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2 Article

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Development of piezoelectric harvesters with 4 integrated trimming devices

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15 **Featured Application:** Trimming devices integrated with the structural layer of the harvester are
16 proposed for improving energy harvesting at low frequency and in the presence of periodic
17 excitation with multiple harmonics.18 **Abstract:** Piezoelectric cantilever harvesters have a large power output at their natural frequency,
19 but in some applications the frequency of ambient vibrations is different from the harvester's
20 frequency and/or ambient vibrations are periodic with some harmonic components. To cope with
21 these operating conditions harvesters with integrated trimming devices (ITDs) are proposed.
22 Some prototypes are developed with the aid of an analytical model and tested with an impulsive
23 method. Results show that a small trimming device can lower the main resonance frequency of a
24 piezoelectric harvester of the same extent as a larger tip mass and moreover generates at high
25 frequency a second resonance peak. A multi-physics numerical FE model is developed for
26 predicting the generated power and for performing stress-strain analysis of harvesters with ITDs.
27 The numerical model is validated on the basis of experimental results. Several configurations of
28 ITDs are conceived and studied. Numerical results show that harvesters with ITDs are able to
29 generate relevant power at two frequencies owing to the particular shape of the modes of
30 vibration. The stress in the harvesters with ITDs is smaller than the stress in the harvester with a
31 tip mass trimmed to the same frequency.32 **Keywords:** harvester; piezoelectric; dynamic vibration absorber; trimming.
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1. Introduction

35 In recent years there has been a great development of energy harvesting techniques based on
36 piezoelectric devices. Piezoelectric cantilever harvesters have been successfully used for
37 transforming ambient vibration energy into electrical energy for feeding sensors, biomedical
38 equipment and small electronic devices [1-3]. The trimming of the harvester to the vibration source
39 is essential to improve the vibration-to-electric energy conversion. In the presence of harmonic
40 vibrations optimum trimming is achieved when the natural frequency of the harvester is set equal
41 to vibration frequency [4-5].42 When the difference between the frequency of ambient vibrations and the resonance frequency
43 of the harvester is not very large (some tens of Hz) the problem of trimming is solved by adding a

44 tip mass [5-7], which increases the excitation of the harvester, because the inertia force due to the
45 lumped mass adds to the inertia forces due to the distributed harvester mass. This phenomenon has
46 a beneficial effect on the generated voltage, but it increases the stress inside the harvester, which
47 may be damaged.

48 When the frequency of ambient vibrations is very low (e.g. in energy harvesting from human
49 motion) the tip mass able to trim the harvester would be impractically large; therefore several
50 frequency-up strategies have been proposed: they are based on specific devices like a flexible
51 stopper [8], a non-linear plectrum [9] or a pendulum magnetically coupled with the harvester [10].

52 The problem of harvester trimming is complex when ambient vibrations are characterized by
53 variable frequency, by multiple harmonic components and by broadband spectrum as well.

54 Several technical solutions have been developed to cope with these working conditions. Some
55 researchers have proposed dynamically tunable harvesters, in which tuning can be achieved both
56 by means of the interface circuit [11] [12] and by mini-actuators that modify the mechanical system
57 [13]. Other researchers have developed wideband harvesters. One approach consists in exploiting
58 several modes of vibration of the cantilever harvester (making use of segmented electrodes) [14] or
59 of more complex structures, like L-shaped harvesters [15]. Another approach consists in making use
60 of non-linear components like bi-stable cantilevers or ropes [16-18]. The possibility of widening the
61 band of the cantilever harvester by increasing the number of degrees of freedom (DOFs) has been
62 analyzed in some researches. In particular in [19-20] two DOFs harvesters have been developed
63 connecting cantilever harvesters with tip masses and the use of arrays of cantilever harvesters is
64 reported in [5] and in [21-22].

65 The problems of trimming and increasing the bandwidth of a piezoelectric harvester by
66 increasing the number of DOFs can be analyzed with the theory of the Dynamic Vibration Absorber
67 (DVA) [23]. Since the beginning of 20th century DVAs have been successfully used for controlling
68 vibrations of many mechanical systems including crankshafts, ships, engines and machine tools
69 [24]. In the field of acoustics the DVA is named Helmholtz resonator [25] and it has been used in
70 mufflers [26] and cavities [27].

71 Some studies dealing with the application of DVAs in the field of energy harvesting have been
72 already carried out. For example [28] proposed to attach lead zirconate/lead titanate (PZT) patches
73 (which harvest energy from vibrations) to an auxiliary structure that acts as a DVA for the main
74 vibrating structure. Several researchers studied dynamic magnifiers, which are spring-mass
75 systems placed between the harvester and the moving base [29]. In [30] the dynamic magnifier was
76 added to a piezoelectric stack harvester and the natural frequency of the dynamic magnifier was set
77 equal to the one of the harvester. Numerical results showed the appearance of two resonance peaks,
78 a large increment in the generated power and a significant widening of the bandwidth of the
79 harvester. In [31] the concept of dynamic magnifier was applied to cantilever harvesters. In [32] a
80 cantilever harvester with tip mass was mounted on a supporting beam for lowering the first natural
81 frequency of the integrated system.

82 These concepts were further extended in [33], where all the modes of a cantilever harvester
83 were trimmed to those of a continuous (beam-type) auxiliary structure. Experimental results
84 showed a doubling of resonance peaks in a wide frequency band.

85 A double-mode harvester was developed in [34]. In this case the harvester does not behave like
86 a DVA, but a DVA, composed of a mass and two helicoidal springs, is used for trimming the
87 harvester and for making possible the exploitation of two modes of vibration. A
88 distributed-parameter model and an experiment showed the potentialities of this harvester, but
89 helicoidal springs are not suited to integration with a harvester. A cantilever DVA, in which the
90 spring is a small cantilever beam, is more suited to this purpose.

91 This research focuses on piezoelectric harvesters having a multi-layer structure with one or
92 more active layers surrounded by layers of structural, conductive and insulating materials. In these
93 harvesters the structural layer (steel or plastic material) can be extended and shaped to create a
94 cantilever DVA, which is named Integrated Trimming Device (ITD). In a simple ITD the structural
95 layer is extended to create a narrow appendix with a final widening: the narrow appendix behaves

96 as a cantilever beam, whereas the final widening behaves as a tip mass. Many alternative designs
 97 are possible in order to shape the extension of the structural layer in such a way that a part basically
 98 behaves as a beam and another part basically behaves as a mass.

99 In order to assess the validity of this concept, analytical calculations and experimental tests
 100 were carried out on prototypes made with a simple technology. Results in the frequency domain
 101 show that the prototypes are able to transform the original resonance peak of the harvester into a
 102 pair of new peaks like in [30]. The former peak has lower frequency than the original peak and is
 103 used for trimming the harvester to the main source of vibrations. The latter peak makes it possible
 104 to collect a significant amount of energy at high frequency as well. This possibility is useful in the
 105 presence of periodic vibrations characterized by several harmonic components.

106 In order to extend the analysis to more complex and realistic ITDs, a multi-physics finite
 107 element (FE) model was developed in COMSOL and validated by means of experimental results.

108 New harvesters equipped with ITDs were designed and modeled in COMSOL extending and
 109 shaping the structural layer of a standard harvester. The potentialities of the harvesters equipped
 110 with ITDs were investigated by means of numerical simulations. Numerical results are presented in
 111 terms of frequency response functions (FRFs) of generated voltage, generated power, stress and
 112 strain distributions inside the piezoelectric material.

113 2. Preliminary design of prototypes

114 Before developing and modeling the harvesters with ITDs, the validity of the basic concept, i.e.
 115 the addition of a cantilever DVA to a piezoelectric harvester, was assessed by means of prototypes
 116 built with a simple technology. The starting point was a PPA 1001 harvester built by MIDE. It is a
 117 general-purpose unimorph harvester having a rectangular shape (length 41.1 mm, width 20.8 mm),
 118 with a PZT 5H piezoelectric layer and a stainless steel structure.

119 Figure 1 shows the cantilever DVAs used in the prototypes. The cantilever beams were made
 120 using harmonic steel wire (radius $r = 0.15$ mm, density $\rho = 7850$ kg/m³); this solution allowed to
 121 obtain a structure with low damping. The mass elements were small brass discs, whose masses
 122 were measured by means of a balance having a resolution of 0.1 mg.

123 The cantilever DVAs were connected to the free end of the harvester with a removable
 124 connection using wax for accelerometers. In some tests the cantilever DVA was joined to the
 125 harvester using epoxy adhesive. No difference was found between the results obtained with the
 126 two joining techniques.

