 doi:10.20944/preprints201804.0348.v1

Article

Application of Self-Similar Symmetry Model to Dark Energy

Tomohide Sonoda

Graduate School of Natural Sciences, International Christian University, Tokyo, Japan

Abstract: Recent observations of the dark energy density demonstrates the fine-tuning problem and challenges in theoretical modelling. In this study, we apply the self-similar symmetry (SSS) model, describing the hierarchical structure of the universe based on the Dirac large numbers hypothesis, to Einstein's cosmological term. We introduce a new similarity dimension, D_B, in the SSS model. Using the D_B SSS model, the cosmological constant, vacuum energy density, and Hubble parameter can be simply expressed as a function of the cosmic microwave background (CMB) temperature. We show that the initial value of the vacuum energy density at the creation of the universe is $\rho_0 = 1/8\pi\alpha_f^6$, where α_f is the fine structure constant. The results indicate that the CMB is the primary factor for the evolution of the universe, providing a unified understanding of the problems of naturalness.

Keywords: Dark energy; Dark matter; Cosmic microwave background; Large numbers hypothesis; Varying fundamental constants; Symmetry

---

1. Introduction

The cosmological constant problem, i.e., the dark energy problem, poses a formidable challenge in physics. In 1998, observations of distant supernovae provided evidence for the acceleration of the expansion of the universe [1,2]. Einstein's cosmological term emerged as the simplest candidate to explain the mechanism of the accelerating universe. However, the inconsistencies between theoretical expectations and observations are extremely problematic, despite many attempts to provide a proper explanation [3–8]. In order to provide insights into this issue, the axiomatic approach has been proposed by Beck [9]. Beck formulated a description of the cosmological constant, Λ, using four statistical axioms: fundamentality (Λ depends only on the fundamental constants of the nature), boundedness (Λ has a lower bound, $0 < \Lambda$), simplicity (Λ is given by the simplest possible formula, consistent with the other axioms), and invariance (Λ values obtained using potentially different values of the fundamental parameters preserve the scale-invariance of the large-scale physics of the universe). Using the four axioms, Beck showed that Λ is given by:

$$\Lambda = \frac{G^2}{\hbar^4}\left(\frac{m_e}{\alpha_f}\right)^6, \tag{1}$$

where G is the gravitational constant, $\hbar$ is the reduced Planck constant, α_f is the fine structure constant, and m_e is the electron mass. The same formula has been proposed using different approaches [10,11], and recently discussed in several reports [12–15].

In this study, we applied the self-similar symmetry (SSS) model [16], that explains the hierarchical structure of the universe based on the Dirac large numbers hypothesis (LNH) [17,18], to Beck's formula. We show that the values of the cosmological constant, vacuum energy density, and Hubble parameter can be simply expressed as a function of the cosmic microwave background (CMB) temperature, and that the initial vacuum energy density is uniquely determined by $\rho_0 = 1/8\pi\alpha_f^6$. These results indicate that the CMB is the primary factor responsible for the evolution of the universe, revealing novel insights into the outstanding challenges.

Preprints (www.preprints.org) | NOT PEER-REVIEWED | Posted: 27 April 2018 doi:10.20944/preprints201804.0348.v1

2. D_B SSS model

The SSS model [16] describes the CMB with a symmetrical self-similar structure. The model consists of dimensionless values because a physical constant with a dimension would not have universality. Therefore, we define the fundamental dimensionless mass ratios of the proton mass $m_{\rm pr}$, electron mass $m_{\rm e}$, and Planck mass $m_{\rm Pl}$ as follows:

$$A = \log\alpha = \log\left(\frac{m_{\rm Pl}}{m_{\rm pr}}\right),\ B = \log\beta = \log\left(\frac{m_{\rm e}}{m_{\rm pr}}\right). \tag{2}$$

We also defined the fundamental dimensionless time and length ratios as follows:

$$T = \log\left(\frac{t}{t_{\rm Pl}}\right),\ L = \log\left(\frac{l}{l_{\rm Pl}}\right), \tag{3}$$

where t and l are the time and length scales of the objects, respectively, and $t_{\rm Pl}$ and $l_{\rm Pl}$ are the Planck time and length, respectively. Using these dimensionless values, we define the similarity dimension D_A as:

$$D_A = \left(\frac{T}{L}\right)^3 = \frac{A}{A+B} \approx 1.20592. \tag{4}$$

A new similarity dimension, D_B, is then introduced:

$$D_B = \frac{A-B}{A+B} \approx 1.41184. \tag{5}$$

The hierarchical structures of the D_B SSS model are constructed according to the following sequences:

$$L_0 = 2(A+B) \approx 31.70089, \tag{6}$$

$$L_n = D_B^n L_0 \quad \text{for} \quad L > L_0, \tag{7}$$

$$L_m = (2 - D_B^m) L_0 \quad \text{for} \quad L < L_0, \tag{8}$$

where n and m are the natural numbers that represent the hierarchical levels. The time scales of each hierarchy are also calculated using Eq. (4).

