

Article

A Scale at 10 MeV, Gravitational Topological Vacuum, and Large Extra Dimensions

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Abstract: We discuss a possible scale of gravitational origin at around 10 MeV, or 10^{-12} cm, arisen in the MacDowell-Mansouri formalism of gravity due to the topological Gauss-Bonnet term in the action, pointed out by Bjorken several years ago. A length scale of the same size emerges also in the Kodama solution in gravity, which is known to be closely related to the MacDowell-Mansouri formulation. We particularly draw attention to the intriguing incident that existence of six compact extra dimensions originated from TeV-scale quantum gravity as well points to a length scale of 10^{-12} cm, as the compactification scale. The presence of six such extra dimensions is also in remarkable consistency with the MacDowell-Mansouri formalism; it provides a possible explanation for the factor of $\sim 10^{120}$ multiplying the Gauss-Bonnet term in the action. We also comment on the relevant implications of such a scale regarding the thermal history of the universe motivated by the fact that it is considerably close to 1 – 2 MeV below which the weak interactions freeze out, leading to Big Bang Nucleosynthesis.

Keywords: MacDowell-Mansouri formalism; Bjorken-Zeldovich scale; Gauss-Bonnet term; Einstein-Cartan formalism; Big Bang Nucleosynthesis; cosmological constant; Kodama wavefunctions

1. Introduction

Bjorken points out in Ref. [1] that the MacDowell-Mansouri (MM) formulation of gravity [2] naturally reveals an induced scale of ~ 10 MeV, or $\sim 10^{-12}$ cm, which he names after Zeldovich, inspired by Zeldovich's seminal papers [3,4]. The MM formulation unifies the tetrad and spin connection of the first order Einstein-Cartan formalism, which take values in $SO(3,1)$, into a grand connection that lives in $SO(4,1)$ (or $SO(3,2)$ for a negative cosmological constant). The resulting action, through breaking the $SO(4,1)$ symmetry down to the $SO(3,1)$, yields the usual Einstein-Hilbert term, a cosmological constant, and the Gauss-Bonnet (GB) term, which is topological in four dimensions [5–7].

Intriguingly, a length scale of 10^{-12} cm, as noted in Ref. [8], is also encountered in the context of so-called the Kodama wavefunction in gravity [9–13], analogous to the Chern-Simons solution in Yang-Mills theory in four dimensions, which is also an important element in Loop Quantum Gravity [14,15]. Actually, there is known to be a connection between the inner product of Kodama states and the MM formalism; Ref. [13] points out that the topological terms arisen in the (extended) MM action and the inner product are the same.

Bjorken, additionally, suggests six extra spatial dimensions, assumed to be compactified on this induced scale of 10^{-12} cm, simply to account for the large factor multiplying the MM action [1]; $\sim 10^{120}$, which, quite remarkably, also happens to be the infamous number often encountered in the cosmological constant problem [16–19].

In this paper, we emphasize that the TeV-scale quantum gravity picture with large extra dimensions (LED) [20–26] naturally reveals a scale of 10^{-12} cm as the compactification scale, provided that the number of extra spatial dimensions is set to six, with no need for an ad-hoc assumption of the corresponding length scale. In order to be consistent with the known physics up to the TeV-scale, we

adopt the well-known approach that only the graviton is allowed to propagate throughout the bulk experiencing the extra dimensions, while the SM fields are localized to the usual 4 dimensions. This, in this scenario, would introduce a deviation in the gravitational interactions on scales smaller than 10^{-12} cm; the gravitational interaction has so far been tested down to the scale of 0.01 cm [27].

Moreover, we notice a combination of the ‘‘Bjorken-Zeldovich (BZ) scale’’ and the TeV scale, $M_{BZ}^3/M_{EW}^2 \sim 10^{-3}$ eV, which is in the order of the observed vacuum energy density in the present universe and in the ballpark of the anticipated neutrino masses [28,29]. Although it is most likely a coincidence, we present several toy models which illustrate its possible role as some sort of a see-saw-type suppression in obtaining the neutrino mass and the cosmological constant.

We also comment on possible other implications in cosmology. This scale is considerably close to $1 - 2$ MeV below which the weak interactions freeze out, leading to Big Bang Nucleosynthesis (BBN). Premised on our current understanding of BBN, it is in general supposed that any deviation from the known radiation density around the decoupling temperature would change the time scale associated with BBN, and it is thus tightly constrained from the observations on the primordial abundances of light elements [30–32].¹

2. The MacDowell-Mansouri formalism and the Bjorken - Zeldovich scale

Bjorken, in Ref. [1], discusses how a scale of ~ 10 MeV is induced in the MM formalism through the GB topological term arisen naturally in the formalism in addition to the usual Einstein-Hilbert action and a cosmological constant term.

