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Article

The Geometrization of Maxwell's Equations and the Emergence of Gravity

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ABSTRACT

Assuming the geometry of nature is Riemannian with four dimensions, the classical Maxwell equations are shown to be a derivable consequence of a single equation that couples the Maxwell tensor to the Riemann-Christoffel curvature tensor. This geometrization of the Maxwell tensor extends the interpretation of the classical Maxwell equations, for example, giving physical quantities such as charge density a geometric definition. Including a conserved energy-momentum tensor, the entirety of classical electromagnetism is shown to be a derivable consequence of the theory. The coupling of the Riemann-Christoffel curvature tensor to the Maxwell tensor also leads naturally to the emergence of gravity which is consistent with Einstein's equation of General Relativity augmented by a term that can mimic the properties of dark matter and/or dark energy in the context of General Relativity. In summary, the proposed geometrization of the Maxwell tensor puts both electromagnetic and gravitational phenomena on an equal footing, with both being tied to the curvature of space-time. Using specific solutions to the proposed theory, the unification brought to electromagnetic and gravitational phenomena as well as the relationship of those solutions with the corresponding solutions of the classical Maxwell and Einstein field equations are examined.

Keywords: Maxwell's equations; General Relativity; unification; dark matter; dark energy; electromagnetic radiation; gravitational radiation; antimatter; antigravity; quantization; superluminal transport

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1. INTRODUCTION

Electromagnetic and gravitational fields have long range interactions characterized by speed of light propagation; similarities that suggest these fields should be coupled together at the classical physics level. Although this coupling or unification is a well-worn problem with many potential solutions having been proposed, it is fair to say that there is still no generally accepted classical field theory that can explain both electromagnetism and gravitation in a coupled or unified framework.[i] Today, the existence of electromagnetic and gravitational fields are generally understood to be distinct and independent with electromagnetic fields described by Maxwell's equations and gravitational fields described by Einstein's equation of General Relativity. The purpose of

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this manuscript is to assess a recently proposed field equation that geometricizes the Maxwell tensor and leads to a geometricized version of Maxwell's equations from which gravity then emerges.

Assuming the geometry of nature is Riemannian with four dimensions, the classical Maxwell equations will be shown to be a derivable consequence of,[ii]

$$F_{\mu\nu;\kappa} = a^{\lambda} R_{\lambda\kappa\mu\nu}\,, \tag{1}$$

where $F_{\mu\nu}$ is the Maxwell tensor, $R_{\lambda\kappa\mu\nu}$ is the Riemann-Christoffel (R-C) curvature tensor, and a^{λ} is a four-vector related to the familiar vector potential A_{μ} of classical electromagnetism. Including the conserved energy-momentum tensor for matter and electromagnetic fields,

$$\left(\rho_m u^{\mu} u^{\nu} + F^{\mu}{}_{\lambda} F^{\nu\lambda} - \frac{1}{4} g^{\mu\nu} F^{\rho\sigma} F_{\rho\sigma}\right)_{;\nu} = 0\,, \tag{2}$$

where u^{λ} is the four-velocity, ρ_m is the scalar mass density, and $g_{\mu\nu}$ is the metric tensor, all the equations of classical electromagnetism will be shown to be a consequence of the equations (1) and (2). Notably, only equation (1) which couples the derivatives of the Maxwell tensor to the R-C tensor through the vector field a^{λ} is new; equation (2), the conserved energy-momentum for matter and electromagnetic fields is already a well-established foundational equation of classical physics.

Beyond the succinct framework for the classical Maxwell equations provided by equation (1), its coupling of the R-C tensor to the Maxwell tensor introduces gravitational effects into any solution of (1) and (2). While the emerging gravitational fields due to this coupling are not identical to those predicted by Einstein's equation of General Relativity, they are, as will be shown, consistent with Einstein's equation of General Relativity augmented by a term that can mimic the properties of dark matter and/or dark energy.

The goal of this manuscript is to show through an axiomatic development that the continuous field theory based on equations (1) and (2) covers classical electromagnetism and the emergence of gravitational phenomena in a unified manner with both tied to the curvature of space-time. Throughout the manuscript, geometric units will be used with a metric tensor having signature [+, +, +, -] in which spatial indices run from 1 to 3 and 4 is the time index. The notation within uses commas before tensor indices to indicate ordinary derivatives and semicolons before tensor indices to indicate covariant derivatives. For the definitions of the R-C curvature tensor and the Ricci tensor, the conventions used by Weinberg[iii] are followed.

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2. CONSEQUENCES OF THE FIELD EQUATIONS (1) AND (2)

Here I give a short derivation of the classical equations of electromagnetism in the framework of the proposed theory. The point in going through this purely formal development is to show that the classical equations of electromagnetism are derivative only to equations (1) and (2) and the algebraic properties of the R-C tensor. After developing the classical equations of electromagnetism from equations (1) and (2), I go on to describe the emergence of gravity that is forced by them.

2.1 The equations of electromagnetism

Maxwell's homogeneous equation and gauge invariance

To begin, I demonstrate that equation (1) forces both the antisymmetry of $F_{\mu\nu}$ and the vanishing of its anti-symmetrized derivative, *i.e.*, $F_{[\mu\nu,\kappa]} = 0$. The antisymmetry of $F_{\mu\nu}$ follows from equation (1) and the algebraic property of the R-C tensor,

$$R_{\lambda\kappa\mu\nu} = -R_{\lambda\kappa\nu\mu}. \qquad (3)$$

Contracting (3) with a^{λ} and using equation (1) gives,

$$a^{\lambda} R_{\lambda\kappa\mu\nu} = -a^{\lambda} R_{\lambda\kappa\nu\mu} \quad \rightarrow \quad F_{\mu\nu;\kappa} = -F_{\nu\mu;\kappa} \quad \rightarrow \quad F_{\mu\nu} = -F_{\nu\mu} \ , \qquad (4)$$

thus, establishing the antisymmetry of $F_{\mu\nu}$. Next, I derive Maxwell's homogeneous equation using the algebraic property of the R-C tensor,

$$R_{\lambda\kappa\mu\nu} + R_{\lambda\mu\nu\kappa} + R_{\lambda\nu\kappa\mu} = 0. \qquad (5)$$

Contracting (5) with a^{λ} and again using equation (1) gives,

$$a^{\lambda} R_{\lambda\kappa\mu\nu} + a^{\lambda} R_{\lambda\mu\nu\kappa} + a^{\lambda} R_{\lambda\nu\kappa\mu} = 0 \quad \rightarrow \quad \begin{cases} F_{\mu\nu;\kappa} + F_{\nu\kappa;\mu} + F_{\kappa\mu;\nu} = 0 \\ \qquad or \\ F_{\mu\nu,\kappa} + F_{\nu\kappa,\mu} + F_{\kappa\mu,\nu} = 0 \end{cases} \qquad (6)$$

thus, establishing Maxwell's homogeneous equation. The switch from the covariant to ordinary derivatives of $F_{\mu\nu}$ in (6) is justified by the antisymmetry of $F_{\mu\nu}$.

Having established the antisymmetry of $F_{\mu\nu}$ and the vanishing of its anti-symmetrized derivative, which is just a statement of Maxwell's homogeneous equation (6), the converse of Poincaré's lemma establishes that $F_{\mu\nu}$ can itself be expressed as the anti-symmetrized derivative of a vector function, *i.e.*,

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$$F_{\mu\nu} = A_{\nu,\mu} - A_{\mu,\nu} \quad , \tag{7}$$

where A_μ is the classical electromagnetic vector potential. Because $F_{\mu\nu}$ can be expressed as the anti-symmetrized derivative of the vector potential A_μ, its value will be unaffected by a gauge transformation in which a gradient field is added to A_μ,

$$A_\mu \rightarrow A_\mu + \partial_\mu \varphi \quad . \tag{8}$$

Maxwell's inhomogeneous equation and the definitions of charge density and four-velocity

Next, Maxwell's inhomogeneous equation and the definitions of charge density ρ_c and four-velocity u^λ are derived using equation (1). Contracting the μ and κ indices in equation (1) gives,

$$F^{\mu\nu}{}_{;\mu} = a^\lambda R_{\lambda\mu}{}^{\mu\nu} = -a^\lambda R_\lambda{}^\nu , \tag{9}$$

where $R_\lambda{}^\nu$ is the Ricci tensor. To establish the connection between equation (9) and Maxwell's inhomogeneous equation, I use the following identity,

$$W^\mu = \sqrt{\left|W^\rho W_\rho\right|} \frac{W^\mu}{\sqrt{\left|W^\sigma W_\sigma\right|}} \quad , \tag{10}$$

which is valid for any non-null four-vector W^μ. With the aid of (10), any W^μ satisfying $W^\mu W_\mu \neq 0$ can be recast as a current density, *i.e.*, the product of a scalar density ρ and a four-velocity u^μ,

$$W^\mu = \rho u^\mu \quad , \tag{11}$$

where the scalar density is defined as,

$$\rho \equiv \pm\sqrt{\left|W^\rho W_\rho\right|} \quad , \tag{12}$$

and the four-velocity as,

$$u^\mu \equiv \frac{W^\mu}{\pm\sqrt{\left|W^\sigma W_\sigma\right|}} \quad . \tag{13}$$

Equation (13) leads to different normalizations for subluminal and superluminal four-velocities,

$$u^\mu u_\mu = \begin{cases} -1 & \text{For subluminal four-velocities} \\ +1 & \text{For superluminal four-velocities} \end{cases} \tag{14}$$

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where subluminal velocities corresponding to time-like W^{μ} ($W^{\mu}W_{\mu} < 0$), and superluminal velocities corresponding to space-like W^{μ} ($W^{\mu}W_{\mu} > 0$).

Using the definitions given in (12) and (13), and identifying W^{ν} with $a^{\lambda}R_{\lambda}{}^{\nu}$ in equation (9) gives,

$$a^{\lambda}R_{\lambda}{}^{\nu} = \rho_c u^{\nu} \tag{15}$$

where the charge density ρ_c is defined by,

$$\rho_c \equiv \pm\sqrt{\left|a^{\rho}R_{\rho}{}^{\kappa}a^{\sigma}R_{\sigma\kappa}\right|} \;, \tag{16}$$

and the four-velocity u^{ν} by,

$$u^{\nu} \equiv \pm\frac{a^{\lambda}R_{\lambda}{}^{\nu}}{\sqrt{\left|a^{\rho}R_{\rho}{}^{\kappa}a^{\sigma}R_{\sigma\kappa}\right|}} \;. \tag{17}$$

Using these definitions for charge density and four-velocity, equations (9) and (15) can then be combined to give Maxwell's inhomogeneous equation,

$$F^{\mu\nu}{}_{;\mu} = -\rho_c u^{\nu} \,. \tag{18}$$

Equations (16) and (17) emphasize the underlying geometric character of the theory of electromagnetism being proposed here. Both the charge density ρ_c and the four-velocity field u^{ν} are defined by the metric tensor $g_{\mu\nu}$ which determines the Ricci tensor $R_{\lambda}{}^{\nu}$, and the four-vector a^{λ}. In the development leading up to Maxwell's inhomogeneous equation (18), I have not imposed the usual restriction that the four-velocity u^{λ} be limited to being subluminal. I have dropped this requirement because I am attempting to develop a theory that flows axiomatically from equations (1) and (2) and there is nothing *a priori* that requires that $a^{\lambda}R_{\lambda}{}^{\nu}$ be time-like.