127 A lumped-element approach was adopted for trimming the natural frequency (f_t) of the
 128 cantilever DVA (having stiffness k_a and equivalent mass m_{eq}) to first natural frequency of the
 129 PPA 1001 harvester:

$$130 \quad \omega_t = 2\pi f_t = \sqrt{\frac{k_a}{m_{eq}}} \quad (1)$$

131 Stiffness k_a depends on cantilever length L_a , Young modulus E and moment of inertia I of the
 132 wire cross-section:

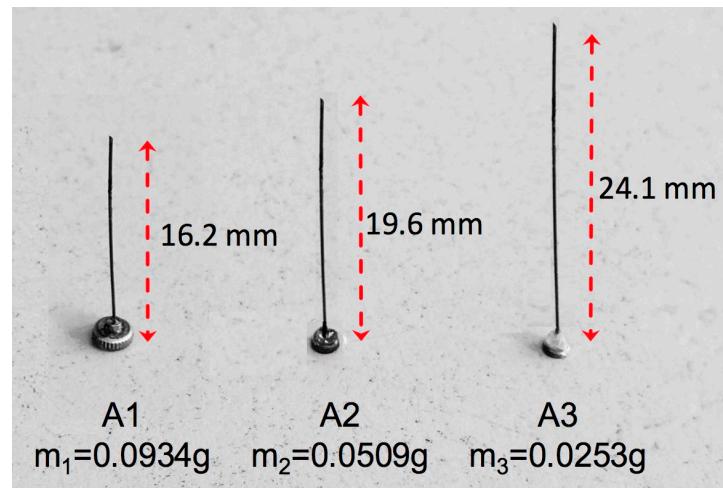
$$133 \quad k_a = \frac{3EI}{L_a^3} \quad (2)$$

134 Equivalent mass takes into account the lumped tip mass m_a and a fraction of the mass of the wire
 135 (having density ρ), making use of the Rayleigh method [23]:

$$136 \quad m_{eq} = m_a + \frac{33}{140} \rho \pi r^2 L_a \quad (3)$$

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Figure 1. Cantilever DVA for harvesters.

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Equations (1), (2) and (3) show that it is possible to trim a cantilever DVA with assigned tip mass m_a and wire radius r by varying the length (L_a) of the cantilever beam. The theory of DVA [23] shows that an increment in the tip mass (m_a) widens the frequency interval between the resonance peaks that substitute for the original resonance peak. Therefore, three different cantilever DVA trimmed to the same frequency ($f_t = 125$ Hz) but with different tip masses were developed.

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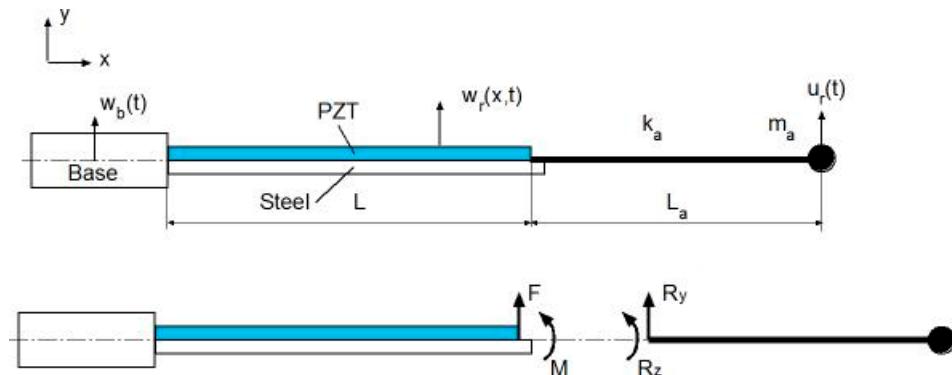
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3. Analytical model of prototypes

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The prototypes are equipped with cantilever DVAs, which are beams with tip masses and can be modeled with a lumped element approach. Therefore, it is possible to develop an analytical model of the prototype coupling the distributed parameter model of the harvester [6-7] with the lumped element model of the cantilever DVA. Figure 2 shows a scheme of the model.

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Figure 2. Model of the coupled system.153
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$w_b(t)$ is the displacement of the base of the harvester in the transverse direction (y), base rotation is neglected since the testing equipment was designed in order to generate only base displacement. $w_r(x,t)$ is the displacement of any point x along the harvester axis with respect to the base, $u_r(t)$ is the displacement of the mass of the cantilever DVA with respect to the harvester tip ($w_r(L,t)$).

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The partial differential equation of the harvester (having length L) is:

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$$(EI)_{eq} \frac{\partial^4 w_r(x,t)}{\partial x^4} + c_s I \frac{\partial^5 w_r(x,t)}{\partial x^4 \partial t} + m \frac{\partial^2 w_r(x,t)}{\partial t^2} + \vartheta v(t) = -m \frac{\partial^2 w_b(t)}{\partial t^2} + F(t) \delta(x - L) + M(t) \frac{\delta(x-L) - \delta(x-(L-h))}{h} \quad (4)$$

160 In equation (4) $(EI)_{eq}$ is the equivalent bending stiffness of the composite cross-section, m is the
 161 mass per unit length of the harvester and c_s is the strain-rate damping coefficient [6], air damping
 162 is neglected because usually it has a small effect on harvester performance [7]. ϑ is the piezoelectric
 163 coupling term [6] and $v(t)$ is generated voltage. At the right-hand side of equation (4) there are the
 164 forcing terms. The first term is due to base acceleration, whereas the last terms are caused by the
 165 force $F(t)$ and the torque $M(t)$ exerted by the cantilever DVA on the harvester; $\delta(x)$ is the Dirac
 166 delta function. It is worth noticing that the loads due to the cantilever harvester are calculated with
 167 the same approach adopted in [7] to calculate the inertia load caused by a tip mass.

168 The piezoelectric layer can be considered a current ($i(t)$) source in parallel with its internal
 169 capacitance C_{pu} [6] and it is connected to an electrical load which can be represented by a
 170 resistance R . The application of the Kirchoff's laws to this circuit gives the following differential
 171 equation:

$$172 \quad C_{pu} \frac{dv(t)}{dt} + \frac{v(t)}{R} = i(t) \quad (5)$$

173 The ordinary differential equation of the DVA is:

$$174 \quad m_{eq} \left(\frac{d^2u_r(t)}{dt^2} + \frac{\partial^2 w_r(L,t)}{\partial t^2} + \frac{d^2w_b(t)}{dt^2} \right) = -k_a u_r(t) - c_a \frac{du_r(t)}{dt} \quad (6)$$

175 constant c_a in equation (6) is DVA damping.

176 The inertia force and torque exerted by the cantilever DVA on the harvester are:

$$177 \quad F(t) = -m_{eq} \left(\frac{d^2u_r(t)}{dt^2} + \frac{\partial^2 w_r(L,t)}{\partial t^2} + \frac{d^2w_b(t)}{dt^2} \right) \quad (7)$$

$$178 \quad M(t) = -m_{eq} L_a \left(\frac{d^2u_r(t)}{dt^2} + \frac{\partial^2 w_r(L,t)}{\partial t^2} + \frac{d^2w_b(t)}{dt^2} \right) \quad (8)$$

180 According to the proportional damping assumption, the equations of motion can be solved
 181 with the modal expansion approach [7]. Displacement $w_r(L,t)$ is represented by a series of
 182 eigenfunctions:

$$183 \quad w_r(L,t) = \sum_{i=1}^{\infty} \phi_i(x) \eta_i(t) \quad (9)$$

185 In equation (9) $\phi_i(x)$ is the i^{th} mass-normalized eigenfunction (mode of vibration) and $\eta_i(t)$ is the
 186 i^{th} modal coordinate.

187 Modal expansion transforms equation (4) into this set of ordinary differential equations:

$$188 \quad \frac{d^2\eta_i(t)}{dt^2} + 2\zeta_i \omega_i \frac{d\eta_i(t)}{dt} + \omega_i^2 \eta_i(t) + \chi_i v_i(t) \\ 189 \quad = \int_0^L -m \frac{d^2w_b(t)}{dt^2} \phi_i(x) dx + F(t) \phi_i(L) + M(t) \frac{\phi_i(L) - \phi_i(L-h)}{h} \quad i = 1 \dots \infty \quad (10)$$

191 In equation (10) ω_i is the undamped natural frequency of the i^{th} mode of vibration and ζ_i is
 192 modal damping. Coefficient χ_i is the backward modal electromechanical coupling term [6].