The $SO(3, 1)$ MM action, obtained through breaking the $SO(4, 1)$ symmetry, is given as [1,2,5–7]

$$\mathcal{S}_{MM} = \frac{M_{Pl}^2}{64\pi H_0^2} \int d^4x \sqrt{-g} \frac{1}{4} F_{\mu\nu}^{ab} F_{\lambda\sigma}^{cd} \epsilon_{abcd} \epsilon^{\mu\nu\lambda\sigma}, \quad (1)$$

where the ϵ symbols denote Levi-Civita tensors, $F_{\mu\nu}^{ab} = R_{\mu\nu}^{ab} - H_0^2 (e_\mu^a e_\nu^b - e_\nu^a e_\mu^b)$, $R_{\mu\nu}^{ab} = R_{\mu\nu}^{\rho\sigma} e_\rho^a e_\sigma^b$ is the Riemann tensor, e_μ^a is the tetrad (vielbein), and $a, \mu = 0, 1, 2, 3$ are the indices of the internal $SO(3, 1)$ space and the four dimensional space-time, respectively. H_0 is the Hubble constant.

Note that $F_{\mu\nu}^{ab}$ is the $SO(3, 1)$ projection of the curvature $F_{\mu\nu}^{AB}$, constructed from the generalized connection A_μ^{AB} ($A = 0, 1, 2, 3, 4$) that lives in a local $SO(4, 1)$. A_μ^{AB} takes the following form. $A_\mu^{4a} \equiv H_0 e_\mu^a$ and $A_\mu^{ab} \equiv w_\mu^{ab}$, where w is the spin connection which lives in the $SO(3, 1)$ group.

The action in Eq. (1) yields

$$\mathcal{S}_{MM} = \frac{M_{Pl}^2}{8\pi} \int d^4x \sqrt{-g} \left(\frac{1}{32H_0^2} R_{\mu\nu}^{\alpha\beta} R_{\lambda\sigma}^{\gamma\delta} \epsilon_{\alpha\beta\gamma\delta} \epsilon^{\mu\nu\lambda\sigma} + \frac{1}{2} R - \Lambda \right), \quad (2)$$

where the cosmological constant $\Lambda = 3H_0^2$ as it ought to be, and the first two terms are the GB and the Einstein-Hilbert terms, respectively. Notice the factor $\sim 10^{120}$ in front of the GB term in Eq. (2), also multiplying the total MM action in Eq. (1), which happens to be the infamous number in the cosmological constant problem ($\frac{M_{Pl}^4}{64\pi^2 \rho_\Lambda} \sim 10^{120}$). The possible role of this factor of the MM action in the resolution of the cosmological constant problem has not been demonstrated yet, to the best of our knowledge.

In the Friedmann-Robertson-Walker (FRW) background, where $ds^2 = -dt^2 + a^2(t) dx_i dx_i$,

¹ Recently, Atomki group in Hungary has reported an anomaly in the ^8Be nuclear decay by internal e^+e^- formation at an invariant mass $m_{e^+e^-} \cong 17$ MeV, with a statistical significance of 6.8σ [33]. See also Refs. [34–42] for the previous studies relevant to this observation. The observation has ignited interest in the high energy physics community to suggest explanations some of which consider a hidden sector at around this energy scale whose effects have so far remained unnoticed [43–61].

69 the GB term in Eq. (2) becomes

$$70 \quad S_{GB} = -\frac{M_{Pl}^2 V(0)}{8\pi H_0^2} \int_0^t dt \frac{d}{dt} \dot{a}^3, \quad (3)$$

where $V(0)$ is given through time dependent volume of region of interest dominated by dark energy, $V(t) = V(0)\dot{a}^3 = V(0)e^{3H_0 t}$. Since in the semiclassical approximation the action is just the phase of the wavefunction, and for a topological term like the GB term the phase takes values in units of 2π , we can write the total amount of the action contributed by the GB term at time t , from Eq. (3), as

$$|\mathcal{S}_{GB}| = \frac{M_{Pl}^2 V(0)\dot{a}^3}{8\pi H_0^2} \Bigg|_0^t \equiv 2\pi(N(t) - N(0)). \quad (4)$$

Then, some sort of number density can be defined as

$$n \equiv \frac{N(t)}{V(t)} = \frac{M_{Pl}^2}{16\pi^2 H_0^2} \left(\frac{\dot{a}}{a}\right)^3 = \frac{H_0 M_{Pl}^2}{16\pi^2} \equiv \Lambda_{BZ}^3, \quad (5)$$

71 which is time independent for the cosmological constant dominated space. Bjorken uses the term
72 “darkness” for the quantity $N(t)$; we prefer to use the “Gauss-Bonnet number”.

