

1 Article

2 Integrating Monte Carlo and the Hydrodynamic 3 Model for Predicting Extreme Water Levels in River 4 Systems

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9 **Abstract:** Estimates of extreme water level return periods in river systems are crucial for hydraulic
10 engineering design and planning. Recorded historical water level data of Taiwan's rivers are not
11 long enough for traditional frequency analyses when predicting extreme water levels for different
12 return periods. In this study, the integration of a one-dimensional flash flood routing
13 hydrodynamic model with the Monte Carlo simulation was developed to predict extreme water
14 levels in the Danshuei River system of northern Taiwan. The numerical model was calibrated and
15 verified with observed water levels using four typhoon events. The results indicated a reasonable
16 agreement between the model simulation and observation data. Seven parameters, including the
17 astronomical tide and surge height at the mouth of the Danshuei River and the river discharge at
18 five gauge stations, were adopted to calculate the joint probability and generate stochastic
19 scenarios via the Monte Carlo simulation. The validated hydrodynamic model driven by the
20 stochastic scenarios was then used to simulate extreme water levels for further frequency analysis.
21 The design water level was estimated using different probability distributions in the frequency
22 analysis at five stations. The design high-water levels for a 200-year return period at Guandu
23 Bridge, Taipei Bridge, Hsin-Hai Bridge, Da-Zhi Bridge, and Chung-Cheng Bridge were 2.90 m, 5.13
24 m, 6.38 m, 6.05 m, and 9.94 m, respectively. The estimated design water levels plus the freeboard
25 are proposed and recommended for further engineering design and planning.

26 **Keywords:** extreme water level, hydrodynamic model, Monte Carlo, joint probability, model
27 calibration and verification, Danshuei River system
28

29 1. Introduction

30 Extreme water levels in tidal rivers can cause overbanking and levee breaking, which results in
31 economic and human life losses and social impacts. The water level in this type of transitional area
32 is affected by upstream river flow, downstream tidal levels, and the operation of existing
33 controllable structures [1]. Tidal rivers have negative impacts on surrounding areas where residents
34 live, but these rivers also contribute several benefits. The astronomical tide at the mouth of tidal
35 rivers obstructs fluvial flows flowing towards downstream reaches. The water level increases due to
36 the interaction between tides and floods in tidal rivers [2,3]. Moreover, it is vital to assess extreme
37 water level frequency for flood risks and future flood defense designs. The resistance of river levees
38 against flooding is captured in the design water level.

39 There are two major approaches when determining extreme water levels depending on the
40 availability of the observed water level data at a site. The first approach is a numerical simulation
41 method. It can be used when there is not an adequate observational record for flooded water levels.
42 A hydrodynamic model provides the link between the known statistics of the generating forces and
43 the desired statistics of the water levels. These simulation methods, such as the joint probability
44 method [4,5] and the Monte Carlo simulation [6], can be used to accurately predict extreme water
45 levels. The second approach is a frequency analysis of annual maximum water levels that results
46 from the combination of several forcing factors [7,8]. This approach can be used when reasonably

47 long observation data (greater than 60 years) are available [9,10]. Xu and Huang [11] used a 91-year
48 data set at the Wusong station near Shanghai in the Yangtze estuary to estimate the 100-year annual
49 maximum water level (AMWL) using a general extreme value (GEV) model to support coastal
50 hazard mitigation planning. To examine the effect of a shorter data set, a 59-year data set was
51 investigated. They found that model predictions using the 59-year data set underestimated the
52 observed 60-year AMWL.

53 It is difficult to find long-term datasets greater than 60 years from gauge stations at Taiwanese
54 river mouths. Therefore, a traditional frequency analysis is not appropriate for gauge stations in
55 Taiwan. The alternative numerical simulation method is suitable for analyzing extreme water levels.
56 This method can be used to simulate extreme water levels using a hydrodynamic model driven by
57 discharge upriver and tidal levels at the river mouth. The hydrodynamic model provides a link
58 between a generating forcing with a return period and the expected extreme water levels. Mantz
59 and Wakeling [12] and Samuels and Burt [13] used a very limited number of sampling scenarios,
60 but they adopted accurate Monte Carlo simulations to produce a very large number of stochastic
61 sampling scenarios. A simplified one-dimensional hydrodynamic model of the lower Rhine delta
62 was applied to simulate extreme water levels. Zhong et al. [6] established a joint probability analysis
63 on astronomical tides, wind-induced storm surge, the Rhine flow and the Muse flow at the river
64 boundaries to produce a joint probability distribution. A one-dimensional hydrodynamic model
65 was then applied to estimate high water-level frequencies in the estuary delta. The sampled Monte
66 Carlo simulation method was used to generate a large number of stochastic scenarios as inputs for
67 the one-dimensional hydrodynamic model.

68 Rivers in Taiwan are short with steep slope and meandering that is favorable to the occurrence
69 of flooding at midstream/downstream reaches when heavy rainfalls occur in the basin during
70 typhoon events. In recent years, the rapid urbanization without sufficient floodplain management
71 resulted in highly developed and densely populated zones over riversides. The existent hydraulic
72 facilities were unable to provide enough flood protection. Therefore, flooding is one of the worst
73 natural hazards causing serious economic and social impacts in Taiwan [14,15].

74 The objective of this study is to integrate a one-dimensional flash flood routing hydrodynamic
75 model with a Monte Carlo simulation to estimate extreme water levels in the Danshuei River system
76 of northern Taiwan. The hydrodynamic model was first calibrated and verified with measured
77 water levels using several typhoon events. The validated model was then applied to calculate water
78 levels in the river system. Different water level scenarios at the downstream boundary and
79 discharges at upstream boundaries were yielded from the Monte Carlo simulation to drive the
80 hydrodynamic model. The desired extreme water levels with return periods of 50, 100, and 200 years
81 are proposed in this study.

82 2. Materials and Methods

83 2.1. Description of Study Site

84 The Danshuei River system is located in northern Taiwan (Figure 1). It covers three major
85 tributaries, the Dahan River, the Xindian River, and the Keelung River, and its watershed has a
86 population of over six million people [16]. Therefore, effective management and flood control of the
87 Danshuei River system is one of the most important tasks in Taiwan. The Erchung flood diversion
88 channel, built near the confluence of the Dahan River and the Xindian River in 1984, was used to
89 divert flood flow during typhoon events. The Danshuei watershed area is 2,726 km², with a mean
90 annual precipitation of 3001 mm. The total channel length is 327.6 km, and the channel slope ranges
91 from 0.015° to 0.0027°. The peak discharge of a 200-year flood reaches 25,000 m³/s. The major forcing
92 mechanisms of flood flows are astronomical tides at the river mouth and river discharges at the
93 upriver boundary. Semi-diurnal tides are represented by principal tidal constituents, with a mean
94 tidal range of 2.21 m and a spring tidal range of 3.1 m. The downstream reaches of all three
95 tributaries are affected by tides [17,18].

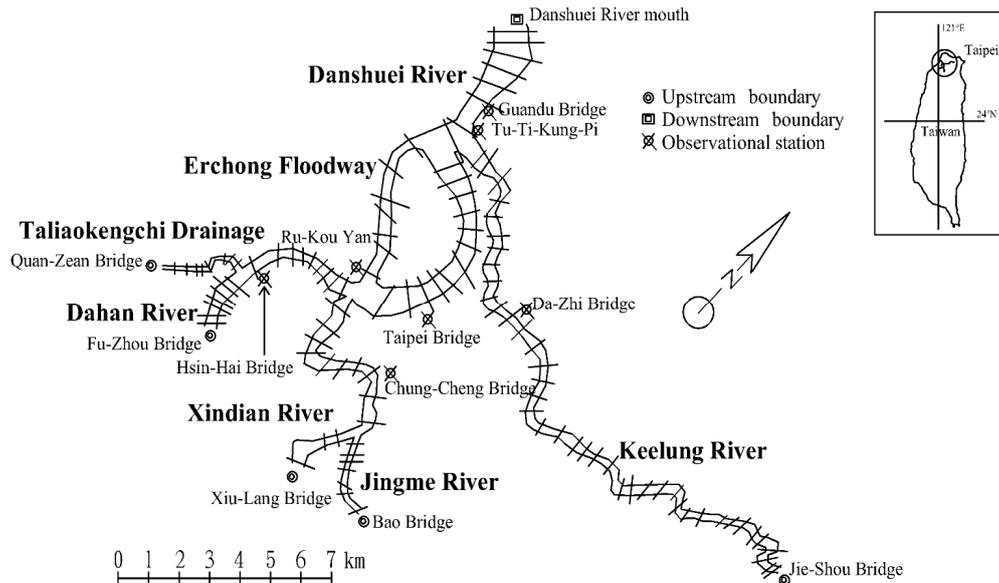


Figure 1. Map of the Danshuei River system in northern Taiwan.

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98 2.2. Governing Equations of the One-Dimensional Model

99 The flash flood routing hydrodynamic model is based on a dynamic wave theory using
100 Saint-Venant equations, which consist of one-dimensional continuity and momentum equations:

$$\frac{\partial A}{\partial t} + \frac{\partial Q}{\partial x} - q_1 + q_2 + q_3 = 0 \quad (1)$$

$$\frac{\partial Q}{\partial t} + \frac{\partial}{\partial x} \left(\frac{Q^2}{A} \right) - gA \left(S_0 - \frac{\partial Y}{\partial x} - S_f \right) - q_1 V_1 + q_2 \left(\frac{Q}{A} \right) + q_3 \left(\frac{Q}{A} \right) = 0 \quad (2)$$

101 where A represents the cross-sectional area, Y represents the water depth, Q represents the
102 discharge, q_1 represents the lateral inflow per unit channel length, q_2 represents the levee-break
103 flow per unit channel length, q_3 represents the lateral outgoing overbank flow per unit channel
104 length, S_0 represents the bottom slope of the channel, S_f represents the friction slope, V_1
105 represents the longitudinal velocity component of the lateral inflow, g represents the gravitational
106 acceleration, t represents time, and x represents the distance along the channel. Because the
107 cross-sectional area can be written as a function of water depth, only two flow variables, Q and Y ,
108 must be solved in Eqs. (1) and (2). The four-point implicit finite difference approximation scheme
109 [19,20] was used in this study to solve for variables Q and Y . The detailed algorithms of the solution
110 can be found in Liu and Wu [21].