193 The electrical equation and the DVA equation after expansion in modal coordinates become:

$$194 \quad C_{pu} \frac{dv(t)}{dt} + \frac{v(t)}{R} = \sum_{i=1}^{\infty} \varphi_i \frac{d\eta_i(t)}{dt} \quad (11)$$

$$196 \quad m_{eq} \left(\frac{d^2u_r(t)}{dt^2} + \sum_{i=1}^{\infty} \phi_i(L) \frac{d^2\eta_i(t)}{dt^2} + \frac{d^2w_b(t)}{dt^2} \right) = -k_a u_r(t) - c_a \frac{du_r(t)}{dt} \quad (12)$$

198 In equation (11) φ_i is the modal electromechanical coupling term [6].

199 The forced response of the harvester with DVA can be calculated by solving the system of
 200 ordinary differential equations (6), (10) and (11) considering a finite number of modes. On the one
 201 hand the aim of this analytical model is to give a first prediction of the response of the harvesters
 202 with trimming devices, more detailed analyses will be carried out with the FE approach. On the
 203 other hand some preliminary results [35] showed that the second mode of the harvester has a
 204 natural frequency much higher than the first and a very small effect on low frequency dynamics.
 205 For these reasons only the first mode of vibration ($i=1$) is considered in equations (10-12). The
 206 frequency response functions (FRFs) of the prototype can be calculated considering harmonic base
 207 excitation:

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$$\frac{d^2w_b(t)}{dt^2} = a_{b0}e^{i\omega t} \quad (13)$$

209 and looking for harmonic solutions for the first mode of the harvester, DVA displacement and
210 voltage:

$$211 \quad \eta_1(t) = \eta_{10}e^{i\omega t} \quad u_r(t) = u_{r0}e^{i\omega t} \quad v(t) = v_0e^{i\omega t} \quad (14)$$

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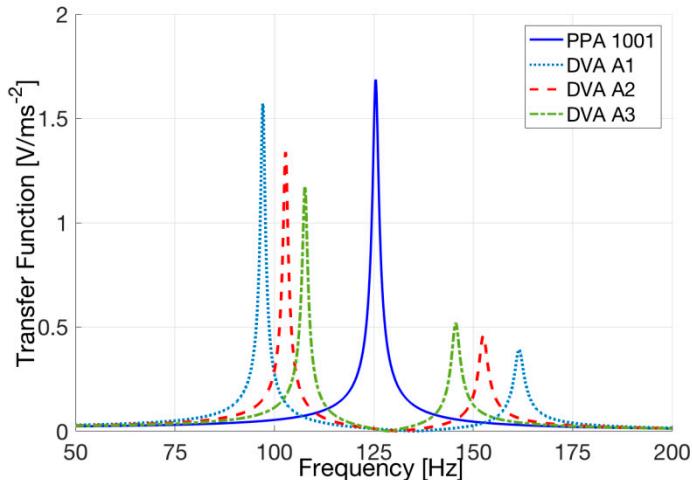
213 a_{b0} , η_{10} , u_{r0} and v_0 are complex constants. The system composed of equations (10-12) becomes a
214 linear-algebraic system, which can be solved numerically by means of MATLAB.

215 At first the analytical model was used to calculate the FRFs between the open circuit voltage
216 and base acceleration of the three prototypes, because the open circuit voltage is an important
217 figure for the design of the energy conversion system of the harvester [36]; results are presented in
218 Figure 3.

219 When the most massive cantilever DVA (A1) is added to the harvester, the original resonance
220 peak (125 Hz) is substituted by two new resonance peaks at 97.1 and 161.5 Hz respectively, the
221 interval between the two resonances is 64.4 Hz. The first peak is slightly lower than the peak of PPA
222 1001 alone, whereas the second peak is rather small. If the DVAs with smaller masses are
223 considered (A2 and A3), the frequency interval between the two new peaks decreases, the height of
224 the first peak decreases, whereas the height of the second peak increases. Table 1 summarizes the
225 resonance frequencies and the corresponding amplitudes in resonance of the three prototypes.

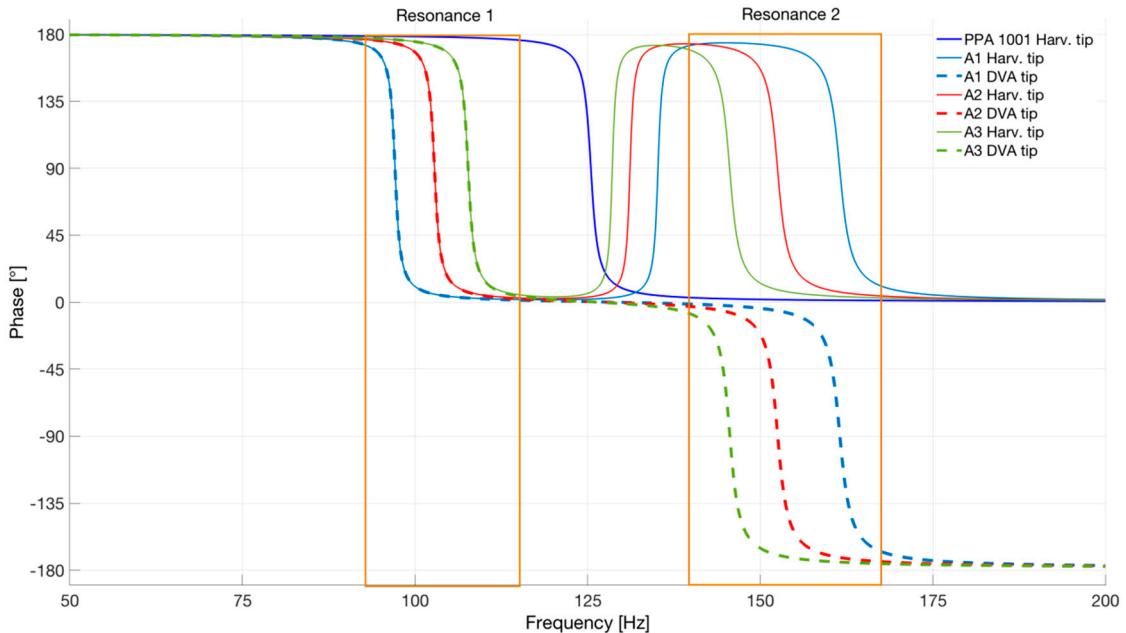
226 The phases of the modal displacement of the harvester (η_{10}) and of the displacement of the
227 DVA (u_{r0}) with respect to base acceleration give useful information about the functioning of the
228 system. Figure 4 shows that at the frequency of the first resonance peak of the harvester with DVA
229 the two displacements essentially have the same phase with respect to base acceleration, therefore
230 the harvester and the DVA move in phase when the first mode of vibration is excited in resonance.
231 At the frequency of the second resonance peak there is a phase shift of about 180° between the
232 modal response of the harvester and the displacement of the DVA, therefore the DVA moves in
233 opposition with respect to the harvester when the second mode of vibration is excited in resonance.

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Figure 3. Open circuit FRFs of the prototypes calculated by means of the analytical model.



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Figure 4. Phases of the displacements of the harvester and of the DVA calculated by means of the analytical model.

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Table 1. Analytical results: resonance frequencies and corresponding amplitudes of the prototypes in open circuit condition.

Harvester	1st Mode		2nd Mode		ΔF [Hz]
	Frequency f_1 [Hz]	Peak amplitude [V/ms ⁻²]	Frequency f_2 [Hz]	Peak amplitude [V/ ms ⁻²]	
PPA 1001	125.5	1.69	-	-	-
PPA1001 + A1	97.1	1.57	161.5	0.39	64.4
PPA1001 + A2	102.8	1.34	152.6	0.46	49.8
PPA1001 + A3	107.7	1.18	145.6	0.52	37.9

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The analytical model makes it possible to analyze the effect of the trimming frequency (f_t) of the cantilever DVA on the performance of the harvester. Two new versions of A1 are considered: the former with increased length ($L_a = 17.11$ mm) and decreased trimming frequency ($f_t = 108.3$ Hz), the latter with decreased length ($L_a = 14.00$ mm) and increased trimming frequency ($f_t = 146.4$ Hz). Figure 5 shows that, when the trimming frequency decreases, both the peaks move towards lower frequency, and the amplitude of the second peak significantly increases. Whereas, when the trimming frequency increases, both the peaks move towards higher frequencies and the amplitude of the second peak decreases.

