

1 Article

2 Transducer Placement Option of Lamb Wave SHM 3 System for Hotspot Damage Monitoring

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12 **Abstract:** In this paper, we investigated transducer placement strategies for detecting crack in
13 primary aircraft structures using ultrasonic Structural Health Monitoring (SHM). The approach
14 developed is for an expected damage location based on fracture mechanics, for example fatigue
15 crack growth in a high stress location.

16 To assess the performance of the developed approach, finite-element (FE) modelling of a
17 damage tolerant aluminum fuselage has been performed by introducing an artificial crack at a rivet
18 hole into the structural FE model and assessing its influence on the Lamb wave propagation,
19 compared to a baseline measurement simulation. The efficient practical sensor position was
20 determined from the largest change in area that is covered by reflected and missing wave scatter
21 using an additive color model. Blob detection algorithms were employed in order to determine the
22 boundaries of this area and to calculate the blob centroid. To demonstrate that the technique can be
23 generalized, the results from different crack lengths and from tilted crack are also presented.

24 **Keywords:** sensor placement option; hotspot damage, lamb wave, Structural Health Monitoring
25 (SHM), Finite Element Modelling, image processing, additive color model

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28 1. Introduction

29 Non-Destructive Testing (NDT) [1] has been implemented in many industries to ensure
30 structural safety and reliability. Structural Health Monitoring (SHM) as a complementary solution
31 with respect to the already existing NDT techniques has been a subject of interest in the last decade
32 due to its potential economic benefit, particularly in structural maintenance [2, 3].

33 Along with fiber-optic techniques such as Fiber-Bragg-Grating sensors [4, 5], an ultrasonic
34 guided waves-based solution such as those making use of Lamb waves is one of the promising
35 techniques for SHM due to its long-range inspection capability up to several meters [6, 7], which is
36 suitable for monitoring large structures like an aircraft fuselage. Also, in comparison to the FBG
37 technique, Lamb wave SHM sensors typically require less complicated installations. This is
38 particularly useful for monitoring aircraft which are already in-service, since aircraft operators
39 generally tend to be reluctant to make larger modifications.

40 Lamb wave damage monitoring relies on the detection of interactions between the Lamb wave
41 and the damage [8]. Various properties of propagating wave are modified by the crack and various
42 properties of the signal such as amplitude, frequency, phase, etc. are assessed to calculate the
43 damage index (DI). The most commonly used sensors for generating and capturing Lamb waves are
44 piezoelectric transducers (PZT) [9]. Since PZTs are permanently attached to the surface of the
45 structure, sensor placement becomes tremendously important because the quantification of DI relies

46 heavily on the sensor positions. A poor transducer placement will result in weak or undetected
47 capture of wave scatter which in turn decreases the damage detectability of the SHM system.

48 The literature describes different approaches for sensor placement options, such as prioritizing
49 the sensor location based on a detectability limit [10], using a modal analysis parameter
50 determination for damage localization assessment on a truss structure [11], and by using global
51 search and greedy algorithms [12]. Li et al. [13] proposed a sensor optimization algorithm based on
52 maximum energy consumption on sensor candidate location of a civil engineering structure.

53 Fendzi et al. [14] proposed a novel approach for sensor placement by using geometric dilution
54 of precision (GDOP), which is based on Lamb wave ray tracing method for known damage locations.
55 Haynes [15] proposed sensor placement by minimizing the Bayesian cost to select locally optimal
56 sensor locations. However, if the damage occurs outside of the designated area, the system might
57 fail to detect it. A similar approach by using Bayesian experimental design has been conducted by
58 Flynn and Todd [16].

59 Furthermore, Sun et al. [17] performed discrete optimization by using the artificial bee colony
60 algorithm in order to optimize their objective function which is based on modal assurance criterion
61 (MAC). Similar approaches by using search metaheuristics were also delivered by Yi et al. [18], Zhao
62 et al. [19], and Shan et al. [20]. In more recent study, Capellari et al. [21] proposed an optimal sensor
63 placement by employing Polynomial Chaos Expansion and stochastic optimization to maximize the
64 gain in Shannon information.

65 Thiene et al. [22] introduced the DI-free sensor placement optimization based on the fitness
66 function which maximizes the coverage area of the sensor network. They calculated the coverage of
67 each pixel in the geometry based on pitch-catch technique, so that every pixel that contributes to the
68 probability that a damage in random location is being detected is counted. Venkat et al. [23] used a
69 Finite Element (FE) simulation platform to build differential images between the undamaged and
70 damaged structure. They plotted the summed-up energy captured by all sensors and determined the
71 most optimal sensor location by the highest captured energy.

72 From all these studies, we concluded that the majority of the sensor placement strategies are for
73 civil engineering structures and that one of the most common features used for the objective function
74 is the MAC, which is typically applicable for low-frequency vibrations. However, we have not seen
75 any application using MAC in Lamb wave SHM frequency, which is normally above 100 kHz. Only
76 [14 – 16] and [22 – 23] are related to sensor placement for guided wave SHM.

77 Summarizing references [14 – 16] and [22 – 23], it can be seen that there are two streams sensor
78 placement research that are specialized for guided wave based SHM: 1). The approach exhibiting
79 known area of damage location, and 2). The random damage location approach. If a damage location
80 can already be approximated by analytical methods from fracture mechanics, FE simulation and
81 fatigue testing, then the optimum sensor location can be determined by maximizing the DI of the
82 sensor network around the predicted damage location. However, for random location damage such
83 as hail impact, a different objective is needed.

84 From this current state of the art, our work is performed to develop the work presented in two
85 most recent articles [22 – 23]. For this paper, the objective is to propose an image processing for
86 known damage location approach, known as hotspot SHM [22].

87 Due to the expected number of different damage locations occurring in an aircraft fuselage
88 multiplied by the numerous possibilities for sensor placement, conducting an experiment or even a
89 numerical simulation to track the wave scatter is not practical. To address this issue, we propose a
90 design for a sensor placement in a hotspot SHM system based on an additive color model and blob
91 detection algorithm that can detect residual wave scatter from simulation data.

92 After the introduction in section 1, we have structured the paper as follows: In section 2, we
93 briefly review the Lamb wave propagation theory whose effect is employed in section 3 for
94 determining the SHM sensor positions by using image processing for predictable critical crack size,
95 location, and orientation. The results and concluding remark are presented in sections 4 and 5,
96 respectively.

97 2. Lamb Wave Propagation

98 First described by English mathematician Horace Lamb in 1917 [24], the description of acoustic
 99 wave propagation in solid plates has been evolving ever since. A theoretical analysis of Lamb wave
 100 propagation in metallic materials, composite, and hybrid materials is described in [25 – 27]. The
 101 elastodynamic wave equation for an anisotropic inhomogeneous medium in a d -dimensional
 102 bounded domain $\Omega \subset \mathbb{R}^d$ ($d = 2,3$) is given in by:
 103

$$\rho \frac{\partial^2 u(x,t)}{\partial t^2} - \nabla \cdot \left(\sum_{k=1}^d \sum_{l=1}^d C_{ijkl} \varepsilon_{kl} \right) = f(x,t) \quad (1)$$

104 where $u(x,t)$ is the time and space dependent displacement, ρ the material density, C_{ijkl} the material
 105 stiffness tensor, ε_{kl} the strain tensor, and $f(x,t)$ the source function, respectively. After applying
 106 boundary conditions of two parallel surfaces, there are two solutions to Eq. (1) for wave propagation
 107 in homogeneous material of given density, described by:
 108
 109

$$\frac{\tan(qh)}{\tan(ph)} = \frac{-4k^2 \cdot (pq)}{(q^2 - k^2)^2} \quad (2a)$$

$$\frac{\tan(qh)}{\tan(ph)} = \frac{-(q^2 - k^2)^2}{4k^2 \cdot (pq)} \quad (2b)$$