111 2.3. Hydrodynamic Model Setup

112 In this study, the model transects were established with measured cross-sectional profiles at
113 approximately 0.5 km intervals along the river. The model transects in the computational domain
114 included 310 transects and covered 11 river reaches (shown in Table 1). The upstream boundaries
115 are specified at Fu-Zhou Bridge (Dahan River), Quan-Zean Bridge (Taliaokengchi drainage, which
116 is a tributary of the Dahan River), Xiu-Lang Bridge (Xindian River), Bao Bridge (Jingme River,
117 which is a tributary of the Xindian River), and Jie-Shou Bridge (Keelung River). The downstream
118 boundary is located at the mouth of the Danshuei River. The upstream and downstream boundaries
119 are shown in Figure 1. The upstream boundary conditions of the dynamic flood routing
120 hydrodynamic model are represented by the discharges and tidal levels specified at the mouth of
121 the Danshuei River (Figure 1). Figure 2 illustrates the layout of the Danshuei River system for
122 model simulation.

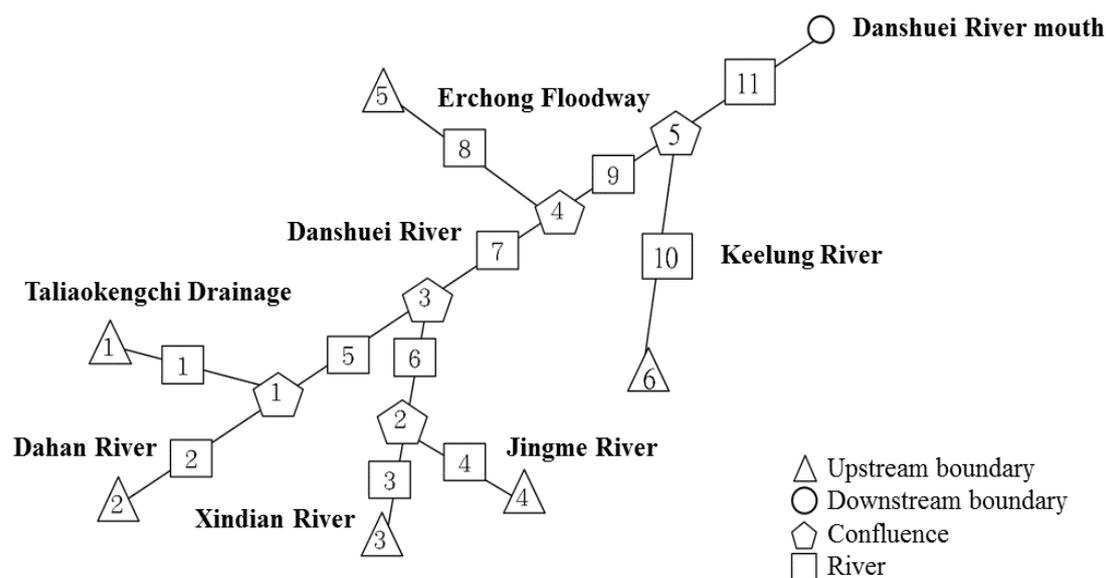
123 **Table 1.** Cross-sectional number of river reaches and Manning coefficients (n) used in the
 124 computational domain.

River reach number	1	2	3	4	5	6
Number of cross section	71	8	3	13	9	22
Manning friction n	0.025	0.033	0.035	0.040	0.033	0.027
River reach number	7	8	9	10	11	
Number of cross section	22	10	2	137	13	
Manning friction n	0.033	0.033	0.033	0.050	0.023	

125 **Table 2.** Data description.

Station	Data	Time	Data description
Danhuei River mouth	Observed tidal level	1994~2015	Hourly tidal level
Quan-Zean Bridge (Taliaokengchi Drainage)	Observed river discharge	1994~2014	Daily-average discharge
Fu-Zhou Bridge (Dahan River)	Observed river discharge	1994~2014	Daily-average discharge
Xiu-Lang Bridge (Xindian River)	Observed river discharge	1994~2014	Daily-average discharge
Bao Bridge (Jingme River)	Observed river discharge	1994~2014	Daily-average discharge
Jie-Shou Bridge (Keelung River)	Observed river discharge	1994~2014	Daily-average discharge

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Figure 2. Layout of the Danshuei River system for the model simulation.

129 2.4. Data Collection and Analysis

130 Observational data, including tidal levels at the mouth of the Danshuei River and discharges
 131 upstream of the Taliaokengchi drainage, Dahan River, Xindian River, Jiangme River, and Keelung

132 River, were collected from the Water Resource Agency of Taiwan. The data are shown in Table 2.
 133 Surge heights during typhoon periods were calculated as the difference between the observed tidal
 134 level and the astronomical tide. Observational data for astronomical tides, surge heights, and
 135 discharges were used to analyze and yield the probability of each parameter.

136 2.5. Frequency Analysis Methods

137 Several researchers have used frequency analyses to investigate extreme water levels [22-24].
 138 For example, the Federal Emergency Management Agency (FEMA) recommends the use of the GEV
 139 model for extreme value analyses of annual maxima to investigate coastal flood insurance along the
 140 west coast of the USA near the Pacific Ocean [25]. There are many peer-reviewed articles regarding
 141 the application of GEV distributions described by a shape factor parameter [9,26,27]. However,
 142 other types of probability distributions for frequency analyses should be used to select the best
 143 distribution for extreme values. In this study, different types of distributions were used to analyze
 144 extreme values. Table 3 shows the frequency analysis probability distributions for extreme values,
 145 including the Gumbel, Weibull, Frechet, normal, lognormal, and log-Pearson 3 distributions.

146 The standard error (SE) and the correlation coefficient (CC) were used as indicators when
 147 evaluating the performance of each extreme value distribution. The most adaptive distribution (i.e.,
 148 the lowest SE and the highest CC) will be selected for further applications. The expressions for the
 149 SE and the CC are given as follows:

$$SE = \sqrt{\frac{\sum_{i=1}^n (x_i - \hat{x}_i)^2}{n}} \quad (3)$$

$$CC = \frac{\sum_{i=1}^n (x_i - \bar{x})(\hat{x}_i - \bar{\hat{x}})}{\sqrt{\sum_{i=1}^n (x_i - \bar{x})^2} \sqrt{\sum_{i=1}^n (\hat{x}_i - \bar{\hat{x}})^2}} \quad (4)$$

150 where x_i represents the observed value, \hat{x}_i represents the estimated value, \bar{x} represents the
 151 mean of the sampling data, $\bar{\hat{x}}$ represents the mean of the estimated data, and n represents the
 152 number of data.

153 2.6. Monte Carlo Simulation

154 Monte Carlo methods are a broad class of computational algorithms that rely on repeated
 155 random sampling to obtain numerical results. The essential idea is the use of randomness to solve
 156 problems that might be deterministic in principle. They are often used in physical and
 157 mathematical problems and are most useful when it is difficult or impossible to use other
 158 approaches [28]. Monte Carlo simulations have been widely used in engineering for sensitivity
 159 analyses and quantitative probabilistic analyses in process design. For example, they are applied in
 160 coastal flood damage estimates and coastal hydrodynamic modeling [29-32], flood propagation
 161 modeling [33,34], rainfall hyetograph design [35], and water quality modeling [36-39].

162 In this study, a joint probability analysis of astronomical tides, surge heights, and river
 163 discharges at five upstream boundaries is established to produce a joint probability distribution of
 164 potential flood events. Then, importance sampling with the Monte Carlo simulation method is used
 165 to help generate a large number of stochastic scenarios as the inputs for the one-dimensional flash
 166 flood routing hydrodynamic model. A set of 2000 scenarios is generated from the Monte Carlo
 167 simulations to compute extreme water levels in a river system.

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Table 3. Probability distributions for the frequency analysis of extreme values.

Probability model	Range	$f(x)$ =PDF $F(x)$ =CDF
Gumbel	$-\infty < x < +\infty$; $\sigma > 0$	$f(x) = \frac{1}{\sigma} \exp\left(-\left(\frac{x-\mu}{\sigma}\right) - \exp\left(-\left(\frac{x-\mu}{\sigma}\right)\right)\right)$ $F(x) = \exp\left(-\exp\left(-\left(\frac{x-\mu}{\sigma}\right)\right)\right)$
Weibull	$\gamma \leq x < +\infty$; $\alpha, \beta > 0$	$f(x) = \frac{\alpha}{\beta} \left(\frac{x-\gamma}{\beta}\right)^{\alpha-1} \exp\left(-\left(\frac{x-\gamma}{\beta}\right)^\alpha\right)$ $F(x) = 1 - \exp\left(-\left(\frac{x-\gamma}{\beta}\right)^\alpha\right)$
Frechet	$\gamma < x < +\infty$; $\alpha, \beta > 0$	$f(x) = \frac{\alpha}{\beta} \left(\frac{\beta}{x-\gamma}\right)^{\alpha+1} \exp\left(-\left(\frac{\beta}{x-\gamma}\right)^\alpha\right)$ $F(x) = \exp\left(-\left(\frac{\beta}{x-\gamma}\right)^\alpha\right)$
Normal	$-\infty < x < +\infty$; $\sigma > 0$	$f(x) = \frac{\exp\left(-\frac{1}{2}\left(\frac{x-\mu}{\sigma}\right)^2\right)}{\sigma\sqrt{2\pi}}$ $F(x) = \Phi\left(\frac{x-\mu}{\sigma}\right)$
Lognormal	$\gamma < x < +\infty$; $\sigma, \mu > 0$	$f(x) = \frac{\exp\left(-\frac{1}{2}\left(\frac{\ln(x-\gamma)-\mu}{\sigma}\right)^2\right)}{(x-\gamma)\sigma\sqrt{2\pi}}$ $F(x) = \Phi\left(\frac{\ln(x-\gamma)-\mu}{\sigma}\right)$
Log-Pearson 3	$0 < x \leq e^\gamma$ $\beta < 0$; $e^\gamma \leq x < +\infty$ $\beta > 0$; $\alpha > 0, \beta \neq 0$	$f(x) = \frac{1}{x \beta \Gamma(\alpha)} \left(\frac{\ln(x)-\gamma}{\beta}\right)^{\alpha-1} \exp\left(-\frac{\ln(x)-\gamma}{\beta}\right)$ $F(x) = \frac{\Gamma(\ln(x)-\gamma/\beta)(\alpha)}{\Gamma(\alpha)}$

170 Note: PDF: Probability density function; CDF: Cumulative density function

171 2.7. Indices of the Hydrodynamic Simulation Performance

172 To evaluate the performance of the one-dimensional flash flood routing hydrodynamic model,
 173 two criteria were used to compare the simulated results and the measured data: the mean absolute
 174 error (MAE) and the root mean square error (RMSE). The error indices can be defined by Eqs. (5)
 175 and (6), respectively:

$$MAE = \frac{1}{N} \sum_{i=1}^N |H_{sim,i} - H_{mes,i}| \quad (5)$$

$$RMSE = \left[\frac{1}{N} \sum_{i=1}^N [H_{sim,i} - H_{mes,i}]^2 \right]^{1/2} \quad (6)$$

176 where H_{mes} represents the measured water level, H_{sim} represents the simulated water level, and
 177 N represents the number of time measurements.

