

1 Article

2

A three phase model characterizing low-velocity impact

3

response of SMA reinforced composites under

4

vibrating boundary condition

5 Mengzhou Chang ¹, Fangyun Kong ¹, Min Sun ¹ and Jian He ^{1,*}6 ¹ College of Aerospace and Civil Engineering, Harbin Engineering University, Harbin 150001, China;
7 changmengzhou@hrbeu.edu.cn; kongfangyun@hrbeu.edu.cn; sunmin@hrbeu.edu.cn8 * Correspondence: hejian@hrbeu.edu.cn; Tel.: +86 0451 8252 5056

9

10 **Abstract:** Structural vibration induced by dynamic load or natural vibration is a nonnegligible
11 factor in failure analysis. Based on vibrating boundary condition, impact resistance of shape
12 memory alloy reinforced composites is investigated. In this investigation, modified Hashin's
13 failure criterion, Brinson's model and visco-hyperelastic model are implemented into the numerical
14 model to characterizing the mechanical behavior of glass fiber/epoxy resin laminates, SMAs and
15 interphase, respectively. First, fixed boundary condition is maintained in simulation to verify the
16 accuracy of material parameters and procedures by comparing with experimental data. Then, a
17 series of vibrating boundaries with different frequencies and amplitudes are applied during the
18 simulation process to reveals the effect on impact resistances. The statistics of absorbed energy and
19 contact force indicate that impact resistance of the composite under high frequency and large
20 amplitude is lower than that under low frequency and small amplitude, and summarized by a
21 mathematical expression.22 **Keywords:** SMA reinforced composite; low-velocity impact; vibrating boundary; numerical
23 analysis

24

25

1. Introduction

26 Fiber reinforced composite materials have been widely used and investigated in recent years
27 due to their unique properties, such as high stiffness, high strength and low density [1]. However,
28 the applications of the composites are limited by their weak impact resistance, especially for the
29 unidirectional cross-ply fiber/matrix laminates. Obviously, the reason for this disadvantage can be
30 traced to the low strength, stiffness between the adjacent layers which will result in delamination
31 and fiber breakage [2].32 This disadvantage can be overcome by changing the composition of material: for example,
33 using short fiber instead of long fiber. The mechanical properties of short fiber reinforced composites
34 are strongly influenced by the manufacturing process, such as injection position, sample geometry,
35 pressure and temperature during the molding process [3-4]. Other factors, such as fiber location,
36 length, diameter and orientation that affect the mechanical properties, have been studied by
37 Thomason [5-7], Wang [8] and Mallick [9] investigate the tensile properties of composite machined
38 with different angles to mould flow direction (MFD). The results indicate that tensile strength (and
39 elastic modulus) of samples machined perpendicular to MFD are nearly 40% lower than that of
40 samples machined parallel to MFD.41 Functional materials and new structures have been used to improve the mechanical property of
42 the composites. Shape memory alloys (SMAs) have been embedded between the adjacent layers for
43 the reinforcement purpose considering their unique properties: changing shape (elastic modulus) in

44 accordance with temperature and stress [10-11]. In our previous research, the effect of SMAs
 45 positions on damage behavior and impact response (including peak force, displacement and energy)
 46 of laminates subjected to low-velocity impact have been investigated [12]. In Khalili's research, the
 47 effect of SMAs type (wires, plates, strips, tubes or layers) on dissipation of the impact energies has
 48 been studied [13-14]. According to Shariyat's research, a higher order global-local hyperbolic plate
 49 theory aimed at studying the asymmetric displacement fields has proposed, the related results
 50 indicate that SMAs have the ability to change shape, to repair damage and to improve impact
 51 property of composites [15-16]. The new structure bounding of the conventional fiber reinforced
 52 polymer with metal layer is also found has the advantage when subjecting to impact loading [17].
 53 Other structures, such as particle reinforced [18-19], sandwich plate [20-21] and 3D fabric [22-23]
 54 have been developed in recent years.

55 We note that the fixed boundary is widely used in the experimental analysis. Except for low and
 56 high velocity impact, the mechanical properties of composite under eccentric impact [24],
 57 multi-impact [25] and compression-after-impact [26] has received much attention due to engineering
 58 practices. However, the vibrating boundary condition, which is unavoidable when membrane
 59 structures subjected to impact [27] or wind flow [28], has rarely discussed due to the difficulties
 60 associated with real experimental condition. Only vibration response during impact and after
 61 impact can be observed to evaluate the impact resistance and damage state [29-30]. In Pérez MA's
 62 work, damage of CFRP induced by low velocity impact and its effect on vibration response is
 63 investigated by a micro-mechanical approach [31].

64 The interfacial debonding between SMA and the matrix, one common failure model, is still a
 65 key problem in the composites [32]. As reported in our previous work, a three phase model-matrix,
 66 reinforcer and interphase is introduced to evaluating the mechanical behavior of fiber reinforced
 67 composite [33]. In this paper, this model is further developed to matching the SMA reinforced
 68 composites. Based on this model, the effect of vibration on impact resistance of SMAs reinforced
 69 composites is mainly investigated through a series of frequencies and amplitudes.

70 2. The three phase model

71 2.1. Material property of glass fiber reinforced composites

72 Glass fiber and the matrix used in this paper are regarded as homogeneous isotropic materials
 73 on microscale. The constitutive model can be expressed as:

$$74 \quad \sigma_{ij} = 2G\varepsilon_{ij} + \lambda\varepsilon_{kk}\delta_{ij} \quad (1)$$

75 where ε_{ij} and σ_{ij} are nominal strain and stress in case of $i = j$, shear strain and shear stress in
 76 case of $i \neq j$ ($i, j = x, y$ and z are the reference coordinates X), respectively. $\lambda = E\mu/(1+\mu)(1-2\mu)$ is
 77 the Lame's constant and $G = E/2(1+\mu)$ is the shear modulus, E and μ are the elastic modulus and
 78 Poisson's ratio, $\varepsilon_{kk} = \varepsilon_{xx} + \varepsilon_{yy} + \varepsilon_{zz}$ is the volumetric strain, δ_{ij} is the Kronecker delta ($\delta_{ij} = 1$ if
 79 $i = j$, and $\delta_{ij} = 0$ otherwise). The relationship between displacement field u_i and strain field ε_{ij}
 80 defined as:

$$81 \quad \varepsilon_{ij} = \frac{1}{2} \left(\frac{\partial u_i}{\partial X_j} + \frac{\partial u_j}{\partial X_i} \right) \quad (2)$$

82 However, it's quite unrealistic modeling each glass fiber in ABAQUS considering the
 83 complexity. A ply including glass fiber and epoxy resin is modeled as anisotropic material. The
 84 stacking sequence is $[0^\circ, 90^\circ]_8$, and the total thickness of the sample is about 3.2 mm (0.2 mm for each
 85 ply). The related material parameters are obtained according to the experimental work of our
 86 research group [12], as shown in Table 1.

88 For the undamaged state, the stress strain relationship can be rewritten as:

$$89 \quad \left\{ \begin{array}{c} \sigma_{11} \\ \sigma_{22} \\ \sigma_{33} \\ \sigma_{23} \\ \sigma_{31} \\ \sigma_{12} \end{array} \right\} = \left[\begin{array}{cccccc} c_{11} & c_{12} & c_{13} & 0 & 0 & 0 \\ c_{21} & c_{22} & c_{23} & 0 & 0 & 0 \\ c_{31} & c_{32} & c_{33} & 0 & 0 & 0 \\ 0 & 0 & 0 & c_{44} & 0 & 0 \\ 0 & 0 & 0 & 0 & c_{55} & 0 \\ 0 & 0 & 0 & 0 & 0 & c_{66} \end{array} \right] \left\{ \begin{array}{c} \varepsilon_{11} \\ \varepsilon_{22} \\ \varepsilon_{33} \\ \varepsilon_{23} \\ \varepsilon_{31} \\ \varepsilon_{12} \end{array} \right\} \quad (3)$$

90 where c_{ij} are the stiffness coefficients which can be derived from G and λ . At elastic state, the
91 specified damage variables d_i are equal to 0.

92 The 3D Hashin's failure criterion accounting for fiber failure and matrix failure is embedded in
93 ABAQUS using subroutine-VUMAT. In this part, two failure models, tensile failure and
94 compressive failure, are mainly considered for the fiber with related damage variables which can be
95 expressed as [34-35]:

$$96 \quad \text{Tensile failure of fiber, } d_{fi} = 1 : \begin{cases} \left(\frac{\sigma_{11}}{X_T} \right)^2 + \left(\frac{\sigma_{12}}{S_{12}} \right)^2 + \left(\frac{\sigma_{13}}{S_{13}} \right)^2 \geq 1 \\ \sigma_{11} > 0 \end{cases} \quad (4)$$

$$97 \quad \text{Compressive failure of fiber, } d_{fc} = 1 : \begin{cases} \left| \frac{\sigma_{11}}{X_C} \right| \geq 1 \\ \sigma_{11} < 0 \end{cases} \quad (5)$$

98 where d_{fi} and d_{fc} are the damage variables for evaluating tensile failure and compressive damage
99 of fiber, respectively.