110 which are well-known as symmetrical (S-Mode) and anti-symmetrical (A-Mode) Lamb wave
 111 propagation modes, respectively. The definitions of p and q are given by:
 112
 113

$$p^2 = \left(\frac{\omega^2}{c_L^2} - k^2 \right) \quad (3a)$$

$$q^2 = \left(\frac{\omega^2}{c_T^2} - k^2 \right) \quad (3b)$$

114 where ω is the frequency, c_L is the longitudinal bulk wave velocity, c_T is the transversal bulk wave
 115 velocity, and k is the wavenumber. The numerical solution of Eq. (2a) and (2b) can be drawn as a
 116 dispersion curve, which describes the wave velocity as a function of frequency-thickness product
 117 [28].
 118

119 As they introduce potentially overlapping signals, higher-order Lamb modes are generally
 120 undesirable, and the maximum cutoff frequency is normally determined as before the A1 mode
 121 appears.
 122

123 3. Sensor Positioning Approach for Hotspot SHM

124 This section describes the developed methodology for sensor placement in hotspot SHM. It
 125 considers: 1). Crack growth and critical crack size in a damage tolerant aircraft substructure, 2).
 126 Numerical simulation of Lamb wave propagation in a plate-like structure, and 3). Image processing
 127 that involves displacement subtraction of damaged from undamaged structures.
 128

129 3.1. Crack Growth in Damage Tolerance Structure

130 To understand deterministic sensor positioning for a predictable crack location, firstly, it is
 131 important to understand the concept of damage tolerant design. The definition of damage tolerance
 132 is “the ability of the structure to sustain design limit loads in the presence of damage caused by fatigue,
 133 corrosion and other sources until such damage is detected and repaired” [29]. The important key elements in
 134 damage tolerance design are: 1). the assumption of initial damage existence, 2). damage growth in
 135 the material due to structural loading, and 3). The critical damage size up to which the structure
 136 endures the loading before catastrophic failure. These three key elements are synchronous to the
 137 regions I, II, and III in a typical da/dN curve [30], which describes crack propagation rate as a
 138 function of stress intensity factor (SIF) range during fatigue cycling (ΔK).

139 The assumption of initial crack existence falls in region I, where the crack growth is typically
 140 slow. This region can be covered by some advanced (but also large and expensive) NDT and
 141 material characterization techniques such as X-Ray computer tomography [31] or scanning electron
 142 microscopy (SEM) [32]. After passing the threshold stress intensity factor range ΔK_{th} , region II begins
 143 where the crack propagation rate is stable, and crack growth normally follows the Paris-Erdogan
 144 law [30, 33], which has Paris-Erdogan constants C and m . In this region, the crack becomes much
 145 larger and sometimes can be seen with naked eye [34]. After transition between region II and III, the
 146 crack propagation rate becomes rapid and unstable until final failure.

147 Lamb waves can interact with a crack which has a size at least the half of its wavelength [35, 36],
 148 and since the critical crack tolerant size (which can be up to several hundred millimeters [22]) is
 149 generally larger than the wavelength (which is typically less than 100 mm), it is safe to assume that
 150 Lamb waves can also interact with the critical crack. Fracture mechanics and fatigue analysis can
 151 predict the most probable crack location for certain loading conditions [37].

152 The SIF is generally larger in the area around the notch, thus one can expect that a crack will
 153 occur around a notch. For example, the changes in cabin pressure in an aircraft fuselage can be
 154 approximated by the internal pressure in a thin-walled cylinder where the hoop stress is two times
 155 larger than the axial stress [38]. Therefore, the crack orientation is expected to be orthogonal to the
 156 direction of the hoop stress. By knowing the most probable crack orientation and location and the
 157 critical crack size for a certain geometry, two FE simulations scenarios of wave propagation can be
 158 performed [39]: 1). Lamb wave propagation in an undamaged structure as a baseline and 2). Lamb
 159 wave propagation in a critically damaged structure.

160 3.2. Simulation of Lamb Wave Propagation

161 The FE formulation for Lamb wave propagation is based on Hamilton’s principle [40] as
 162 described in:

$$0 = \left(- \int_{\Phi} \left[\rho \cdot \delta \underline{u}^T \cdot \ddot{\underline{u}} + \delta \underline{\varepsilon}^T \cdot \underline{C}_{ijkl} \cdot \underline{\underline{\varepsilon}} \right] d\Phi \right) + \left(\int_{\Phi} \left(\delta \underline{u}^T \cdot \underline{F}_V \right) d\Phi + \int_{\Gamma} \left(\delta \underline{u}^T \cdot \underline{F}_S \right) \right) \quad (4)$$

164 where Γ and Φ are the surficial and volumetric integral areas, ρ is the material density, \underline{u} and $\ddot{\underline{u}}$ are
 165 the particle displacement vectors in the material and their corresponding accelerations, respectively,
 166 $\underline{\varepsilon}$ is the strain tensor and \underline{C}_{ijkl} is the stiffness matrix. The external forces can be classified as surface
 167 load \underline{F}_S and volume load \underline{F}_V . To numerically solve Eq. (4), the geometry involved is divided into
 168 discrete mesh elements over which the equation can be approximated [41]. For a 3D-problem, there
 169 are four element types: prism, pyramid, brick and tetrahedron [42]. To reliably model an ultrasonic
 170 signal, the recommended number of mesh elements is 4 elements per A0- or 8 per S0-wavelength
 171 [43].

172 Beside the spatial discretization, time incrementation of Eq. (4) is needed as well. The minimum
 173 requirement to ensure numerical stability of time integration is the Courant-Friedrich-Lewy (CFL)
 174 condition [40, 43 – 45].
 175

176 3.3. Additive Color Model

177 Mathematical representations of color can be summarized as additive and subtractive color
 178 models [46]. The additive color model is used for combining light of different color and the
 179 subtractive color model is used for combining pigments and dyes, which are typically represented in
 180 4 basis colors: cyan, magenta, yellow, and black (CMYK).

181 Both additive and subtractive color models are typically represented in vector form. Both color
 182 models relate to the perception of the human visual system. In the additive color model, the
 183 background is black and color is composed of a red, green, and blue (RGB) tuple as depicted in Fig.
 184 1. This relates to the three cone cells of the human retina (long, medium, and short) which have peak
 185 sensitivities to light of wavelengths 559 nm, 531 nm, and 419 nm. These correspond to red, green,
 186 and blue light, respectively [46]. Normally, red, green, and blue color may be written in 8-bit form as
 187 [255 0 0], [0 255 0], and [0 0 255], respectively or in normalized form as [1 0 0], [0 1 0], and [0 0 1],
 188 respectively.
 189