While the forgoing development leads to Maxwell's inhomogeneous equation in its familiar form (18), it goes further than the usual classical interpretation in that the charge density ρ_c and the four-velocity u^{ν} are both tied to the value of the Ricci tensor through (16) and (17), respectively. This geometrization of ρ_c and u^{ν} hints at the emergence of gravity in the proposed theory and

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is reminiscent of classical General Relativity's geometric interpretation of the mass density ρ_m in terms of the curvature scalar R.

The conservation of charge

Next, I derive the conservation of charge. Returning to the antisymmetry of $F_{\mu\nu}$ which was established in equations (3) and (4), it follows that,

$$F^{\mu\nu}{}_{;\mu;\nu} \equiv 0\,, \tag{19}$$

which is an identity for all antisymmetric tensors. Comparing (19) to Maxwell's inhomogeneous equation (18) then gives,

$$F^{\mu\nu}{}_{;\mu;\nu} = -\left(\rho_c u^\nu\right)_{;\nu} = 0\,, \tag{20}$$

thus, establishing the conservation of charge,

$$\left(\rho_c u^\nu\right)_{;\nu} = 0 \quad . \tag{21}$$

The Lorentz force law and the conservation of mass

Having already established Maxwell's homogeneous equation (6) and Maxwell's inhomogeneous equation (18) as a consequence of equation (1), here I establish that the Lorentz force law and the conservation of mass equation are both a consequence of the conserved energy-momentum tensor, equation (2). To see this, distribute the covariant derivative in (2),

$$\left(\rho_m u^\nu\right)_{;\nu} u^\mu + \rho_m u^\mu{}_{;\nu} u^\nu + F^{\mu\lambda} F^\nu{}_{\lambda;\nu} + F^{\mu\lambda}{}_{;\nu} F^\nu{}_\lambda - \frac{1}{2} g^{\mu\nu} F^{\rho\sigma} F_{\rho\sigma;\nu} = 0\,. \tag{22}$$

With some substitutions and rearrangements using Maxwell's homogeneous equation (6) and inhomogeneous equation (18), equation (22) can be re written as,

$$\left(\rho_m u^\nu\right)_{;\nu} u^\mu + \rho_m u^\mu{}_{;\nu} u^\nu - \rho_c F^\mu{}_\lambda u^\lambda = 0\,. \tag{23}$$

Contracting (23) with u_μ, the 2nd and 3rd terms on the LHS are zeroed due to the normalization of u_μ (14) and the antisymmetry of $F_{\mu\nu}$ (4), respectively, leaving,

$$\left(\rho_m u^\nu\right)_{;\nu} = 0 \quad , \tag{24}$$

thus, establishing the conservation of mass equation. Using (24) to zero out the conservation of mass term in (23) then leaves the Lorentz force law,

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$$\rho_m \frac{Du^\mu}{D\tau} = \rho_c F^\mu{}_\lambda u^\lambda \quad , \tag{25}$$

where $\frac{Du^\mu}{D\tau} \equiv u^\mu{}_{;\nu} u^\nu$.

Relationship of a^λ to the classical electromagnetic vector potential A^λ

The vector field a^λ and equation (1) are the only truly new pieces of physics that have been introduced in the forgoing development. In some respects a^λ plays a role similar to the vector potential A^λ of classical electromagnetism, and if fact the two are closely related to each other. To see this, take the covariant derivative of both sides of (7) giving,

$$F_{\mu\nu;\kappa} = A_{\nu;\mu;\kappa} - A_{\mu;\nu;\kappa} \quad . \tag{26}$$

Now compare (26) to equation (1) rewritten as,

$$F_{\mu\nu;\kappa} = a^\lambda R_{\lambda\kappa\mu\nu} = -a_{\kappa;\mu;\nu} + a_{\kappa;\nu;\mu} \quad , \tag{27}$$

where the RHS of (27) follows from the commutation property of covariant derivatives. Equating the RHS's of equations (26) and (27) gives,

$$a_{\kappa;\nu;\mu} - a_{\kappa;\mu;\nu} = A_{\nu;\mu;\kappa} - A_{\mu;\nu;\kappa} \, , \tag{28}$$

establishing a connection between a^λ and A^λ in the theory of electromagnetism being proposed here.

In summary, the theory of electromagnetism based on equations (1) and (2) in no way changes the traditional equations of classical electromagnetism although their derivations are very different. In the theory being proposed here, Maxwell's equations are derivative only to equation (1). Then using Maxwell's equations and the conserved energy-momentum tensor (2), the Lorentz force law and the conservation of mass are derived. Although the equations of electromagnetism derived correspond one-to-one with the classical equations of electromagnetism, adopting equation (1) as the starting point does introduce conceptual changes to electromagnetic theory that go beyond the classical interpretations. Notably, the charge density ρ_c is no longer an externally inserted field as it is in the classical physics picture, but instead is determined by a^λ and the Ricci tensor. The same comment applies to the four-velocity u^μ which describes the motion of both the mass density ρ_m and charge density ρ_c . These dependencies intermingle electromagnetic and gravitational phenomena in a fundamentally new way. In subsequent sections, the

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consequences of equations (1) and (2) will be developed further using specific solutions to show that electromagnetic and gravitational phenomena are effectively described in a unified manner and put on an equal footing, with both being tied to nonzero space-time curvatures.

2.2 A theory of gravitation

The preceding discussion established that the equations of classical electromagnetism follow directly from equations (1) and (2). With the R-C tensor coupled to the Maxwell tensor as it is in equation (1), some form of gravitation can be expected to emerge. The question that naturally arises is this: Will this emergent gravity be equivalent to Einstein's General Relativity,

$$G^{\mu\nu} = -8\pi T^{\mu\nu}, \tag{29}$$

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where $G^{\mu\nu} \equiv R^{\mu\nu} - \frac{1}{2} g^{\mu\nu} R$ is the Einstein tensor?

As will be shown using the specific example of a spherically symmetric, non-rotating, charged particle, the Reissner-Nordström metric is an exact solution of equations (1) and (2), thus establishing that the emergent gravity in the proposed theory and classical General Relativity (29) support the same gravitational metric field solutions, at least in the case of spherical symmetry. However, one must go further to determine if Einstein's field equations are a derivable consequence of (1) and (2). Here I investigate this issue starting with the conserved energy-momentum tensor given in equation (2) and note an immediate consequence of $G^{\mu\nu}$ and $T^{\mu\nu}$ being both symmetric and independently conserved (independently conserved because the Bianchi identity gives $G^{\mu\nu}{}_{;\nu} = 0$ and equation (2) gives $T^{\mu\nu}{}_{;\nu} = 0$) is that for any constant α, one can define a tensor field $\Lambda^{\mu\nu}$ by,

$$\Lambda^{\mu\nu} \equiv G^{\mu\nu} - \alpha T^{\mu\nu}. \tag{30}$$

With this definition, $\Lambda^{\mu\nu}$ is constrained to be both symmetric,

$$\Lambda^{\mu\nu} = \Lambda^{\nu\mu}, \tag{31}$$

and conserved,

$$\Lambda^{\mu\nu}{}_{;\nu} = 0 \quad . \tag{32}$$

The value of the constant α in (30) is completely arbitrary and without physical significance because $\Lambda^{\mu\nu}$ as defined can absorb any change in α such that (30) remains satisfied. Taking advantage of this arbitrariness and setting the value of the constant $\alpha = -8\pi$ then gives with a slight rearrangement of (30),

$$G^{\mu\nu} = -8\pi\, T^{\mu\nu} + \Lambda^{\mu\nu} \quad , \tag{33}$$

which is recognized as Einstein's equation of General Relativity (29) augmented on its RHS by the term $\Lambda^{\mu\nu}$. From the perspective of classical General Relativity, $\Lambda^{\mu\nu}$ mimics the properties of the energy-momentum tensor for dark matter and/or dark energy, *viz.*, it is a conserved and symmetric tensor field, it is a source of gravitational fields in addition to energy-momentum tensor $T^{\mu\nu}$ for normal matter and normal energy, and it has no interaction signature beyond the gravitational fields it sources.

It is important to recognize that (33) is a trivial result with no physical significance in the proposed theory based on equations (1) and (2). This follows because any solution of the equations (1) and (2) must necessarily be a solution of (33) for some choice $\Lambda^{\mu\nu}$. In fact, the validity of (33) rests only on the existence of a conserved energy-momentum tensor and the properties of the R-C tensor, and so will be true in any physical theory that has a conserved energy-momentum tensor. However, the interesting point in the context of the proposed theory is that the value of $\Lambda^{\mu\nu}$ can be calculated from solutions to (1) and (2) without postulating the existence of dark matter and/or energy. This feature will be investigated further in subsequent sections in which specific solutions to equations (1) and (2) will be developed.

In the view being put forth here, gravitation emerges as a manifestation of the geometricized theory of electromagnetism based on equations (1) and (2), *i.e.*, a theory of gravitation is self-contained within equations (1) and (2). Specifically, it is the coupling of the derivatives of the Maxwell tensor to the R-C tensor in (1) that brings gravitation into the picture. Importantly, the gravitational theory that emerges does not obey the classical General Relativity field equations (29), although any solution of equations (1) and (2) must necessarily be a solution of equation (33) for some choice of $\Lambda^{\mu\nu}$. While viewing gravitation as a manifestation of electromagnetism and vice versa is not new [iv, v, vi, vii, viii], the specific approach being followed here with equation (1) is new.

3. MATHEMATICAL STRUCTURE OF THE FIELD EQUATIONS

3.1 Symmetries of equations (1) and (2)

Listed in Table I are the continuous field variables that the theory based on equations (1) and (2) solves for. Also included in Table I are the charge density ρ_c and the four-velocity field u^λ that are defined for any solution in terms of $g_{\mu\nu}$ and a^λ as given by (16) and (17), respectively.