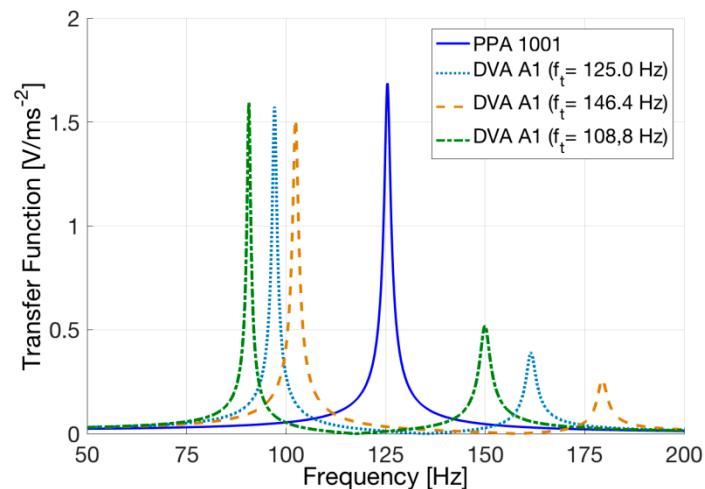
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When the harvester is connected to a resistive load, the FRF between the generated voltage and base acceleration (FRF_L) is related to the generated power. For this reason FRF_L was calculated by means of the analytical model considering the optimal load resistance [36], which maximizes the generated power at the frequency of the first resonance (ω_r):

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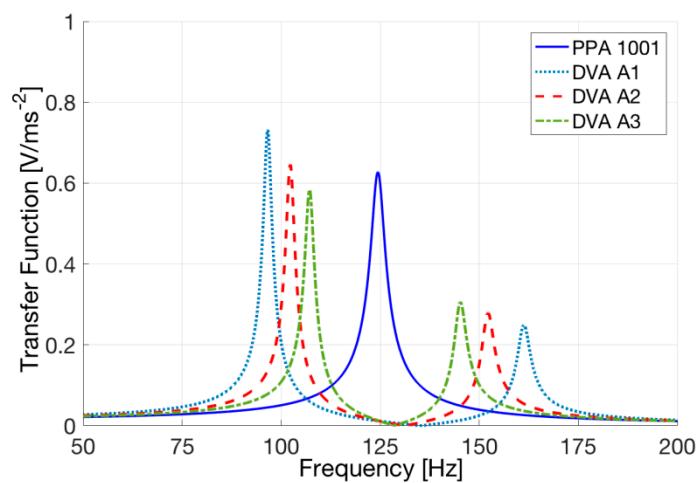
$$R_{opt} = \frac{1}{\omega_r C_{pu}} \quad (15)$$

255 Figure 6 shows the predicted FRFs. The load resistance significantly decreases the amplitudes
 256 of the resonance peaks. If PPA 1001 alone is considered the peak frequency slightly decreases (from
 257 125.5 to 124.6 Hz), this effect is in agreement with the results presented in [6] and [38]. Also when
 258 the harvester is coupled with the trimming device the frequencies of the resonance peaks decrease
 259 owing to the resistive load. This effect is more important for the first peak than for the second.



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261 **Figure 5.** Open circuit FRFs of the prototypes. Effect of trimming frequency.



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263 **Figure 6.** FRFs of the prototypes with optimal resistance calculated by means of the analytical
 264 model.

265 **4. Experimental tests and results**

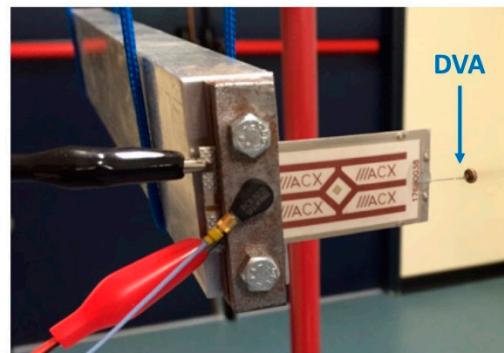
266 A specific test rig was developed to excite by means of a hammer for modal testing a small
 267 harvester equipped with a cantilever DVA, which is a simple prototype of the harvester with ITD.
 268 Impulsive excitation instead of shaker excitation was adopted, because the testing equipment is
 269 cheaper and allows obtaining the same Frequency Response Functions (FRFs) [35]. The harvester
 270 base was clamped at one end of a suspended aluminum bar and the hammer hit was exerted at the
 271 opposite end of the bar, see Figure 7. In this way the hammer impact generated longitudinal
 272 vibrations inside the bar that in turn generated the base motion of the harvester.

273 The aluminum bar was suspended from a frame by means of ropes in order to isolate the
 274 system from sources of vibrations other than the hammer impact. The pendular motion caused by
 275 the ropes showed a natural frequency about 50 times smaller than the frequencies of the harvester
 276 with DVA predicted by the analytical model, and did not influence measurements. The dimensions

277 of the bar were selected in order to avoid the presence of bending modes of the bar in the range of
 278 frequencies of interest.

279 The measurement system included a piezoelectric accelerometer mounted on the clamped base
 280 of the harvester and a piezoelectric load cell mounted on the head of the hammer for modal testing.
 281 The signals of the sensors and the voltage generated by the harvester were acquired by means of a
 282 NI 9234 board. Digital signals were analyzed in time and frequency domain by means of NI Signal
 283 Express.

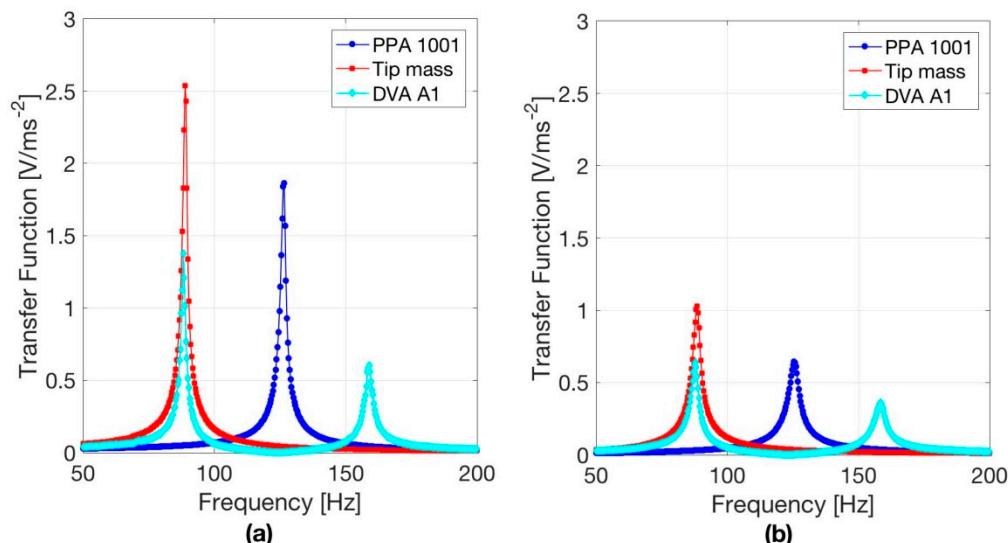
284 Two electrical parameters were measured to evaluate the performance of the harvester: the
 285 open circuit voltage and the load voltage in the presence of the optimal load resistance [36].
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288 **Figure 7.** Testing equipment for impulsive testing.

289 The FRFs between open circuit voltage and base acceleration were measured in order to
 290 highlight the effect of the cantilever DVAs on harvester trimming. Five measurements were carried
 291 out in each configuration, then the mean values of resonance frequencies and peak amplitudes were
 292 calculated. The repeatability of resonance frequencies resulted about two times the frequency
 293 resolution, which is 0.33 Hz. The repeatability of peak amplitudes resulted of about $\pm 5\%$.



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295 **Figure 8.** Measured FRFs of the harvester alone, with tip mass and with cantilever DVA. (a) Open
 296 circuit. (b) Optimal load resistance.

297 Figure 8a shows an example of measured results. Three configurations are considered:
 298 harvester PPA 1001 alone, harvester PPA 1001 with A1, harvester PPA1001 with a tip mass.

299 As foreseen by the analytical model, the introduction of the cantilever DVA splits the original
 300 resonance peak into two new peaks, the former at lower frequency, the latter at higher frequency.

301 The amplitude of the first peak is slightly lower than the amplitude of the original peak, Table 2
 302 shows that it increases if the mass of the DVA increases. The amplitude of the second peak is about
 303 1/3 of the one of the original peak and it decreases if the mass of the DVA increases. The frequency
 304 interval (Δf) between the two peaks increases when the mass of the DVA increases. The comparison
 305 between Table 1 and Table 2 highlights that the analytical model, which considers only one mode of
 306 vibration of the cantilever with DVA, predicts with sufficient accuracy the behavior of the harvester
 307 with DVA, therefore in the next future this simple model will be used for optimization purposes.

308 Modal damping was calculated by means of the half-power-points method [37]. Results, which
 309 are summarized in Table 2, highlight that the damping ratio ζ of the first mode of vibration of the
 310 coupled system is essentially equal to the one of the fundamental mode of the harvester alone.