178 3. Model Calibration and Verification

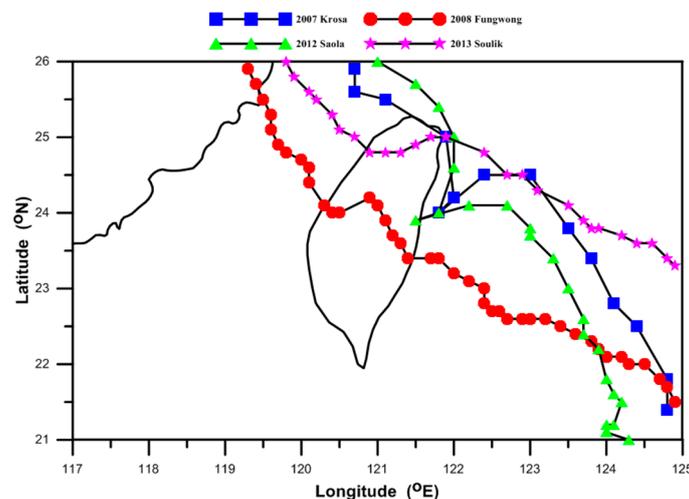
179 To ascertain the model accuracy for further practical applications, the observational data are
 180 used to calibrate and verify the model and to validate its capability when predicting water levels in
 181 the river system.

182 The friction coefficient (n) is an important parameter affecting the calculation of the water level
 183 in the Danshuei River system during flooding periods. Two typhoon events, Typhoon Krosa (2007)
 184 and Typhoon Fungwong (2008), were used for the model calibration, and two other typhoon events,
 185 Typhoon Saola (2012) and Typhoon Soulik (2013), were used for model verification. The tracks of
 186 these typhoon events are illustrated in Figure 3.

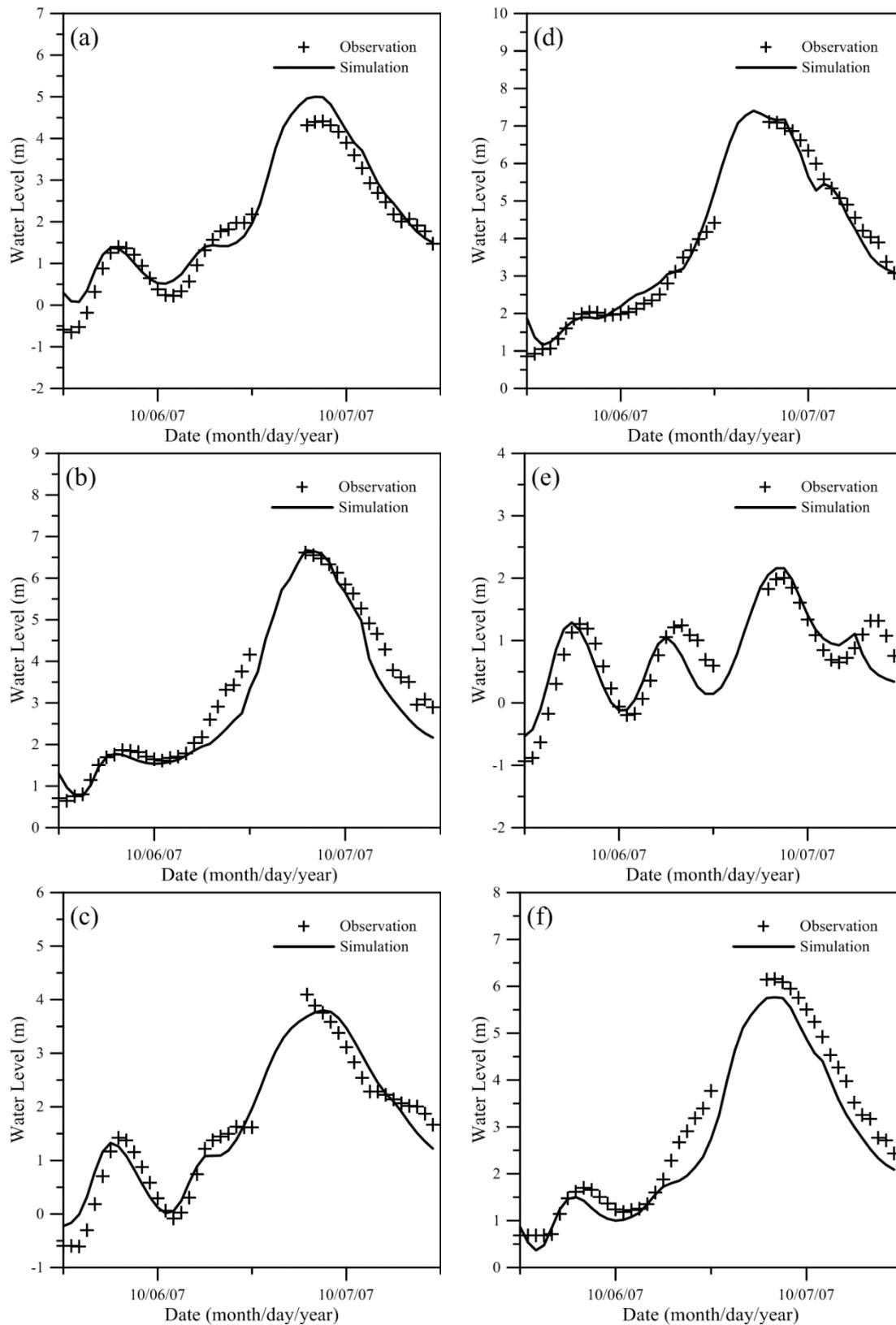
187 Figure 4 and Figure 5 present the calibration results for Typhoon Krosa (2007) and Typhoon
 188 Fungwong (2008), respectively. They indicate that the computed water level hydrograph is similar to
 189 the observed water level hydrograph. The mean absolute errors (MAEs) of the events at Taipei
 190 Bridge, Hsin-Hai Bridge, Da-Zhi Bridge, Chung-Cheng Bridge, Tu-Ti Kung Pi, and Ru-Kou Yan
 191 (locations are shown in Figure 1) during Typhoon Krosa are 0.30 m, 0.30 m, 0.23 m, 0.22 m, 0.23 m,
 192 and 0.33 m, respectively. In addition, the mean absolute errors (MAEs) of the events at Taipei Bridge,
 193 Hsin-Hai Bridge, Da-Zhi Bridge, Chung-Cheng Bridge, Tu-Ti Kung Pi, and Ru-Kou Yan during
 194 Typhoon Fungwong are 0.19 m, 0.15 m, 0.24 m, 0.22 m, 0.07 m, and 0.18 m, respectively. It can be
 195 seen that the calibrated results for Typhoon Fungwong are better than those for Typhoon Krosa.

196 Figure 6 and Figure 7 present the verification results for Typhoon Saola (2012) and Typhoon
 197 Soulik (2013), respectively. These figures also reveal that the computed water levels mimic the
 198 observed water levels at different gauge stations. The mean absolute errors (MAE) of the events at
 199 Taipei Bridge, Hsin-Hai Bridge, Da-Zhi Bridge, Chung-Cheng Bridge, Tu-Ti Kung Pi, and Ru-Kou
 200 Yan during Typhoon Saola are 0.24 m, 0.06 m, 0.21 m, 0.11 m, 0.17 m, and 0.40 m, respectively. In
 201 addition, the mean absolute errors (MAEs) of the events at Taipei Bridge, Hsin-Hai Bridge, Da-Zhi
 202 Bridge, Chung-Cheng Bridge, and Tu-Ti Kung Pi during Typhoon Soulik are 0.19 m, 0.18 m, 0.13 m,
 203 0.17 m, and 0.10 m, respectively. There are no observational data at Rou-Kou Yan during Typhoon
 204 Soulik for model verification.

205 Table 1 presents the Manning friction coefficient (n) adopted in the model. Table 4 presents the
 206 MAE and RMSE values at each station for the four typhoon events. The results show a reasonable
 207 agreement between the model predictions and the measured data.



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 209 **Figure 3.** Paths of the four typhoon events.