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Table I. Dynamic fields

Field	Description
$g_{\mu\nu}$	Metric tensor
$F_{\mu\nu}$	Maxwell tensor
ρ_m	Mass density scalar field
a^{λ}	Four-vector coupling electromagnetism to gravitation
ρ_c	Charge density scalar field
u^{λ}	Four-velocity vector field

Three important global symmetries of equations (1) and (2) that are shared by all their solutions are reviewed here. The first of these global symmetries corresponds to charge-conjugation,

$$\begin{pmatrix} u^{\lambda} \\ a^{\lambda} \\ F^{\mu\nu} \\ g_{\mu\nu} \\ \rho_c \\ \rho_m \end{pmatrix} \rightarrow \begin{pmatrix} u^{\lambda} \\ -a^{\lambda} \\ -F^{\mu\nu} \\ g_{\mu\nu} \\ -\rho_c \\ \rho_m \end{pmatrix}, \tag{34}$$

the second corresponds to a matter-antimatter transformation as will be discussed in section 5.4,

$$\begin{pmatrix} u^{\lambda} \\ a^{\lambda} \\ F^{\mu\nu} \\ g_{\mu\nu} \\ \rho_c \\ \rho_m \end{pmatrix} \rightarrow \begin{pmatrix} -u^{\lambda} \\ -a^{\lambda} \\ -F^{\mu\nu} \\ g_{\mu\nu} \\ \rho_c \\ \rho_m \end{pmatrix}, \tag{35}$$

and the third to the product of the first two,

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$$\begin{pmatrix} u^{\lambda} \\ a^{\lambda} \\ F^{\mu\nu} \\ g_{\mu\nu} \\ \rho_c \\ \rho_m \end{pmatrix} \to \begin{pmatrix} -u^{\lambda} \\ a^{\lambda} \\ F^{\mu\nu} \\ g_{\mu\nu} \\ -\rho_c \\ \rho_m \end{pmatrix} . \tag{36}$$

All three transformations leave equations (1) and (2) unchanged. Adding the identity transformation to these symmetries forms the Klein-4 group, with the product of any two of the symmetries (34) through (36) giving the remaining symmetry. Note that among the fields of the theory, only $g_{\mu\nu}$ and ρ_m are unchanged by all the symmetry transformations, a fact that will be useful in section 5.6 for defining boundary conditions that lead to quantized mass, charge, and angular momentum of particle-like solutions as well as for the treatment of antimatter. Finally, in addition to the proposed theory's general covariance and global symmetries (34) through (36), it also exhibits the electromagnetic gauge covariance of classical electromagnetism as detailed in equations (7) and (8).

3.3 Do solutions exist to equation (1)?

Equation (1) represents a mixed system of first order partial differential equations for $F_{\mu\nu}$ and illustrates one of the mathematical complexities of equations (1) and (2) that must be dealt with when attempting to find solutions.[ix] Specifically, mixed systems of first order partial differential equations must satisfy integrability conditions if solutions are to exist.[x] Although there are several ways of stating what these integrability conditions are, perhaps the simplest is given by,

$$F_{\mu\nu;\kappa;\lambda} - F_{\mu\nu;\lambda;\kappa} = -F_{\mu\sigma}R^{\sigma}{}_{\nu k\lambda} - F_{\sigma\nu}R^{\sigma}{}_{\mu\kappa\lambda} \;, \tag{37}$$

which can be derived using the commutation relations for covariant derivatives. Using (1) to substitute for $F_{\mu\nu;\kappa}$ in (37) gives,

$$\left(a^{\rho}R_{\rho\kappa\mu\nu}\right)_{;\lambda} - \left(a^{\rho}R_{\rho\lambda\mu\nu}\right)_{;k} = -F_{\mu\sigma}R^{\sigma}{}_{\nu k\lambda} - F_{\sigma\nu}R^{\sigma}{}_{\mu\kappa\lambda} \;, \tag{38}$$

which can be interpreted as conditions that are automatically satisfied by any solution consisting of expressions for $g_{\mu\nu}$, a^{λ} and $F_{\mu\nu}$ that satisfy (1). With (38) as integrability conditions that must be satisfied by any solution of (1), the question that naturally arises is this: Are these integrability conditions so restrictive that perhaps no solution to the proposed theory exists? Although this view could be construed as making the proposed field theory uninteresting because perhaps no solutions exist, it will be shown that solutions that are consistent with known solutions of the classical Maxwell and Einstein Field Equations (M&EFEs) can be found.

Additionally, equation (38), which is linear in $F_{\mu\nu}$ is often useful in developing solutions to equations (1) and (2), an approach that will be used in the solution to be found in section 4.1. Finally, to further elucidate questions regarding solutions of the proposed theory, an outline showing how the field equations can be solved numerically is given in the appendix where an analysis is presented of equations (1) and (2) in terms of a Cauchy initial value problem.

4. SOLUTIONS TO EQUATIONS (1) AND (2)

In this section three solutions to equations (1) and (2) are presented. The first solution is spherically symmetric, representing the electric and gravitational fields of a non-rotating, charged particle. The second solution is radiative with two distinct sub solutions, one with electromagnetic radiation in the presence of gravitational radiation and the other with standalone gravitational radiation. The third solution has a maximally symmetric 3-dimensional subspace, for example, representing an isotropic and homogenous universe. The purpose in developing these solutions is twofold: First, to provide a comparison of solutions to equations (1) and (2) with those corresponding to the classical M&EFEs, and second to demonstrate that the solutions to equations (1) and (2) go further than the classical M&EFEs by uniting electromagnetic and gravitational phenomena.

4.1 Spherically symmetric solution

Here a solution representing a non-rotating, spherically symmetric charged mass is investigated. It is demonstrated that the Reissner-Nordström metric with an appropriate choice for the fields $F_{\mu\nu}, a^{\lambda}, u^{\lambda}, \rho_c$ and ρ_m satisfies equations (1) and (2). Although the presentation in this section is purely formal, it is included here for several reasons. First, if the theory could not describe the asymptotic electric and gravitational fields of a charged particle it would be of no interest on physical grounds. Second, as already discussed, equation (1) requires the solution of a mixed system of first order partial differential equations, a system that may be so restrictive that no solutions exist, and so an outline of at least one methodology to a solution is warranted.

To proceed, I draw on a solution for a spherically symmetric charged particle that was previously derived.[xi] Starting with the Reissner-Nordström metric[xii],

$$g_{\mu\nu} = \begin{pmatrix} \dfrac{1}{1+\dfrac{q^2}{r^2}-\dfrac{2m}{r}} & 0 & 0 & 0 \\ 0 & r^2 & 0 & 0 \\ 0 & 0 & r^2\mathrm{Sin}^2(\theta) & 0 \\ 0 & 0 & 0 & -1-\dfrac{q^2}{r^2}+\dfrac{2m}{r} \end{pmatrix}, \tag{39}$$

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and the Ricci tensor that follows from it,

$$R_\lambda{}^\nu = \frac{q^2}{r^4}\begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & -1 & 0 & 0 \\ 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}, \tag{40}$$

I investigate a trial solution for a^λ,

$$a^\lambda = (0,0,0,c_1), \tag{41}$$

where c_1 is a yet to be determined constant. Using (41), ρ_c can be determined from (16) as,

$$\rho_c = \pm \frac{q^2\sqrt{q^2 + r(r-2m)}}{r^5}\left|c_1\right|, \tag{42}$$

and u^λ from (17) as,

$$u^\lambda = \left(0,0,0,\pm\frac{r}{\sqrt{q^2 + r(r-2m)}}\frac{c_1}{\left|c_1\right|}\right). \tag{43}$$

The next step is to satisfy (1) by solving for $F_{\mu\nu}$. Rather than tackle this head on by directly trying to find a solution to the mixed system of first order partial differential equations that is (1), I instead solve the integrability equations (38) which are linear in $F_{\mu\nu}$ for $F_{\mu\nu}$. Proceeding in this manner I find that all the integrability equations are satisfied for $F_{\mu\nu}$ given by,

$$F_{\mu\nu} = \begin{pmatrix} 0 & B_\phi & -B_\theta & E_r \\ -B_\phi & 0 & B_r & E_\theta \\ B_\theta & -B_r & 0 & E_\phi \\ -E_r & -E_\theta & -E_\phi & 0 \end{pmatrix} = \begin{pmatrix} 0 & 0 & 0 & \frac{(mr-q^2)}{r^3}c_1 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ -\frac{(mr-q^2)}{r^3}c_1 & 0 & 0 & 0 \end{pmatrix}. \tag{44}$$

By direct substitution it is easily verified that $F_{\mu\nu}$ as given in (44) is a solution of (1).[xiii] Choosing the value of the undetermined constant $c_1 = \pm q/m$ then gives an electric field which agrees with the Coulomb field of a point charge to leading order in $1/r$. Finally, the remaining unknown field, the scalar mass density ρ_m is found using the conserved energy-momentum tensor (2). Substituting the known fields into (2) and then solving for ρ_m gives,

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$$\rho_m = \frac{q^4(q^2 - 2mr + r^2)}{m^2 r^6}. \tag{45}$$

To summarize, the following expressions for $g_{\mu\nu}$, $F_{\mu\nu}$, a^λ, u^λ, ρ_c and ρ_m are an exact solution to equations (1) and (2):

$$g_{\mu\nu} = \begin{pmatrix} \frac{1}{1-\frac{2m}{r}+\frac{q^2}{r^2}} & 0 & 0 & 0 \\ 0 & r^2 & 0 & 0 \\ 0 & 0 & r^2\mathrm{Sin}^2(\theta) & 0 \\ 0 & 0 & 0 & -\left(1-\frac{2m}{r}+\frac{q^2}{r^2}\right) \end{pmatrix}$$

$$F_{\mu\nu} = \begin{pmatrix} 0 & B_\phi & -B_\theta & E_r \\ -B_\phi & 0 & B_r & E_\theta \\ B_\theta & -B_r & 0 & E_\phi \\ -E_r & -E_\theta & -E_\phi & 0 \end{pmatrix} = s\begin{pmatrix} 0 & 0 & 0 & \frac{q}{r^2} - \frac{q^3/m}{r^3} \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ -\frac{q}{r^2} + \frac{q^3/m}{r^3} & 0 & 0 & 0 \end{pmatrix}$$

$$u^\lambda = s\left(0,0,0,\frac{1}{\sqrt{1-\frac{2m}{r}+\frac{q^2}{r^2}}}\right)$$

$$a^\lambda = s\left(0,0,0,\frac{q}{m}\right) \tag{46}$$

$$\rho_c = \frac{q^3}{m}\frac{\sqrt{1-\frac{2m}{r}+\frac{q^2}{r^2}}}{r^4}$$

$$\rho_m = \frac{q^4}{m^2}\frac{(1-\frac{2m}{r}+\frac{q^2}{r^2})}{r^4}$$

In (46), the multiplicative parameter s in the solutions for u^λ, a^λ and $F_{\mu\nu}$ takes on the values ±1 and corresponds to the global matter-to-antimatter symmetry transformation (35) which will be further discussed in section 5.4. Except for the possibility of both matter and antimatter solutions, the physical interpretation of solution (46) is almost identical to that of the classical M&EFEs, *i.e.*, a non-rotating, spherically symmetric particle having charge q and mass m. The metric tensor which is identical to the Reissner-Nordström metric establishes that the new theory and Einstein's General Relativity predict the same gravitational fields. However, solution (46) does differ from the classical picture in several ways. For example, the mass and