311 The FRFs of Figure 8a show that harvester with cantilever DVA has two modes of vibration
 312 that can be excited by a uniform distribution of base acceleration. Conversely, even if the range of
 313 frequency of the FRF is extended, harvester PPA 1001 alone shows only a very small peak at 763 Hz
 314 caused by the excitation of the second bending mode.

315 From the point of view of trimming, the cantilever DVA is able to lower the frequency of the
 316 main resonance peak by some tens of Hz. It should be noted that the same effect of the cantilever
 317 DVA can be obtained by means of a tip mass. Figure 8a shows that with a tip mass of 0.591 g (much
 318 larger than the one of A1) the natural frequency is roughly equal to the first natural frequency of
 319 PPA 1001 equipped with A1. With this solution the generated voltage is larger, but the stress inside
 320 the piezoelectric material increases as well.

321 **Table 2.** Resonance frequencies of the coupled systems and corresponding amplitudes of the
 322 prototypes. Mean values of 5 tests.

Harvester	1st Mode			2nd Mode			ΔF [Hz]
	Frequency f_1 [Hz]	Peak amplitude [V/ms ⁻²]	Damping Ratio ζ	Frequency f_2 [Hz]	Peak amplitude [V/ms ⁻²]	Damping Ratio ζ	
PPA 1001	126.4	1.85	0.0072	-	-	-	-
PPA1001 + Tip Mass	90.0	2.48	0.0087	-	-	-	-
PPA1001 + A1	88.2	1.35	0.0087	158.9	0.55	0.0085	70.7
PPA1001 + A2	93.2	1.33	0.0080	156.4	0.61	0.0077	63.2
PPA1001 + A3	97.4	1.25	0.0073	148.3	0.79	0.0071	50.9

323
 324 The same series of measurements was carried out considering for each harvester configuration
 325 the optimal load resistance for the first resonance peak at frequency ω_r , which is given by equation
 326 (15). Figure 8b shows an example of experimental results. In all the cases here considered the
 327 amplitudes in resonance are significantly lower than the ones in open circuit condition due to the
 328 effect of the resistance [6]. The resonance frequency of PPA 1001 and of PPA 1001 with tip mass
 329 slightly decreases passing from the open circuit condition to the optimal resistance condition; this
 330 result is in agreement with [6], [38] and analytical results. In the harvester equipped with DVA the
 331 introduction of the optimal resistance lowers the frequency of first resonance peak by about 1 Hz
 332 and has a very small effect on the frequency of the second peak. Therefore, the frequency shifts
 333 caused by the cantilever DVAs in the presence of optimal load are similar to those of Table 2.

334 The power (P) generated by the harvester in resonance can be calculated from the voltage (V_L)
 335 measured in loaded condition:

336

$$P = \frac{V_L^2}{2R_{opt}} = \frac{|FRF_L|^2 a_{b0}^2}{2R_{opt}} \quad (16)$$

337 FRF_L is the FRF measured in loaded condition.

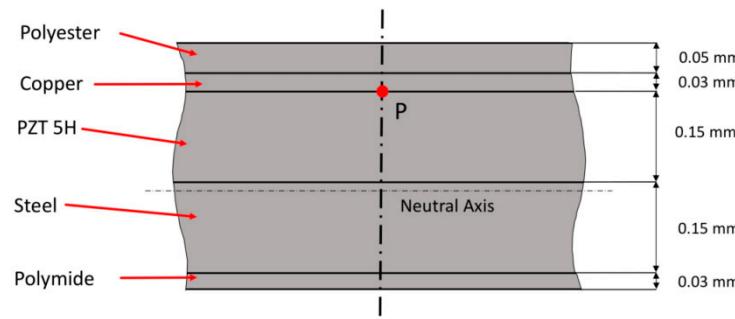
338 If base acceleration a_{b0} is set to 10 ms^{-2} , harvester PPA 1001 generates in resonance 1.61 mW ,
 339 whereas harvester PPA 1001 with A1 generates 0.97 mW at the first resonance and 0.37 mW at the
 340 second resonance.

341

342 **5. Numerical model and validation**

343 The PPA 1001 unimorph cantilever is composed of 5 layers of different materials, see Figure 9.
 344 The active layer is made of PZT 5H (0.15 mm), it is partially covered by a copper electrode (0.03 mm)
 345 and by a polyester layer (0.05 mm). The structure is an AISI 304 stainless steel layer (0.15 mm),
 346 the bottom layer is made of polyimide (0.03 mm).

347 A numerical FE model was developed with the COMSOL software. The piezoelectric material
 348 was modeled as transversely isotropic linear elastic material with polarization axis perpendicular to
 349 the layer. The other materials were simulated as linear elastic materials with isotropic properties.
 350 The characteristics of the harvester's materials can be found in [39]. A mapped mesh of second
 351 order hexahedral elements with 27 nodes was used. The piezoelectric layer was modeled by means
 352 of 5 elements in the thickness direction and by 30 elements and 10 elements in the direction of
 353 length and width respectively. Geometrical details, like the edges of structural material that
 354 surround the piezoelectric material, were taken into account.



355

356 **Figure 9.** Layers of PPA 1001.

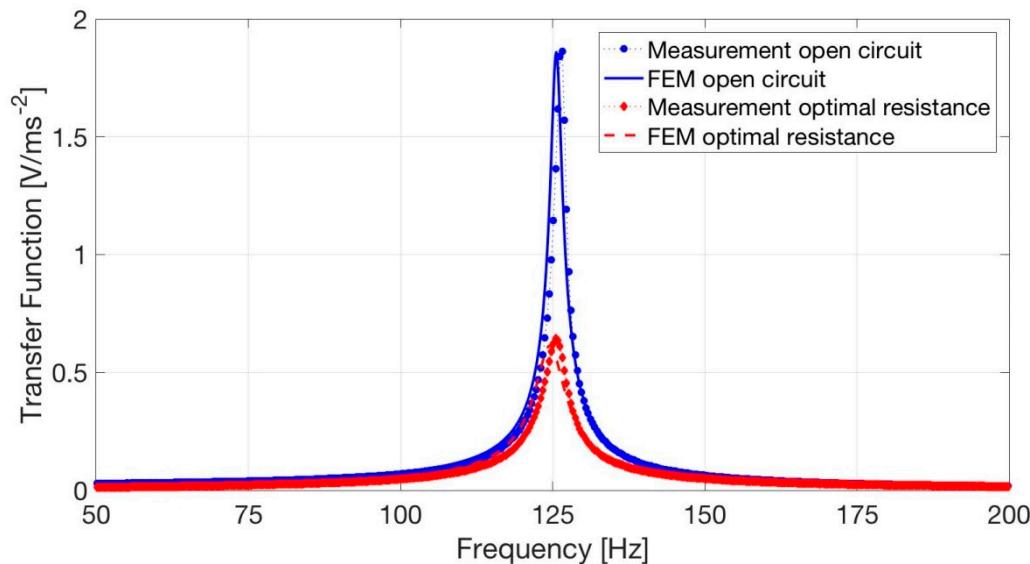
357 The steady-state harmonic response was calculated in the multi-physics domain, which
 358 includes mechanical, electrostatic and electrical equations. In the simulations a hysteretic damping
 359 model with $\eta=2\zeta=0.017$ (where ζ is damping ratio) was used, in agreement with experimental
 360 results of Table 2.

361 First the numerical model was validated comparing the numerical FRFs between generated
 362 voltage and base acceleration with the experimental FRFs. Both open circuit condition and optimal
 363 load resistance were considered.

364 Figure 10 shows very small differences both in the values of the resonance frequency and in
 365 the values of the resonance peak. In open circuit condition the numerical resonance peak is 1.82
 366 V/ms^{-2} and takes place at 125.6 Hz , the experimental values being 1.85 V/ms^{-2} at 126.4 Hz . With
 367 optimal load resistance ($12.7 \text{ k}\Omega$) the numerical resonance peak is 0.60 V/ms^{-2} and takes place at
 368 124.5 Hz , the experimental values being 0.64 V/ms^{-2} at 125.6 Hz .

369 Then, the numerical model was extended in order to allow the simulation of the harvester
 370 equipped with A1. A beam of hexahedral elements simulated the steel wire of A1 and a lumped
 371 mass simulated the brass disk.