3. Verification of the D_B SSS model

In order to verify the proposed D_B SSS model, we compared the model values with reference values. Table 1 and 2 summarize the length and time scales of the Planck, weak, solar, and universe hierarchies. The values obtained using the D_B SSS model agree well with the reference values. Figure 1 shows the hierarchy time scale as a function of length scale. The coincidences seen in the figure confirm the validity of the SSS model.

4. Discussion

Using the gravitational coupling constant $\alpha_{\rm G} = Gm_{\rm pr}^2/\hbar c$ and Eq. (2), $2A = -\log\alpha_{\rm G}$ is obtained. The following relations are satisfied:

$$L_{n=1} + L_0 = 3L_0 - L_{m=1} = 4A, \tag{9}$$

$$L_{m=1} - L_0 = L_0 - L_{n=1} = 4B. \tag{10}$$

Therefore, $\alpha_{\rm G}$ and β are important in forming the hierarchical structure of the universe. Regarding the similarity dimension, $D_A = (r_a - r_b)/(1 - r_b)$, (where $r_a = (D_A^3 + D_A^2 - 2)/D_A$ and $r_b = (2 -$

 doi:10.20944/preprints201804.0348.v1

Table 1. Length scales of the hierarchies of the universe.

Hierarchy	l (m)	L	D_B SSS model	Error (%)
Planck scale[a]	1.6×10^{-35}	0	0.21 ($m = 2$)	-
Weak scaleb[b]	10^{-16}	18.79	18.65 ($m = 1$)	-0.8
Solar scalec[c]	1.4×10^{9}	43.93	44.76 ($n = 1$)	1.8
Universe scaled[d]	4.1×10^{28}	63.40	63.19 ($n = 2$)	-0.3

[a] Planck length $l_{\text{Pl}} = \sqrt{\hbar G/c^3}$, where c is the speed of light in vacuum.
[b] Experimental results show that the range of the weak interaction is $r_w \leq 10^{-16}$m [19].
[c] Diameter of the sun, based on the nominal solar radius defined by the International Astronomical Union [20].
[d] Upper bound of the universe derived from the D_A SSS model [16].

Table 2. Time scales of the hierarchies of the universe.

Hierarchy	t (s)	T	D_B SSS model	Error (%)
Planck scale[a]	5.4×10^{-44}	0	0.23 ($m = 2$)	-
Weak scale[b]	6.6×10^{-27}	17.09	19.85 ($m = 1$)	13.9
Solar scale[c]	2.3×10^{5}	48.63	47.64 ($n = 1$)	-2.1
Universe scale[d]	1.7×10^{24}	67.49	67.26 ($n = 2$)	-0.3

[a] Planck time $t_{\text{Pl}} = \sqrt{\hbar G/c^5}$.
[b] The electromagnetic and weak forces unify at 100 GeV; [21] $t = \hbar/10^{11}$ s.
[c] Sun's rotational period; [22] $t = 2.32 \times 10^5$ s.
[d] Time scale of the universe derived from the D_A SSS model [16].

$D_A^3)/(D_A^2 - D_A)$ are the ratios of the length scales of the hierarchies [16]) can be used to obtain a simple relation between r_a and r_b:

$$(\alpha\beta)^{r_a} = \alpha\beta^{r_b}. \tag{11}$$

Equation (11) can be interpreted as the basic formula for the similarity dimension and indicates the correlation between the cosmic structure and fundamental dimensionless mass ratios [1]. Using Eq. (11), we obtain:

$$D_B = \frac{2r_a - r_b - 1}{1 - r_b}. \tag{12}$$

However, the numerical relation between D_B and r_a is:

$$\frac{r_a D_B}{\sqrt{2}} \approx 1.000009. \tag{13}$$

Equation (13) indicates the validity of D_B; if the D_B value is substituted into Eq. (8), $L_{m=2}$ is consistent with the Planck length.