100 Similarly, tensile failure and compressive failure of the matrix can also be obtained [34-35].

$$101 \quad \text{Tensile failure of matrix, } d_{mt} = 1 : \begin{cases} \left(\frac{\sigma_{11}}{X_T} \right)^2 + \left(\frac{\sigma_{12}}{S_{12}} \right)^2 + \left(\frac{\sigma_{22}}{Y_T Y_C} \right)^2 + \frac{\sigma_{22}}{Y_T} + \frac{\sigma_{22}}{Y_C} \geq 1 \\ \sigma_{22} + \sigma_{33} > 0 \end{cases} \quad (6)$$

$$102 \quad \text{Compressive failure of matrix, } d_{mc} = 1 : \begin{cases} \left(\frac{\sigma_{11}}{X_T} \right)^2 + \left(\frac{\sigma_{12}}{S_{12}} \right)^2 + \left(\frac{\sigma_{22}}{Y_T Y_C} \right)^2 + \frac{\sigma_{22}}{Y_T} + \frac{\sigma_{22}}{Y_C} \geq 1 \\ \sigma_{22} + \sigma_{33} < 0 \end{cases} \quad (7)$$

103 where d_{mt} and d_{mc} are the damage variables for evaluating tensile failure and compressive
104 damage of matrix, respectively.

105 Using the damage variables and related parameters, the stress is decreased linearly to zero once
106 the failure criterion is reached. The stiffness coefficients obtained from Equation 3 should be
107 re-calculated during damage process, as:

$$108 \quad c_{11} = E_1 (1 - \nu_{23} \nu_{32}) \Gamma (1 - d_f) \quad (8a)$$

$$109 \quad c_{22} = E_2 (1 - \nu_{13} \nu_{31}) \Gamma (1 - d_f) (1 - d_m) \quad (8b)$$

$$110 \quad c_{33} = E_3 (1 - \nu_{12} \nu_{21}) \Gamma (1 - d_f) (1 - d_m) \quad (8c)$$

$$111 \quad c_{12} = E_1 (\nu_{21} + \nu_{31} \nu_{23}) \Gamma (1 - d_f) (1 - d_m) \quad (8d)$$

$$112 \quad c_{23} = E_2 (\nu_{32} + \nu_{12} \nu_{31}) \Gamma (1 - d_f) (1 - d_m) \quad (8e)$$

$$113 \quad c_{13} = E_1 (\nu_{31} + \nu_{21} \nu_{32}) \Gamma (1 - d_f) (1 - d_m) \quad (8f)$$

$$114 \quad c_{44} = E_{12} (1 - d_f) (1 - s_{mt} d_{mt}) (1 - s_{mc} d_{mc}) \quad (8g)$$

$$115 \quad c_{55} = E_{23} (1 - d_f) (1 - s_{mt} d_{mt}) (1 - s_{mc} d_{mc}) \quad (8h)$$

116
$$c_{66} = E_{13} (1 - d_f) (1 - s_{mt} d_{mt}) (1 - s_{mc} d_{mc}) \quad (8i)$$

117
$$\Gamma = 1 / (1 - v_{12} v_{21} - v_{23} v_{32} - v_{13} v_{31} - 2 v_{21} v_{32} v_{13}) \quad (8j)$$

118 where s_{mt} and s_{mc} are the factors that control the reduction in shear stiffness according to tensile
 119 and compressive failure, respectively. Parameters $d_f = 1 - (1 - d_{f1})(1 - d_{fc})$ and $d_m = 1 - (1 - d_{mt})$
 120 $(1 - d_{mc})$ are the global damage variables characterizing fiber and matrix, respectively.

121 **Table 1.** Material parameters of unidirectional glass fiber/epoxy composite laminates.

Mechanical constants	Values
Young's modulus/GPa (E_1, E_2, E_3)	55.2, 18.4, 18.4
Poisson's ratio (v_{12}, v_{13}, v_{23})	0.27, 0.27, 0.43
Shear modulus/GPa (E_{12}, E_{13}, E_{23})	13.8, 13.8, 13.8
Ultimate tensile stress/MPa (X_T, Y_T, Z_T)	1656, 73.8, 73.8
Ultimate compressive stress/MPa (X_C, Y_C, Z_C)	1656, 91.8, 91.8
Ultimate shear stress/MPa (S_{12}, S_{13}, S_{23})	117.6, 117.6, 117.6

122

2.2. Material property of SMA

123 A similar stress-strain relationship as Equation 1 can be observed if the reinforced material is
 124 isotropic elastic, i.e. glass fiber or carbon fiber. Differently, the constitutive model of SMA is sensitive
 125 to temperature and stress state. Brinson's model [36-37] has been referred to most often in
 126 subsequent studies due to its high accuracy. Here, the constitutive law of SMA based on energy
 127 balance equations is derived considering the effect of temperature T and phase conversion between
 128 martensite and austenite can be denoted as:

129
$$d\sigma_s = E_s (\varepsilon_s, \xi, T) d\varepsilon_s + \Omega (\varepsilon_s, \xi, T) d\xi + \Theta (\varepsilon_s, \xi, T) dT \quad (9)$$

130 where σ_s and ε_s are the Piola-Kirchhoff stress and Green strain of SMA; ξ is the martensite
 131 fraction characterizing the phase conversion; also, E_s , Ω and Θ are the elastic modulus,
 132 transformation coefficient and thermal coefficient, respectively.

133 Some assumptions are taken to simplify the application, as follows: the elastic modulus D is a
 134 function of martensite fraction ξ ; the transformation coefficient is also connected with elastic
 135 modulus:

136
$$E_s (\varepsilon, \xi, T) = E_s (\xi) = E_a + \xi (E_m - E_a) \quad (10a)$$

137
$$\Omega (\xi) = -\varepsilon_L E (\xi) \quad (10b)$$

138 where E_m and E_a are elastic moduli for martensite and austenite state, ε_L is the maximum
 139 residual strain. The equations of the phase conversion between martensite and austenite can be
 140 expressed as (determined by temperature and stress):

141 (a) Phase conversion to martensite

142 If $T > M_s$ and $\sigma_s^{cr} + C_m (T - M_s) < \sigma < \sigma_f^{cr} + C_m (T - M_s)$

143
$$\xi_s = \frac{1 - \xi_{S_0}}{2} \cos \left\{ \frac{\pi}{\sigma_s^{cr} - \sigma_f^{cr}} (\sigma - \sigma_f^{cr} - C_m (T - M_s)) \right\} + \frac{1 + \xi_{S_0}}{2} \quad (11a)$$

144
$$\xi_T = \xi_{T_0} - \frac{\xi_{T_0}}{1 - \xi_{S_0}} (\xi_s - \xi_{S_0}) \quad (11b)$$

145 If $T < M_s$ and $\sigma_s^{cr} < \sigma < \sigma_f^{cr}$

146
$$\xi_s = \frac{1 - \xi_{S_0}}{2} \cos \left\{ \frac{\pi}{\sigma_s^{cr} - \sigma_f^{cr}} (\sigma - \sigma_f^{cr}) \right\} + \frac{1 + \xi_{S_0}}{2} \quad (11c)$$

147
$$\xi_T = \xi_{T_0} - \frac{\xi_{T_0}}{1 - \xi_{S_0}} (\xi_S - \xi_{S_0}) + \Delta_{T\xi} \quad (11d)$$

148
$$\Delta_{T\xi} = \begin{cases} \frac{1 - \xi_{T_0}}{2} [\cos(a_m(T - M_f)) + 1] & : M_f < T < M_s \text{ and } T < T_0 \\ 0 & : \text{else} \end{cases} \quad (11e)$$

149 (b) Phase conversion to austenite

150 If $T > A_s$ and $C_a(T - A_f) < \sigma < C_a(T - A_s)$

151
$$\xi = \frac{\xi_0}{2} \left\{ \cos \left[a_a \left(T - A_s - \frac{\sigma}{C_a} \right) \right] + 1 \right\} \quad (12a)$$

152
$$\xi_S = \xi_{S_0} - \frac{\xi_{S_0}}{\xi_0} (\xi_0 - \xi) \quad (12b)$$

153
$$\xi_T = \xi_{T_0} - \frac{\xi_{T_0}}{\xi_0} (\xi_0 - \xi) \quad (12c)$$

154 In Equation 11 and 12, parameters $a_m = \pi/(M_s - M_f)$ and $a_a = \pi/(A_f - A_s)$. Four important
155 parameters are introduced to characterize the phase conversion temperature: M_s - start temperature of
156 martensite phase; M_f - finish temperature of martensite phase; A_s - start temperature of austenite
157 phase; A_f - finish temperature of austenite phase. The stress-strain relationship at arbitrary
158 temperature T can be obtained according to Equation 9-12. Debonding behavior of interface between
159 SMA and matrix is carried out by use of cohesive zone model.