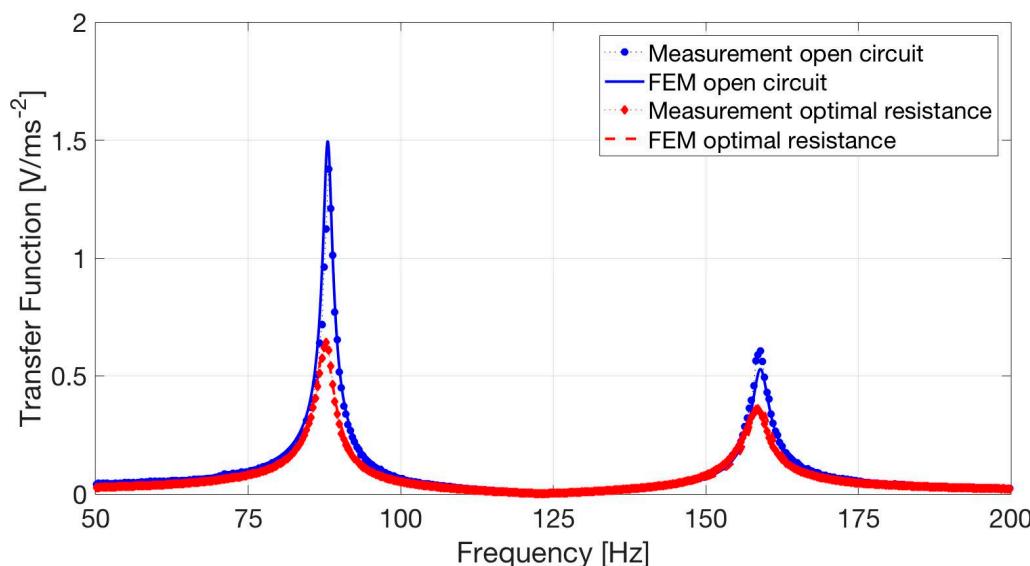
372 Figure 11 shows both the numerical and experimental FRFs between generated voltage and
 373 base acceleration of PPA 1001 equipped with A1. In open circuit condition there is a small
 374 difference (about 7 %) in the height of the first resonance peak. With optimal load resistance the
 375 difference is smaller.



376

377
378

Figure 10. Validation of the numerical model, FRFs of PPA 1001 in open circuit and with optimal load resistance.



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Figure 11. Validation of the numerical model, FRFs of PPA 1001 with A1 in open circuit and with optimal load resistance.

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6. Numerical simulation of harvesters with ITDs

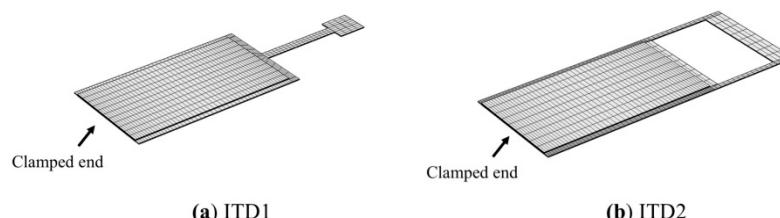
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The prototypes were built with a simple technology, which is not suited to mass production. After the validation of the numerical model, the FE analysis was used for predicting the performances of trimming devices integrated with the harvester structure (ITDs). The ITDs were built by extending and shaping the structural layer of PPA 1001. The first design, which is named ITD1, is a development of the tested prototypes (Figure 12a). In this case the cantilever beam of the DVA is a narrow extension of the structural layer of the harvester having width $b=2.16$ mm and length $L_a=21.4$ mm. The tip mass is a square patch of structural layer having a side length of 6.5 mm, the mass is 0.05 g. Evidently, ITD1 is a 3D structure and the tip mass cannot be considered a point mass. The analytical model can give only a rough approximation of the behavior of this

393 device, nevertheless equations (1-3) were used for obtaining a first indication of trimming
 394 frequency.

395 The second design, which is named ITD2, is represented in Figure 12b. In this case the
 396 structural layer is extended for 23.5 mm and then a rectangular hole is made. It is worth noticing
 397 that this shape could be obtained by means of a simple manufacturing process. The two lateral
 398 sides of the rectangular hole are two cantilever beams, whereas the final edge of the rectangular
 399 hole is the tip mass. This mass (0.12 g) is larger than the one of ITD1, because ITD2 is designed in
 400 order to show the possibility of trimming the harvester to two harmonics of ambient vibrations.
 401 This possibility is useful in the presence of periodic vibrations.

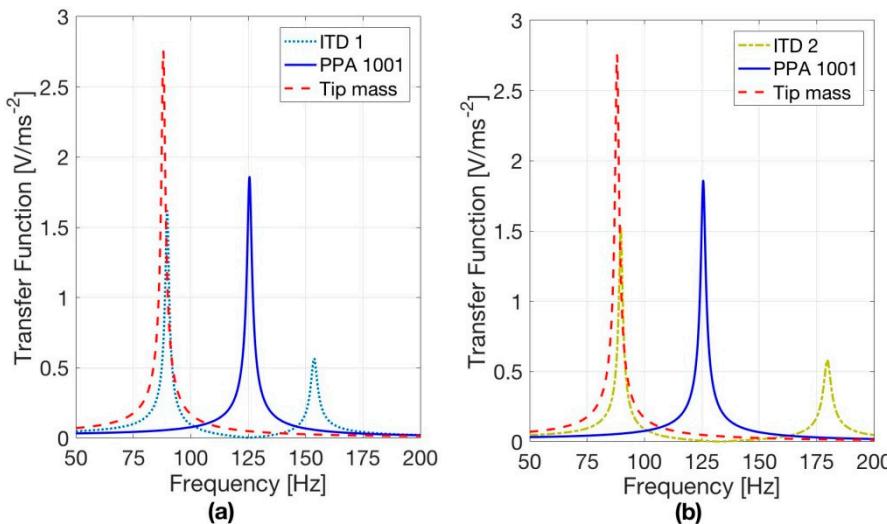
402 In the following of the paper results dealing with ITD1 and ITD2 are presented, but other
 403 kinds of ITDs were conceived and simulated in the framework of this research.



404
 405 **Figure 12.** Numerical models of the harvester with (a) ITD1, (b) ITD2.

406 *6.1. Open circuit voltage*

407 First, the FRFs of the harvesters equipped with ITDs were calculated to assess the trimming
 408 capabilities of the ITDs. The open circuit condition was considered, since analytical results and
 409 experimental tests on prototypes showed that the resistive load causes only minor frequency shifts
 410 of the peaks.

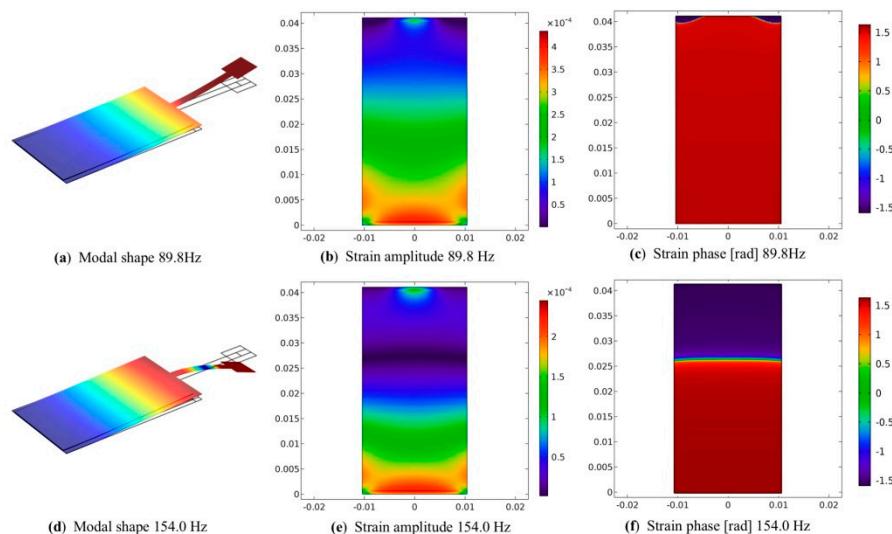


411
 412 **Figure 13.** Numerical results, FRFs of the harvester with ITDs, with tip mass and alone; open circuit condition.
 413 (a) ITD1, (b) ITD2.

414
 415 Figure 13a shows that ITD1 trims the main resonance to 89.8 Hz with a peak value (1.6 V/ms^{-2})
 416 a bit smaller than the one of PPA 1001 alone. The second resonance peak, which appears at 154.0 Hz
 417 is much lower than the main peak and is caused by the excitation of the second mode of vibration
 418 generated by ITD1. For comparison Figure 13a shows the effect of a large tip mass (0.59 g). In this
 419 case the trimming frequency is 88.1 Hz and the peak value is 2.7 V/ms^{-2} .