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charge are not localized, with both ρ_m and ρ_c having a spatial extent with wings that fall off as $1/r^4$. Also, the radial electric field,

$$E_r = \frac{q}{r^2} - \frac{q^3/m}{r^3} = \frac{q}{r^2}\left(1 - \frac{q^2/m}{r}\right) , \tag{47}$$

which agrees with the Coulomb field q/r^2 to leading order in $1/r$ does have a higher order term. This next term in the radial electric field depends on both the charge and mass of the particle. Taking an electron as an example, its electric field as given by (47) would be,

$$E_r = \frac{q_e}{r^2}\left(1 - \frac{r_e}{r}\right) \approx \frac{q_e}{r^2}\left(1 - \frac{2.82x10^{-15}m}{r}\right) , \tag{48}$$

where $r_e = q_e^{\,2}/m_e$ is recognized as the classical radius of an electron (~2.82x10^{-15} m). Although the correction term to the Coulomb field is small, being only ~53 ppm at a Bohr radius, it may have interesting consequences in various situations because it depends on both the charge and the mass of the particle.

The gravitational field predicted by the solution investigated here agrees with the corresponding solution to Einstein's General Relativity (29), with both described by the Reissner-Nordström metric. However, it is important to note that the classical General Relativity field equations (29) are not satisfied using the energy-momentum tensor,

$$T^{\mu\nu} = \rho_m u^\mu u^\nu + F^\mu{}_\lambda F^{\nu\lambda} - \frac{1}{4} g^{\mu\nu} F^{\rho\sigma} F_{\rho\sigma} , \tag{49}$$

when evaluated using the solution given in (46). However, Einstein's equation of General Relativity augmented by the $\Lambda^{\mu\nu}$ term on its RHS in equation (33) is trivially satisfied. For completeness, the values of $G^{\mu\nu}, T^{\mu\nu}$ and $\Lambda^{\mu\nu}$ that go with solution (46) are given here:

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$$G^{\mu\nu}=\begin{pmatrix} \frac{q^2\left(q^2+r(-2m+r)\right)}{r^6} & 0 & 0 & 0 \\ 0 & -\frac{q^2}{r^6} & 0 & 0 \\ 0 & 0 & -\frac{q^2\csc^2(\theta)}{r^6} & 0 \\ 0 & 0 & 0 & -\frac{q^2}{r^2\left(q^2+r(-2m+r)\right)} \end{pmatrix}$$

$$T^{\mu\nu}=\begin{pmatrix} -\frac{q^2\left(q^2-mr\right)^2\left(q^2+r(-2m+r)\right)}{2m^2r^8} & 0 & 0 & 0 \\ 0 & \frac{q^2\left(q^2-mr\right)^2}{2m^2r^8} & 0 & 0 \\ 0 & 0 & \frac{q^2\left(q^2-mr\right)^2\csc^2(\theta)}{2m^2r^8} & 0 \\ 0 & 0 & 0 & \frac{3q^6+m^2q^2r^2+2q^4r(-3m+r)}{2m^2r^4\left(q^2+r(-2m+r)\right)} \end{pmatrix} \quad (50)$$

$$\Lambda^{\mu\nu}=G^{\mu\nu}+8\pi T^{\mu\nu}$$

In the context of classical General Relativity (29), the interpretation of $\Lambda^{\mu\nu}$ is that of the energy-momentum tensor for dark matter and/or dark energy, which serves as a source term for gravitational fields in addition to $T^{\mu\nu}$. However, in the context of the proposed theory, $\Lambda^{\mu\nu}$ depends only on the existence of normal matter and normal energy and is a consequence of equations (1) and (2), again emphasizing that theory of gravitation emerging from (1) and (2) differs from that of classical General Relativity as given by (29).

4.2 Radiative solutions in the weak field limit

Electromagnetic radiation

Working in the weak field limit, I derive expressions for a propagating electromagnetic plane wave in terms of the vector field a^{λ} and the metric tensor $g_{\mu\nu}$. This example establishes a fundamental relationship between electromagnetic and gravitational radiation imposed by equation (1), predicting that both are manifestations of wave propagation of the underlying metric $g_{\mu\nu}$. To begin, consider an electromagnetic plane wave having frequency ω, propagating in the +*z*-direction and polarized in the *x*-direction. The Maxwell tensor for this field is given by,

$$F_{\mu\nu} = \begin{pmatrix} 0 & 0 & -B_y & E_x \\ 0 & 0 & 0 & 0 \\ B_y & 0 & 0 & 0 \\ -E_x & 0 & 0 & 0 \end{pmatrix} e^{i\omega(t-z)} \tag{51}$$

where E_x and B_y are the constant field amplitudes of the electromagnetic wave. Next, assume a near-Minkowski weak field metric given by,

$$\begin{gathered} g_{\mu\nu} = \eta_{\mu\nu} + h_{\mu\nu} e^{i\omega(t-z)} \\ |h_{\mu\nu}| \ll 1 \end{gathered} \tag{52}$$

where $\eta_{\mu\nu} = diag[1,1,1,-1]$, $h_{\mu\nu}$ are complex constants, and the vector field a^λ is assumed to be constant and given by,

$$a^\lambda = \left(a^1, a^2, a^3, a^4\right) \ . \tag{53}$$

I proceed by substituting for $F_{\mu\nu}$ from (51), for $g_{\mu\nu}$ from (52), and for a^λ from (53) into (1), and then only retain terms to first order in the fields $h_{\mu\nu}$ and $F_{\mu\nu}$, both of which are assumed to be small and of the same order.[xiv] Doing this leads to a set of 8 independent linear equations for the 16 unknown constants: $h_{\mu\nu}$, a^λ, E_x and B_y. Solving these 8 independent equations gives 8 field components E_x, B_y, h_{13}, h_{22}, h_{23}, h_{34}, a^2 and a^3 in terms of 8 free constants a^1, a^4, h_{11}, h_{12}, h_{14}, h_{24}, h_{33}, and h_{44}:

$$\begin{aligned} E_x &= i\,\omega \frac{\left(h_{11}{}^2 + h_{12}{}^2\right)}{2\,h_{11}} a^1 \quad , \\ B_y &= E_x \end{aligned} \tag{54}$$

$$g_{\mu\nu} = \eta_{\mu\nu} + \begin{pmatrix} h_{11} & h_{12} & -h_{14} & h_{14} \\ h_{12} & -h_{11} & -h_{24} & h_{24} \\ -h_{14} & -h_{24} & h_{33} & -\frac{1}{2}(h_{33}+h_{44}) \\ h_{14} & h_{24} & -\frac{1}{2}(h_{33}+h_{44}) & h_{44} \end{pmatrix} e^{i\omega(t-z)} \ , \tag{55}$$

and

$$a^{\lambda}=\left(a^{1},a^{1}\frac{h_{12}}{h_{11}},a^{4},a^{4}\right) \quad . \tag{56}$$

This solution illustrates several ways in which the new theory departs from the classical physics view of electromagnetic radiation. Of most significance, the undulations in the electromagnetic field are due to undulations in the underlying metric field $g_{\mu\nu}$ given in (55). This result also underscores that the existence of electromagnetic radiation is forbidden in strictly flat space-time. An interesting aspect of this solution is that while electromagnetic radiation necessitates the presence of an underlying gravitational radiation field, the underlying gravitational radiation is not completely defined by the electromagnetic radiation. The supporting gravitational radiation has 6 undetermined constants $(h_{11},h_{12},h_{14},h_{24},h_{33},h_{44})$ with the only restriction being $|h_{\mu\nu}|\ll 1$ and $h_{11}\neq 0$ as required by (54). Further insight into the physical content of the metric (55) is evident after making the infinitesimal coordinate transformation from $x^{\mu}\rightarrow x'^{\mu}$ given by,

$$\begin{pmatrix} x \\ y \\ z \\ t \end{pmatrix} \rightarrow \begin{pmatrix} x' \\ y' \\ z' \\ t' \end{pmatrix} = \begin{pmatrix} x-\dfrac{i}{\omega}h_{14}\,e^{i\omega(t-z)} \\ y-\dfrac{i}{\omega}h_{24}\,e^{i\omega(t-z)} \\ z+\dfrac{i}{2\omega}h_{33}\,e^{i\omega(t-z)} \\ t+\dfrac{i}{2\omega}h_{44}\,e^{i\omega(t-z)} \end{pmatrix} \quad , \tag{57}$$

and only retaining terms to first order in the *h's*. Doing this, the metric (55) is transformed to,

$$g'_{\mu\nu}=\eta_{\mu\nu}+\begin{pmatrix} h_{11} & h_{12} & 0 & 0 \\ h_{12} & -h_{11} & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix} e^{i\omega(t-z)} \quad , \tag{58}$$

while E'_{x} and B'_{y}, the transformed electric and magnetic field amplitudes, respectively, are identical to E_{x} and B_{y} given in (54). Note, only the h_{11} and h_{12} components of the metric (58) have an absolute physical significance and $h_{22}=-h_{11}$ which makes the gravitational plane wave solution (58) identical to the gravitational plane wave solution of the classical Einstein field equations.[xv], [xvi]

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Gravitational radiation

The forgoing analysis demonstrated the necessity of having an underlying gravitational wave to support the presence of an electromagnetic wave, but the converse is not true and gravitational radiation can exist independent of electromagnetic radiation. The following analysis demonstrates this by solving for the structure of gravitational radiation in the absence of electromagnetic radiation. Following the same weak field formalism for the unknown fields $h_{\mu\nu}$ given in (52), but this time zeroing out E_x and B_y in (51), leads to the following solutions for $g_{\mu\nu}$ and a^{λ},

$$g_{\mu\nu} = \eta_{\mu\nu} + \begin{pmatrix} h_{11} & h_{12} & -h_{14} & h_{14} \\ h_{12} & \dfrac{h_{12}{}^{2}}{h_{11}} & -h_{24} & h_{24} \\ -h_{14} & -h_{24} & h_{33} & -\dfrac{h_{33}+h_{44}}{2} \\ h_{14} & h_{24} & -\dfrac{h_{33}+h_{44}}{2} & h_{44} \end{pmatrix} e^{i\omega(t-z)} \tag{59}$$

and,

$$a^{\lambda} = \left(a^{1}, -a^{1}\frac{h_{11}}{h_{12}}, a^{4}, a^{4} \right) . \tag{60}$$

Both $g_{\mu\nu}$ given by (59) and a^{λ} given by (60) are modified from their solutions in the presence of an electromagnetic wave as given by (55) and (56), respectively. Performing a transformation to the same primed coordinate system as given in (57), here gives the metric field,

$$g'_{\mu\nu} = \eta_{\mu\nu} + \begin{pmatrix} h_{11} & h_{12} & 0 & 0 \\ h_{12} & \dfrac{h_{12}{}^{2}}{h_{11}} & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix} e^{i\omega(t-z)} , \tag{61}$$

illustrating again that only the h_{11} and h_{12} components have an absolute physical significance. Of particular note is the change in the value of the h_{22} component depending on whether the gravitational wave supports an electromagnetic wave as in (58) or is standalone as in (61).