160 2.3. *Material property of interphase*

161 According to our research in glass fiber reinforced polymer composite, the time (or strain rate)
162 depended stress-strain relationship has been investigated [33]. In this model, the time effect is
163 existing in interphase part and can be expressed as stress $\sigma_{m,ij}(t)$ to strain relationship experiencing
164 some continuous strain history given by the function $\sigma_{m,ij}(t)$:

165
$$\sigma_{in}(t) = \int_0^t g(t-t') \dot{\varepsilon}_m(t') dt' \quad (13)$$

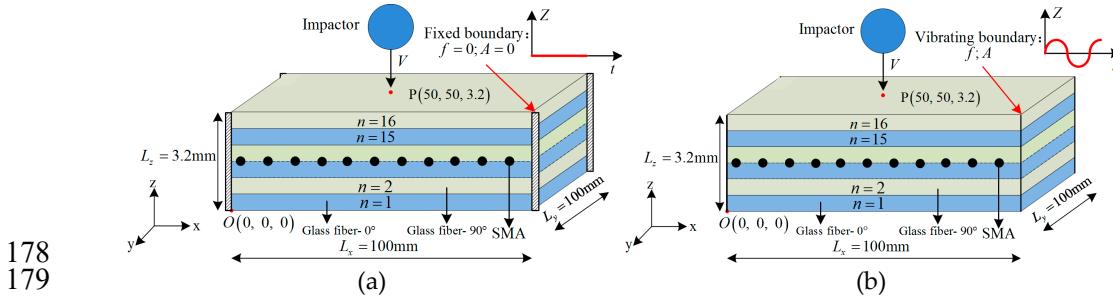
166 where g is the relaxation modulus, t' is new time variable, $\dot{\varepsilon}_m(t) = \partial \varepsilon_{in}(t') / \partial t'$ is strain rate. The
167 strain history $\varepsilon_m(t)$ can be obtained using Boltzmann superposition principle. Also, the relaxation
168 modulus g can be expressed using discrete relaxation spectrum as follows:

169
$$g(t) = g_0 + \sum_{i=1}^n g_i e^{-\frac{t}{t_i}} \quad (14)$$

170 where g_0 , g_i and t_i are the related parameters which can be obtained from relaxation tests. The
171 parameters in interphase part can be obtained by fitting with tensile or pull-put tests with different
172 loading speed [33].

173 2.4. *Boundary condition*

174 For a sample with dimension $L_x \times L_y \times L_z = 100\text{mm} \times 100\text{mm} \times (n \times 0.2\text{mm})$, $n=16$ for our
175 experiment, the fixed boundary is employed to investigate the accuracy of numerical model.
176 Frequency and amplitude of the four boundaries $x=0$, $x=L_x$, $y=0$ and $y=L_y$ are zero, and
177 shown in Figure 1a.



178
179 **Figure 1.** Schematic of the SMA reinforced composite samples: (a) Fixed boundary; (b) Vibrating
180 boundary.
181

182 As for vibrating boundary condition (Figure 1b), the movement of the boundaries can be
183 expressed as:

$$z = A \sin(2\pi ft) \quad (13)$$

184 where A and f are the amplitude and the frequency of the vibration, respectively.

185 3. Effect of fixed boundary condition on impact resistance

186 The fixed boundary condition is kept in the experiment process.

187 3.1. Composite laminates

188 Simulation results of the model under fixed boundary are investigated and compared with the
189 experimental results. Stacking sequence of the laminate is $[0^\circ, 90^\circ]_8$, 0° and 90° are the layer angles
190 (glass fiber) to X- direction. The sample ($L_x \times L_y \times L_z = 100\text{mm} \times 100\text{mm} \times 3.2\text{mm}$) is impacted by a rigid
191 half ball-cylinder on the center of the top surface, as shown in Figure 1. Diameter of the half ball of
192 the impactor is 14mm, and the mass of the impactor is 8kg (Steel). Two impact energies are
193 considered in the tests, 32J and 64J, and the corresponding impact velocities are 2.83m/s and 4m/s.
194 The SMA wires (diameter 0.2mm) have been embedded in the middle layer laminate (between layer
195 8 and 9) with distance 5mm (total 21 wires for a model). In summary, four types of experiments have
196 been conducted, as shown in Table 2.
197

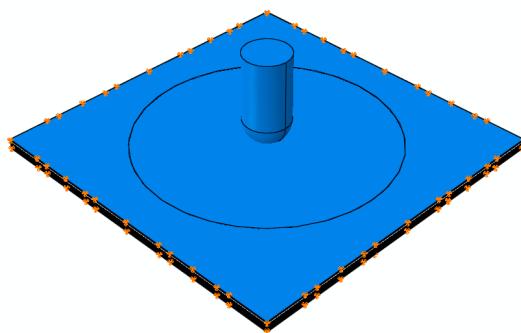
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Table 2. Experimental groups.

	Stacking sequence	Impact energy/J
Group A1	$[0^\circ, 90^\circ]_8$	32
Group A2	$[0^\circ, 90^\circ]_8$	64
Group A3	$[(0^\circ, 90^\circ)_4, \text{SMA}, (0^\circ, 90^\circ)_4]$	32
Group A4	$[(0^\circ, 90^\circ)_4, \text{SMA}, (0^\circ, 90^\circ)_4]$	64

200 3.2. Simulation result: damage state during impact process

201 The model has been built in ABAQUS according to the details mentioned before, as shown in
202 Figure 2.



203
204
205
Figure 2. Modeling the impact test of the composite laminate.

206 **Group A1:** The simulation results of the composite laminate under impact at different time ($0.0015 \text{ s} < t < 0.009 \text{ s}$) are shown in Figure 3 and Figure 4. Under lower impact energy, 32J, a generally elastic
207 behavior of the composite laminate can be drawn as: deformation is increased with time t from
208 initial state ($t = 0$) to the maximum deformation ($t \approx 0.0045 \text{ s}$), then the deformation is decreased.
209

210 As shown in Figure 3, it is clear that the impactor has been bounced back by the composite
211 laminate. Three layers have been chosen to investigate the fracture morphology during impact as
212 shown in Figure 4: layer 16, 8 and 1. A hole shape damage can be founded on several layers,
213 especially layer 16, however the damage region on the layer 8 and layer 1 are smaller, as shown in
214 Figure 4.

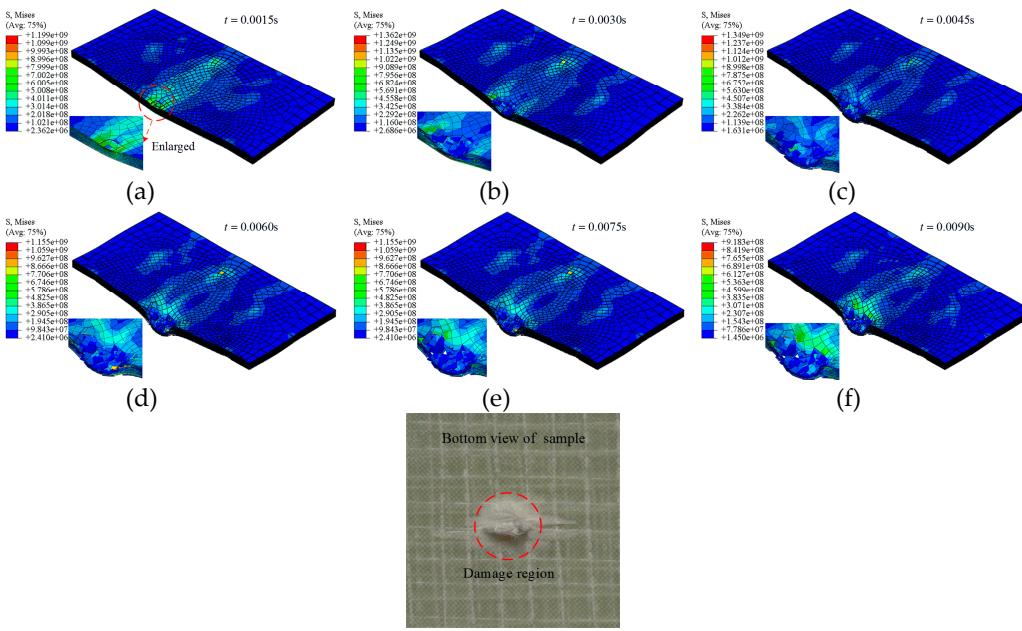
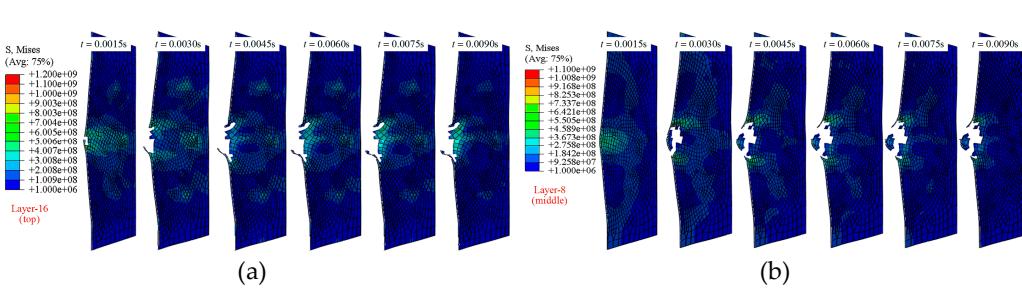
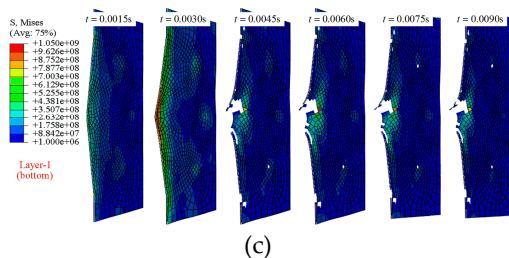


Figure 3. Simulation results of the composite laminates at different time, group A1. and the bottom view of the real sample after impact.