420 Figure 13b shows that ITD2 is able to lower the frequency of the main resonance peak to 89.8
 421 Hz and to create a second resonance peak at double frequency (179.5 Hz). The heights of the two
 422 peaks are 1.5 V/ms^{-2} and 0.57 V/ms^{-2} respectively.

423 When the frequency of base excitation coincides with one of the resonance frequencies and the
 424 deformed shape is dominated by the mode of vibration excited in resonance, the strain distribution
 425 inside the PZT layer, gives useful information about the effect of the ITD on the harvester.

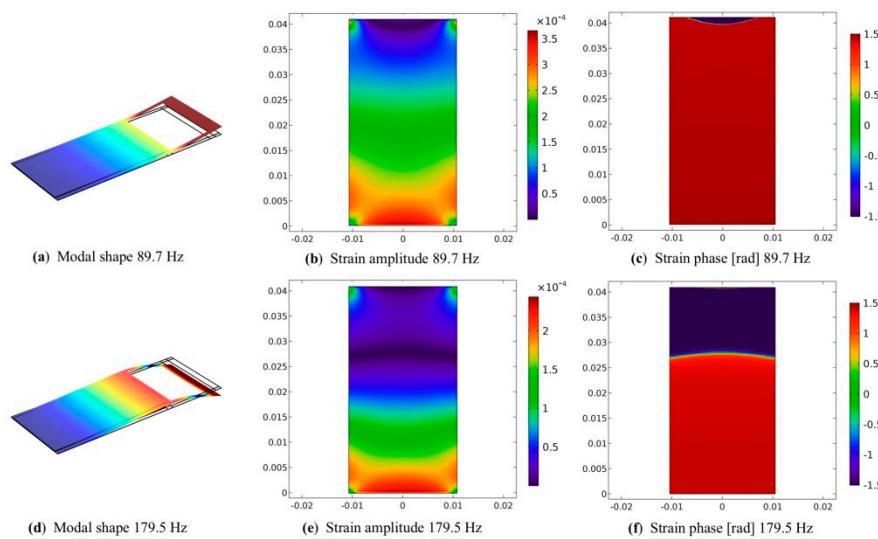


426
 427 **Figure 14.** Numerical results, strains in the piezo layer of the harvester with ITD1. (a) modal shape at the first
 428 resonance (89.8 Hz). (b) strain modulus at the first resonance (89.9 Hz). (c) strain phase at the first resonance
 429 (89.8 Hz). (d) Modal shape at the second resonance (154.0 Hz). (e) strain modulus at the second resonance
 430 (154.0 Hz). (f) strain phase at the second resonance (154.0 Hz).

431
 432 Figure 14 shows the strain in the upper plane of the PZT layer of the harvester with ITD1. This
 433 active layer is surrounded by edges of structural material. The strain component in the longitudinal
 434 direction is represented, since it is the most important strain component caused by harvester
 435 bending. The color maps show both the amplitude and phase of strain. Strain node lines [38], which
 436 are the lines on the surface of the PZT layer where the strain changes sign, are represented by light
 437 lines in the phase plots.

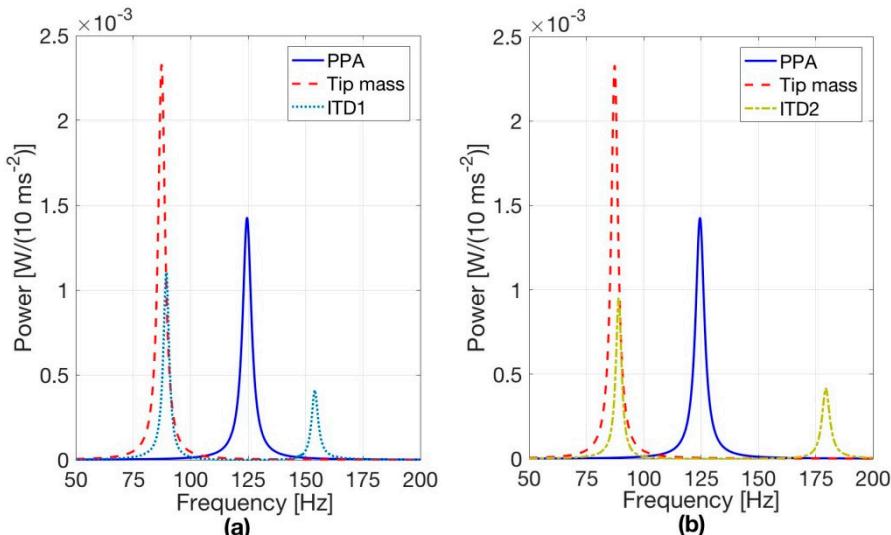
438 At the first resonance the ITD moves in phase with the harvester. The amplitude of strain
 439 decreases rather regularly from the clamped-end to the free-end. There are changes in the sign of
 440 strain near the corners of the free-end of the layer. This is a 3D effect, since the ITD is attached at the
 441 middle of the free-end of the harvester, see Figure 12a. With this strain distribution cancellations in
 442 generated voltage [38] are very small and the first peak is high. At the second resonance the ITD
 443 moves in phase opposition with respect to the harvester tip (Figure 14d). The strain distribution in
 444 this condition is very different with a strain node line at about 6/10 of the harvester's length. There
 445 is a partial cancellation in the generated voltage, which leads to a small resonance peak.

446 The strain distributions in the PZT layer of the harvester equipped with ITD2 are represented
 447 in Figure 15. At the first resonance there is only a strain node line near the central part of the
 448 free-end of the layer, because the ITD is attached to the harvester in correspondence of the side
 449 edges of structural material that surround the active layer (Figure 15a). Owing to the presence of
 450 this strain node line, a cancellation in the generated voltage takes place, but this effect is small,
 451 because the negative strain area is small and negative strains have small amplitudes. When the
 452 harvester with ITD2 is excited at the frequency of the second resonance peak, the side beams of
 453 ITD2 alter the strain distribution inside the PZT layer and there is a partial cancellation in generated
 454 voltage.

455
456457 **Figure 15.** Numerical results, strains in the piezo layer of the harvester with ITD2. (a) modal shape at the first
458 resonance (89.7 Hz). (b) strain modulus at the first resonance (89.7 Hz). (c) strain phase at the first resonance
459 (89.7 Hz). (d) modal shape at the second resonance (179.5 Hz). (e) strain modulus at the second resonance
460 (179.5 Hz). (f) strain phase at the second resonance (179.5 Hz).461 *6.2. Generated power*462 The most important characteristic of a piezoelectric harvester is the power that it can generate
463 at the various frequencies. In order to calculate the generated power, load resistance was set equal
464 to the optimal value for the first resonance peak (Equation (15)). The harmonic response with a base
465 acceleration of 10 ms^{-2} was simulated. Calculated results are represented in Figure 16, and the
466 powers generated by the harvesters with ITDs are compared with the powers generated by the
467 harvester alone and by the harvester with a tip mass that trims the harvester to the same frequency.468 The power generated by the harvester with ITD1 at the low frequency resonance (89.3 Hz) is
469 1.10 mW. This value is lower than the power generated by PPA 1001 alone at 124.5 Hz, but the
470 harvester with ITD1 moreover generates 0.41 mW in correspondence of the high frequency
471 resonance (154.0 Hz).472 The harvester with ITD2 is suited to operate in the presence a periodic excitation with the first
473 harmonic at 89.5 Hz (fundamental frequency), because it generates 0.95 mW at the fundamental
474 frequency and 0.42 mW at the frequency of the second harmonic.

475 Finally, the harvester with tip mass generates the largest power at 87.3 Hz.

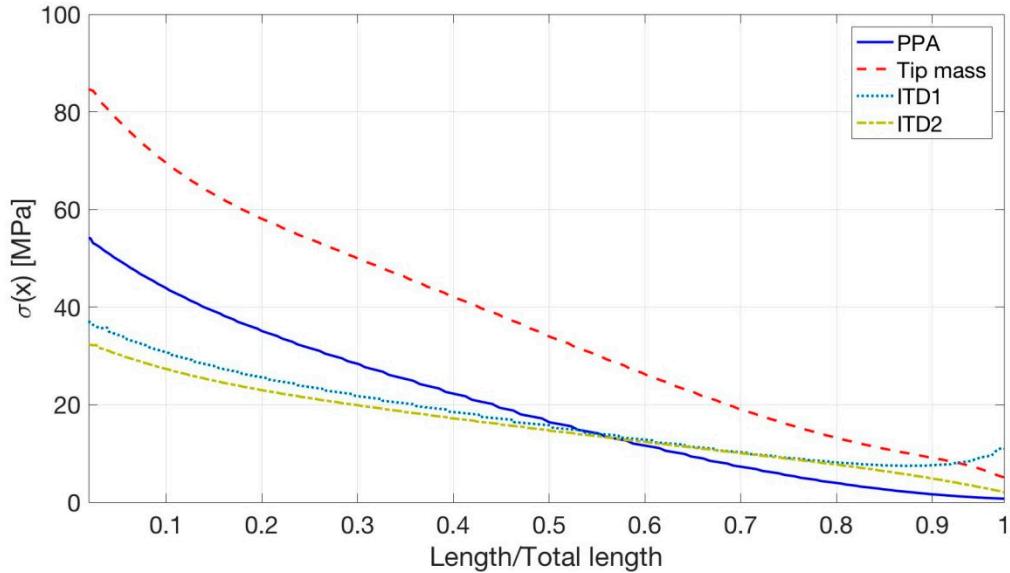
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477

478 **Figure 16.** Numerical results, power generated by PPA 1001 equipped with trimming devices, optimal load
 479 resistance, base acceleration 10 ms^{-2} (1g). (a) ITD1, (b) ITD2.