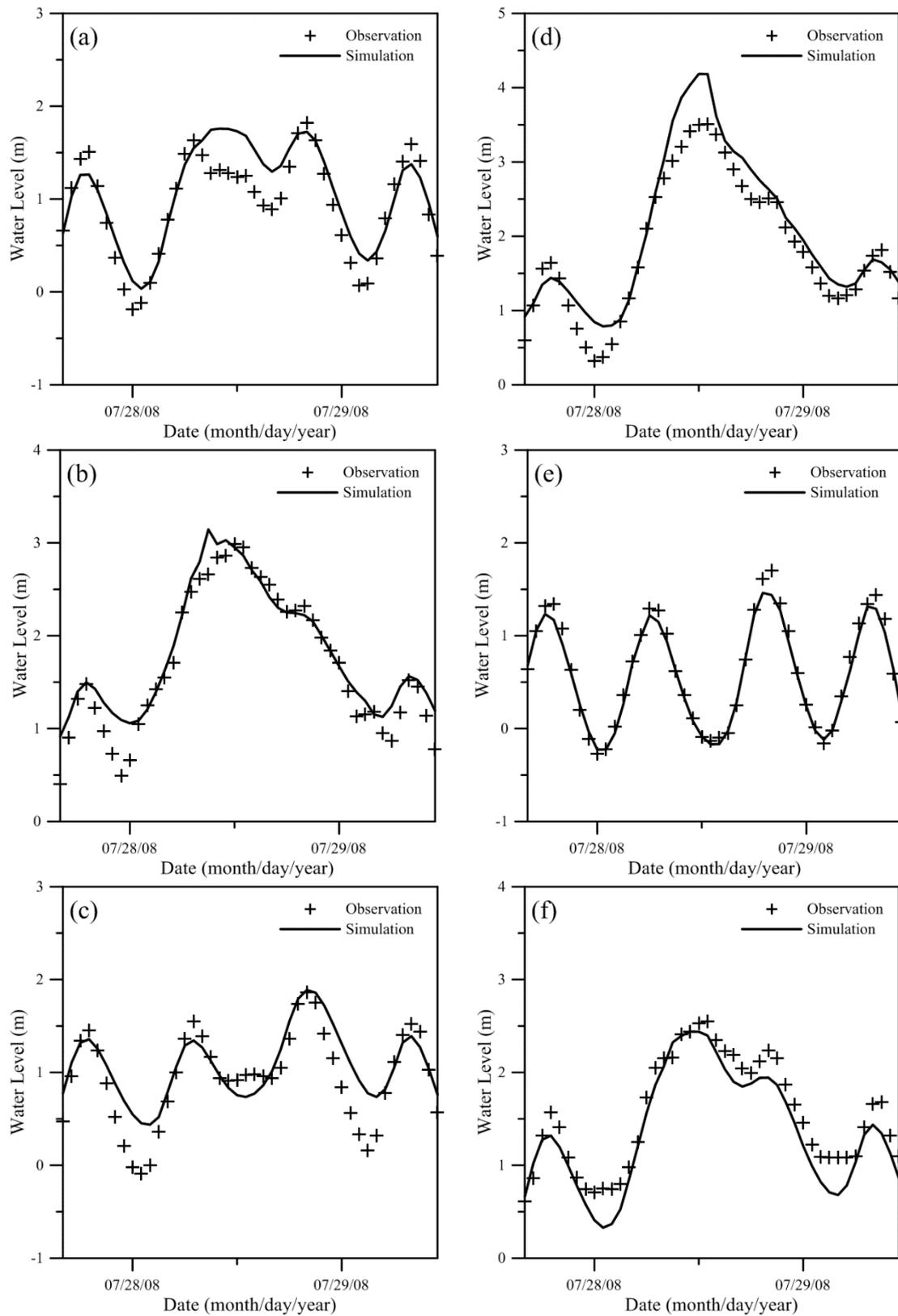


210 **Figure 4.** Model calibration results for Typhoon Krosa (2007) at (a) Taipei Bridge, (b) Hsin-Hai
 211 Bridge, (c) Da-Zhi Bridge, (d) Chung-Cheng Bridge, (e) Tu-Ti-Kung-Pi, and (f) Ru-Kou Yan.

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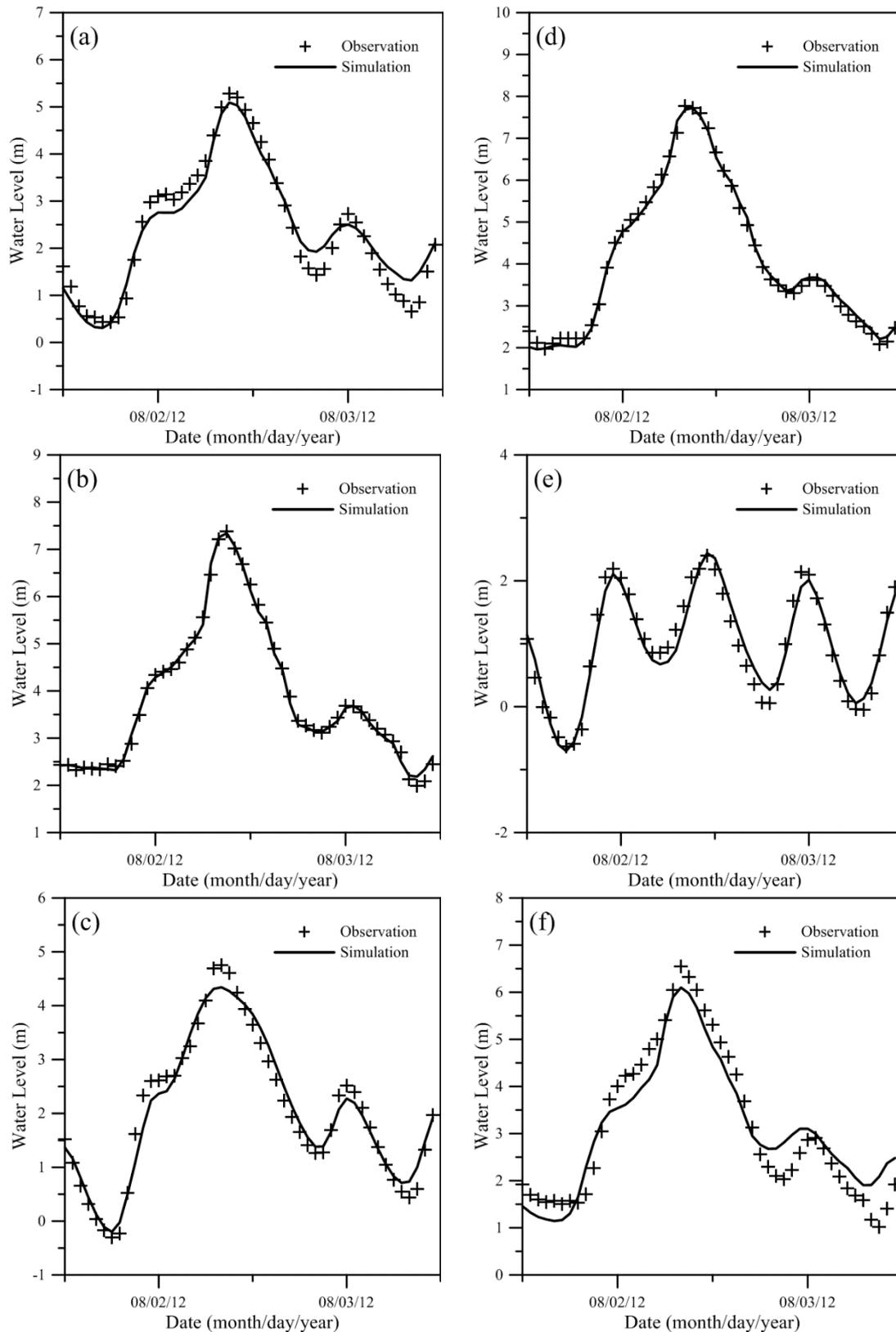
215 **Figure 5.** Model calibration results for Typhoon Fungwong (2008) at (a) Taipei Bridge, (b) Hsin-Hai
 216 Bridge, (c) Da-Zhi Bridge, (d) Chung-Cheng Bridge, (e) Tu-Ti-Kung-Pi, and (f) Ru-Kou Yan.

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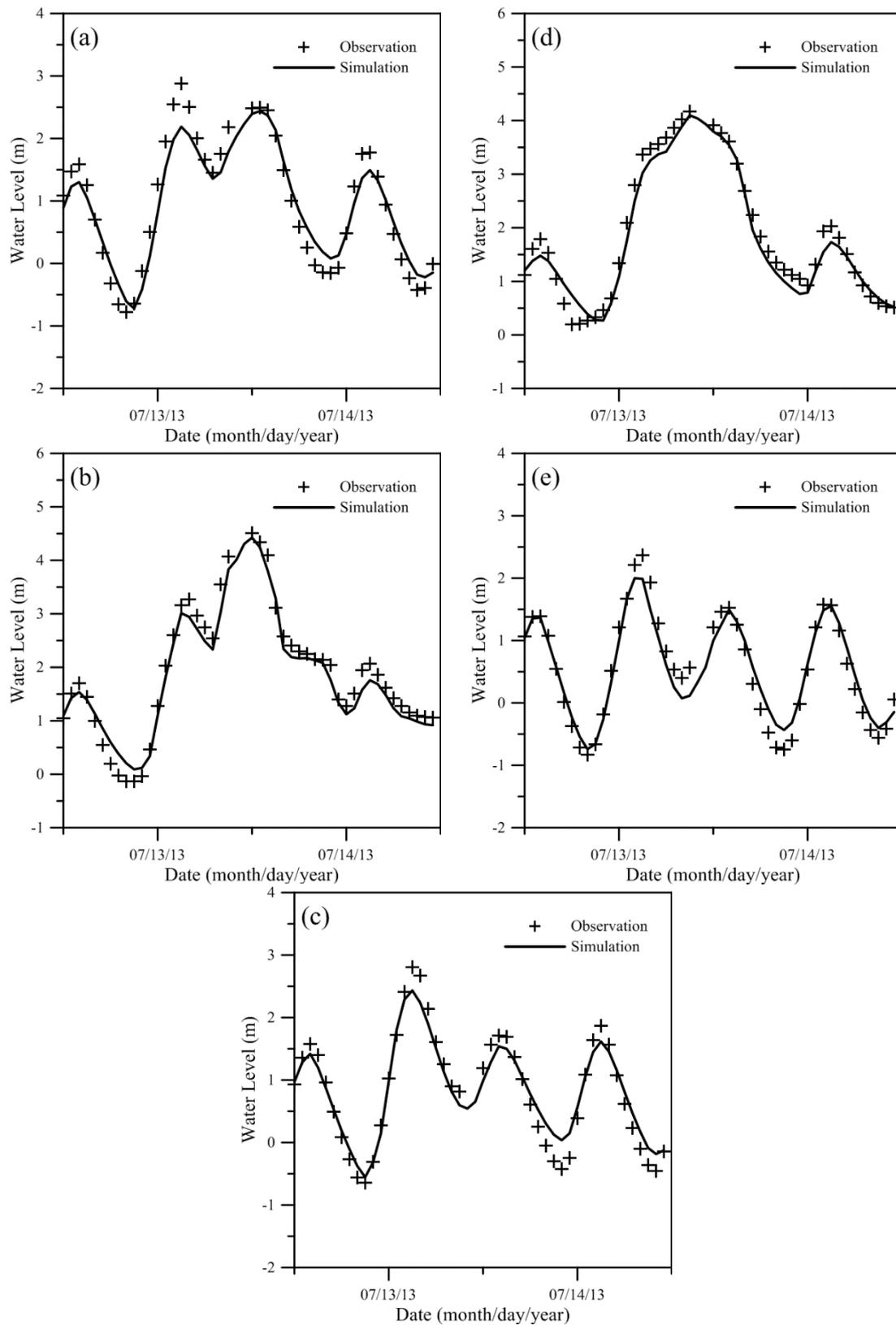
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221 **Figure 6.** Model verification results for Typhoon Saola (2012) at (a) Taipei Bridge, (b) Hsin-Hai
 222 Bridge, (c) Da-Zhi Bridge, (d) Chung-Cheng Bridge, (e) Tu-Ti-Kung-Pi, and (f) Ru-Kou Yan.