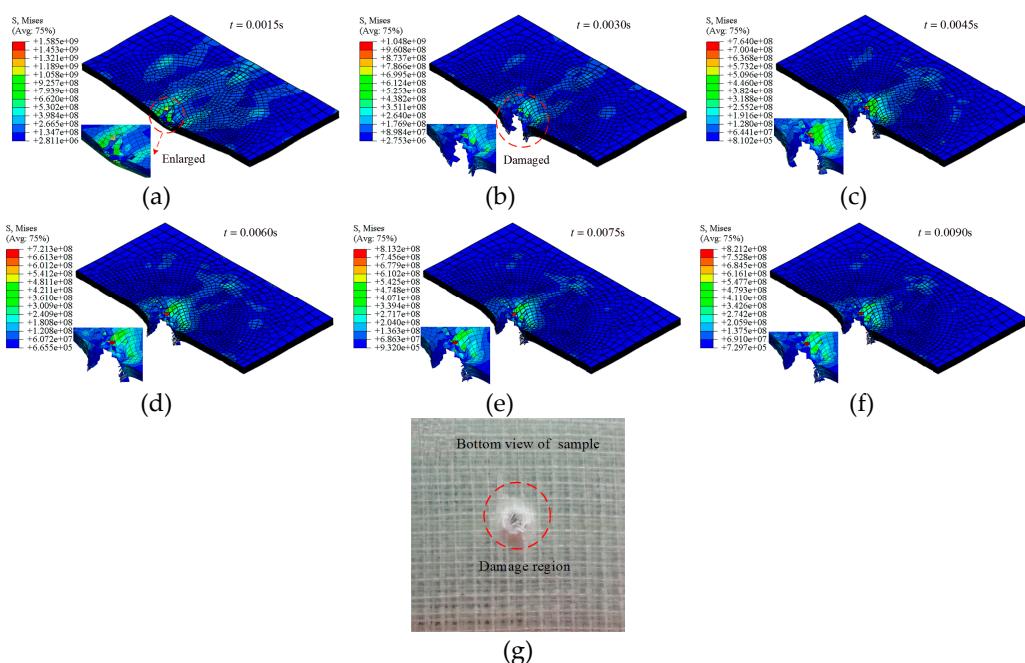




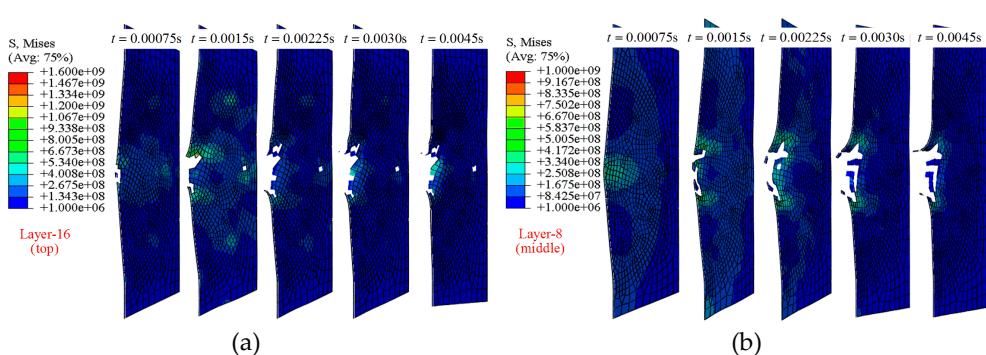
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227
228 **Figure 4.** Cross-section of different layers during impact, group A1: (a) layer 16; (b) layer 8; (c) layer
229 1.
230

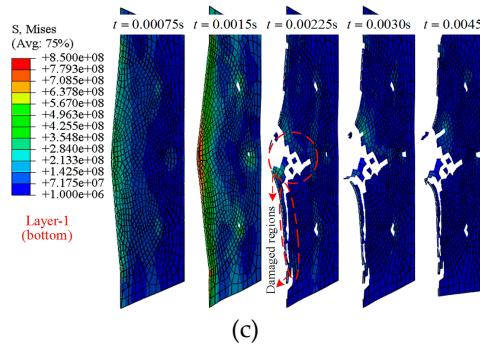
231 **Group A2:** Different from group A1, the composite is destroyed completely under higher energy,
232 64J, as shown in Figure 5. During simulation process, the impactor is founded moving along the top
233 layer to the bottom layer of the composite without bounced back ($t > 0.0030s$), this can also be
234 confirmed by the experimental result, as shown in Figure 5g. The velocity of impactor is reduced in
235 the breakdown process then remained as a constant.
236

237 From Figure 6c, a larger damage region is founded at the final state. This is different from group
238 A1, and can be explained by delamination due to the friction between impactor and layer 1 during
239 penetration.



240
241 **Figure 5.** Simulation results of the composite laminates at different time, group A2.
242
243





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Figure 6. Cross-section of different layers during impact, group A2: (a) layer 16; (b) layer 8; (c) layer 1.

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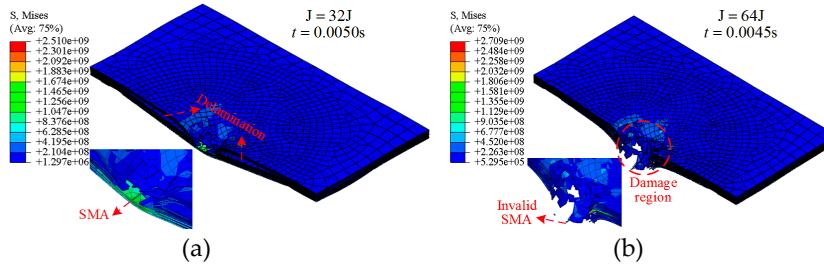
Group A3 and A4: Embedding SMA alloys is effective to improve the impact resistance of composite laminates. As shown in Figure 7a, SMAs were stretched to a larger strain in the case of 32J. In Figure 7b, a broken or invalid state of SMA is obtained due to the larger strain which is beyond critical value. More specific, 5 SMAs in the center region are chosen to demonstrate the working mechanism, as shown in Figure 8c and 8d.

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Beyond that, the defect is obviously: the damage region of layer 8 (contact with SMA) is larger than that of group A1 and A2, as shown in Figure 8a and 8b.



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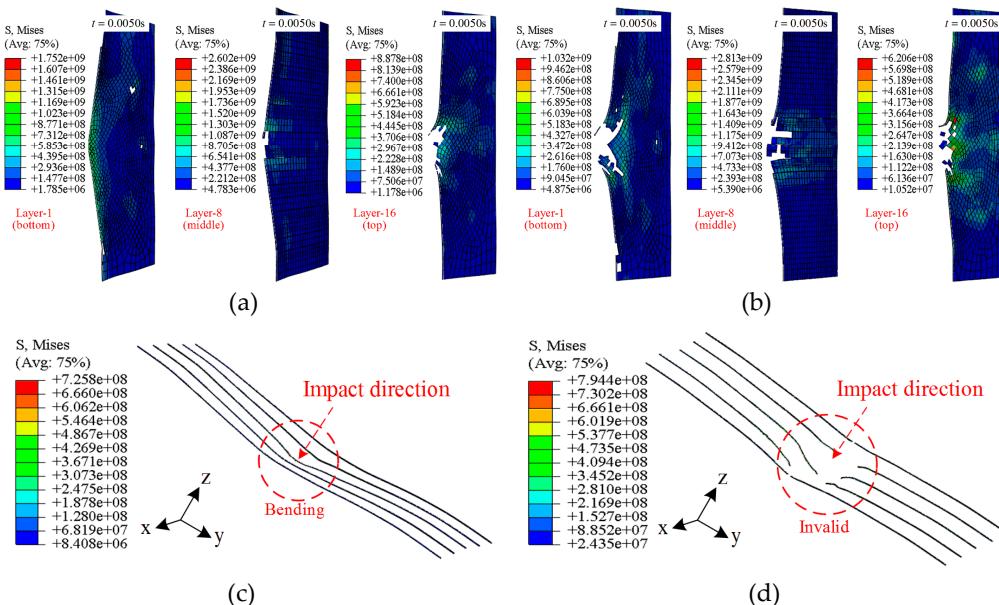
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Figure 7. Simulation results of the SMA reinforced composite laminates at different time: (a) group A3, $t = 0.0050s$; (b) group A4, $t = 0.0045s$.