480 6.3. Stress analysis



481

482 **Figure 17.** Numerical results, maximum stress in the piezoelectric layer of the harvesters.

483

484 The last numerical analysis here presented deals with the stress analysis inside the
 485 piezoelectric material. Four configurations were considered: harvesters equipped with ITD1 and
 486 ITD2, PPA 1001 alone and with tip mass. The harvesters were excited by a base acceleration of 10
 487 ms^{-2} at the main resonance frequency, which coincides with the low frequency peak of the
 488 harvesters with ITDs and with the main peak of the others.

489 Normal stress in the longitudinal direction σ_x was calculated, since it is the most important
 490 stress component caused by harvester bending. Stress σ_x was evaluated along the centerline of the
 491 upper surface of the PZT layer, which is at the largest distance from the neutral axis of the
 492 composite cross-section. The trace of this line is point P of Figure 9.

493 Figure 17 shows that the introduction of the tip mass does not considerably modify the trend
 494 of stress distribution along the span of the harvester, which is characterized by the largest values
 495 near the clamp. But the tip mass leads to a large increase in the values, which almost double.

496 Conversely, when ITD1 is introduced, the trend of stress distribution along the span of the
497 harvester changes. In particular, the stress decreases less when moving from the clamp to the tip of
498 the harvester and it increases sharply in the proximity of the connection with the ITD. The
499 maximum value is reached at the clamp and it is smaller than the one of PPA 1001 alone. The
500 insertion of ITD2 lowers the stress at the clamped-end as well, but with ITD2 the stress near the
501 free-end of the PZT layer increases less than with ITD1, since ITD2 is connected to the harvester in
502 correspondence of the side edges of the structural layer that surround the PZT layer, see Figure 12b.

503 It should be noted that the bending breaking strength of PZT 5H is in the range 100 - 140 MPa
504 [40-41] and it considerably reduces in the presence of cyclic loading (see Figure 3 of [41]). Hence,
505 with high acceleration levels the harvester with tip mass may be closer to the failure condition than
506 the harvester with ITDs.

507 7. Discussion

508 The introduction of an ITD chiefly has two effects on the harvester's performance: the lowering
509 of the main resonance frequency and the appearance of a new resonance peak that can be exploited
510 in order to harvest energy.

511 The first effect could be achieved by adding a tip mass, but the tip mass is much larger than the
512 ITD mass and it leads to a large increase in the inertia forces. This fact increases the generated
513 power (Figure 16) but also the stress inside the piezoelectric layer (Figure 17). Hence, an ITD is
514 useful for trimming an harvester when vibration levels are high and stress inside an harvester
515 equipped with a tip mass could reach dangerous values.

516 The lowering of the resonance frequency could be attained also by adjusting geometric and
517 inertial parameters of the harvester without inserting an ITD. The basic theory of vibrating beams
518 [23] shows that this effect could be attained by increasing the length and decreasing the width of
519 the rectangular cantilever harvester (slender harvester), but the results of numerical simulations
520 (Table 3) show that the stress near the clamped end increases. The reduction in the equivalent
521 bending stiffness of the composite cross section could reduce the natural frequency, but also in this
522 case the harvester becomes rather weak and it could be plastically deformed and/or damaged
523 during manipulations. Another possibility is the modification of the shape of the harvester, even if
524 the deposition of the various layers on a non-rectangular base could lead to practical problems. To
525 make a comparison an inverse-tapered harvester [42] having the same cross section and the same
526 volume of piezo material as PPA 1001 was investigated. Numerical results, which are reported in
527 Table 3, highlight that also in this case there is a large increase in stress near the clamped end.

528 The second effect of the ITDs (the presence of two resonance peaks) presents some limits and
529 some potentialities as well. The main limit is the large frequency interval (Δf) between the peaks
530 (64 Hz in ITD1). Therefore the harvesters with these ITDs cannot be considered broadband
531 harvesters. The analytical model and experimental results on the prototypes (Table 2) actually show
532 that Δf can be decreased by decreasing the mass of the trimming device; nevertheless it is rather
533 difficult to obtain two close peaks ($\Delta f < 10$ Hz). The potentialities of the harvesters with ITDs are
534 related to the fact that periodic excitation with relatively large frequency intervals between the
535 harmonic components is rather common in machines. Typical examples of periodic excitation can
536 be found in rolling bearings [43], electrical machines [43], piston machines [44] (engines and
537 compressors), fans and blowers [43] (harmonics of the blade passing frequency) and in geared
538 systems [45] (harmonic of the mesh frequency). In these cases the harvesters with ITD can be
539 trimmed to a couple of sharp and stable peaks which are characteristic of the spectra of these
540 sources of excitation. Numerical simulations show that ITD2 can be trimmed to 90 Hz and 180 Hz
541 with an overall harvested power of 1.37 mW.

542

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547**Table 3.** Dimensional characteristics of the PZT layer, generated power ad maximum stress of PPA 1001 and various harvesters trimmed to the same frequency, base acceleration is 10 ms⁻².

Harvester	L [mm]	b _e [mm]	b _c [mm]	Frequency [Hz]	Peak power [mW]	σ [MPa]
PPA 1001	40.1	20.8	20.8	124.5	1.42	55
PPA + ITD1	40.1	20.8	20.8	89.3	1.10	37
PPA + ITD2	40.1	20.8	20.8	89.5	0.95	33
PPA + tip mass	40.1	20.8	20.8	87.3	2.33	85
Slender-rectangular	48.8	17.5	17.5	87.6	1.95	70
Inverse tapered	41.1	31.6	10.0	87.4	2.30	118

548

549 8 Conclusions

550 Analytical calculations and experimental results show that it is possible to develop cantilever
551 DVAs that behave as reactive mechanical loads and are able to modify the natural frequencies and
552 modes of vibration of the harvester, without increasing the damping of the system.

553 Usually vibration energy is available at frequencies lower than the fundamental frequency of a
554 piezoelectric harvester. Experimental results show that decrements in the fundamental frequency of
555 about 30% can be achieved by means of a simple cantilever DVA.

556 These concepts, which have been validated by means of prototypes built with a simple
557 technology, are then extended to develop the ITDs. The integration of the trimming device with the
558 structural layer of the harvester is a promising technology, since the extension of the structural
559 layer (steel or plastic material) would be rather inexpensive and the extension could be cut into the
560 desired shape by means of consolidated technologies such as punching or laser cutting. The
561 harvester with ITD appears simpler and cheaper than other promising solutions.

562 In the framework of this research the potentialities of harvesters equipped with ITDs have
563 been investigated by means of FE simulation. Numerical results show that the ITDs basically have
564 the same properties of the cantilever DVA, but some different designs are possible, which
565 correspond to different performances of the harvester.

566 The first integrated trimming device (ITD1) is suited to lower the natural frequency of the
567 harvester of about 35 Hz and to collect power at low frequency (1.1 mW). The second integrated
568 trimming device (ITD2) is characterized by two resonance peaks with multiple frequencies and it is
569 suited to collect energy from periodic excitation.

570 The main limit of an ITD seems the increased length of the device, but this limit could be
571 overcome with a folded design in which the mass lies near the harvester base.

572 Future research directions are testing and numerical simulation of harvesters equipped with
573 more complex ITDs, which behave as dynamic systems with many DOFs.

574

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579 devices; A. Doria and C. Medè performed the experiments; C. Medè performed the simulations with the
580 supervision of F. Moro; all the authors analyzed and discussed the results, and revised the paper written by A.
581 Doria.

582 **Conflicts of Interest:** The authors declare no conflict of interest.

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