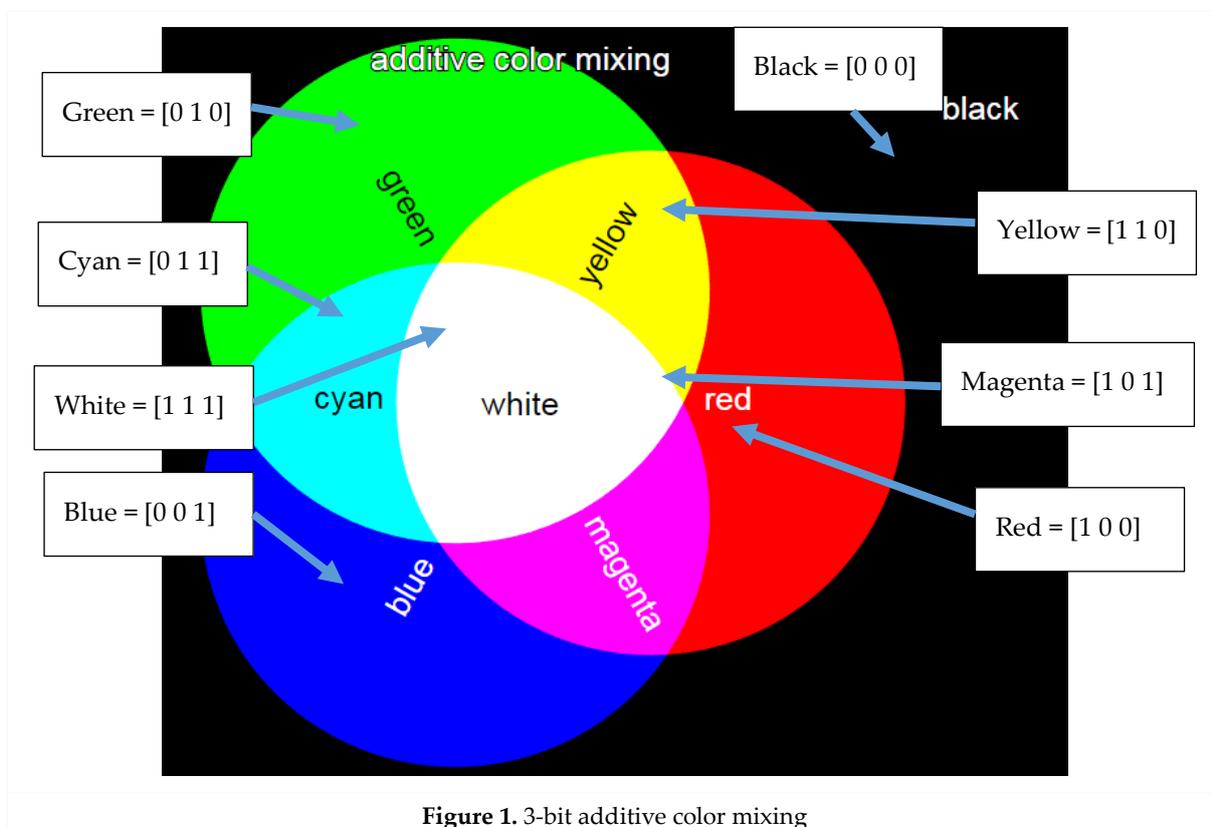


Figure 1. 3-bit additive color mixing

190
 191 When working with digital imaging, the additive color model (i.e. RGB) is a natural choice
 192 because what humans see in a measurement device / computer screen is normally encoded in RGB
 193 values. Computer scientist and electrical engineers have selected to represent the digital vision (e.g.
 194 in computer monitor) as an RGB array because of the trichromatic cone cells in human eye which are
 195 sensitive to the frequency spectrum of red, green, and blue photons that are falling into human
 196 retina. Therefore, the origins of the additive color model is physical and biological than purely
 197 technical.

198 In the 3-bit model, shown in Fig. 1, other colors such as cyan, magenta, yellow, and white are
 199 obtained by adding basic colors. For instance, cyan is the addition of blue and green, while white is
 200 the addition of red, green, and blue. Colors may also be subtracted from one another, as given in
 201 Table 1. Note that in the convention used negative values are set at zero, as negative light amplitude
 202 has no physical meaning. In this paper, material displacements are represented as colors in the
 203 additive color model and mathematical operations are performed to identify and visualize the
 204 optimum sensor positions.

Table 1. Subtractive mathematical operations for a 3-bit additive color model

y - x	R	G	B	C	M	Y	K
R	K	R	R	R	K	K	R
G	G	K	G	K	G	K	G
B	B	B	K	K	K	B	B
C	C	B	G	K	G	B	C
M	B	M	R	R	K	B	M
Y	G	R	Y	R	G	K	Y
K	K	K	K	K	K	K	K

205

206

R = Red, G = Green, B = Blue, C = Cyan, M = Magenta, Y = Yellow, K = Black

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4. Results and Discussion

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4.1. Simulation

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The method for choosing the simulation parameters has been described in [23, 43]. The plate dimensions are 600 mm x 400 mm x 2 mm. For this work, the following parameters are used: ABAQUS explicit, aluminum properties (Young's modulus of 70 GPa, Poisson ratio of 0.33, density of 2700 kg/m³), quadratic brick mesh (C3D20) with a global mesh size of 1 mm, single node out-of-plane excitation with windowed 5 sine-cycle of central frequency of 250 kHz with 1 N concentrated force, dynamic implicit step, no boundary conditions imposed, time increment of 0.1 μs (which means a sampling frequency of 10 MHz), total time period of 500 μs and a single nodal output precision. The specification of the computer where the simulation was run was: Intel Xeon E5-1620 3.5 GHz (Quad-core 8-Threads), 32 GB DDR3-RAM, and NVidia NVS310M Graphic card (GPU acceleration was not activated).

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4.2. Data Extraction

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To capture the wave propagation image, the screenshot program which is called Lightshot can be downloaded from prntscr.com [47]. In the program settings, the option "*Keep the selected area position*" must be selected so that Lightshot remembers the X-Y position of the captured screenshot for every time increment. As mentioned previously, this technique is less exhaustive and more memory efficient as every saved image has a size of only around 500 KB. For 20-time increments and 2 cases (uncracked and cracked plate), only around 20 MB of space is needed in comparison to ODB extraction which takes around 3.2 GB of space.

In a computer, the color in a single pixel is represented as an RGB array and this can be used to represent different Lamb wave displacement amplitudes. Fig. 2 shows an example simulation of Lamb wave propagation 30 μs after excitation. The displacement magnitude U is defined as:

$$U = \sqrt{U_x^2 + U_y^2 + U_z^2} \quad (5)$$

231

232

233

where U_x , U_y , and U_z are the displacements in the x, y, and z-directions, respectively. It is obvious that no displacement ($U = 0$ nm) is shown as a blue pixel, while a displacement of 2.5 nm is shown in

234 green, and a displacement of 5 nm is shown in red. The values in-between such as 1.25 nm and 3.75
 235 nm are shown in cyan and yellow, respectively.
 236