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225 **Figure 7.** Model verification results for Typhoon Soulik (2013) at (a) Taipei Bridge, (b) Hsin-Hai
 226 Bridge, (c) Da-Zhi Bridge, (d) Chung-Cheng Bridge, and (e) Tu-Ti-Kung-Pi.

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Table 4. Statistical errors of the simulations using four typhoon events.

Station	Typhoon Krosa		Typhoon Fungwong		Typhoon Saola		Typhoon Soulik	
	MAE (m)	RMSE (m)	MAE (m)	RMSE (m)	MAE (m)	RMSE (m)	MAE (m)	RMSE (m)
Taipei Bridge	0.30	0.37	0.19	0.24	0.24	0.29	0.19	0.24
Hsin-Hai Bridge	0.30	0.42	0.15	0.21	0.06	0.08	0.18	0.21
Da-Zhi Bridge	0.23	0.28	0.24	0.33	0.21	0.26	0.13	0.16
Chung-Cheng Bridge	0.22	0.31	0.22	0.29	0.11	0.14	0.17	0.21
Tu-Ti Kung Pi	0.23	0.28	0.07	0.10	0.17	0.22	0.10	0.12
Ru-Kou Yan	0.33	0.42	0.18	0.21	0.40	0.44	---	---

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Table 5. Statistical errors between the predicted and observed cumulative probability functions.

Distribution model	Surge height		Astronomical tide		River discharge at Quan-Zean Bridge		River discharge at Fu-Zhou Bridge	
	SE (m)	CC	SE (m)	CC	SE (m ³ /s)	CC	SE (m ³ /s)	CC
Gumbel	0.0346	0.9806	0.0397	0.9754	6.5460	0.9695	95.6662	0.8605
Weibull	0.0323	0.9848	0.0128	0.9975	2.6015	0.9953	71.7064	0.9354
Frechet	0.1339	0.8880	0.2388	0.8410	10.2649	0.9617	96.7808	0.9557
Normal	0.0663	0.9258	0.0138	0.9971	11.2177	0.9041	122.8602	0.7713
Lognormal	0.0262	0.9901	0.0132	0.9974	7.1395	0.9795	61.7958	0.9578
Log-Pearson 3	0.0268	0.9889	0.0129	0.9975	2.7390	0.9952	68.8232	0.9532

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Table 5. Statistical errors between the predicted and observed cumulative probability functions.

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(Continue)

Distribution model	River discharge at Xiu-Lang Bridge		River discharge at Bao Bridge		River discharge at Jie-Shou Bridge	
	SE (m ³ /s)	CC	SE (m ³ /s)	CC	SE (m ³ /s)	CC
Gumbel	325.3179	0.9761	69.6968	0.9744	109.3666	0.9695
Weibull	321.7177	0.9849	49.0460	0.9895	43.4613	0.9953
Frechet	455.4074	0.9613	945.5690	0.8449	171.4874	0.9617
Normal	548.7033	0.9221	117.6550	0.9195	187.4202	0.9041
Lognormal	403.8747	0.9718	143.3619	0.9739	119.2626	0.9795
Log-Pearson 3	200.0291	0.9911	49.0987	0.9866	45.7697	0.9952

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4. Results and Discussion

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4.1. Selecting the Best Distribution Function for the Statistical Parameters

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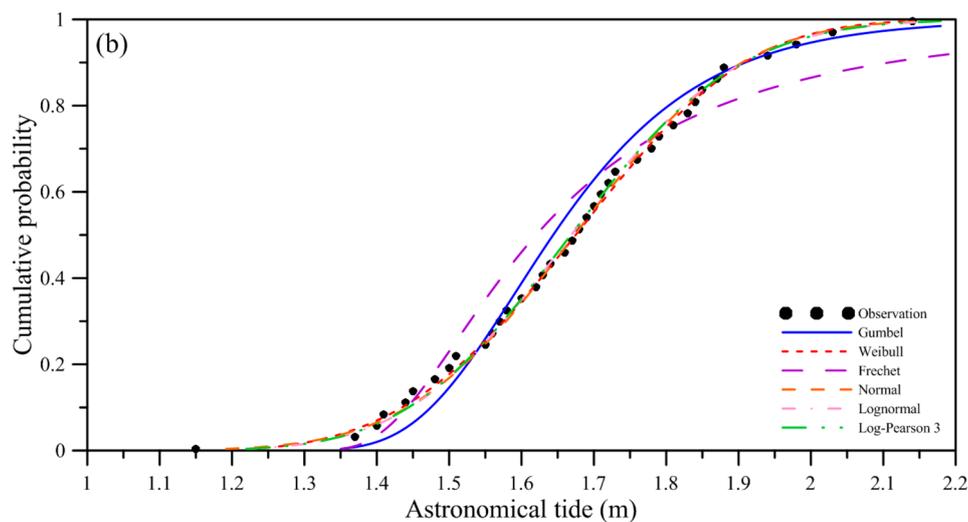
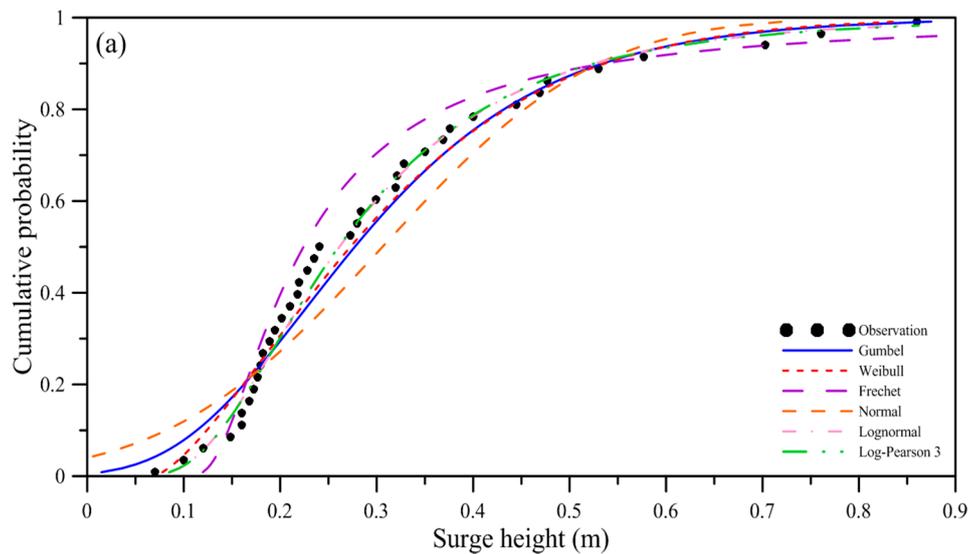
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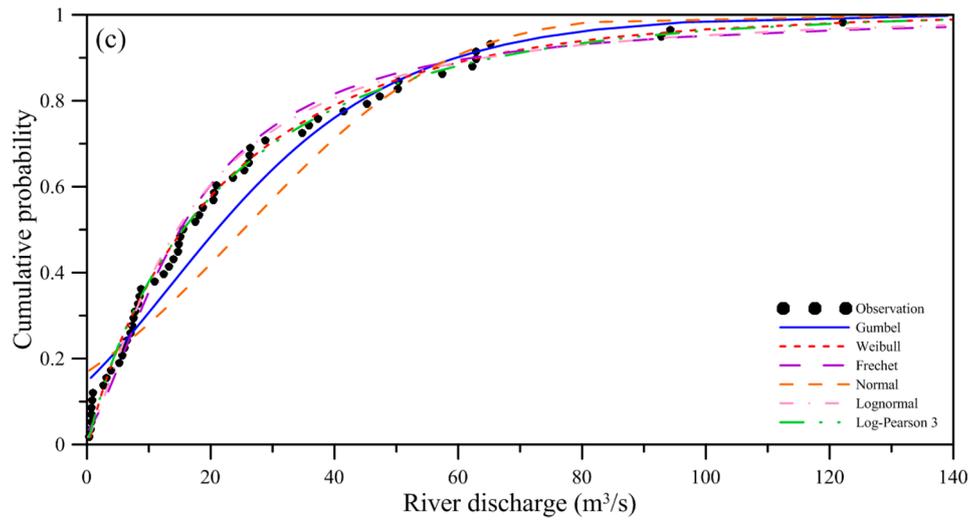
Seven parameters, including the astronomical tide and surge height at the mouth of the Danshuei River and the river discharges at the Quan-Zean Bridge, Fu-Zhou Bridge, Xiu-Lang Bridge, Bao Bridge, and Jie-Shou Bridge, are selected for the frequency analysis. The annual maximum for each parameter was adopted to establish the probability density function. The cumulative probability function of each parameter was then determined.

Figure 8 presents the predicted and observed cumulative probability function for each parameter. It shows the predicted cumulative probabilities using different frequency analysis methods, including the Gumbel, Weibull, Frechet, normal, lognormal, and log-Pearson 3

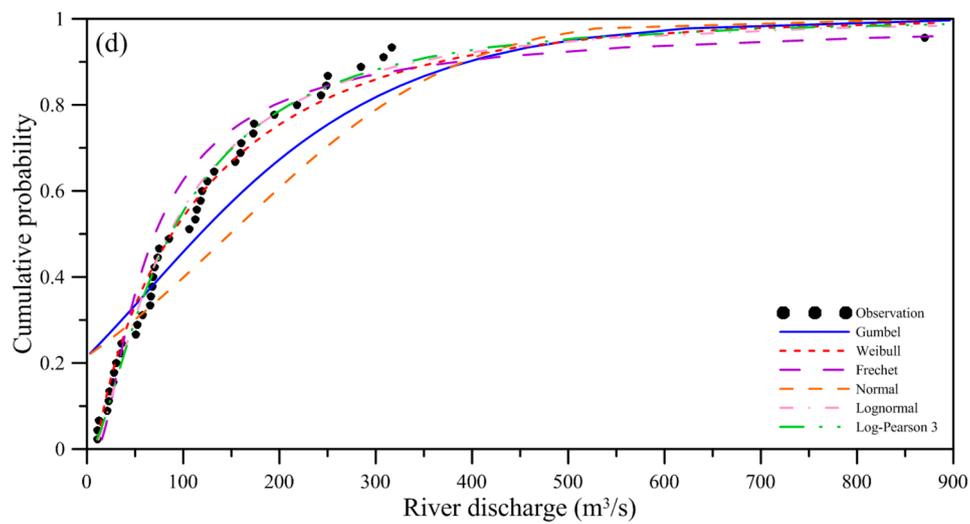
244 distributions (shown in Table 3). Two indices (i.e., the standard error (SE) and the correlation
 245 coefficient (CC)) were used to determine the optimal frequency analysis for each parameter. Table 5
 246 shows statistics for the comparison between the predicted and observed cumulative probabilities
 247 for different frequency analysis methods. We found that the optimal frequency analysis methods
 248 were the lognormal distribution for surge height, the Weibull distribution for astronomical tide, the
 249 Weibull distribution for river discharge at the Quan-Zean Bridge (Taliaokengchi drainage), the
 250 lognormal distribution for river discharge at the Fu-Zhou Bridge (Dahan River), the log-Pearson 3
 251 distribution for river discharge at the Xiu-Long Bridge (Xindian River), the Weibull distribution for
 252 river discharge at Bao Bridge (Jingme River), and the Weibull distribution for river discharge at the
 253 Jie-Shou Bridge (Keelung River).