1 Using Eq. (11), we can derive another similarity dimension, $D_C = -B/(A+B) \approx 0.20592$.

Preprints (www.preprints.org) | NOT PEER-REVIEWED | Posted: 27 April 2018 doi:10.20944/preprints201804.0348.v1

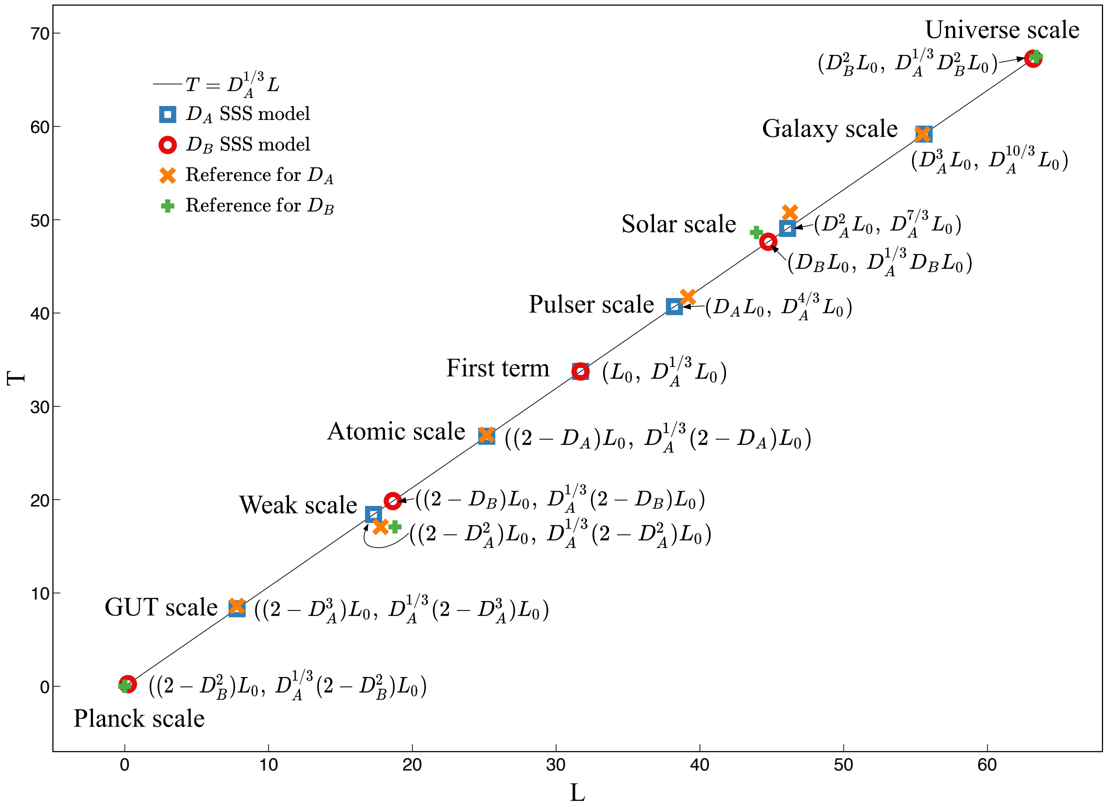

Figure 1. Time scale as a function of length scale for the SSS model and reference values. The reference values for the D_A SSS model are taken from Ref. [16]. The lower and upper bounds of the universe are interpolated in the D_B SSS model. Note the symmetry of the first term L_0, which corresponds to the CMB temperature. This symmetry indicates that each hierarchy is self-similar to the CMB temperature.

Regarding Λ, Eq. (1) can be written in an equivalent dimensionless form using $G = \hbar c / m_{\text{Pl}}^2$:

$$l_{\text{Pl}}^2 \Lambda = \alpha_f^{-6} \left(\frac{m_{\text{e}}}{m_{\text{Pl}}} \right)^6 . \tag{14}$$

We employed the following formulas derived from the D_A SSS model [16] in Eq. (14):

$$\alpha_{\text{G}} \simeq \tau_{\text{CMB}}^{D_A}, \tag{15}$$

$$\beta^2 \simeq \tau_{\text{CMB}}^{D_A - 1}, \tag{16}$$

where $\tau_{\text{CMB}} = T_{\text{CMB}} / T_{\text{Pl}}$; T_{CMB} is the CMB temperature and T_{Pl} is the Planck temperature. Then, we obtain:

$$\lambda(\xi) \simeq \xi^3 \tag{17}$$

where λ is the cosmological constant in reduced Planck units, $\lambda = l_{\text{Pl}}^2 \Lambda$, and we defined $\xi \equiv \alpha_f^{-2} \tau_{\text{CMB}}^{D_B}$. Equation (17) is based on the LNH and indicates that the CMB temperature can be considered as a cosmological scalar field.