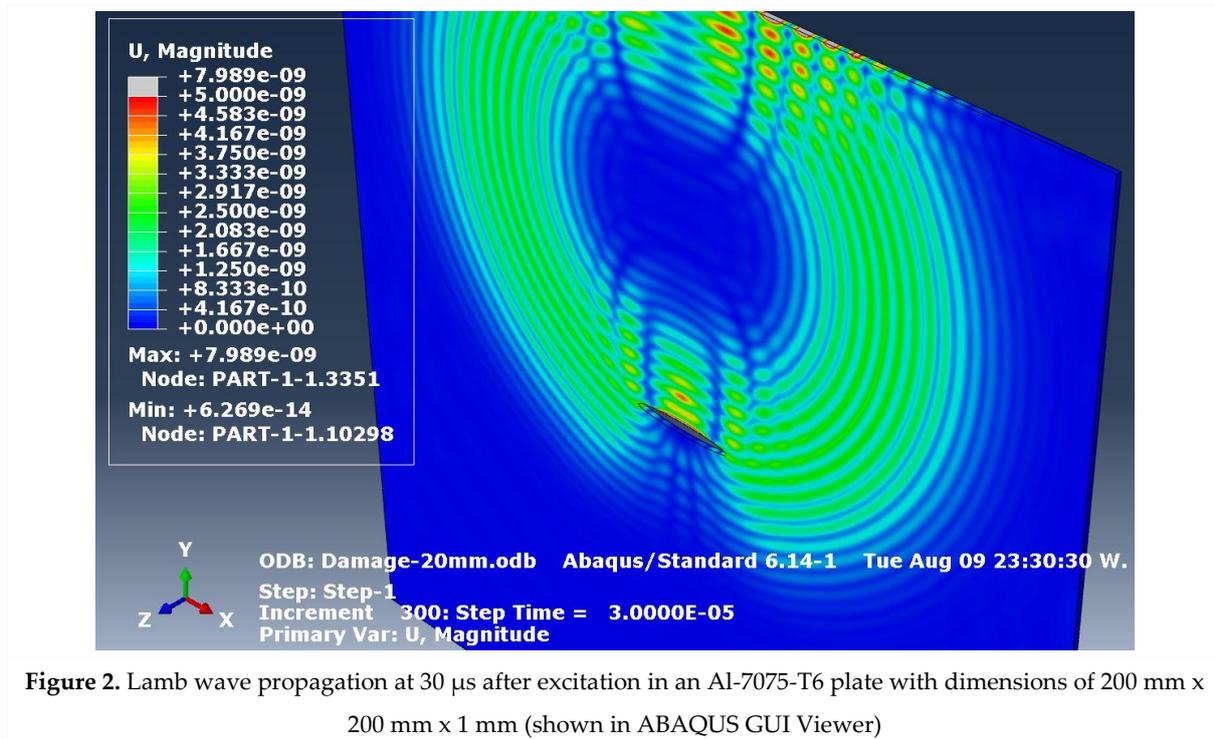


Figure 2. Lamb wave propagation at 30 μ s after excitation in an Al-7075-T6 plate with dimensions of 200 mm x 200 mm x 1 mm (shown in ABAQUS GUI Viewer)

237
 238 This colormap ‘rainbow’ is the default colormap in ABAQUS. Note that this colormap is slightly
 239 different from the basic 3-bit RGB colormap depicted in Fig. 2 as it has more color transitions, i.e.
 240 there are smoother transitions between blue and cyan, cyan and green, and so on. For ease, we
 241 neglected displacements larger than 5 nm (shown in grey color), which are mostly due to the
 242 excitation, because in this simulation this only appears in few locations. While this image processing
 243 procedure offers less displacement information as all displacement values are translated into an RGB
 244 array, we find this alternative procedure much faster, and more memory efficient rather than
 245 extracting data directly from the ABAQUS ODB binary file.

246 4.3. Differential Images

247 The approach is demonstrated by taking screenshots from the ABAQUS Viewer User Interface.
 248 Fig. 4a shows Lamb wave propagation at $t = 100 \mu$ s in an uncracked Al-7075 plate. The simulation
 249 images are actually similar to those which are depicted in Fig. 4 of [16], except that the simulation
 250 platform, color map, excitation signal, and the geometries are slightly different.

251 The plate contains 3 rivet holes as depicted in Fig. 3a. Fig. 3b shows Lamb wave propagation in
 252 the same plate but with a symmetric crack (from tip-to-tip, including the hole diameter of 10 mm) of
 253 28 mm length in the middle of the plate (marked by a yellow rectangle). The images captured have
 254 size of 1210 x 807 pixels, so the resolution is 2 pixel/mm. Images are stored as matrices with a size of
 255 1210 x 807 x 3, where each pixel has 3 arrays, each containing a normalized floating value between 0
 256 and 1 for each of the RGB colors.

257 A similar pattern of Lamb wave propagation can be observed if the crack length differs by +/-
 258 10%, as shown in Fig. 3c. In this case, the crack length is 30 mm instead of 28 mm. However, if the

259 crack is much larger, a notable change in the wave propagation pattern can be observed, as depicted
260 in Fig. 4d. In this case, the crack length is 60 mm. Similarly, the wave propagation at $t = 125 \mu\text{s}$ for an
261 uncracked plate, and for plates with crack lengths of 28 mm, 30 mm, and 60 mm can be observed in
262 Fig. 4e – h, respectively.
263

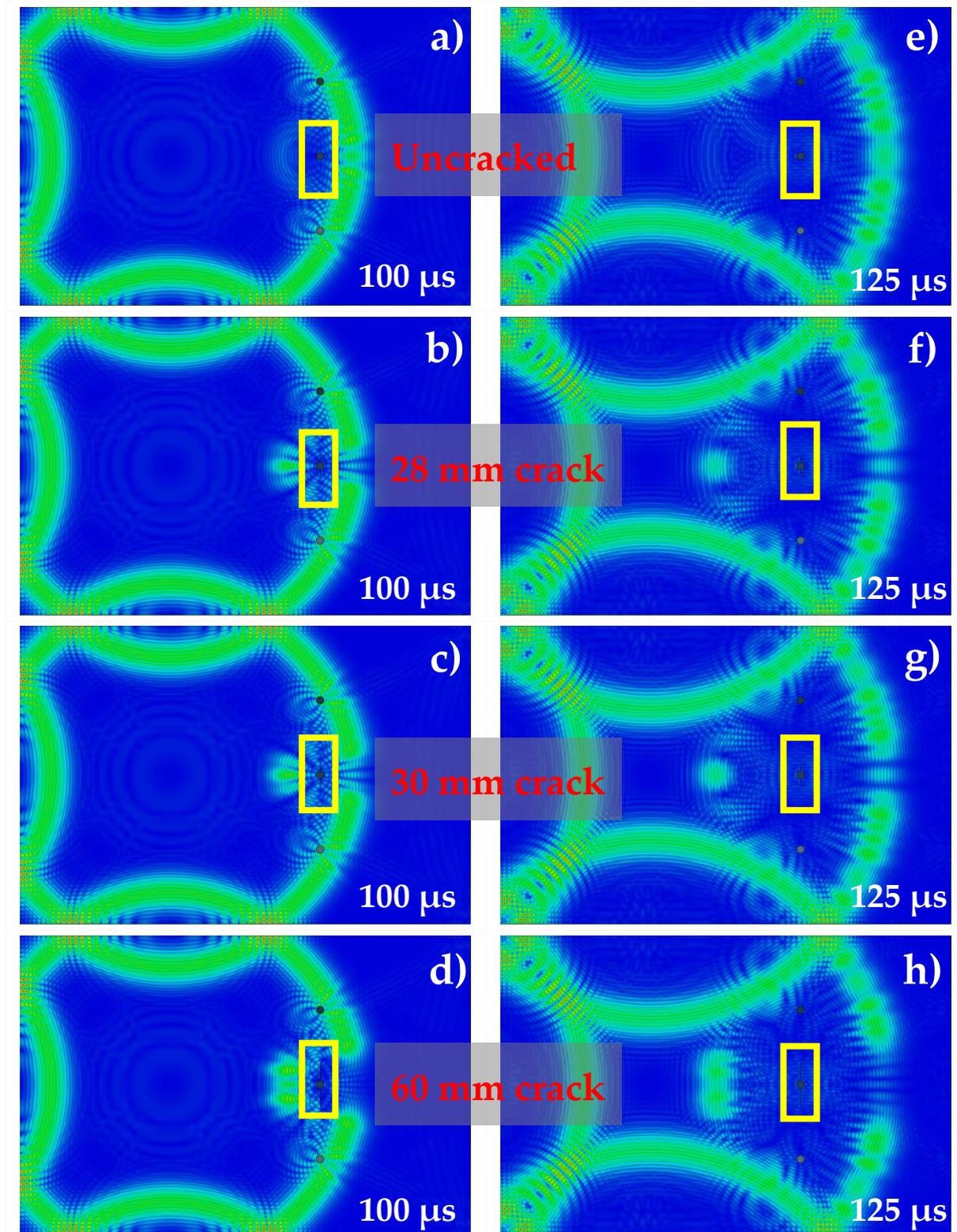


Figure 3 a – h. Lamb wave propagation at $t = 100$ and $125 \mu\text{s}$ in Al 7075 Plate with different crack lengths