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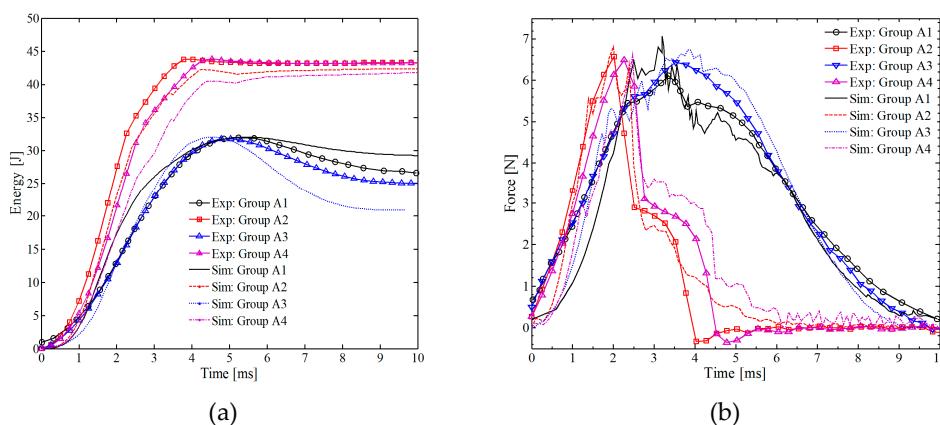
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Figure 8. Cross-section of different layers during impact: (a) layers in group A3; (b) layers in group A4; (c) SMA in group A3; (d) SMA in group A4.

274 3.3. *Simulation result: absorbed energy and contact force*

275 Two important items: absorbed energy and contact force was obtained from ABAQUS and
 276 compared with experimental data, as shown in Figure 9. The relative errors of energy comparison
 277 are 5.7%, 6.3%, 13.8% and 7.3% for groups A1-A4, respectively. As for the force-time curves, more
 278 inflection points are founded on the simulation curves due to the broken and deletion of the element.
 279 Even so, the tendency of simulation curves agrees well with the experimental curves. The relative
 280 errors for force comparison are 7.3%, 8.1%, 8.9% and 9.2% for groups A1-A4, which can be also
 281 accepted.



282
 283 **Figure 9.** Comparison between the simulation results and experimental results. (a) absorbed
 284 energy-time history; (b) contact force-time history.

287 From the comparison in this section, the accuracy of material parameters and model used in
 288 ABAQUS can be tested. Furthermore, the constitutive relationship of SMA and the processing of
 289 interface (between SMA and laminates) are also regarded as appropriate. This is the foundation of
 290 the numerical simulation in section 4.

291 4. Effect of vibrating boundary condition on impact resistance

292 The simulation of the model under vibrating boundary under impact is investigated in this
 293 section. The same model ($L_x \times L_y \times L_z = 100\text{mm} \times 100\text{mm} \times 3.2\text{mm}$, 16 layers) is impacted by a rigid half
 294 ball-cylinder with a fixed energy 32J on the center of the top layer as shown section 3.1. Two types of
 295 composites have been invested: without SMA and with SMA.

296 4.1. Effect of amplitude

297 In order to fully understand the influence of amplitude on the impact resistance, a low
 298 frequency $f = 1000$ cycle/s is kept (10 cycles during simulation process, $t_{\text{tot}} = 0.01\text{s}$). Several
 299 amplitudes, A, are chosen for the study as shown in Table 3.

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Table 3. The amplitudes and frequencies used in the simulation.

	Group B1	Group B2	Group B3	Group B4	Group B5	Group B6	Group B7	Group B8
A(m)	0.0016	-0.0016	0.0032	-0.0032	0.008	-0.008	0.016	-0.016
f(c/s)					1000			
SMA					NO			
Stacking					[0°,90°]s			
	Group C1		Group C2		Group C3		Group C4	
A(m)	0.0016		0.0032		0.008		0.016	
f(c/s)					1000			
SMA					YES			

Stacking	[(0°,90°)4, SMA, (0°,90°)4]				
	Group D1	Group D2	Group D3	Group D4	Group D5
A(m)			0.0032		
f (c/s)	100	200	500	2000	10000
SMA			NO		
Stacking			[0°,90°]8		
	Group E1	Group E2	Group E3	Group E4	Group E5
A(m)			0.0032		
f (c/s)	100	200	500	2000	10000
SMA			YES		
Stacking			[(0°,90°)4, SMA, (0°,90°)4]		

302

303 The positive value in Table 3 means that the moving direction of the boundary is on the contrary
 304 of the impactor's moving direction (+z direction) at initial state. The negative value means that
 305 moving directions of impactor and boundary are same at initial time.

306 When applying amplitude A to control the movement of the boundary, the morphologies of
 307 composite laminate after impact are shown in Figure 10 and 11. As shown in Figure 10a, the damage
 308 region is close to the size of impactor. From Figure 11b-11d, the damage regions are increased with
 309 increasing of the value of applied amplitude, however, general damage with larger area can be
 310 observed in case of A=0.016m, as shown in Figure 10d. It is interesting that the damage state is
 311 depends on the absolute value of amplitude rather than the valueness when comparing related
 312 groups, i.e. Figure 10b and Figure10f. Similar conclusion can be obtained for group C from the
 313 simulation results shown in Figure 11.

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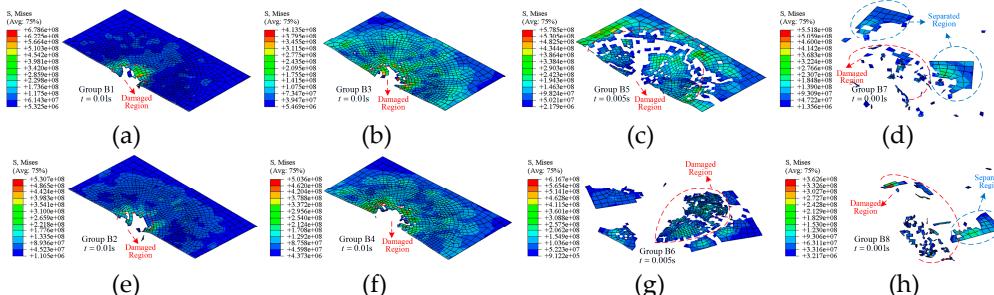
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Figure 10. Fracture morphology of the top layer of composite laminate under different amplitudes, group B.

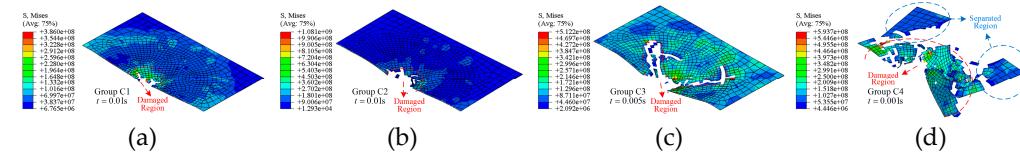
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Figure 11. Fracture morphology of top layer of composite laminate with SMA under different amplitudes, group C.

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The details of impact process of half model in group B are shown in Figure 12. From Figure 12a, a representative impact process is shown: gradually damage with time t similar with group A1. With increasing amplitude, more elements in the center region have been deleted due to the large deformation (or severe vibration), as shown in Figure 12b, 12c and 12d. As for larger amplitude, 0.016m, a larger damage region is observed, even separating from the main model, t = 0.001s, as shown in Figure 12d. More importantly, the time that the clear damage region can be observed has changed from t = 0.0002s to 0.00002s due to the relative movement. The effect of valueness of amplitudes can be further validated comparing Figure 12b and 12e.