Once we put in the numerical factors, the Bjorken-Zeldovich scale yields

$$\Lambda_{BZ} \approx 10 \text{ MeV} \quad \text{or} \quad l_{BZ} = \frac{1}{\Lambda_{BZ}} \approx 2 \times 10^{-12} \text{ cm}. \quad (6)$$

73 Λ_{BZ} appears to be the scale up to which the MM formalism is valid. Next, we will see how
74 a length scale of the same size comes about as the compactification scale of six extra dimensions
75 originated from TeV-scale gauge-gravity unification. Considering this as the picture above Λ_{BZ} , $N(t)$
76 can be interpreted as an effective quantity, revealed below Λ_{BZ} upon integrated-over extra dimensions.
77 This scenario, as we will see, accurately explains the factor 10^{120} in the MM action, as well.

78 3. Bjorken-Zeldovich scale from large extra dimensions

79 In this section, we draw attention to an interesting incident regarding the onset of the scale
80 of 10^{-12} cm from six compact extra (spatial) dimensions originated from TeV-scale gauge-gravity
81 unification. If one imposes gauge-gravity unification at the TeV scale, the weakness of gravitational
82 interactions can be explained via the existence of compact extra dimensions, large compared to the
83 (inverse) TeV-scale [20–26].

84 For two test objects placed within a distance $r \gg R$, the gravitational potential becomes $V(r) \sim$
85 $\frac{m_1 m_2}{M_U^{n+2} R^n} \frac{1}{r}$, where M_U is the unification scale of gauge and gravitational interactions, and R is the
86 compactification scale of the extra dimensions. Imposing the requirement to get the right (reduced)
87 Planck mass through the identification $M_U^{n+2} R^n = \bar{M}_{Pl}^2$, and assuming $M_U \sim 1$ TeV, we obtain

$$88 \quad R \sim \frac{l_U^{1+2/n}}{l_{Pl}^{2/n}} \cong 2.7 \times 10^{30/n-17} \text{ cm}, \quad (7)$$

89 where l_U and l_P are corresponding length scales for the TeV-scale and (reduced) Planck masses,
90 respectively. As can be seen in Eq. (7), for $n=6$, we have

$$91 \quad R \sim 2.7 \times 10^{-12} \text{ cm} \cong l_{BZ}, \quad (8)$$

92 a remarkable agreement with the Bjorken-Zeldovich length scale, given in Eq. (6), revealed in the MM
93 formalism, discussed previously.

94 Remarkably, existence of six extra spatial dimensions compactified on a scale of 10^{-12} cm in
95 the MM framework, as also noted in Ref. [1], could also explain the factor $M_{Pl}^2/64\pi H_0^2 = 10^{120}$,
96 multiplying both the MM action given in Eq. (1) and the GB term in the action given in Eq. (2).

Since we take $M_U \sim 1$ TeV and the LHC has not observed anything new at around this energy scale, one wonders then how robust is the numerical agreement, given in Eq. (8), against the choice of M_U . As can be seen in Eq. (7), the outcome has some sensitivity against the value of M_U . Nevertheless, with a value of M_U up to around 10 TeV, we still get required length scale up to an order of magnitude, which is generally acceptable when a scale is under discussion, as displayed in Figure 1. If, for instance, we take $M_U \sim 5$ TeV, we have $R \cong 0.3 \times 10^{-12}$ cm. For $M_U \sim 10$ TeV, we have $R \cong 0.13 \times 10^{-12}$ cm $\sim d_{\text{proton}}$. On the other hand, for values below TeV down to the Higgs mass, R goes up to $R \cong 43 \times 10^{-12}$ cm. The current upper bound for the size of such extra dimensions, through the deviation in the effective gravitational interaction, is 0.01 cm [27].

4. A "see-saw" relation for the small cosmological constant and the neutrino mass?

In the effort to understand the smallness of the cosmological constant, several numerical relations among the energy scales have been noticed (or proposed) in the literature that mimic a see-saw-type suppression mechanism [62–77].