265 By subtracting the image of the cracked plate (Fig. 3h) from that of uncracked plate (Fig. 3e), the
 266 reflected wave scatter image can be obtained, as shown in Fig. 4a. Note the RGB values are
 267 subtracted rather than the displacement values. Pixels, for which there is no change in wave scatter
 268 are shown as black. In this figure, the reflected wave scatter is highlighted by the yellow rectangle,
 269 while the corruptly transmitted wave is obtained as well (in the red rectangle) but is not clearly
 270 visible.

271 In an analogous way, we can subtract Fig. 3e from Fig. 3h, and in this case, the corruptly
 272 transmitted wave scatter image is highlighted more (red rectangle) as depicted in Fig. 4b, while the
 273 reflected wave scatter can still be seen (yellow rectangle) but is less visible. For the further sections in
 274 this paper, only the results from the uncracked plate and the cracked plate of 60 mm crack will be
 275 shown for conciseness. To highlight both the reflected and corrupted wave scatter, the two images
 276 shown in Fig. 4a – b can be joined to form a composite image.

277 Two common ways combining images are described here, the first one is called image addition.
 278 In this case, both image matrices are just mathematically added. The second one is called image
 279 fusion and uses the '*imfusion*' function in MATLAB, where the two images are firstly converted into
 280 greyscale mode, given a false color and then added mathematically. Furthermore, it is possible to
 281 invert the composite images to obtain the inverted images. This step is not necessary, but some
 282 people subjectively may find the colored wave scatters easier to track if the background is white. The
 283 inverted images from image addition and image fusion are depicted in Fig. 4c – d, respectively. For
 284 the sensor placement procedure described in Section 4.4, the inverse fused image (Fig. 4d) is used
 285 because we can still see the representation of reflected and missing wave scatter and that enables us
 286 to cross check the results with the simulation data.
 287

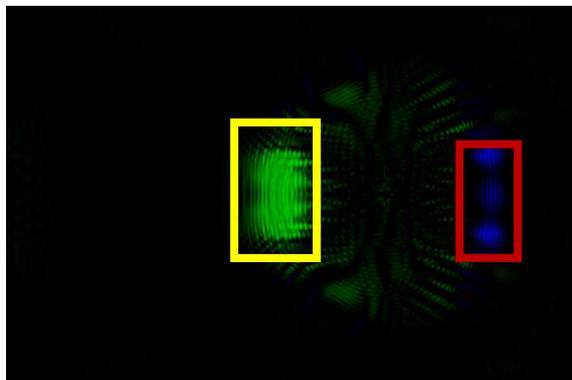


Figure 4a. Differential image of (3h – 3e)

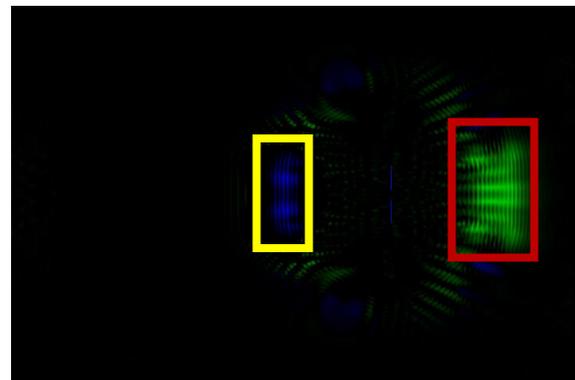


Figure 4b. Differential image of (3e – 3h)

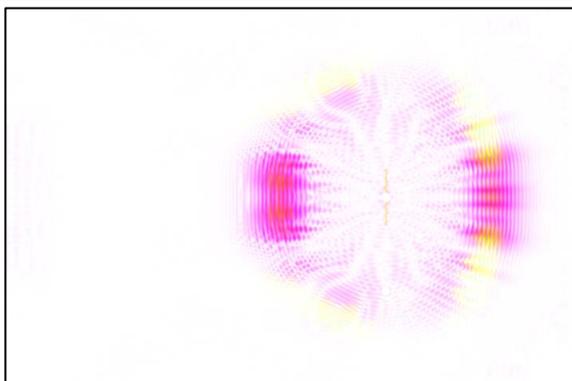


Figure 4c. Inverse of added image (4a + 4b)

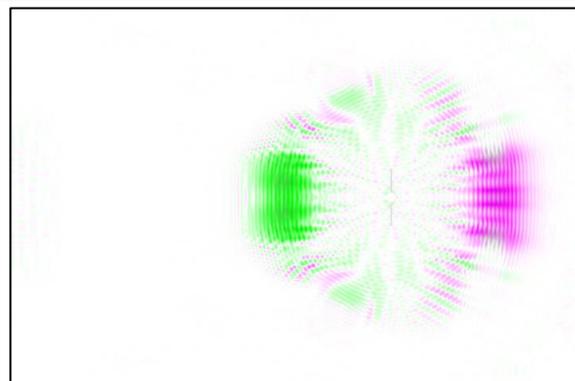


Figure 4d. Inverse of fused image (4a + 4b)

288 4.4. Sensor Placement

289 The wave scatter which was caused by both reflections and corrupted transmissions due to the
 290 crack front are represented by false color (i.e. green and magenta pixels in Fig. 4d). These pixels have
 291 a different color from the background (white). Regions of interest are determined by using the
 292 Matlab blob detection function to locate areas of adjacent green and magenta pixels. The larger the
 293 blob is, the larger the area of wave scatter. The sensor should be placed in the center of the largest
 294 blob, so that it will have a high probability of capturing a portion of wave scatter from cracks.

295 The blob detection algorithm is based on the Laplacian of Gaussian [48 – 50] with a kernel of
 296 8-pixel connectivity. Given the Gaussian function G of an input image $f(x,y)$ and feature scaling σ as
 297 given in Eq. (6). The Laplacian operator ∇^2 is given in Eq. (7).

298

$$G(x, y, \sigma) = \frac{1}{\sqrt{2\pi} \cdot \sigma^2} \cdot \exp\left(-\frac{x^2 + y^2}{\sigma^2}\right) \quad (6)$$

$$\nabla^2 = \frac{\partial^2 f}{\partial x^2} + \frac{\partial^2 f}{\partial y^2} \quad (7)$$

299

300 By applying the Laplacian operator in Eq. (7) to the Gaussian function in Eq. (6), one obtains the
 301 Laplacian of Gaussian, commonly known as LoG, as described in Eq. (8). Hence, the blob centroid \hat{x} ,
 302 \hat{y} with the scale $\hat{\sigma}$ is the simultaneously local extremum of the LoG in Eq. (9).