Using the relation between the vacuum energy density ρ_Λ and Λ in Einstein's field equation, we obtain: $\rho_\Lambda = c^2 \Lambda / 8\pi G$. Therefore, the dimensionless vacuum energy density can be expressed as:

$$\rho(\xi) \equiv \frac{\rho_\Lambda}{\rho_{\text{Pl}}} \simeq \frac{\xi^3}{8\pi}, \tag{18}$$

where $\rho_{\rm Pl}$ is the Planck density. The solution of the Friedmann equation for a flat universe reveals the Hubble parameter H:

$$H^2 = \frac{8\pi G}{3}\frac{\rho_\Lambda}{\Omega_\Lambda}, \tag{19}$$

where Ω_Λ is the normalized vacuum energy density with respect to the critical density. Then, we obtain:

$$h^2(\xi) \simeq \frac{\xi^3}{3\Omega_\Lambda}, \tag{20}$$

where h is the Hubble parameter in reduced Planck units, $h = t_{\rm Pl}H$.

If we employ $T_{\rm CMB} = 2.725{\rm K}$ and $\Omega_\Lambda = 0.691$ [23] as the current parameters in Eqs. (18) and (20), we obtain $\rho_{current} \approx 1.22 \times 10^{-123}$ and $H_{current} \approx 69.69({\rm km/s})/{\rm Mpc}$, consistent with the latest observational data [23].

If we employ $T_{\rm CMB} = T_{\rm Pl}$ for the universe initial condition and substitute it into Eqs. (15), (16), and (18), we obtain $\alpha = \beta = 1$ and $\rho_0 = 1/8\pi\alpha_f^6$, which implies that the entire hierarchy was contained in a single point and that a high-energy density ρ_0 can trigger the cosmic inflation. The value of ρ decreases with the decrease of $T_{\rm CMB} \ll T_{\rm Pl}$, while the size of the universe L expands according to $L \sim \log(T_{\rm Pl}/T_{\rm CMB})$. Assuming that $T_{\rm CMB} \to 0$ is the ultimate fate of the universe, $L \to \infty$ and $\rho \to 0$. This indicates that the universe falls into an inactive state as it expands to infinity.

The SSS model can be evaluated by investigating the precise values of $\alpha_{\rm G}$ and β for the region that exhibits CMB anisotropy [24]. The model predicts that a higher temperature region yields a larger G and $m_{\rm e}$. This can be identified as the reason for the formation of the large-scale structure in the universe. An alternative is to measure the precise CMB temperature in the region where dark matter is considered to exist [25,26]. The model predicts that the CMB temperature in that region is higher than elsewhere because larger values of G and $m_{\rm e}$ can be identified as dark matter.

5. Conclusions

We have demonstrated that the D_B SSS model offers the simplest solution to the fine-tuning problem or the problems of naturalness. The dynamical vacuum energy that can be simply expressed as a function of the CMB temperature can cause inflation, and thus facilitates the evolution of the universe. We suggested a testable prediction to verify the hypothesis. Therefore, it is desired to perform observational investigations using the SSS model in the future.

Acknowledgments: The author thanks M. B. Greenfield and K. Kitahara for helpful discussions.

Author Contributions: T.S. conceived the study and prepared the manuscript.

Conflicts of Interest: The author declares that the research was conducted in the absence of any commercial or financial relationships that could be construed as a potential conflict of interest.

1. Riess, A.G.; Filippenko, A.V.; Challis, P.; Clocchiatti, A.; Diercks, A.; Garnavich, P.M.; Gilliland, R.L.; Hogan, C.J.; Jha, S.; Kirshner, R.P.; others. Observational evidence from supernovae for an accelerating universe and a cosmological constant. *The Astronomical Journal* **1998**, *116*, 1009.
2. Perlmutter, S.; Aldering, G.; Goldhaber, G.; Knop, R.; Nugent, P.; Castro, P.; Deustua, S.; Fabbro, S.; Goobar, A.; Groom, D.; others. Measurements of Ω and Λ from 42 high-redshift supernovae. *The Astrophysical Journal* **1999**, *517*, 565.
3. Weinberg, S. The cosmological constant problem. *Reviews of Modern Physics* **1989**, *61*, 1.
4. Sahni, V.; Starobinsky, A. The case for a positive cosmological Λ-term. *International Journal of Modern Physics D* **2000**, *9*, 373–443.
5. Carroll, S.M. The cosmological constant. *Living Reviews in Relativity* **2001**, *4*, 1.
6. Padmanabhan, T. Cosmological constant—the weight of the vacuum. *Physics Reports* **2003**, *380*, 235–320.
7. Peebles, P.J.E.; Ratra, B. The cosmological constant and dark energy. *Reviews of Modern Physics* **2003**, *75*, 559.