254 Zhong et al. [6] found that the characteristics of high astronomical tide levels at the Hook of
 255 Holland could be captured in a normal distribution. The Gaussian copula function presented a
 256 dependent structure between the Rhine discharge and the Meuse discharge, where the marginal
 257 distributions fit a lognormal distribution for Rhine discharge and a Gamma distribution for Meuse
 258 discharge. The best fit for the frequency analysis method is based on the characteristics of each
 259 parameter. Therefore, the distribution models for the frequency analysis of astronomical tide and
 260 river discharge used in Zhong et al. [6] are quite different from the distribution models adopted in
 261 our study.

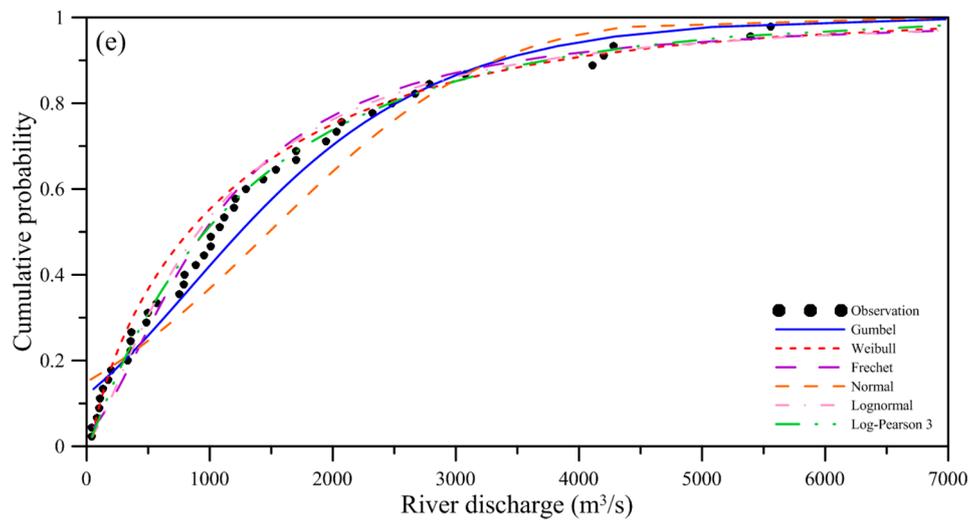




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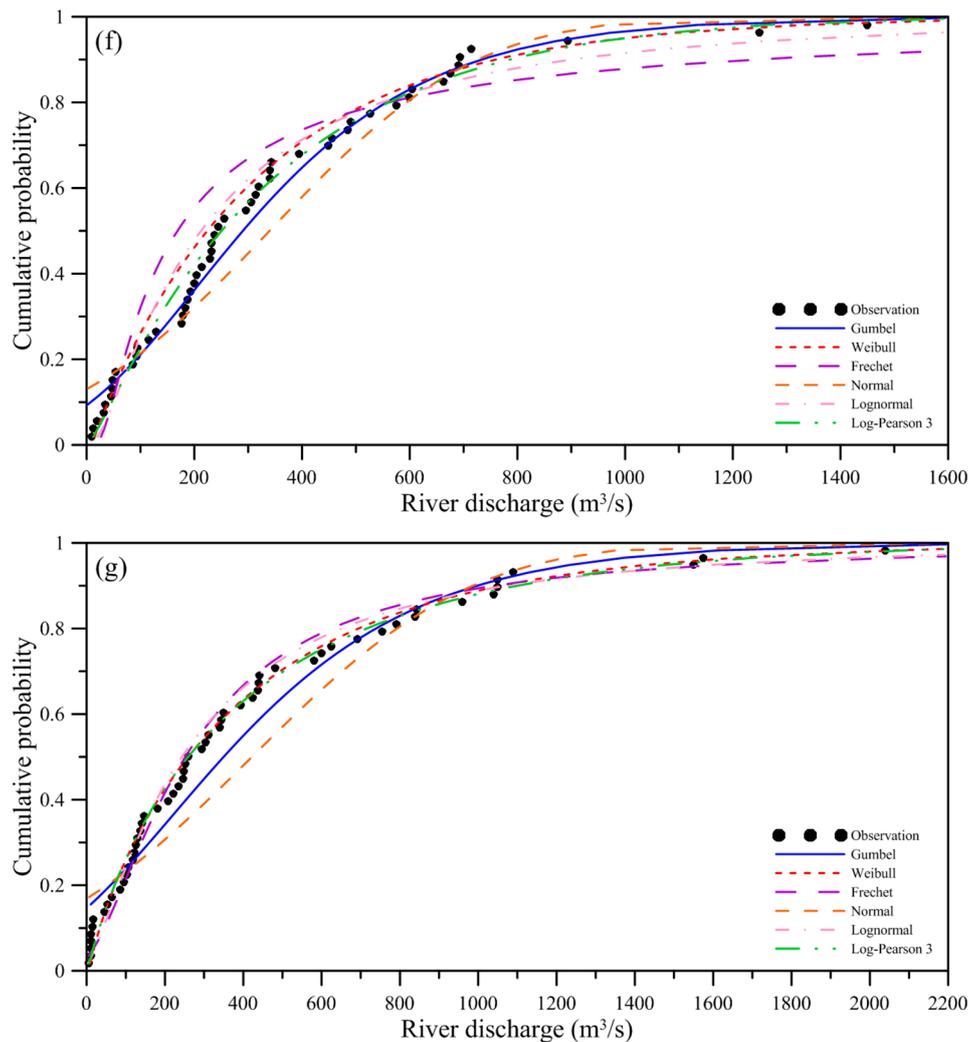


Figure 8. Predicted and observed cumulative probability functions for each of the following parameters: (a) surge height at the mouth of the Danshuei River, (b) astronomical tide at the mouth of the Danshuei River, (c) river discharge at Quan-Zean Bridge (Taliaokengchi drainage), (d) river discharge at Fu-Zhou Bridge (Dahan River), (e) river discharge at Xiu-Lang Bridge (Xindian River), (f) river discharge at Bao Bridge (Jingme River), and (g) river discharge at Jie-Shou Bridge (Keelung River).

4.2. Integrating the Monte Carlo Simulation and the Hydrodynamic Model to Calculate Extreme Water Levels

The joint probability of these seven parameters was used to generate different simulation scenarios. Two thousand scenarios based on the sampled Monte Carlo simulation were generated to drive the validated one-dimensional hydrodynamic model. This means that the Manning friction coefficients (n) in the one-dimensional hydrodynamic model were adopted as shown in Table 1.

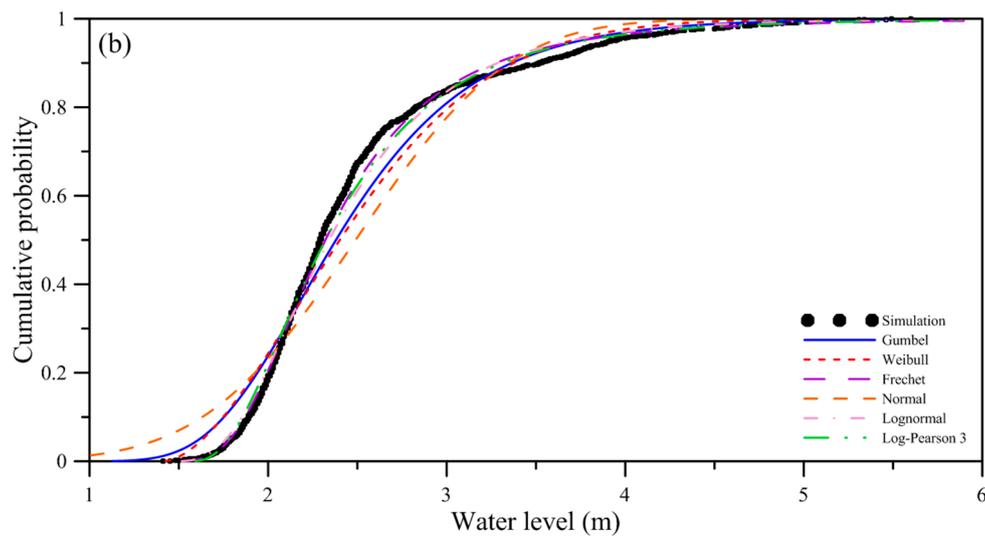
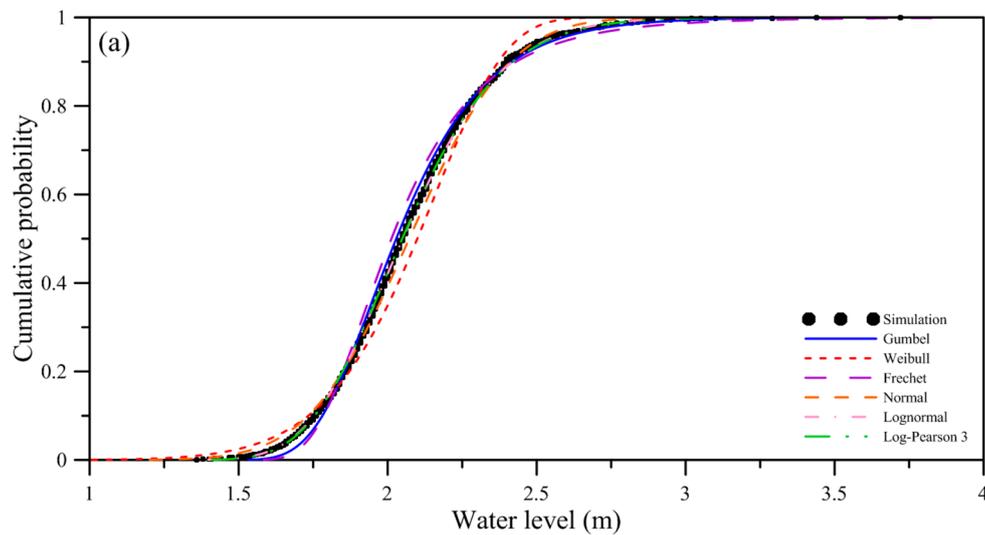
Figure 9 presents the cumulative probability function of the water level for different distribution methods at different gauge stations. Gauge stations in the Danshuei River include Guandu Bridge, Taipei Bridge, Hsin-Hai Bridge, Da-Zhi Bridge, and Chung-Cheng Bridge. Table 6 shows statistics for the comparison between the predicted and observed cumulative probabilities at different stations for different frequency analysis methods. The results indicate that the optimal frequency analysis methods are the log-Pearson 3 distribution at Guandu Bridge, the lognormal distribution at Taipei Bridge, the lognormal distribution at Hsin-Hai Bridge, the lognormal distribution at Da-Zhi Bridge, and the Weibull distribution at Chung-Cheng Bridge.