303

$$\nabla^2 G(x, y) = \left(\frac{x^2 + y^2 - 2\sigma^2}{\pi \cdot \sigma^4} \right) \cdot \exp\left(-\frac{x^2 + y^2}{2\sigma^2}\right) \quad (8)$$

$$(\hat{x}, \hat{y}, \hat{\sigma}) = \arg \max \min_{(x,y,z)} (\nabla^2 G(x, y)) \quad (9)$$

304

305 For the blob detection, the kernel of 8-pixel connectivity is used because it is more suitable for a
 306 larger area since the diagonal neighbor is counted as well, see Fig. 5a, while 4-pixel connectivity (Fig.
 307 5b) is typically used for line and corner detection. In Fig. 5a – b, the meaning of -1 and 0 are pixels
 308 which are counted and are not counted as a neighbor of the center pixel, respectively.

309

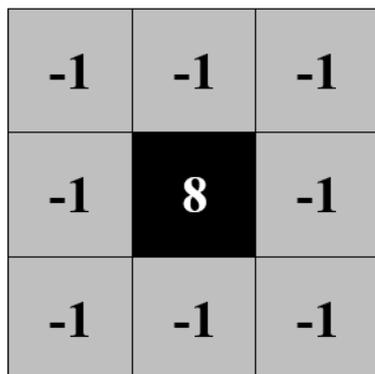


Fig. 5a: 8-pixel connectivity

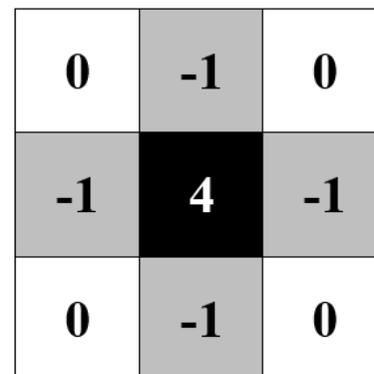


Fig. 5b: 4-pixel connectivity

310 The blob detections from various time increments are depicted in Fig. 6a – 6d. As mentioned
 311 before, we would like to place the sensor in the location with the largest change in signal over time,
 312 i.e. the largest blob. Since it generally contains more than a single pixel, the centroid can be
 313 calculated to determine the average pixel location that would receive wave scatter. In Fig. 6a – 6d,
 314 the largest and the second-largest blob centroids are marked by red and green dots, respectively.
 315 Blob boundaries are marked by the yellow polylines.

316 The rest of the centroids are marked by blue dots. In order not lose the overview, the reader is
 317 encouraged to compare Fig. 6a with Fig. 3a and 3d, as well as Fig. 6b with Fig. 3e and 3h. The X-Y
 318 coordinates of the blob centroid and the blob size are summarized in Table 2. The detailed procedure
 319 in MATLAB to find to trace the blob is described in Algorithm 1.
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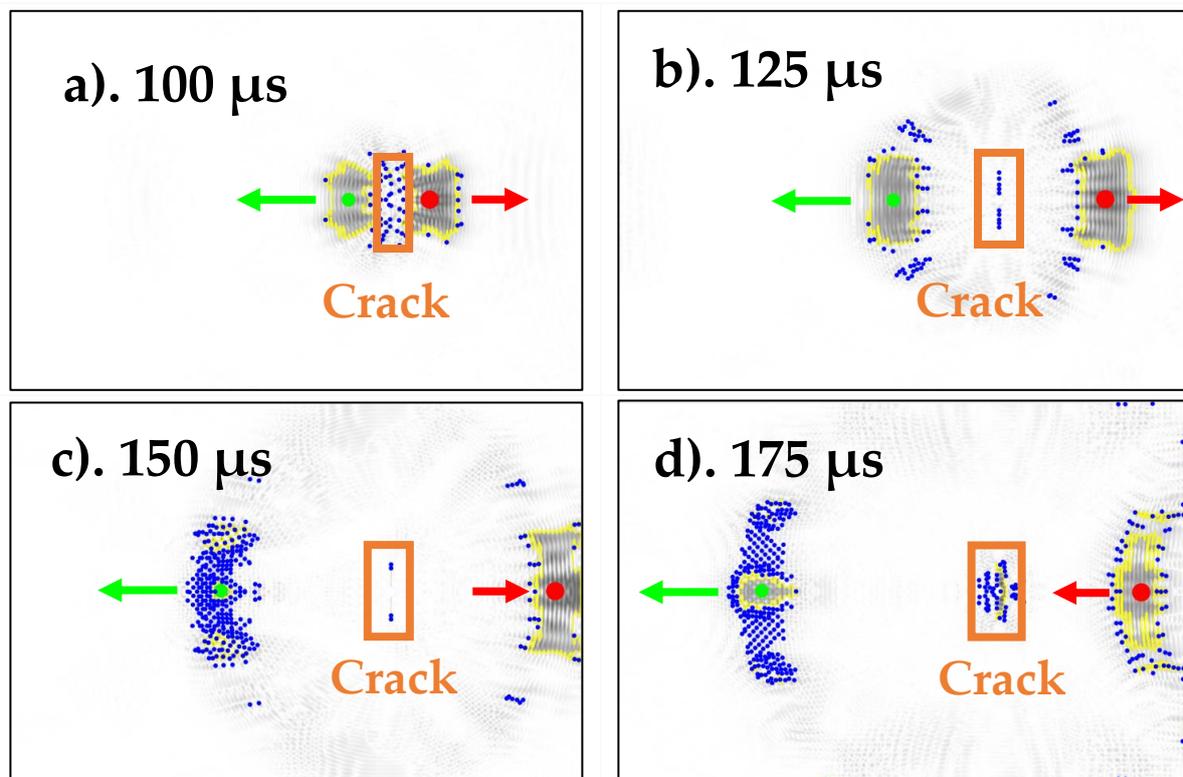


Figure 6. Detected blobs at a). 100 μ s, b). 125 μ s, c). 150 μ s, and d). 175 μ s. The largest and second-largest blobs are marked in red and green, respectively. Arrows indicate the direction of movement of the blobs.

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Table 2. X,Y – Area and coordinates of the largest and second largest centroid

Time frame	Largest centroid			Second-largest centroid		
	[pixel]	[mm]	Area [pixel]	[pixel]	[mm]	Area [pixel]
100 μ s	888,403	440,200	15910	716,403	355,200	12917
125 μ s	1031,402	511,199	29535	582,404	289,200	18794
150 μ s	1154,402	572,199	27067	445,401	221,200	8808
175 μ s	1111,405	551,201	24214	304,402	151,199	7949

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Units are in pixel and mm. Total area is in pixel. Average resolution is 2 pixel / mm.

Algorithm 1:	Meaning:
1 j ← number of available image files	Assign j as the number of available images
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3 for i ← 1:j	Loop over images from 1 to j:
4 img[i] ← imread(image[i])	Store the image in matrix 'img'
5 I[i] ← mat2gray(rgb2gray(img[]))	Convert the RGB array into greyscale
6 BW[i] ← I[i] < threshold	Set the pixel intensity threshold
7 B{i} ← bwboundaries(BW,8)	Trace region with 8-pixel connectivity
8 s(i) ← regionprops(BW)	Open the region properties
9 end	End the loop
10	
11 n ← number of detected blobs	Assign n as the number of detected blobs
12	
13 for m ← 1:n	Loop over all traced region from 1 to n:
14 S[m] ← s[m].Area	Store the area information in matrix 'S'
15 [val ind] ← sort(S,'descend')	Sort from the largest to smallest blob
16 boundary{m} ← B{ind(m)}	Store the blob boundaries
17 centroids[m] ← mean(boundary{m})	Calculate blob centroids
18 X ← [X centroids(2)]	Assign X-coordinate from the centroid
19 Y ← [Y centroids(1)]	Assign Y-coordinate from the centroid
20 end	End the loop

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After a certain time, the wave pattern becomes more chaotic due to multiple reflections from the crack front, rivet holes, and plate edges so that more smaller centroids will be born that are not exactly aligned with the mid Y-axis anymore. The smaller centroids imply that the potential energy capture by PZT is getting smaller and this will be aggravated by Lamb wave attenuation. This is the reason we recommend 'early wave scatter capture' for hotspot SHM design. Typically, one can decide the best sensor position by considering the movement of the centroid per time increment, also known as ray tracing [51].