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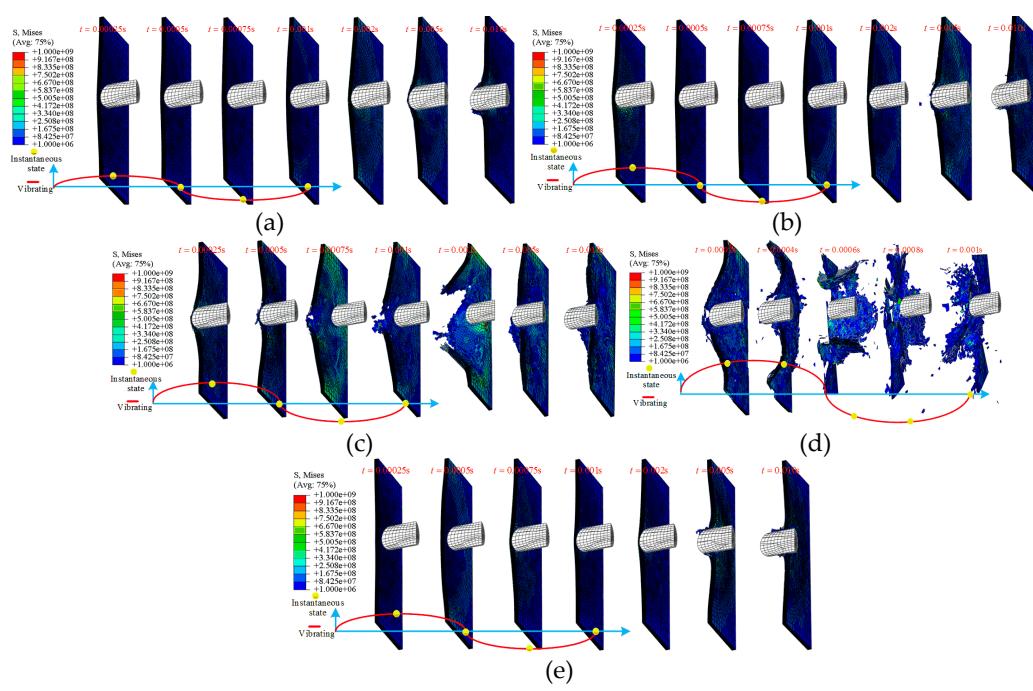
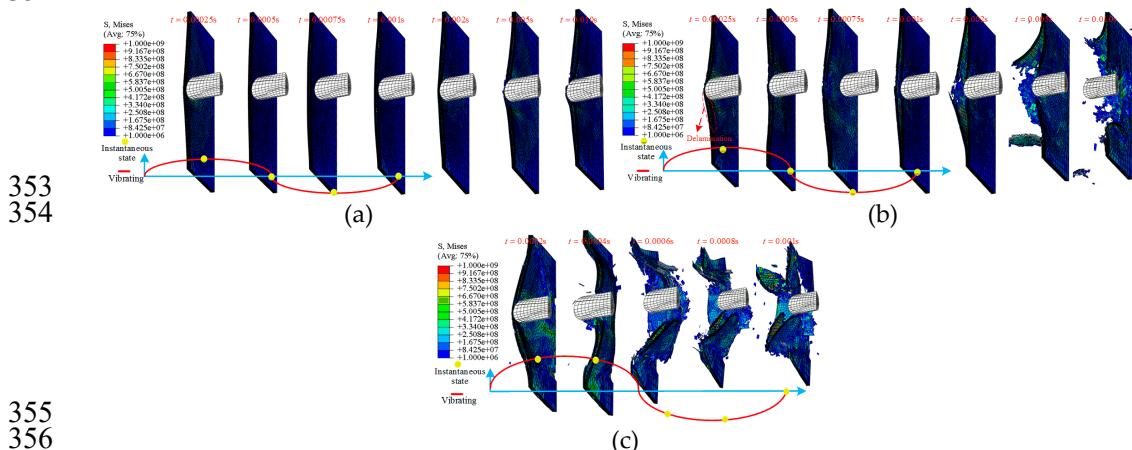
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Figure 12. Cross-section of half model during impact: (a) group B1; (b) group B3; (c) group B5; (d) group B7; (e) group B4.

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Figure 13. Cross-section of half model during impact: (a) group C2; (b) group C3; (c) group C4.



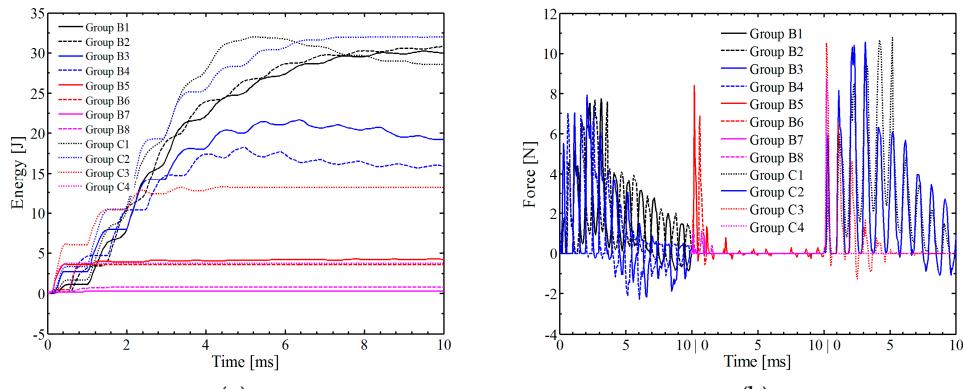


Figure 14. Analysis of the impact resistance of composite laminate in group B and C: (a) absorbed energy; (b) contact force.

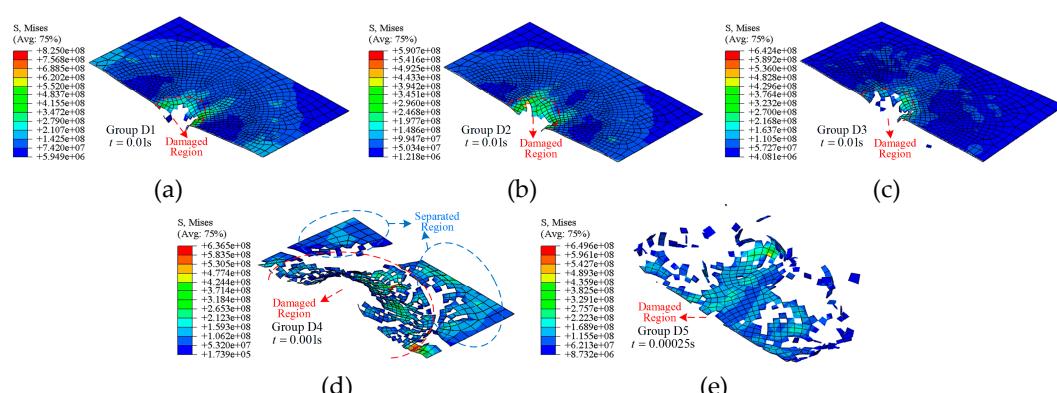
With different amplitudes, the absorbed energy and contact force are plotted against time, as shown in Figure 14. As for group B1 and C1, the maximum value of absorbed energies is 32J, however, the values at $t = 0.01s$ are 30.04J and 28.6J, respectively. With increasing the value of amplitude the absorbed energy is decreased. As for group B3, B5 and B7, the maximum value of absorbed energies are 9.24J, 4.24J and 0.30J, respectively. The maximum value of absorbed energies in related opposite direction groups, i.e. B4, B6 and B8, kept as same level and about 20% lower. As for related group C, the absorbed energy is 3.5 times larger. As for high amplitude, the absorbed energy is close to zero, as group B7 and B8. From $t = 0$ to $0.004s$, effect of vibrations on the energy-time curve can be observed, then, the energy is kept as constant, as shown in Figure 14a. In Figure 14b, the force is also affected by the vibration and shown more dramatic changes comparing with section 3.

4.2. Effect of frequency

In order to fully understand the influence of frequency on the impact resistance, a small amplitude $A = 0.0032m$ is kept considering the small influence of this level. Several frequencies, f , are chosen for the study as shown in Table 3.

During the simulation $f = 100$ cycle/s to 500 cycle/s in group D1 to D3, the damage state is close to each other at time $t = 0.01s$, only a small hole can be founded according to the fracture morphology shown in Figure 15a, 16b and 16c. As for higher frequency $f = 2000$ cycle/s to 10000 /s, the damage states show randomicity with larger area, as shown in Figure 15d and 16e.

Damage states of SMA reinforced composite laminates are shown in Figure 15f to 15j, also. Overall, the effect of frequency on the damage state is similar with that in group D. It should be noted that the separated region in group E5 maintains a more complete shape, as shown in Figure 15j.



