and V. Verela. "Negative energy density and classical electron models." *Physics Letters A* 189, no. 1-2 (1994): 11-14.
- ^{xiv} This calculation was presented in reference [i] but contained an error that is corrected here. In that original publication, the electric and magnetic fields were not restricted to the same weak field approximation as the h 's.
- ^{xv} S. Weinberg, *Gravitation and Cosmology*, John Wiley & Sons, New York, NY 1972, Section 10.2.
- ^{xvi} L.D. Landau and E.M. Lifshitz, *The Classical Theory of Fields*, Fourth Edition, p. 235, Pergamon Press, New York, NY 1975, Section 107.
- ^{xvii} S. Weinberg, *Gravitation and Cosmology*, John Wiley & Sons, New York, NY 1972, Equation 15.1.21.
- ^{xviii} Glenn D. Starkman, "Modifying Gravity: You Can't Always Get What You Want," [v1] Mon, 9 Jan 2012 06:46:26 GMT (34kb) [arXiv:1201.1697v1](https://arxiv.org/abs/1201.1697v1) [gr-qc].
- ^{xix} Troisi, Antonio. "Higher-order gravity in higher dimensions: geometrical origins of four-dimensional cosmology?." *The European Physical Journal C* 77, no. 3 (2017): 171.
- ^{xx} Li, Li-Xin. "Electrodynamics on cosmological scales." *General Relativity and Gravitation* 48, no. 3 (2016): 1-36.
- ^{xxi} L.P. Eisenhart, *An Introduction to Differential Geometry*, Chapter 23, "Systems of Partial Differential Equations of the First Order, Mixed Systems," Princeton University Press 1947.
- ^{xxii} For more explanation see the description in reference [i], Section 4. Integrability Conditions.