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In this section, equation (1) has been shown to have a weak field electromagnetic plane wave solution identical to that of the classical Maxwell equations. Additionally, the gravitational radiation solution that underpins this electromagnetic plane wave is identical to the weak field gravitational wave solution of classical General Relativity. The solutions of equation (1) are again seen to be consistent with those of the classical M&EFEs but to go further by providing an underlying unification between electromagnetic and gravitational phenomena.

4.3 Solution with a maximally symmetric 3-dimensional subspace

Next, I consider the time-dependent Friedmann–Lemaître–Robertson–Walker (FLRW) metric,

$$g_{\mu\nu} = \begin{pmatrix} \frac{R_s^2(t)}{1-kr^2} & 0 & 0 & 0 \\ 0 & R_s^2(t)r^2 & 0 & 0 \\ 0 & 0 & R_s^2(t)r^2\,\mathrm{Sin}^2(\theta) & 0 \\ 0 & 0 & 0 & -1 \end{pmatrix} \tag{62}$$

where k equals +1, 0 or -1 depending on whether the spatial curvature is positive, zero or negative, respectively, and $R_s(t)$ is a time-dependent scale factor. Just as in the case of General Relativity where the FLRW metric is a cosmological solution representing a homogeneous and isotropic universe it is the same for equation (1) with an appropriate choice for the time development of $R_s(t)$. To derive the time dependence of $R_s(t)$, I note the 3-dimensional spatial subspace of (62) is maximally symmetric and so any tensor fields that inhabit that subspace must also be maximally symmetric.[xvii] Specifically, this restricts the form of a^μ to be,

$$a^\mu = \left(0,0,0,a^4(t)\right), \tag{63}$$

and forces the antisymmetric Maxwell tensor to vanish,

$$F_{\mu\nu} = 0 \ . \tag{64}$$

Because $F_{\mu\nu}$ vanishes so must $F_{\mu\nu;\kappa}$,

$$F_{\mu\nu;\kappa} = 0 \ , \tag{65}$$

which on substitution in (1) forces,

$$a^\lambda R_{\lambda\kappa\mu\nu} = 0 \, . \tag{66}$$

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This in turn forces,

$$a^{\lambda} R_{\lambda}^{\ \nu} = 0 \, , \qquad (67)$$

which gives $\rho_c = 0$ by equation (16). Substituting a^{μ} given by (63), and the FLRW metric given by (62) into (66) then leads to the following set of equations to be satisfied,

$$\begin{aligned} a^4(t) R_{4114} &= a^4(t) \left(\frac{R_s(t)}{k\,r^2 - 1} \frac{d^2 R_s(t)}{dt^2} \right) = 0 \\ a^4(t) R_{4224} &= a^4(t) \left(-r^2 \, R_s(t) \frac{d^2 R_s(t)}{dt^2} \right) = 0 \\ a^4(t) R_{4334} &= a^4(t) \left(-r^2 \, R_s(t) \, \mathrm{Sin}^2(\theta) \frac{d^2 R_s(t)}{dt^2} \right) = 0 \end{aligned} \qquad (68)$$

with all other components of (66) not listed in (68) being trivially satisfied, *i.e.*, $0 = 0$. The nontrivial components given in equations (68) are all satisfied if,

$$\frac{d^2 R_s(t)}{dt^2} = 0 \, , \qquad (69)$$

or,

$$R_s(t) = R_{s\,0} + v_s \, t \ \ , \qquad (70)$$

where $R_{s\,0}$ is the scale factor at time *t=0*, and v_s is its constant rate of change.

Summarizing, the predictions of the new theory for a homogeneous and isotropic solution are:

1. It must be charge neutral, $\rho_c = 0$.

2. The scale factor $R_s(t)$ changes linearly with time.

3. The spatial curvature of the solution can be positive, negative or 0.

The second prediction above regarding the time dependence of the scale factor differs from the predictions of the Friedmann models of General Relativity and again emphasizes that the theory of gravitation emerging from equations (1) and (2) differs from that described by the General Relativity. In fact, equation (70) for the time rate of change of the scale factor $R_s(t)$ as determined here depends only on equation (1), the geometricized version of Maxwell's equations.

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5. DISCUSSION

5.1 The classical Maxwell's field equations from $F_{\mu\nu;\kappa} = a^{\lambda}R_{\lambda\kappa\mu\nu}$

As developed within, equation (1) leads directly to the classical Maxwell equations. In fact, equation (1) was empirically chosen to reproduce the classical Maxwell equations under the assumption that the geometry of nature is Riemannian with four dimensions. Going further, equation (1) extends the interpretation of the classical Maxwell equations in that both the charge density ρ_c and the four-velocity u^{λ} are defined in terms of the Ricci tensor and the vector field a^{λ} as given by (16) and (17), respectively. In this sense, the fields ρ_c and u^{λ} are not fundamental, but rather are determined from the other fields listed in Table I. Finally, the vector field a^{λ}, which is not familiar to classical physics and here serves to couple the Maxwell tensor to the R-C tensor is directly relatable to the familiar vector potential A^{λ} of classical electromagnetism through equation (28).

It is an unusual circumstance that equation (1), the geometricized version of Maxwell's equations contains both the four-vector a^{λ} and the R-C tensor explicitly, while the classical Maxwell equations which are derivative to (1) contain neither,

$$\text{Eq. (1)} \quad \rightarrow \quad \text{Classical Maxwell Eqs.}$$

$$F_{\mu\nu;\kappa} = a^{\lambda}R_{\lambda\kappa\mu\nu} \quad \rightarrow \quad \begin{cases} F_{\mu\nu,\kappa} + F_{\nu\kappa,\mu} + F_{\kappa\mu,\nu} = 0 \\ F^{\mu\nu}{}_{;\mu} = -\rho_c u^{\nu} \end{cases} \tag{71}$$

This circumstance conspires to give the classical Maxwell equations the appearance of being valid in flat space-time and motivates their classical interpretation in terms of a conserved charge density source term ρ_c having its motion described by the four-velocity field u^{λ}, both of which are classically assumed to exist in flat space-time. However, in the view of the geometricized theory based on equation (1), the conserved charge density ρ_c and the four-velocity u^{λ} cannot exist in flat space-time. Thus, the classical Maxwell equations and their interpretation in flat space-time are at best an approximation to the geometricized Maxwell equations that are derivative to equation (1).

5.2 Dark matter and dark energy

With General Relativity as the foundation of observational gravitational physics today, dark matter and dark energy have been postulated to exist because of the many galactic and cosmological scale observations that cannot be understood using General Relativity with normal matter and normal energy alone. For example, some of the large-scale gravitational features of galaxies and galactic clusters dating back to Zwicky's observations in the 1930's have been explained using dark matter[xviii], and the acceleration

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of the universe discovered in the 1990's explained using dark energy[xix]. Another example of modifications made to the original General Relativity field equations (29) to satisfy a perceived need was the cosmological constant term $\lambda g_{\mu\nu}$ that Einstein added to their RHS,

$$G_{\mu\nu} = -8\pi T_{\mu\nu} + \lambda g_{\mu\nu} . \tag{72}$$

This was done to enable a static universe solution, but then subsequently dropped after expansion was discovered. Today this term is again in vogue as a possible descriptor of dark energy.

One of the vexing problems facing dark matter and dark energy-based explanations of various observational phenomena today is an ongoing inability to directly detect them. However, equations (1) and (2) offer the prospect that dark matter and dark energy effects can be explained in terms of normal matter and normal energy alone, *i.e.*, the $\Lambda^{\mu\nu}$ term in (33) which represents the energy-momentum tensor of dark matter and/or dark energy in the context of General Relativity is provided with a mechanism for directly calculating its structure using equations (1) and (2) with only normal matter and normal energy. The already investigated spherically symmetric particle-like solution (46) is one example that outlines such a direct calculation of $\Lambda^{\mu\nu}$. With questions today regarding the validity of classical General Relativity beyond the confines of our own solar system[xx] and the inability to directly detect dark matter and dark energy, the possible interpretation of the $\Lambda^{\mu\nu}$ term in (33) using only normal matter and normal energy is an enticing feature of equations (1) and (2). However, it must be acknowledged that one of the challenging tasks facing the theory based on equations (1) and (2), and one well beyond the analysis presented in this manuscript, is that of finding additional solutions that could be interpreted as agreeing with the rapidly developing observational understanding of galactic and cosmological structures.

5.3 The unification of gravitational and electromagnetic radiation

One of the successes of equation (1) is the existence of solutions describing both electromagnetic and gravitational radiation, with both phenomena being unified as undulations of the underlying metric field $g_{\mu\nu}$. Because both gravitational and electromagnetic radiation are due to undulations of the metric field $g_{\mu\nu}$, their speed of propagation is predicted to be identical. This prediction has recently been refined experimentally with observations made during the binary neutron star merger in NGC 4993, 130 million light years from Earth.[xxi] The nearly simultaneous detection, within 2 seconds of each other, of gravity waves[xxii] and a burst of gamma rays[xxiii] from this event experimentally constrain the propagation speed of electromagnetic and gravitational radiation to be the same to better than 1 part in 10^{15}.