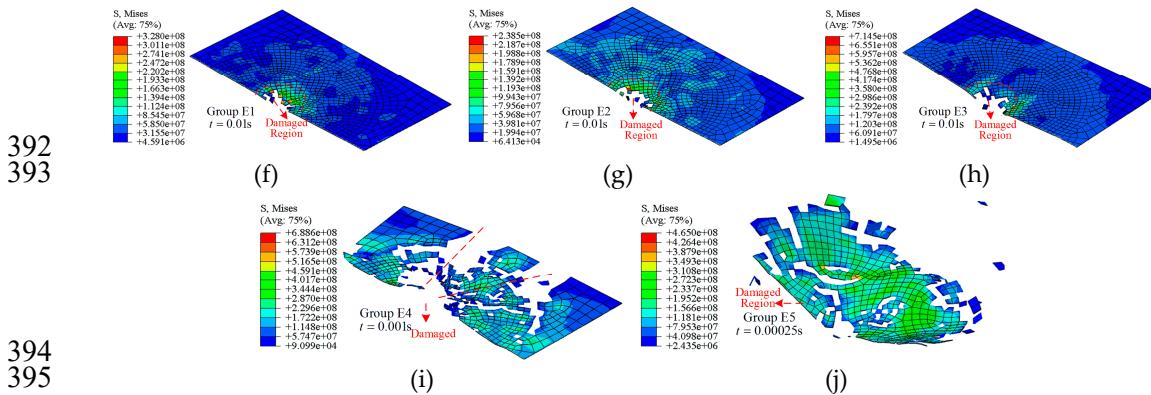


Figure 15. Fracture morphology of the top layer with different frequencies.

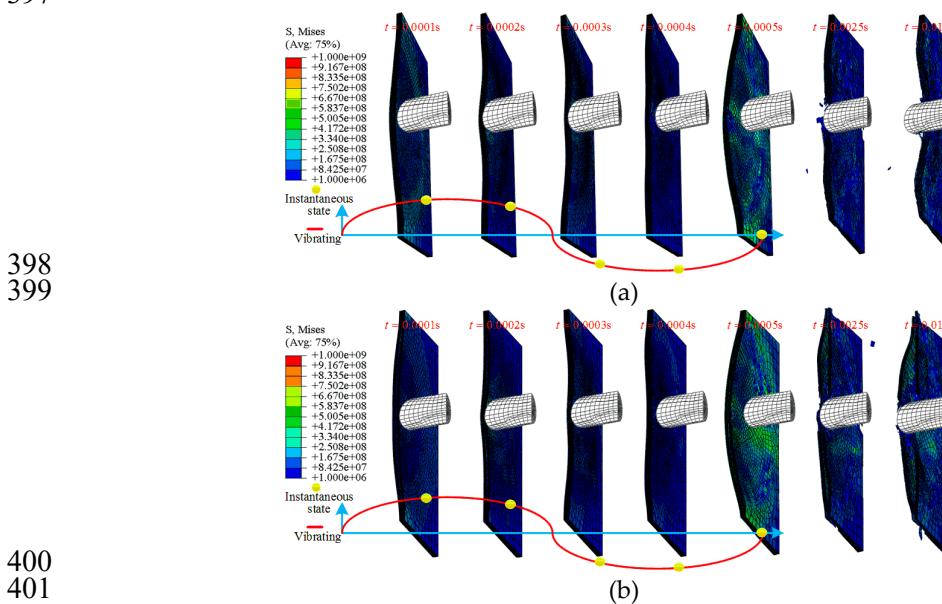


Figure 16. Cross-section of half model during impact: (a) group D4; (b) group E4.

Applying different frequencies f on the boundary, the simulation process of two groups: D5 and E5 are invested to demonstrate the details of damage morphologies of the composite laminate, as shown in Figure 16. As for D4, an hole shape damage region is gradually appear at time $t = 0.0025s$, then increased with impact process, at time $t = 0.01s$ the center region is damaged completely. As for group E5, delamination is observed except the hole shape damage, this is mainly due to the global enhancement effect of SMA.

The relationship between absorbed energy and time or contact force and time can be founded in Figure 17. As shown in Figure 17a, the absorbed energy is decreased with increasing frequency generally. Considering a real low frequency $f = 100/s$, the maximum value of absorbed energy for group D1 and E1 are same, 32J. Considering a high frequency $f = 10000/s$, the absorbed energy for group E5 is 2.36J, as for D5 the absorbed energy is 1.79J, which means the composites can barely bearing impact. As shown in Figure 17b, the maximum value of force in the case of $f < 2000$ cycle/s is under the range 7N to 7.5N for group D, 9N to 12N for group E. As for group D5 and E5, a saltation is observed comparing force with the adjacent groups which is mainly due to the transient change of velocity.

More important, the maximum value of absorbed energy and contact force for SMA reinforced composites is generally 15%-30% larger than that of pure glass fiber reinforced composites under same amplitude or frequency.

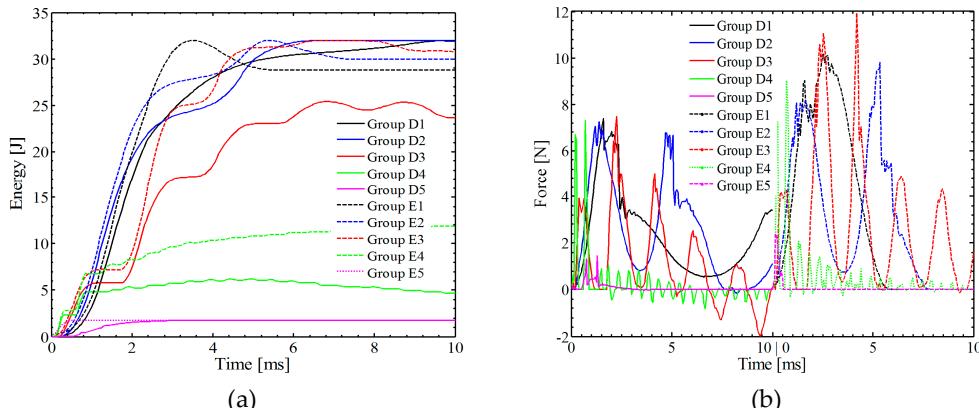


Figure 17. Analysis of the impact resistance of composite laminate in group D and E: (a) absorbed energy; (b) contact force.

4.3. Statistical analysis of damage state

Table 4. Statistics of absorbed energy and contact force.

	A1	B1	B2	B3	B4	B5	B6	A7	B8
E_{\max} (J)	32	32	32	21.67	18.27	4.30	3.72	0.30	0.80
$E_{t=0.01s}$ (J)	28.70	30.04	30.79	9.24	5.90	4.24	3.56	0.30	0.80
F_{\max} (N)	7.04	7.74	7.69	7.92	7.35	8.39	6.86	0.86	1.13
F_{avg} (N)	1.93	1.70	1.81	0.83	0.65	0.16	0.13	0.01	0.02
	A3	C1	C2		C3		C4		
E_{\max} (J)	32	32	32		13.20		3.74		
$E_{t=0.01s}$ (J)	25.14	28.60	31.97		13.17		3.74		
F_{\max} (N)	6.89	10.77	10.56		10.52		8.71		
F_{avg} (N)	3.32	2.98	2.31		0.52		0.12		
	D1	D2	D3	D4	D5				
E_{\max} (J)	32	32	25.38	6.21	1.75				
$E_{t=0.01s}$ (J)	31.98	31.92	23.68	4.69	1.74				
F_{\max} (N)	7.40	7.23	7.45	7.29	1.44				
F_{avg} (N)	2.31	2.36	1.11	0.17	0.06				
	E1	E2	E3	E4	E5				
E_{\max} (J)	32	32	32	11.86	1.79				
$E_{t=0.01s}$ (J)	28.77	29.94	30.70	11.86	1.79				
F_{\max} (N)	10.15	9.79	11.92	9.02	2.36				
F_{avg} (N)	2.95	2.81	2.69	0.46	0.05				

In Table 4, the maximum energy- E_{\max} , energy at time 0.01s- $E_{t=0.01s}$, maximum force- F_{\max} and average force- F_{avg} of different groups are shown. Energy $E_{\max}=32\text{J}$ denotes a rebound behavior of impactor. The average force is defined as:

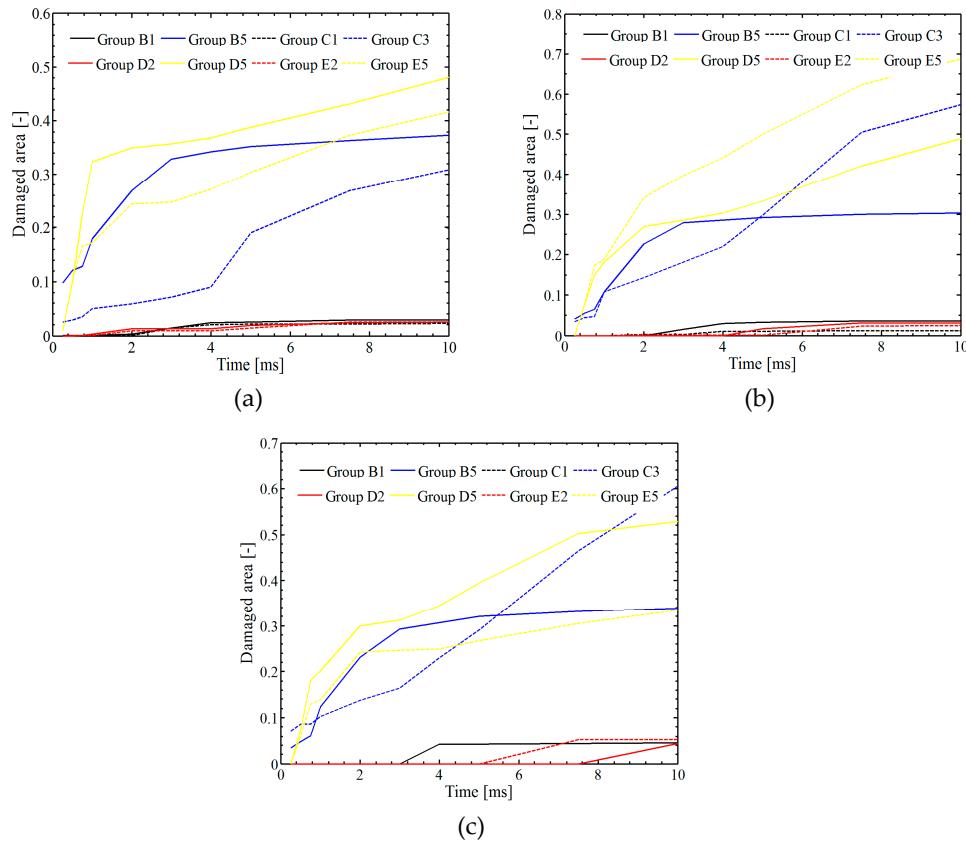
$$F_{\text{avg}} = \frac{\sum F_i}{N} \quad (14)$$

where N is the total number of output data of force F_i within time 0.01s, more important, the average value is still in accordance with A and f .

In Figure 18, the ratio between damaged area and the whole model are plotted against time. Generally, the damaged areas of laminates for $A < 0.0032\text{m}$ or $f < 500$ cycle/s is kept as < 5%. In Figure 18a, the damaged areas of top layer (layer-16) for $A > 0.0032\text{m}$ or $f > 500$ cycle/s is kept as 30-50%.

440 Differently, the damaged area of middle layer (layer-8) and bottom layer (layer-1) under same
 441 conditions are kept as 30-70% and 30-65%, respectively.
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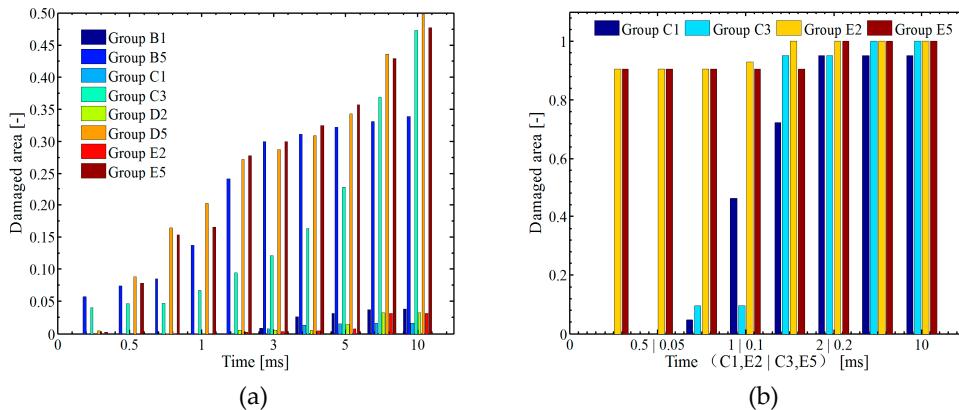
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Figure 18. Statistic of damage area in different groups: (a) layer-16; (b) layer-8; (c) layer-1.

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Figure 19. Statistic of general damage area: (a) laminate; (b) SMA.

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In Figure 19a, the average damage area are investigated by calculating 16 layers, the results indicate that a 4% damage for small amplitude and frequency and a nearly 50% damage for large amplitude or frequency at time 0.01s. In Figure 19b, the damage state of SMA is studied, the damage of SMA can be observed at time earlier than the damage of composite laminates which means that SMA has advantages absorbing energy.

457

4.4. Mathematical expression-effect of amplitude and frequency

458 In this section, the relationship between velocity and amplitude (or frequency) is investigated
 459 firstly. The results showing a clearly inverse proportion, as:

460
$$\Delta v = \frac{k_a}{\sum_{i=0} m_i A^i} \text{ (for given } f \text{)} \quad (15a)$$

461
$$\Delta v = \frac{k_f}{\sum_{i=0} n_i f^i} \text{ (for given } A \text{)} \quad (15b)$$

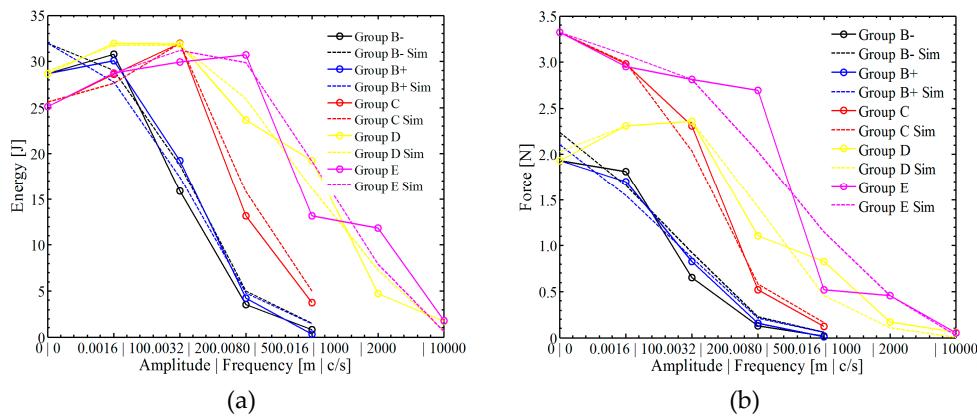
462 where, where k_a and m_i are parameters related to amplitude; k_f and n_i are parameters related to
 463 frequency. Inserting Equation 15 to energy equation, the relationship between absorbed energy and
 464 amplitude (or frequency) is obtained as:

465
$$E = \frac{1}{2} m V_0^2 - \frac{1}{2} m \left(V_0 - \frac{k}{\sum_{i=0} m_i A^i \sum_{i=0} n_i f^i} \right)^2 \quad (16)$$

466 where E and m is the absorbed energy and mass of the impactor, respectively. Considering that
 467 average $F_{avg} = m \times \Delta v / \Delta t$, the average force is also obtained as:

468
$$F_{avg} = \frac{k}{t_{tot} \sum_{i=0} m_i A^i \sum_{i=0} n_i f^i} \quad (17)$$

469
 470 The simulation results using the equations mentioned above are shown in Figure 20. In this
 471 simulation, $i = 3$ is chosen to simplify the expression. The comparison indicate that the both energy
 472 and force can be predicted using Equation 16 and 17.



473
 474 **Figure 20.** Comparison study of statistical results and simulation results: (a) energy- $E_{t=0.01}$; (b) force-
 475 F_{avg} .

477 5. Conclusions

478 The effect of vibrating boundary on the impact resistance of SMA reinforced composite
 479 laminates has been investigated. Two main factors are used to characterizing the vibration of the
 480 boundary: amplitude and frequency. A 3D finite element model based on Hashin's failure criterion
 481 was employed in ABAQUS to study the destruction process. A Mathematical expression is obtained
 482 to characterizing the effect of vibration on energy and force.

483 Comparison between the simulation results and our previous work with fixed boundary shows
 484 that the parameters and the simulation process are acceptable with relative error smaller than 10%
 485 (both for 32J and 64J). As for vibrating boundary condition with different amplitudes and
 486 frequencies, the absorbed energy and the fracture morphology of the composite laminate have been
 487 studied.

488 g Both high frequency and amplitude can weaken the impact resistance of composite laminate,
 489 extensive damage can be observed rather than impact hole.

490 g The absolute value of amplitude has greater influence on the impact resistance rather than the
491 moving direction of laminates at initial time. The absorbed energy and contact force in positive
492 direction is about 20% larger than that in negative direction.

493 g Embedding of SMA can improve the impact resistance of composite laminates due to the
494 superelasticity- with increasing absorbed energy and contact force about 50%; also, embedding of
495 SMA can also change the damage morphology: shape and proportion.

496 **Author Contributions:** conceptualization, Jian He; methodology, Mengzhou Chang; software, Mengzhou
497 Chang; validation, Min Sun; formal analysis, Mengzhou Chang; investigation, Mengzhou Chang; resources,
498 Min Sun; data curation, Min Sun; writing—original draft preparation, Fangyun Kong; writing—review and
499 editing, Fangyun Kong; visualization, Fangyun Kong; supervision, Jian He; project administration, Jian He;
500 funding acquisition, Jian He.

501 **Funding:** This research was funded by National Natural Science Foundation of China, grant number 11872157,
502 11472086 and 11532013.

503 **Conflicts of Interest:** The authors declare no conflict of interest.

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