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Furthermore, it is possible to fuse Fig. 6a – d into a single image. We performed this operation and the result is depicted in Fig. 7, where each pixel contains information about the normalized intensity between 0 and 1 which is then mapped into the rainbow color scale. From Fig. 7, it can be subjectively judged that the best sensor position is between X = 44 cm and 57 cm and the second-best sensor position is between and X = 34 and 38 cm, while the vertical coordinate for both positions remains at Y = 20 cm.

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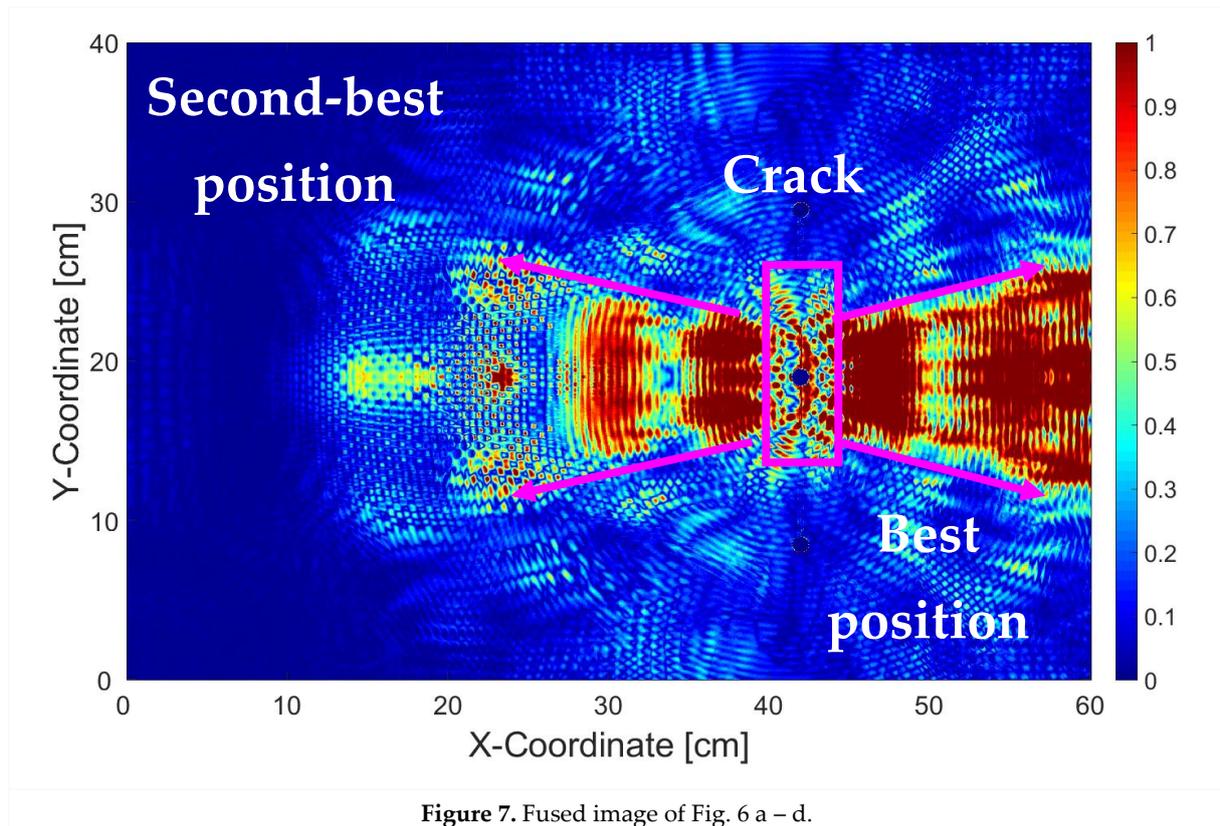
In order to demonstrate that the image processing algorithm also works for a different case of simulations, we slightly modified the case for the a). the critical crack length to 30 mm, and b). the orientation of the crack by 8°. The whole procedure was repeated, and the results are depicted in Fig. 8 and 9, respectively.

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From Fig. 8, it can be seen that the areas with higher pixel intensity (colored in red with value between 0.7 and 1) is smaller than those of Fig. 7. This is absolutely normal, since the wave perturbation at the crack front due to a crack length of 30 mm is smaller than those of 60 mm.



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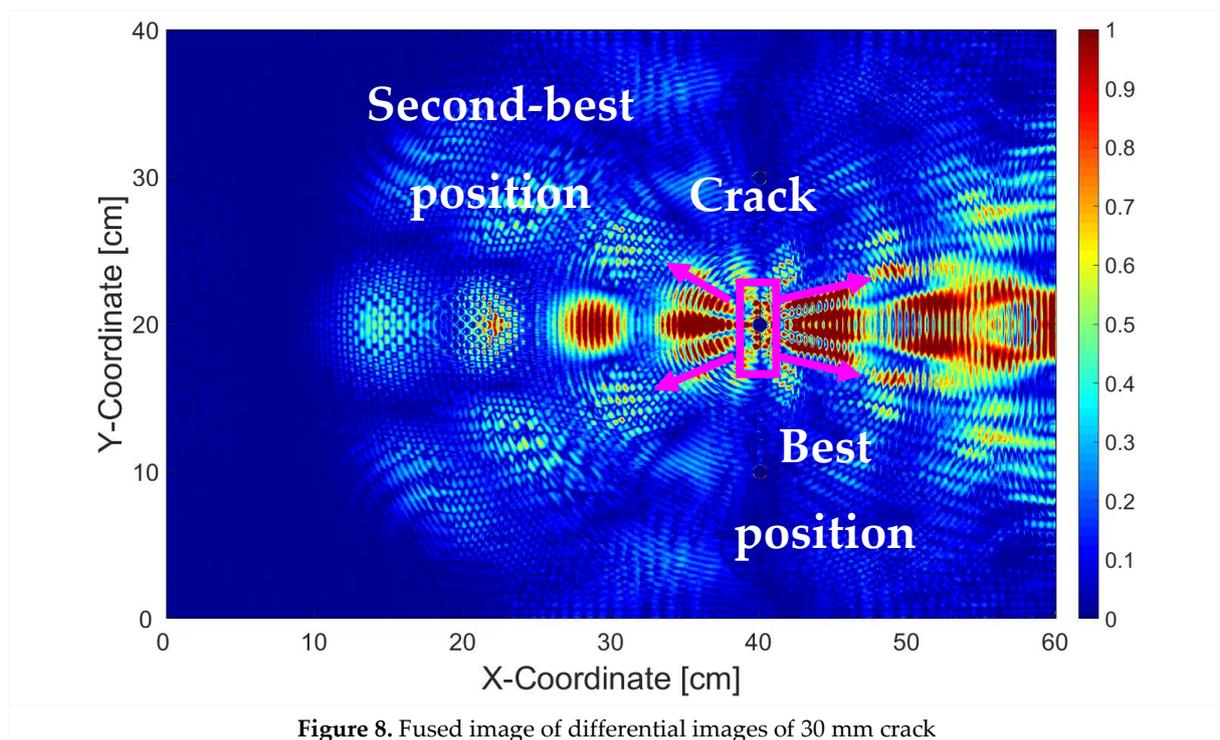
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Meanwhile from Fig. 9, it can be seen that the crack orientation changes the direction of the reflected wave scatter and it is still conform with Snell's law. However it does not change the orientation of areas where the wave scatter is not present. After cross validating with the original simulation data, it can be confirmed that angled crack only heavily influences the reflected wave scatter portion, but not the missing scatter portion (Fig. 10a – b, marked in yellow).