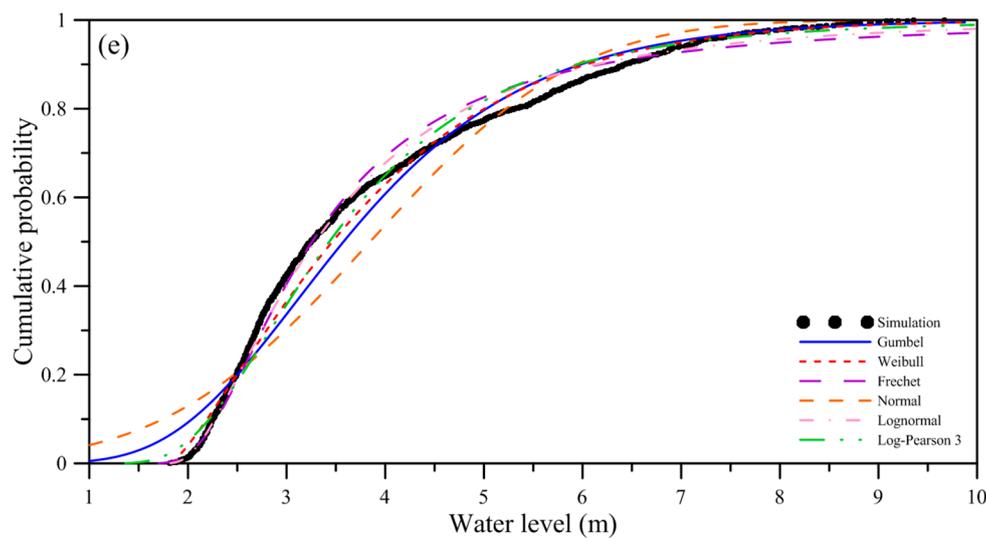
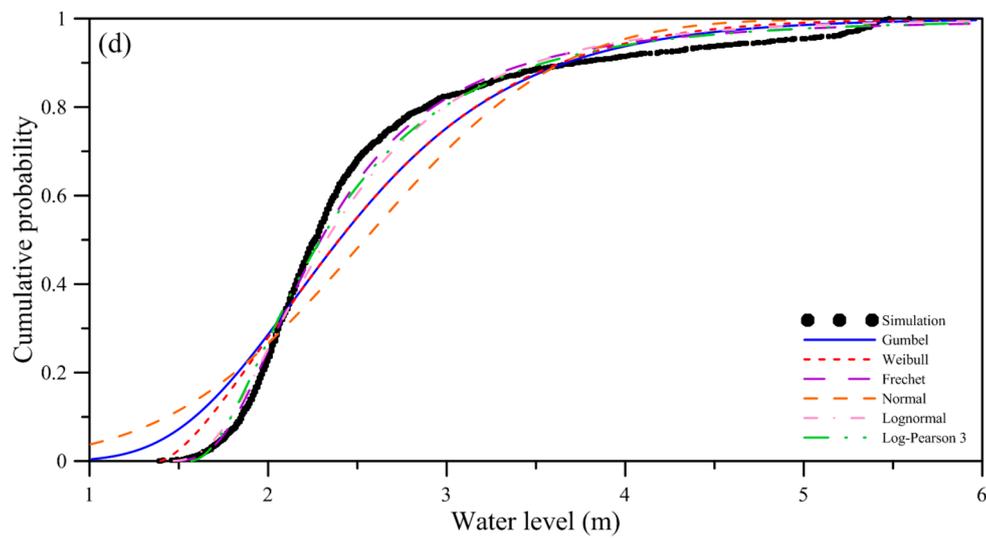
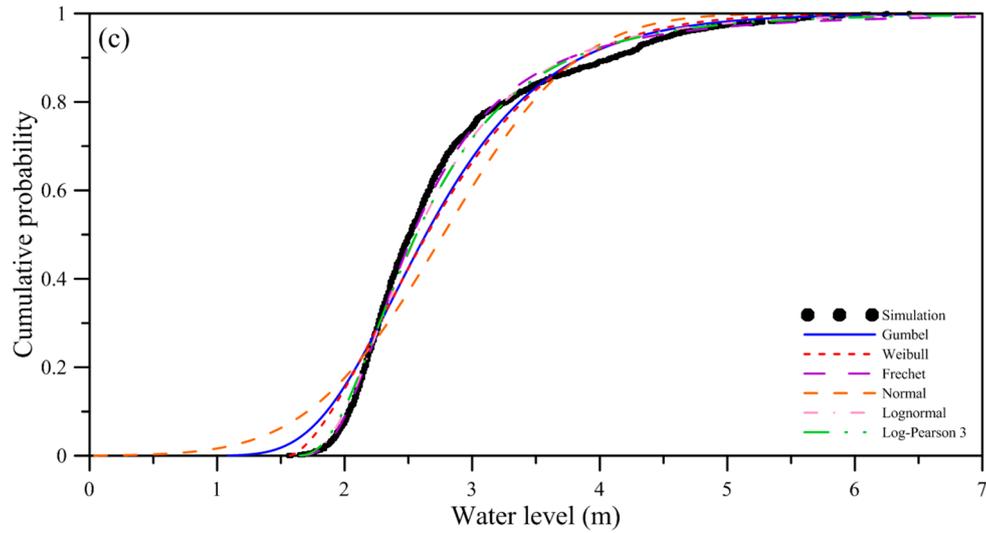
Sindhu and Unnikrishnan [8] estimated the different return periods of extreme events at 26 stations along the east coast of India based on annual maximum sea levels extracted from simulations using a vertically integrated two-dimensional model. The annual maximum sea level fit

290 the Gumbel distribution using the r -largest annual maxima method. In our study, we found that the
 291 Gumbel distribution was not the best fit for water levels at the five stations.

292 **Table 6.** Statistical errors between the predicted and observed cumulative probability functions at
 293 different stations.

Distribution model	Guandu Bridge		Taipei Bridge		Hsin-Hai Bridge		Da-Zhi Bridge		Chung-Cheng Bridge	
	SE (m)	CC	SE (m)	CC	SE (m)	CC	SE (m)	CC	SE (m)	CC
Gumbel	0.0293	0.9939	0.1348	0.9796	0.1704	0.9789	0.2715	0.9504	0.3266	0.9800
Weibull	0.0757	0.9583	0.1650	0.9694	0.1757	0.9783	0.2616	0.9542	0.2355	0.9897
Frechet	0.0590	0.9800	0.1431	0.9778	0.3017	0.9494	0.3565	0.9209	1.7252	0.8140
Normal	0.0416	0.9876	0.2771	0.9142	0.3428	0.9148	0.4296	0.8763	0.5832	0.9365
Lognormal	0.0200	0.9972	0.1061	0.9888	0.1432	0.9852	0.2404	0.9627	0.7784	0.9318
Log-Pearson 3	0.0182	0.9976	0.1105	0.9864	0.1603	0.9817	0.2781	0.9494	0.4557	0.9645





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Figure 9. Predicted and observed cumulative probability functions of the water level at (a) Guandu Bridge, (b) Taipei Bridge, (c) Hsin-Hai Bridge, (d) Da-Zhi Bridge, and (e) Chung-Cheng Bridge.

302 4.3. Potential Water Levels for Different Return Periods

303 The design (extreme) water level is crucial for the design, construction, and maintenance of
 304 flood defense systems. According to the Taipei flood control system, the design water level in the
 305 Danshuei River system is regarded as the water level with an exceedance frequency of 1/200 (i.e., a
 306 200-year return period). Based on the cumulative probability function of water levels from the
 307 frequency analysis, the design water level for different return periods can be yielded. The equation
 308 for calculating the water level that corresponds to each return period can be given as follows:

$$T = \frac{1}{1 - F(x)} \quad (7)$$

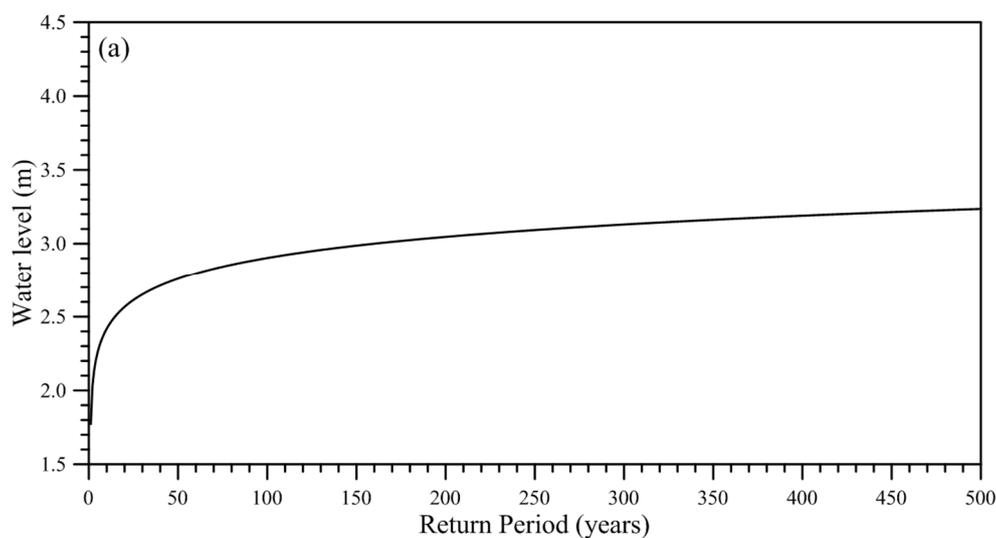
309 where T represents the return period, and $F(x)$ is the cumulative probability function.