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5.4 The emergence of antimatter and its behavior in electromagnetic and gravitational fields

One of the unique features of equations (1) and (2) is that the properties of antimatter emerge naturally in their solutions. Traditionally, these properties emerge in quantum mechanical treatments but here emerge in the context of a classical continuous field theory due to the global symmetry (35) of equations (1) and (2); every matter containing solution has a corresponding antimatter solution generated by the symmetry transformation (35). This is evident in the spherically symmetric, particle-like solution (46) where the multiplicative factor s in the expressions for $F_{\mu\nu}$, a^λ and u^λ is defined by,

$$s = \begin{cases} +1 \text{ for matter} \\ -1 \text{ for antimatter} \end{cases} \tag{73}$$

and accounts for the matter-antimatter symmetry expressed in (35). The physical interpretation of the $s=-1$ solution is that it represents a particle having the same mass but opposite charge and four-velocity to that of the $s=+1$ solution. This is equivalent to the view that a particle's antiparticle is the particle moving backwards through time.[xxiv] Said another way, the time-like component of the four-velocity is positive for matter and negative for antimatter,

$$u^4 \begin{cases} >0 \text{ for matter} \\ <0 \text{ for antimatter} \end{cases} \tag{74}$$

With these definitions for the four-velocity of matter and antimatter, charged mass density can annihilate similarly charged anti-mass density and satisfy both the local conservation of charge (20) and local conservation of mass (24). Additionally, such annihilation reactions must conserve energy by (2).

Building on the distinction between matter and antimatter, their behavior in external electromagnetic and gravitational fields in the context of equations (1) and (2) is briefly reviewed here. As already mentioned, antimatter can be viewed as matter moving backwards through time. To see this more rigorously consider the four-velocity associated with a fixed quantity of charge and mass density,

$$u^\lambda = \frac{dx^\lambda}{d\tau} \quad . \tag{75}$$

Under the matter-antimatter transformation (35), $u^\lambda \to -u^\lambda$, or equivalently $d\tau \to -d\tau$. This motivates the following expression for the four-velocity in terms of the coordinate time in locally inertial coordinate systems,

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$$u^\lambda = \frac{dx^\lambda}{d\tau} = s\gamma \frac{dx^\lambda}{dt} = s\gamma \begin{pmatrix} \vec{v} \\ 1 \end{pmatrix} \,, \tag{76}$$

where s is the matter-antimatter parameter defined in (73), $\vec{v}$ is the ordinary 3-space velocity of the charge and mass density, and $\gamma = 1/\sqrt{1-\vec{v}^2}$. Equation (76) establishes that corresponding matter and antimatter solutions travel in opposite time directions relative to each other, and equation (35), the matter-antimatter transformation requires ρ_c does not change sign under such transformations. To see that these solution characteristics are consistent with the usual view in which antiparticles have the opposite charge of their corresponding particles, I use (76) to illustrate the behavior of a charged matter and antimatter density in an external electromagnetic field. Consider a region with an externally defined electromagnetic field,

$$F_{\mu\nu} = \begin{pmatrix} 0 & B_z & -B_y & E_x \\ -B_z & 0 & B_x & E_y \\ B_y & -B_x & 0 & E_z \\ -E_x & -E_y & -E_z & 0 \end{pmatrix} \tag{77}$$

in a locally inertial coordinate system. Starting with the Lorentz force law (25), and then expanding and rearranging slightly leads to the following development,

$$\rho_p \frac{Du^\mu}{D\tau} = \rho_c F^\mu{}_\lambda u^\lambda$$

$$\downarrow$$

$$\rho_p s\gamma \frac{du^\mu}{dt} = \rho_c F^\mu{}_\lambda u^\lambda$$

$$\downarrow$$

$$\rho_p s\gamma \frac{d}{dt}\begin{pmatrix} s\gamma\vec{v} \\ s\gamma \end{pmatrix} = \rho_c \begin{pmatrix} 0 & B_z & -B_y & E_x \\ -B_z & 0 & B_x & E_y \\ B_y & -B_x & 0 & E_z \\ E_x & E_y & E_z & 0 \end{pmatrix} \begin{pmatrix} s\gamma v_x \\ s\gamma v_y \\ s\gamma v_z \\ s\gamma \end{pmatrix}$$

$$\downarrow$$

$$\rho_p \frac{d}{dt}\begin{pmatrix} \gamma\vec{v} \\ \gamma \end{pmatrix} = s\rho_c \begin{pmatrix} \vec{E} + \vec{v}\times\vec{B} \\ \vec{v}\cdot\vec{E} \end{pmatrix} \tag{78}$$

which on the last line above ends up at the conventional form of the Lorentz force law except for the extra factor of s on the RHS. This factor of s gives the product $s\rho_c$ the appearance that antimatter charge density has the opposite sign of matter charge density when interacting with an external electromagnetic field.

Next, I investigate the behavior of antimatter in an external gravitational field. There is no question about the gravitational fields generated by matter and antimatter, they are identical under the matter-antimatter symmetry (35), as $g_{\mu\nu}$ is unchanged by that transformation. To understand whether antimatter is attracted or repelled by an external gravitational field, I again go to the Lorentz force law (25) but this time assume there is no electromagnetic field present, just a gravitational field given by a Schwarzschild metric generated by a central mass $m>0$ that is composed of either matter or antimatter. I explicitly call out $m>0$ because I am endeavoring to develop a physical theory that flows axiomatically from (1) and (2), and at this point in the development there is nothing to preclude the existence of negative mass density $\rho_m<0$, a consideration I will return to in section 5.5. Placing a test particle having mass m_{test} composed of either matter or antimatter a distance r from the center of the gravitational field and assuming the test particle is initially at rest, the trajectory of the test particle is that of a geodesic given by the following development,

$$\begin{gathered} m_{test}\frac{Du^\mu}{D\tau}=0 \\ \downarrow \\ s\gamma\frac{du^\mu}{dt}=-\Gamma^\mu{}_{\nu\rho}u^\nu u^\rho \\ \downarrow \\ s\gamma\frac{d}{dt}\left(s\gamma\frac{d}{dt}\begin{pmatrix} r\\ \theta\\ \phi\\ t\end{pmatrix}\right)=-\Gamma^\mu{}_{\nu\rho}u^\nu u^\rho\approx-\Gamma^\mu{}_{44}u^4u^4=-\begin{pmatrix}\left(1-\frac{2m}{r}\right)\frac{m}{r^2}\\ 0\\ 0\\ 0\end{pmatrix}\left(\frac{s}{\sqrt{1-\frac{2m}{r}}}\right)^2=-\begin{pmatrix}\frac{m}{r^2}\\ 0\\ 0\\ 0\end{pmatrix}s^2 \end{gathered} \tag{79}$$

where $s=\pm1$ references whether the test particle is composed of matter or antimatter as defined by (73). In the last line of (79), I have approximated the RHS using the initial at rest value of the test particle's four-velocity $u^\mu=\left(0,0,0,s/\sqrt{1-2m/r}\right)$, and additionally used the fact that the only nonzero $\Gamma^\mu{}_{44}$ in a Schwarzschild metric is $\Gamma^1{}_{44}=\left(1-\frac{2m}{r}\right)m/r^2$. Simplifying the LHS of the last line in (79) by noting that initially $\gamma=1$ then gives,

$$\frac{d^2r}{dt^2}\approx-\frac{m}{r^2}, \tag{80}$$

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which is independent of s, and so demonstrates that the proposed theory predicts both matter and antimatter test particles will be attracted by the source of the gravitational field, and this regardless of whether the source of the gravitational field is matter or antimatter. The result that the test particle is attracted toward the source of the gravitational field is also independent of whether the test particle's mass, m_{test}, is positive or negative, this because the geodesic trajectory (80) is independent of m_{test}.

5.5 Possibility of negative mass solutions and antigravity

As already noted, there appears to be nothing in equations (1) and (2) that precludes the possibility of negative mass density $\rho_m < 0$. The existence of negative mass density is equivalent to the existence of antigravity because negative mass density generates gravitational fields that are repulsive, *viz.*, equation (80) with $m < 0$. However, logical inconsistencies are introduced if negative mass density were to exist. As just shown, equation (80) with $m > 0$ predicts a test particle at some distance from the origin will feel an attractive gravitational force regardless of whether the test particle is comprised of matter or antimatter and regardless of whether its mass is positive or negative. Now consider equation (80) with the central mass $m < 0$. Using the same argument as in the previous section, the test particle in this case will feel a repulsive gravitational force regardless of whether it is comprised of matter or antimatter and regardless of whether its mass is positive or negative. These two situations directly contradict each other. For example, in the first case the negative mass test particle is gravitationally attracted toward the positive mass particle located at the origin, but in the second case the positive mass test particle is gravitationally repelled by the negative mass particle located at the origin. This contradiction makes equations (1) and (2) logically inconsistent if negative mass density were to exist. The only way to avoid this logical contradiction is to require that mass density be non-negative always. This condition that mass density ρ_m be non-negative always is also consistent with the global symmetry transformations (34) through (36) where it was noted that the field ρ_m does not change sign under any of the symmetry transformations.

It is interesting to note that the existence of negative mass in the context of classical General Relativity has been extensively studied[xxv], [xxvi] and invoked, particularly when trying to find stable particle-like solutions using the conventional Einstein field equations.[xxvii], [xxviii], [xxix] However, in the context of the present theory the existence of negative mass density leads to a logical contradiction that can only be resolved by requiring mass density be non-negative always, *i.e.*, $\rho_m \geq 0$.

5.6 Conjecture for quantizing the charge and mass of particle-like solutions

Consider particle-like solutions such as (46). Because the mass density and charge density are specified as part of the solution of equations (1) and (2), a self-consistency condition exists for physically allowed solutions that provides a mechanism for quantizing the charge and mass of such solutions. For example, for solution (46) to be self-consistent, the particle's total charge q and total mass m, both parameters of the Reissner-Nordström metric, must agree with the spatially integrated charge and mass density, respectively. For the charge, this amounts to requiring the asymptotic value of the electric field be consistent with the spatially integrated charge density,

$$\lim_{r\to\infty} r^2 F_{14} = \int \rho_c u^4 \sqrt{\gamma_{sp}}\, d^3x = q \quad , \tag{81}$$

where q is the total charge of the particle and given by the asymptotic value of $r^2 F_{14}$ per the solution given in (46), and γ_{sp} is the determinant of the spatial metric defined by,[xxx]

$$\gamma_{sp\; ij} = g_{ij} - \frac{g_{i4}\, g_{j4}}{g_{44}}, \tag{82}$$

where *i* and *j* run over the spatial dimensions 1, 2 and 3. An analogous quantizing boundary condition for the mass of the particle is arrived at by requiring the asymptotic value of its gravitational field be consistent with the spatially integrated mass density of the solution,

$$\lim_{r\to\infty} r\frac{1+g_{44}}{2} = \int \rho_m \left|u^4\right| \sqrt{\gamma_{sp}}\, d^3x = m \quad . \tag{83}$$

The reason for the absolute value of u^4 in the mass boundary condition (83) but not in the charge boundary condition (81) are the global symmetries (34) through (36) exhibited by the theory's equations (1) and (2), and the requirement that the boundary conditions exhibit those same symmetries. The conjecture being put forth here is that boundary conditions (81) and (83) represent self-consistency constraints on the charge and the mass, respectively, that any particle-like solution to equations (1) and (2) must satisfy if the solutions are to be physically realizable.