In the case of the existence of an energy scale of ~ 10 MeV, the relevant combination we notice is $\rho_\Lambda^{1/4} \stackrel{?}{=} M_{BZ}^3/M_{EW}^2 \sim 10^{-3}$ eV, where $M_{EW} \sim 1$ TeV. It is not straightforward to devise a realistic model yielding such a relation, since this requires a contribution in the amount of M_{BZ}^{12}/M_{EW}^8 in Lagrangian. Nevertheless, this type of terms in the context of vacuum energy density contributions may be obtained in models where the cosmological constant problem is addressed by entertaining the possibility that the universe may be stuck in a false vacuum, split from the vanishing global vacuum in the amount of the cosmological constant [70,78].

The relation $M_{BZ}^3/M_{EW}^2 \sim 10^{-3}$ eV is intriguing from this point of view as well, since it is in the vicinity of (at least one of the) the neutrino masses. Next, we will work on a hypothetical scenario just as an illustration of obtaining this combination in a model.

Consider a hidden sector with a QCD-like gauge interaction, where the symmetry group is $\mathcal{G}_h \equiv SU(N)$. Besides the corresponding gauge bosons, consider a real scalar field ϕ and a Dirac fermion F (for each family), both of which transform in some representation of \mathcal{G}_h , where the fermion does so vectorially (non-chirally). We assume that F has a confining scale of order 10 MeV and the SM fields are not charged under \mathcal{G}_h . The SM connects to the hidden sector through a portal coupling between the Higgs and the scalar ϕ . We also assume a discrete \tilde{Z}_2 symmetry that transforms F , ϕ , and the neutrinos in the following way. $F_{L(R)} \rightarrow \pm F_{L(R)}$, $\nu_{L(R)} \rightarrow \pm \nu_{L(R)}$, and $\phi \rightarrow -\phi$. Therefore, there are no mass or Yukawa-type terms (via the Standard Model Higgs) allowed at the tree level. The Yukawa-type terms involving the scalar ϕ do not contribute as mass terms at tree-level either, since we assume that $\langle \phi \rangle = 0$ so that the $SU(N)$ symmetry remains unbroken.

However, a mass term can be induced through the effective operator $\mathcal{L}^{eff} \supset \frac{c_\nu}{\Lambda^2} \bar{\nu} \nu \bar{F} F$, which is induced by integrating out the scalar field. We assume that a condensate forms, due to the possible nonperturbative characteristic of the $SU(N)$ vacuum, at $\sim M_{BZ}$, breaking the \tilde{Z}_2 symmetry; $\langle \bar{F} F \rangle \sim f^3 \sim (a \cdot M_{BZ})^3$, $\Lambda \simeq m_\phi \sim \Lambda_{EW} \sim 1$ TeV. The scalar ϕ gets its mass via the portal coupling to the SM Higgs, *i.e.* $\lambda \phi^2 H^\dagger H$, which justifies its electroweak-scale mass. The extra fermion F acquires its mass via coupling to the condensate through the corresponding dimension-6 operator that yields a mass value on the order of the neutrino mass. The effective neutrino mass in this scenario becomes $m_\nu = (c_\nu a^3) M_{BZ}^3/M_{EW}^2 \lesssim 10^{-2}$ eV, provided that $c_\nu a^3 \lesssim 10$.

5. More on the relevance in cosmology

A scale around 10 MeV might be relevant also in terms of the thermal history of the universe. It is an energy scale considerably close to $T \sim 1 - 2$ MeV below which the weak interactions freeze out; the reaction rate $\Gamma \sim G_F^2 T^5$ drops below the expansion rate $H \sim \sqrt{g^*} T^2/M_{Pl}$, where g^* is given as $g^* = g_b + (7/8)g_f$ and g_b (g_f) denotes the total number of the effective bosonic (fermionic) degrees of freedom at around the background temperature T . Consequently, primordial neutrinos and possibly cold dark matter -if it exists- decouple from the rest of the matter, and the ratio of neutrons

145 to protons freezes out. Any increase from the known radiation density would bring forward the
 146 Big Bang Nucleosynthesis (BBN) and hence would cause a larger Helium abundance in the present
 147 universe [30–32]. Therefore, if there is some unrevealed physics associated with such a scale of 10 MeV,
 148 there may have direct implications on our understanding of BBN, which is consistent with the current
 149 observations on the primordial abundances of light elements.