310 The design water level can be estimated using different probability distributions from the
 311 frequency analysis for different return periods at the five stations based on Eq. (7). Figure 10 presents
 312 the predicted water level for the different return periods at five stations. Table 7 shows the design
 313 water levels for the different return periods (50, 100, and 200 years). The table indicates that the
 314 design water levels for the 200-year return period at Guandu Bridge, Taipei Bridge, Hsin-Hai Bridge,
 315 Da-Zhi Bridge, and Chung-Cheng Bridge are 2.90 m, 5.13 m, 6.38 m, 6.05 m, and 9.94 m, respectively.
 316 It should be noted that the design water level should be added to the 1.5 m freeboard to result in a
 317 projected water level [40].

318 To avoid extremely high water-levels produced from these extreme events, the construction of
 319 new facilities in rivers needs exploration, and the present operational water management system
 320 requires adaptation in the future.

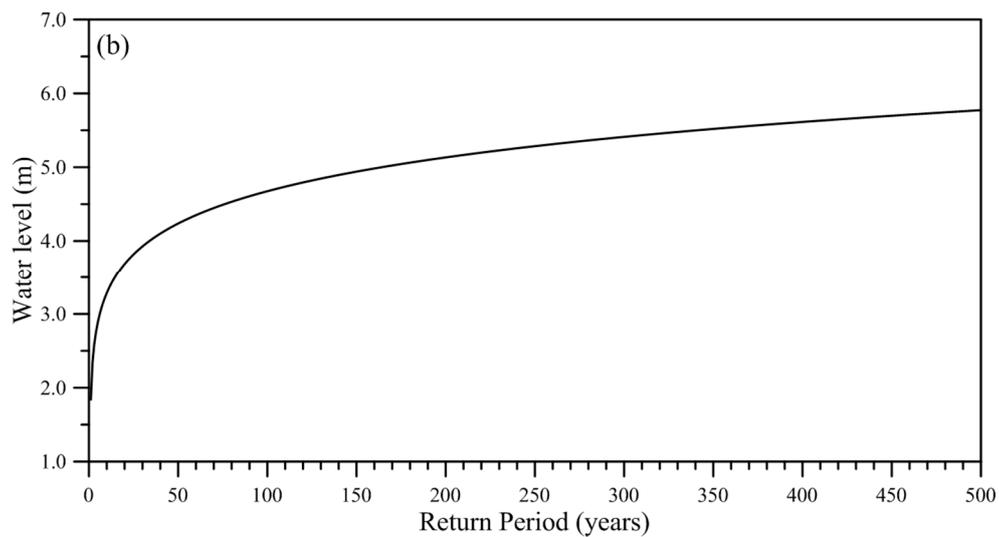
321 **Table 7.** Design water levels for different return periods.

Return Period	Guandu Bridge (m)	Taipei Bridge (m)	Hsin-Hai Bridge (m)	Da-Zhi Bridge (m)	Chung-Cheng Bridge (m)
50	2.69	4.24	5.08	4.77	8.22
100	2.80	4.68	5.71	5.39	9.09
200	2.90	5.13	6.38	6.05	9.94

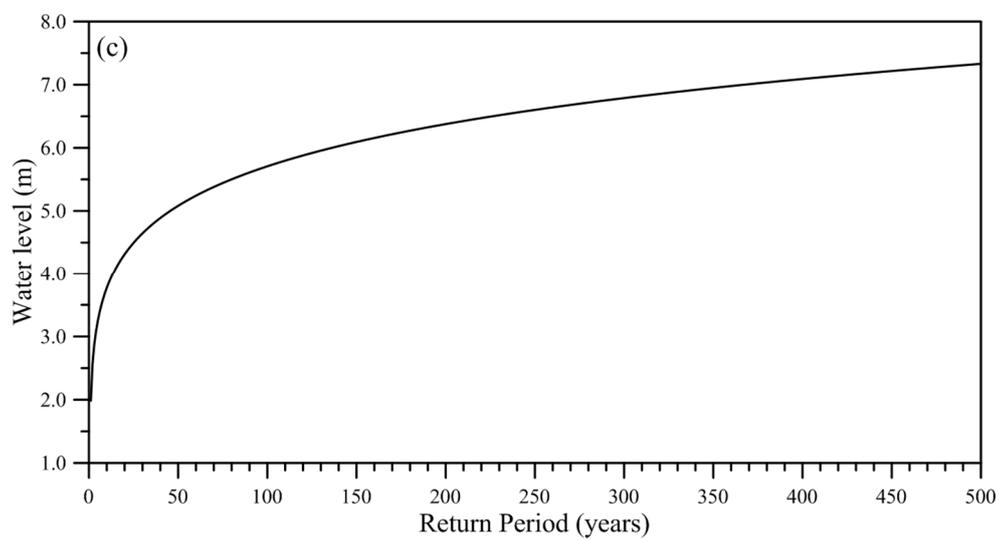


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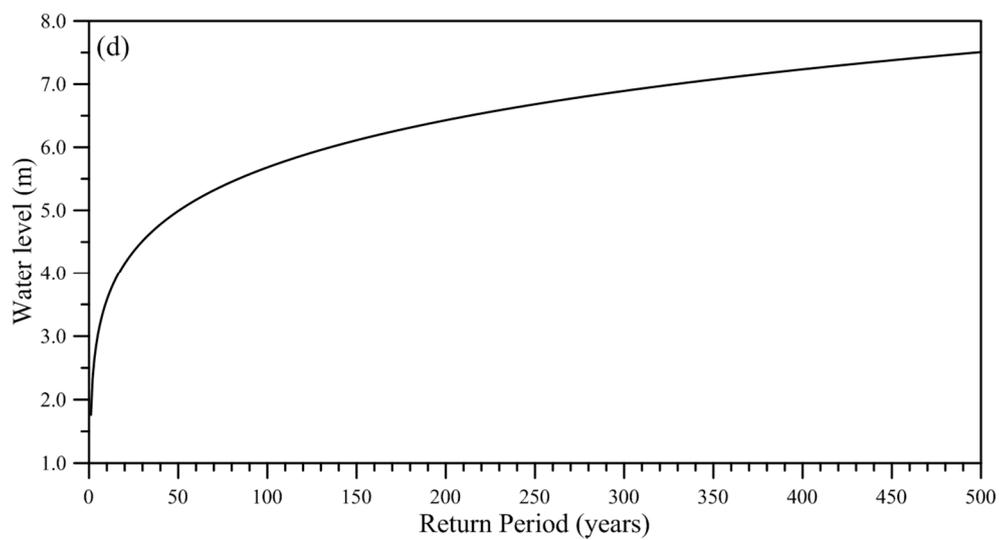
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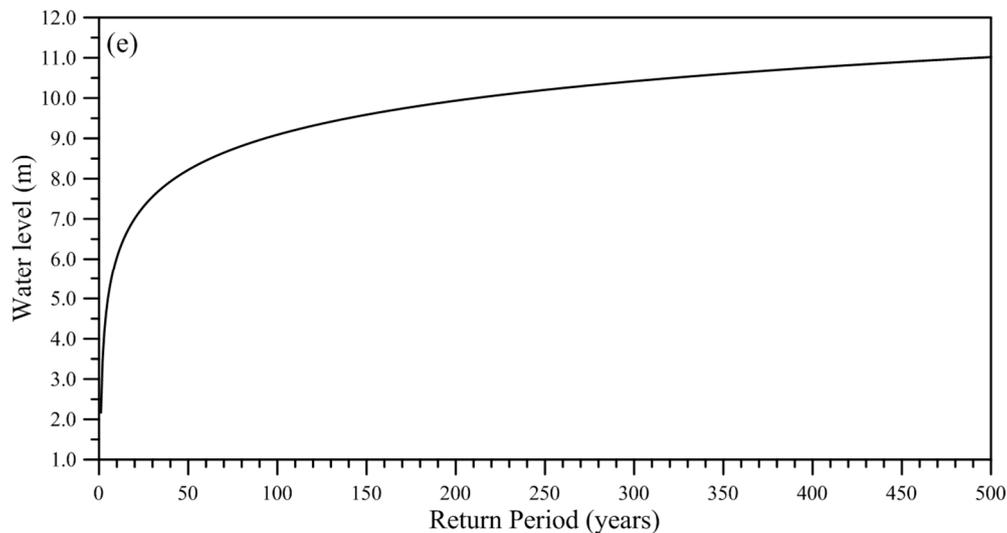
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Figure 10. Predicted water levels for different return periods at (a) Guandu Bridge, (b) Taipei Bridge, (c) Hsin-Hai Bridge, (d) Da-Zhi Bridge, and (e) Chung-Cheng Bridge.

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5. Conclusions

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Recorded historical water levels in the Danshuei River estuarine system of northern Taiwan are not sufficient to perform a traditional frequency analysis directly when predicting extreme water levels. The numerical simulation approach can be adopted when there is not an adequate amount of recorded historical data. This study presents extreme water levels simulated by a stochastic model, which is the integration of the Monte Carlo simulation and a one-dimensional flash flood routing hydrodynamic model. The hydrodynamic model was calibrated and verified with observational water levels using data from four typhoon events. The results indicated a reasonable agreement between the model predictions and recorded observations.

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Seven parameters, including the astronomical tide and surge height at the mouth of the Danshuei River and river discharges at Quan-Zean Bridge, Fu-Zhou Bridge, Xiu-Lang Bridge, Bao Bridge, and Jie-Shou Bridge, were selected for the frequency analysis. The joint probability of these seven parameters was used to produce a large set of stochastic scenarios, which were generated by via importance sampling of the Monte Carol simulation. Two thousand scenarios were generated to drive the validated hydrodynamic model simulations. The design water level was estimated using different probability distributions from the frequency analysis for different return periods at the five stations. We found that the design water levels for a 200-year return period at Guandu Bridge, Taipei Bridge, Hsin-Hai Bridge, Da-Zhi Bridge, and Chung-Cheng Bridge were 2.90 m, 5.13 m, 6.38 m, 6.05 m, and 9.94 m, respectively. The estimated design water levels plus the freeboard should be proposed and recommended for engineering design and planning.

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In future research, different statistical uncertainties must be investigated. One other issue is that the probability of failure for dikes in river systems should be further considered to understand flood risks.

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Conflicts of Interest: The authors declare no conflict of interest.

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