 doi:10.20944/preprints201804.0348.v1

8. Copeland, E.J.; Sami, M.; Tsujikawa, S. Dynamics of dark energy. *International Journal of Modern Physics D* **2006**, *15*, 1753–1935.
9. Beck, C. Axiomatic approach to the cosmological constant. *Physica A: Statistical Mechanics and its Applications* **2009**, *388*, 3384–3390.
10. Nottale, L. Mach's Principle, Dirac's Large Numbers, and the Cosmological Constant Problem. *preprint* **1993**.
11. Boehmer, C.; Harko, T. Physics of dark energy particles. *Foundations of Physics* **2008**, *38*, 216–227.
12. Burikham, P.; Cheamsawat, K.; Harko, T.; Lake, M.J. The minimum mass of a charged spherically symmetric object in D dimensions, its implications for fundamental particles, and holography. *The European Physical Journal C* **2016**, *76*, 106.
13. Eaves, L. The apparent fine-tuning of the cosmological, gravitational and fine structure constants. *Physica A: Statistical Mechanics and its Applications* **2016**, *443*, 355–357.
14. Wei, H.; Zou, X.B.; Li, H.Y.; Xue, D.Z. Cosmological constant, fine structure constant and beyond. *The European Physical Journal C* **2017**, *77*, 14.
15. Lake, M.J. Is there a connection between "dark" and "light" physics? Journal of Physics: Conference Series. IOP Publishing, 2017, Vol. 883, p. 012001.
16. Sonoda, T. Self-Similar Symmetry Model and Cosmic Microwave Background. *Frontiers in Applied Mathematics and Statistics* **2016**, *2*, 5.
17. Dirac, P.A. The cosmological constants. *Nature* **1937**, *139*, 323.
18. Dirac, P.A. A new basis for cosmology. Proceedings of the Royal Society of London A: Mathematical, Physical and Engineering Sciences. The Royal Society, 1938, Vol. 165, pp. 199–208.
19. Barrow, J.; Tipler, F.J., The anthropic cosmological principle; New York: Oxford University, 1986; p. 294.
20. Prša, A.; Harmanec, P.; Torres, G.; Mamajek, E.; Asplund, M.; Capitaine, N.; Christensen-Dalsgaard, J.; Depagne, É.; Haberreiter, M.; Hekker, S.; others. Nominal values for selected solar and planetary quantities: IAU 2015 Resolution B3. *The Astronomical Journal* **2016**, *152*, 41.
21. Bertone, G.; Hooper, D.; Silk, J. Particle dark matter: Evidence, candidates and constraints. *Physics Reports* **2005**, *405*, 279–390.
22. National Astronomical Observatory of Japan., Chronological Scientific Tables; Maruzen Co., Ltd, 2016; p. 97.
23. Ade, P.A.; Aghanim, N.; Arnaud, M.; Ashdown, M.; Aumont, J.; Baccigalupi, C.; Banday, A.; Barreiro, R.; Bartlett, J.; Bartolo, N.; others. Planck 2015 results-XIII. Cosmological parameters. *Astronomy & Astrophysics* **2016**, *594*, A13.
24. Adam, R.; Ade, P.; Aghanim, N.; Akrami, Y.; Alves, M.; Argüeso, F.; Arnaud, M.; Arroja, F.; Ashdown, M.; Aumont, J.; others. Planck 2015 results-I. Overview of products and scientific results. *Astronomy & Astrophysics* **2016**, *594*, A1.
25. Natarajan, P.; Chadayammuri, U.; Jauzac, M.; Richard, J.; Kneib, J.P.; Ebeling, H.; Jiang, F.; Van Den Bosch, F.; Limousin, M.; Jullo, E.; others. Mapping substructure in the HST Frontier Fields cluster lenses and in cosmological simulations. *Monthly Notices of the Royal Astronomical Society* **2017**, *468*, 1962–1980.
26. Oguri, M.; Miyazaki, S.; Hikage, C.; Mandelbaum, R.; Utsumi, Y.; Miyatake, H.; Takada, M.; Armstrong, R.; Bosch, J.; Komiyama, Y.; others. Two-and three-dimensional wide-field weak lensing mass maps from the Hyper Suprime-Cam Subaru Strategic Program S16A data. *Publications of the Astronomical Society of Japan* **2018**, *70*, S26.