For the spherically symmetric solution investigated in (46), the RHS of both (81) and (83) diverge leaving no hope for satisfying these quantization/boundary conditions. The upshot of this observation is that while (46) represents a mathematical solution that describes the gravitational and electrical fields of a particle-like solution that formally satisfies the equations (1) and (2), (46) cannot represent a physically allowed solution. The possibility of finding solutions that satisfy both equations (1) and (2), and the charge and mass boundary conditions (81) and (83) remains an open question at this point. However, interesting possibilities exist beyond the spherically symmetric solution based on the Reissner-Nordström metric investigated within. For example, the modified Reissner-Nordström and modified Kerr-Newman metrics developed by S.M. Blinder[xxxi] give finite values for the RHS of both (81) and (83). Finally, when considering metrics that include nonzero angular momentum, as for example would be

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required for particles having an intrinsic magnetic field, the same approach used here to quantize the particle's mass and charge could be used to quantize its angular momentum. Traditionally the quantization of mass, charge and angular momentum are introduced in quantum mechanical treatments but here are conjectured within the framework of a classical continuous field-theoretic description of nature and are another example of how the proposed theory differs from the classical M&EFEs.

5.7 Possibility of superluminal transport if $a^{\lambda}R_{\lambda}{}^{\nu}$ is space-like

Having chosen the form of equations (1) and (2), all subsequent results presented in this manuscript have been mathematically derivative to them. As an example, after the definitions of the charge density ρ_c and the four-velocity u^{λ} were developed in equations (16) and (17), respectively, Maxwell's inhomogeneous equation (18) was shown to follow from equation (1). Noteworthy in the definition for ρ_c is that in addition to its motion being described in terms of subluminal transport, the development naturally includes the case of superluminal transport. Because I am attempting to develop the theory that flows axiomatically from equations (1) and (2), and because there is nothing *a priori* that precludes the possibility of superluminal transport, I have carried it as a possibility, although one that must be regarded as speculative at this point because the specific solutions investigated within have not exhibited it. Although not pursued here further, the possibility of superluminal transport in the context of a classical field theory may be an interesting and timely avenue of investigation as recent research has suggested the possible existence of nonlocal correlations stronger than those predicted by quantum theory.[xxxii]

6. CONCLUSION

The choice of the equations (1) and (2) was empirically driven by the desire to preserve as much as possible the physics embodied in the classical theory of electromagnetism, while providing that theory with a geometric foundation under the assumption that nature is Riemannian with four dimensions. Using the four-vector field a^{λ} that is related to the familiar vector potential A^{λ} of classical electromagnetism, equation (1) which couples the Maxwell tensor to the Riemann-Christoffel curvature tensor was shown to reproduce the classical Maxwell equations in their entirety. Next, the interpretation of the Maxwell equations based on equation (1) was shown to go further than the classical interpretation of them in that the charge density ρ_c and the four-velocity u^{λ} were given a geometric underpinning with both dependent on the Rici Tensor. It is this geometric underpinning that ties electromagnetism to gravitation. Although the gravity emerging form (1) and (2) is different than that described by General Relativity, it is consistent with Einstein's field equations of General Relativity augmented by a symmetric and conserved tensor field, *i.e.*, a field exhibiting the properties of an energy-momentum tensor for dark matter and/or dark energy. However, in the

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context of equations (1) and (2), and in contrast to that in General Relativity, this augmenting field is determined by conventional matter and energy.

Using specific solutions to the theory based on equations (1) and (2), the unification brought to electromagnetic and gravitational phenomena as well as the relation of these solutions to those of the classical M&EFEs was emphasized throughout. Also discussed were unique features/interpretations of the theory based on equations (1) and (2) that set it apart from the classical M&EFEs. These distinguishing features include the emergence of antimatter and its behavior in electromagnetic and gravitational fields, the emergence of dark matter and dark energy mimicking terms in the context of General Relativity, an underlying relationship between electromagnetic and gravitational radiation, and the impossibility of negative mass solutions that would generate repulsive gravitational fields or antigravity. Although not yet based on specific solutions to the proposed theory, a method for quantizing the charge, mass, and angular momentum of particle-like solutions, as well as the possibility of superluminal transport when $a^{\lambda} R_{\lambda}{}^{\nu}$ is space-like were conjectured.

The genesis of the work presented here was reported in a preliminary form in references [ii]. The same coupling between the Maxwell tensor and the R-C tensor given in equation (1) was first reported there, although in a somewhat modified form. The discussion of systems of first order partial differential equations and the existence of solutions to such systems was also given in reference [ii] but is included here to keep the mathematical description of the proposed theory self-contained. New to this manuscript is the discussion of the global symmetries of equations (1) and (2), and based on those global symmetries the interpretation of the particle-like solution has been advanced, as has the discussion of boundary conditions. The discussion of Einstein's equation of General Relativity augmented by a term that can mimic the properties of dark matter and/or dark energy is also new to this manuscript, as is the discussion of the solution based on the FLRW metric. The present manuscript also corrects an error in the weak field analysis of reference [ii], leading to an expanded discussion of electromagnetic radiation and its underlying gravitational radiation. The discussion of the impossibility of both negative mass solutions and antigravity is new. The speculation on superluminal transport if $a^{\lambda} R_{\lambda}{}^{\nu}$ is space-like is also new. Finally, the appendix containing the analysis of the Cauchy initial value problem as it relates to the theory's equations (1) and (2) is new and included to replace an incorrect discussion of the logical consistency of the fundamental field equations that was given in reference [ii].

7. ACKNOWLEDGEMENTS

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8. APPENDIX - The Cauchy problem applied to the fundamental field equations

One of the unusual features of equations (1) and (2) is the lack of any explicit derivatives of the vector field a^λ, a situation which raises questions about the time dependent development of a^λ. To further elucidate this and other questions regarding solutions of equations (1) and (2), and to outline how they can be solved numerically, they are here analyzed in terms of a Cauchy initial value problem.

Given initial conditions for the fields in Table I at all spatial locations, a procedure is outlined that propagates those fields in time. To begin, assume $g_{\mu\nu}, F_{\mu\nu}, u^\lambda, \rho_c, \rho_m$ and $\frac{\partial g_{\mu\nu}}{\partial t}$ are known at all spatial coordinates at some initial coordinate time t_0. Note that the initial value of field a^λ is not required, rather it will be solved for using equation (1) as described below. Also note that in addition to $g_{\mu\nu}$ the initial values of $\frac{\partial g_{\mu\nu}}{\partial t}$ must be specified because the fundamental field equations are second order in the time derivatives of $g_{\mu\nu}$, a situation analogous to classical General Relativity. The goal of the Cauchy method as it applies here is to start with specified initial conditions for $g_{\mu\nu}, F_{\mu\nu}, u^\lambda, \rho_c, \rho_m$ and $\frac{\partial g_{\mu\nu}}{\partial t}$ at t_0, and then using the equations (1) and (2) solve for $a^\lambda, R_{\lambda\kappa\mu\nu}, \frac{\partial F_{\mu\nu}}{\partial t}, \frac{\partial u^\lambda}{\partial t}, \frac{\partial \rho_m}{\partial t}, \frac{\partial \rho_c}{\partial t}$ and $\frac{\partial^2 g_{\mu\nu}}{\partial t^2}$ at t_0. Armed with these values at t_0, it is straight forward to propagate the fields $g_{\mu\nu}, F_{\mu\nu}, u^\lambda, \rho_c, \rho_m$ and $\frac{\partial g_{\mu\nu}}{\partial t}$ from their initial conditions at t_0 to $t_0 + dt$ and then solve for $a^\lambda, R_{\lambda\kappa\mu\nu}, \frac{\partial F_{\mu\nu}}{\partial t}, \frac{\partial u^\lambda}{\partial t}, \frac{\partial \rho_m}{\partial t}, \frac{\partial \rho_c}{\partial t}$ and $\frac{\partial^2 g_{\mu\nu}}{\partial t^2}$ at $t_0 + dt$ using the same procedure that was used to find them at t_0. Repeating this procedure, values for the fundamental fields of the theory can then be found at all times. One additional requirement on the field values specified by initial conditions is that they must be self-consistent with the equations (1) and (2), *i.e.*, the specified initial conditions must be consistent with a solution existing to equations (1) and (2).

In what follows, Greek indices ($\mu, \nu, \kappa, \ldots$) take on the usual space-time coordinates 1-4 but Latin indices ($i, j, k, \ldots$) are restricted to spatial coordinates, 1-3 only. Since the values of $g_{\mu\nu}$ and $\frac{\partial g_{\mu\nu}}{\partial t}$ are known at all spatial coordinates at time t_0, the

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values of $\frac{\partial g_{\mu\nu}}{\partial x^i}$, $\frac{\partial^2 g_{\mu\nu}}{\partial x^i \partial x^j}$ and $\frac{\partial^2 g_{\mu\nu}}{\partial x^i \partial t}$ can be calculated at all spatial coordinates at time t_0. This leaves the ten quantities $\frac{\partial^2 g_{\mu\nu}}{\partial t^2}$ as the only second derivatives of $g_{\mu\nu}$ not known at t_0. To find the values of $\frac{\partial^2 g_{\mu\nu}}{\partial t^2}$ at t_0 proceed as follows. First find the values of the six $\frac{\partial^2 g_{ij}}{\partial t^2}$ at t_0 using a subset of equations from (1), the subset containing only those equations having spatial derivatives of $F_{\mu\nu}$ on the LHS and at most one time-index in each occurrence of the R-C tensor on the RHS. These equations will be used to solve for the values of a^λ at time t_0. In all there are 12 such equations out of the 24 that comprise (1), as listed here:

$$\begin{aligned}
F_{12;1} &= a^\lambda R_{\lambda 112} \\
F_{13;1} &= a^\lambda R_{\lambda 113} \\
F_{23;1} &= a^\lambda R_{\lambda 123} \\
F_{12;2} &= a^\lambda R_{\lambda 212} \\
F_{13;2} &= a^\lambda R_{\lambda 213} \\
F_{23;2} &= a^\lambda R_{\lambda 223} \\
F_{12;3} &= a^\lambda R_{\lambda 312} \\
F_{13;3} &= a^\lambda R_{\lambda 313} \\
F_{23;3} &= a^\lambda R_{\lambda 323} \\
F_{12;4} &= -F_{24;1} - F_{41;2} = a^\lambda R_{\lambda 412} \\
F_{13;4} &= -F_{34;1} - F_{41;3} = a^\lambda R_{\lambda 413} \\
F_{23;4} &= -F_{34;2} - F_{42;3} = a^\lambda R_{\lambda 423}
\end{aligned} \tag{84}$$