150 Since the effective MM action in 4D reveals the Einstein gravity with a cosmological constant and
 151 the GB term that does not have any effects in the equations of motion in 4D, the formalism at first
 152 sight only defines the graviton. This seemingly does not cause any problem in terms BBN since the
 153 gravitational interaction rate, as well known, is significantly suppressed compared to the expansion
 154 rate, *i.e.* $\Gamma \sim G_p^2 T^5 \ll H$, at $T \sim 1$ MeV. However, this is the case only if there is no any other relevant
 155 degrees of freedom obtained from the original action based on $SO(4,1)$, in addition to the terms given
 156 in Eq. (2). Recall that the generalized connection A_μ^{AB} living in $SO(4,1)$ has 40 components. As also
 157 mentioned in Ref. [1], one may wonder whether some of these degrees of freedom can be identified
 158 with the (bosonic) degrees of freedom of the SM². Then, several leftover terms may possibly define
 159 additional light degrees of freedom. One may expect at first that the relevant interactions are supposed
 160 to be suppressed, similar to the case with gravitons. However, one should not forget the enormous
 161 factor of 10^{120} multiplying the action in 4D, possibly arisen due to the integrated over extra dimensions.
 162 If such identifications related to the SM are possible, then it is probably because of this large factor,
 163 and the same factor may amplify some interactions regarding these new light degrees of freedom,
 164 making them interact frequently enough to be in equilibrium at $T \sim 1$ MeV. Then, the model would be
 165 in tension with the constraints coming from BBN.

166 6. Summary and Discussions

167 In this paper, we aim to bring into attention the possibility of a gravitational scale at around ~ 10
 168 MeV, or 10^{-12} cm, induced in the (MM) extension to the Einstein-Cartan formalism in 4D, due to the
 169 topological GB term in the action, as suggested by Bjorken [1].

170 *First*; we point out that a scale of the same size, 10^{-12} cm, naturally comes about in the context of
 171 large extra dimensions, originated from TeV-scale quantum gravity, as the compactification scale, if the
 172 number of extra spatial dimensions is set to six. Apparently, these two approaches can be combined,
 173 where the 4-dimensional MM formalism is the effective theory after the six extra dimensions are
 174 integrated over, which also explains the factor 10^{120} in the MM action. *Second*; we discuss that
 175 existence of such a scale may play a role in the smallness of the cosmological constant and the neutrino
 176 mass; to this end, we refer to some toy models as illustrations that generate a seesaw-type suppression
 177 mechanism. Note that we do not claim in this paper that this mechanism can directly be accommodated
 178 in the MM formalism. Rather, the main intention of this paper should be taken as an attempt to point
 179 out several coincidences regarding a scale at around 10 MeV, or 10^{-12} cm; the possibility that they are
 180 non-accidental deserves attention. Finally, we comment on possible implications in cosmology in the
 181 context of Big Bang Nucleosynthesis.

182 We note that if Nature contains six extra dimensions with a compactification scale of 10^{-12} cm, it
 183 raises the question why no Kaluza-Klein excitations with masses with a starting value of ~ 10 MeV
 184 have been observed so far. However, their elusiveness would not be unanticipated since these modes
 185 are expected to be relatively suppressed [92]. Note also that existence of six compact extra dimensions
 186 is quite familiar from the string theory perspective. [93–95].

187 If the Bjorken-Zeldovich scale is associated with the gravitational sector only, it is conceivable
 188 that it has not been uncovered yet. In fact, for a possible extra-dimension connection where the effects
 189 are experienced only through gravity, it is necessary to probe the effective gravitational interaction at
 190 distance scales smaller than 10^{-12} cm, which is far from being reached at present [27].

² See, for instance, Refs. [79–91] for various geometric approaches to the Standard Model and beyond.

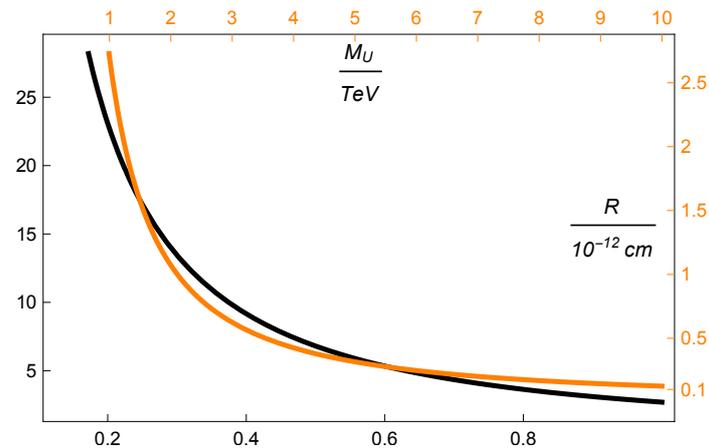


Figure 1. Compactification radius R vs. quantum gravity scale M_U in the case of six large extra dimensions. The black and orange plots denote the cases in which M_U takes values below and above 1 TeV, respectively.

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