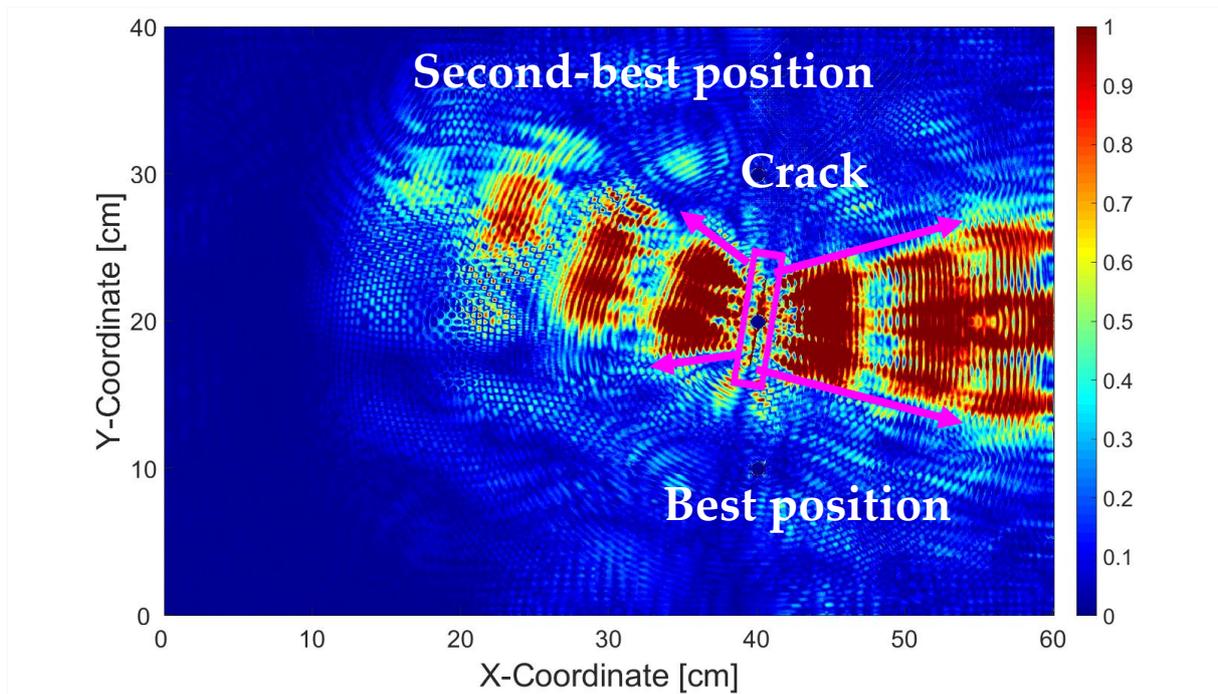


Figure 9. Fused image of differential images of 60 mm crack with 8° orientation

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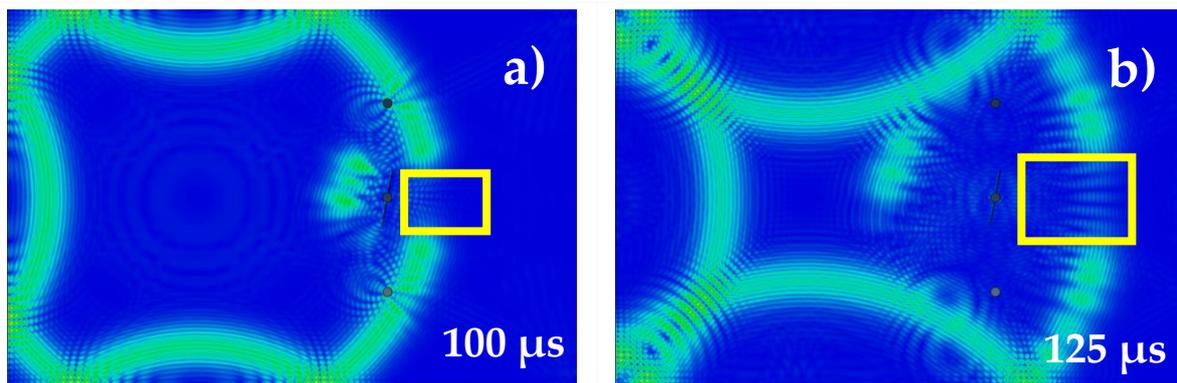


Figure 10. Lamb wave propagation at a). $t = 100$ and b). $125 \mu\text{s}$ in Al 7075 Plate

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For current work, we have demonstrated a new efficient method based on an additive color model to find the best two locations for placing the sensor. However, the number of sensors that are allocated for every case can be adjusted according to the manufacturer and/or operator requirement to achieve required crack detectability. Furthermore, not only the number of sensors, but also the excitation frequency can be changed depending on the size of the critical crack that must be detected. A higher frequency means a shorter wavelength, and this would mean the wave will be able to interact with smaller critical crack. However, such a higher frequency Lamb wave will also be more quickly attenuated than the lower frequency Lamb wave. Therefore, in order to stabilize the SHM network performance, more sensors will be required. Nevertheless, when more sensors are employed, higher procurement costs are also expected due to more weight, more data processing capability, etc. We regard this as the classical trade-off between SHM investment cost and SHM network reliability and would like to pass this decision back to the aircraft manufacturer or operator according to their needs.

369 5. Conclusion

370 In this work, we have demonstrated a novel technique to design the sensor network topology
371 for hotspot SHM by using differential images and blob detection algorithm. While our image
372 processing technique does not allow a quantitative approach to observe the nodal displacement, i.e.
373 displacement from every single FE node, we believe this technique offers a more holistic view (Fig.
374 7) of where to place the PZT sensors on the structure to be monitored.

375 Also, with this technique we think that the sensor placement can be done more quickly without
376 exhaustive data processing from simulation file for each surface while not sacrificing too much
377 spatial resolution. In practice, even the extracted nodal data from the simulation must be
378 interpolated, since in reality a PZT sensor would always occupy more than a single node (e.g. a
379 typical PZT in our lab has a diameter of 1 cm, so would theoretically occupy about 78 FE nodes).
380 Therefore, as concluding remark, we hope this technique will help further research in sensor
381 placement.

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383 **Acknowledgments:** The project is funded by the TKI Smart Sensing for Aviation project of Delft University of
384 Technology, Netherlands sponsored by the Dutch Ministry of Economic Affairs under the 'Topsectoren' Policy
385 for High-Tech Systems and Materials.

386 **Conflicts of Interest:** The author declares no conflict of interest.

387 **Supplementary Materials:** The source code in MATLAB and all the wave propagation images being involved
388 can be downloaded in the author's personal repository and 4TU website. [52 – 53]

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