The last three equations in (84) use (6), Maxwell's homogeneous equation to express the time derivative of a Maxwell tensor component on the LHS as the sum of the spatial derivatives of two Maxwell tensor components. The importance of having only spatial derivatives of the Maxwell tensor components on the LHS of (84) is that they are all known quantities at time t_0, *i.e.*, since all the $F_{\mu\nu}$ are known at time t_0, all $\frac{\partial F_{\mu\nu}}{\partial x^i}$ and $F_{\mu\nu;i}$ can be calculated at time t_0. Equally important is that the RHS of the 12 equations that comprise (84) contain at most a single time index in each occurrence of their R-C tensor and so are also

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known at time t_0. To see that this is so I examine the general form of the R-C tensor in a locally inertial coordinate system where all first derivatives of $g_{\mu\nu}$ vanish, *i.e.*,

$$R_{\lambda\kappa\mu\nu} = \frac{1}{2}\left(\frac{\partial^2 g_{\mu\lambda}}{\partial x^\nu \partial x^\kappa} - \frac{\partial^2 g_{\mu\kappa}}{\partial x^\nu \partial x^\lambda} - \frac{\partial^2 g_{\nu\lambda}}{\partial x^\mu \partial x^\kappa} + \frac{\partial^2 g_{\kappa\nu}}{\partial x^\mu \partial x^\lambda} \right). \quad (85)$$

Note, having at most a single time index on the RHS of (85) means that the R-C tensor is made up entirely of terms from $\frac{\partial^2 g_{\mu\nu}}{\partial x^i \partial x^j}$ and $\frac{\partial^2 g_{\mu\nu}}{\partial x^i \partial t}$, all of which are known at time t_0. Examining the set of equations (84), there are 12 equations for 4 unknowns, the unknowns being the components of a^λ. These 12 equations can be solved for a^λ at time t_0 provided the initial conditions were chosen self-consistently with equations (1) and (2), *i.e.*, chosen such that a solution to the field equations is indeed possible.

Knowing the value of a^λ at time t_0, I now proceed to determine the R-C tensor components with two time indices at time t_0. Going back to the 24 equations that comprise the set of equations (1), here I collect the subset of those equations in which the LHS is known at time t_0, *i.e.*, contains only spatial derivatives of the Maxwell tensor, and the RHS has an R-C tensor component that contains two time indices:

$$\begin{aligned}
F_{14;1} &= a^\lambda R_{\lambda 114} \\
F_{24;1} &= a^\lambda R_{\lambda 124} \\
F_{34;1} &= a^\lambda R_{\lambda 134} \\
F_{14;2} &= a^\lambda R_{\lambda 214} \\
F_{24;2} &= a^\lambda R_{\lambda 224} \\
F_{34;2} &= a^\lambda R_{\lambda 234} \\
F_{14;3} &= a^\lambda R_{\lambda 314} \\
F_{24;3} &= a^\lambda R_{\lambda 324} \\
F_{34;3} &= a^\lambda R_{\lambda 334}
\end{aligned} \quad (86)$$

Each of the equations in (86) contains only one unknown, the R-C component having two time indices. In total, there are six such independent R-C tensor components:

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$$\begin{aligned} &R_{1414} \\ &R_{1424} \\ &R_{1434} \\ &R_{2424} \\ &R_{2434} \\ &R_{3434} \end{aligned} \qquad (87)$$

so the system of nine equations (86) can be algebraically solved for the six unknown R-C components at time t_0. With this I now know the value of all components of the R-C tensor at time t_0. From the t_0 values of the R-C tensor components listed in (87), the values of the six unknown $\frac{\partial^2 g_{ij}}{\partial t^2}$ at t_0 can be found.

There are three remaining equations from the set of equations (1) that have not yet been addressed:

$$\begin{aligned} F_{14;4} &= a^{\lambda} R_{\lambda 414} \\ F_{24;4} &= a^{\lambda} R_{\lambda 424} \\ F_{34;4} &= a^{\lambda} R_{\lambda 434} \end{aligned} \qquad (88)$$

These are the equations for which the temporal derivatives of the Maxwell tensor components are not yet known. Because all values of the R-C tensor and a^{λ} are now known at t_0, these three remaining time-differentiated components of the Maxwell tensor can now be solved for directly using (88), giving complete knowledge of $\frac{\partial F_{\mu\nu}}{\partial t}$ at time t_0.

If the values of the four $\frac{\partial^2 g_{\mu 4}}{\partial t^2}$ could be calculated then all $\frac{\partial^2 g_{\mu\nu}}{\partial t^2}$ would be known and all $\frac{\partial g_{\mu\nu}}{\partial t}$ could be propagated from t_0 to $t_0 + dt$. Just as is the case with classical General Relativity, the four $\frac{\partial^2 g_{\mu 4}}{\partial t^2}$ can be determined from the four coordinate conditions that are fixed by the choice of coordinate system.[xxxiv] Recapping, at t_0 the following quantities are now known: $g_{\mu\nu}, F_{\mu\nu}, u^{\lambda}, \rho_c, \rho_m$ and $\frac{\partial g_{\mu\nu}}{\partial t}$ are defined by initial conditions; $a^{\lambda}, \frac{\partial^2 g_{\mu\nu}}{\partial x^{\kappa} \partial x^{\lambda}}, R_{\lambda\kappa\mu\nu}$, and $\frac{\partial F_{\mu\nu}}{\partial x^{\lambda}}$ are then solved for using those initial conditions, the fundamental field equations, and the four coordinate conditions that are fixed by the choice of coordinate

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system. Still needed to propagate the initial conditions in time from t_0 to $t_0 + dt$ are $\frac{\partial u^\mu}{\partial t}$, $\frac{\partial \rho_m}{\partial t}$ and $\frac{\partial \rho_c}{\partial t}$. Using the Lorentz force law (25), the following development,

$$\begin{gathered}
\rho_m \frac{Du^\mu}{D\tau} = \rho_c u^\lambda F^\mu{}_\lambda \\
\downarrow \\
\rho_m u^\mu{}_{;\nu} u^\nu = \rho_c u^\lambda F^\mu{}_\lambda \\
\downarrow \\
\rho_m u^\mu{}_{;4} u^4 = -\rho_m u^\mu{}_{;i} u^i + \rho_c u^\lambda F^\mu{}_\lambda \\
\downarrow \\
\rho_m \left(\frac{\partial u^\mu}{\partial t} + \Gamma^\mu{}_{4\sigma} u^\sigma \right) u^4 = -\rho_m u^\mu{}_{;i} u^i + \rho_c u^\lambda F^\mu{}_\lambda
\end{gathered} \tag{89}$$

shows on the last line above that $\frac{\partial u^\mu}{\partial t}$ can be solved for at t_0 in terms of knowns at t_0. Next, using the conservation of mass equation (24) and knowing $\frac{\partial u^\mu}{\partial t}$ at t_0, the following development,

$$\begin{gathered}
\left(\rho_m u^\nu \right)_{;\nu} = 0 \\
\downarrow \\
\left(\rho_m u^4 \right)_{;4} = -\left(\rho_m u^i \right)_{;i} \\
\downarrow \\
\frac{\partial \rho_m}{\partial t} u^4 = -\rho_m u^4{}_{;4} - \left(\rho_m u^i \right)_{;i}
\end{gathered} \tag{90}$$

shows on the last line above that $\frac{\partial \rho_m}{\partial t}$ can be solved for at t_0 in terms of knowns at t_0. Following an analogous development for ρ_c using the charge conservation equation (20), $\frac{\partial \rho_c}{\partial t}$ can be solved for at t_0 in terms of knowns at t_0. With these, the values of $a^\lambda, R_{\lambda\kappa\mu\nu}, \frac{\partial F_{\mu\nu}}{\partial t}, \frac{\partial u^\lambda}{\partial t}, \frac{\partial \rho_m}{\partial t}, \frac{\partial \rho_c}{\partial t}$ and $\frac{\partial^2 g_{\mu\nu}}{\partial t^2}$ are all known at t_0 and can be used to propagate the initial conditions $g_{\mu\nu}, F_{\mu\nu}, u^\lambda, \rho_c, \rho_m$ and $\frac{\partial g_{\mu\nu}}{\partial t}$ at t_0 to time $t_0 + dt$. Iterating the process, the values of the fundamental fields can be determined at all times.

9. REFERENCES AND END NOTES

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[iii] S. Weinberg, *Gravitation and Cosmology*, John Wiley & Sons, New York, NY 1972. The definition of the Riemann-Christoffel curvature tensor is $R^{\lambda}{}_{\mu\nu\kappa} \equiv \partial_{\kappa}\Gamma^{\lambda}{}_{\mu\nu} - \partial_{\nu}\Gamma^{\lambda}{}_{\mu\kappa} + \Gamma^{\eta}{}_{\mu\nu}\Gamma^{\lambda}{}_{\kappa\eta} - \Gamma^{\eta}{}_{\mu\kappa}\Gamma^{\lambda}{}_{\nu\eta}$, and the definition of the Ricci tensor is $R_{\mu\kappa} \equiv R^{\lambda}{}_{\mu\lambda\kappa}$.

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[ix] This section follows the discussion given in reference [ii] and is included here to keep the mathematical description of the proposed theory self-contained.

[x] L.P. Eisenhart, *An Introduction to Differential Geometry*, Chapter 23, "Systems of Partial Differential Equations of the First Order, Mixed Systems," Princeton University Press 1947.

[xi] The interpretation of the solution here is different than in the discussion given in reference [ii]. Specifically, in reference [ii] the charge density was restricted to be positive, a restriction that is lifted here.

[xii] C. Misner, K. Thorne, and J. Wheeler, *Gravitation*, p. 840, W.H. Freeman and Company, San Francisco, CA 1970.

[xiii] Calculations within were performed in Mathematica: Wolfram Research, Inc., Mathematica, Version 12.1, Champaign, IL (2020).

[xiv] This calculation was presented in reference [ii] but contained an error that is corrected here. In [ii], the electric and magnetic fields where not restricted to the same weak field approximation as the *h's*.

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