

1 Article

# 2 Life Cycle Modeling of Structural Defects via 3 Computational Geometry and Time Series 4 Forecasting

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13 **Abstract:** The evaluation of geometric defects is necessary in order to maintain the integrity of  
14 structures over time. These assessments are designed to detect damages of structures and ideally  
15 help inspectors to estimate the remaining life of structures. Current methodologies for monitoring  
16 structural systems, while providing useful information about the current state of a structure, are  
17 limited in the monitoring of defects over time and in linking them to predictive simulation. This  
18 paper presents a new approach to the predictive modeling of geometric defects. A combination of  
19 segmented from point clouds are parametrized using the convex hull algorithm to extract features  
20 from detected defects, and a stochastic dynamic model is then adapted to these features to model  
21 the evolution of the hull over time. Describing a defect in terms of its parameterized hull enables  
22 consistent temporal tracking for predictive purposes, while implicitly reducing data dimensionality  
23 and complexity as well. In this study, 2D point clouds analogous to information derived from point  
24 clouds were first generated over simulated life-cycles. The evolutions of point cloud hull  
25 parameterizations were modeled as stochastic dynamical processes via autoregressive integrated  
26 moving average (ARIMA) and vectorized autoregression (VAR) and compared against ground  
27 truth. The results indicate that this convex hull approach provides consistent and accurate  
28 representations of defect evolution across a range of defect topologies and is reasonably robust to  
29 noisy measurements, however assumptions regarding the underlying dynamical process play a  
30 significant the role in predictive accuracy. The results were then validated on experimental data  
31 from fatigue testing with high accuracy. Longer term, the results of this work will support finite  
32 element model updating for predictive analysis of structural capacity.

33 **Keywords:** Remote sensing; Photogrammetry; Life cycle modeling; Time series forecasting;  
34 Structural damage; Stochastic modeling; Convex Hull; ARIMA; VAR; Fatigue crack prediction  
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## 36 1. Introduction

37 This paper presents a method for modelling the time-history evolution of defects quantified  
38 through remote sensing technologies such as laser scanning, photogrammetry, or digital imaging.  
39 The goal is to expand the use of structural assessment information commonly collected during  
40 routine inspections and improve the structural life-cycle assessment process.

41 Conventional structural assessments are typically based on visual inspections, embedded sensor  
42 systems, nondestructive evaluation (NDE) techniques, or some combination thereof [1–7]. In almost  
43 all cases, the goal is the nondestructive detection and identification of structural performance changes  
44 and damage, as well as to assess the reliability and safety of monitored structures[8,9]. In addition to  
45 immediate structural evaluation, these assessments ideally help engineers to estimate the remaining  
46 life of structures. This is commonly done by reviewing historical performance records and holistically

47 identifying temporal trends from the assessment data. However, most assessment data are not  
48 structured in a way that explicitly captures the life cycle performance of a structure, and it is therefore  
49 challenging to quantitatively evaluate the evolution of inspection data over time and carry out a  
50 predictive analysis of the future state of the structure. Life-cycle data modelling advancements could  
51 provide more precise and robust structural information, leading to better system asset management  
52 decision-making, with apparent safety and financial benefits.

53

#### 54 *1.1. Prior Work*

55 Damage identification, localization and quantification have been extensively studied in the last  
56 decades. Using state-of-the-art NDE methods, crack can be identified and located using vibration-  
57 based methods [10–12], also large cracks and voids in concrete as well as corrosion and cracks in steel  
58 can be identified and located using ultrasonic tests[13–15]. Acoustic wave techniques can also be used  
59 to identify and locate imperfections such as the initiation of crack and the growth rate of fatigue  
60 cracks and corrosion [16], to classify crack modes in concrete[17–19] and to quantify the severity of  
61 damage [20,21]. Furthermore, it is possible to detect and locate hidden and subsurface defects using  
62 radiographic tests [22,23] and electromechanical method such as ground penetration radar [24].  
63 Although these studies have shown the capacity to identify and assess damage to structures, the long  
64 - term contribution to structural evaluation is limited. They focus primarily on improving the  
65 accuracy of the measurement of defects and efforts have not been made to model these measured  
66 changes/defects as time-dependent phenomena.

67 In addition to the development of NDE and SHM techniques, there are increasing studies on the  
68 use of remote sensing and imaging technologies such as LiDAR or photogrammetry to provide new  
69 sources of inspection information. These remote sensing technologies provide high resolution 2D  
70 images or 3D point cloud models of structures, and can capture the small scale defects that are critical  
71 to understanding structural performance [25–29]. In complement to the expanding use of these  
72 technologies, there are now a variety of methods for isolating and extracting defects from 2D or 3D  
73 images[30,31], and advancements in deep machine learning methods portend future improvements  
74 [32,33]. A key advantage of these data sources is the direct link between quantified geometric changes  
75 and changes in the underlying mechanical performance that can be captured in finite element  
76 analysis, as has been evidenced by a variety of prior work [34–36]. While such capabilities provide  
77 valuable tools for structural assessment, they do not explicitly quantify life-cycle dynamics and  
78 forecasts of future defect conditions.

79 The question of the reliability of an engineered system has led researchers to investigate the  
80 growth of defects such as fatigue cracks and corrosion over the life-cycle of the systems. To study  
81 fatigue crack growth, model-based estimation methods such as Bayesian methods [37], Extended  
82 Kalman filtering [38] and Monte Carlo sampling [39] have been used for quantification of the  
83 estimation uncertainty. For corrosion, theoretical models and simulation tools have been developed  
84 for a better understanding of the nature of the pitting corrosion process, to allow prediction of the  
85 time evolution of maximum pit depth in corroding structures. In recent studies, stochastic approaches  
86 have also been applied to simulate corrosion [40–43]. All these efforts primarily focus on estimating  
87 the reliability of a system with given estimates of the current state of a defect, rather than quantifying  
88 the future state of defect in a way that provides a support for predictive capacity assessments.

89

#### 90 *1.2. Contribution of This Research*

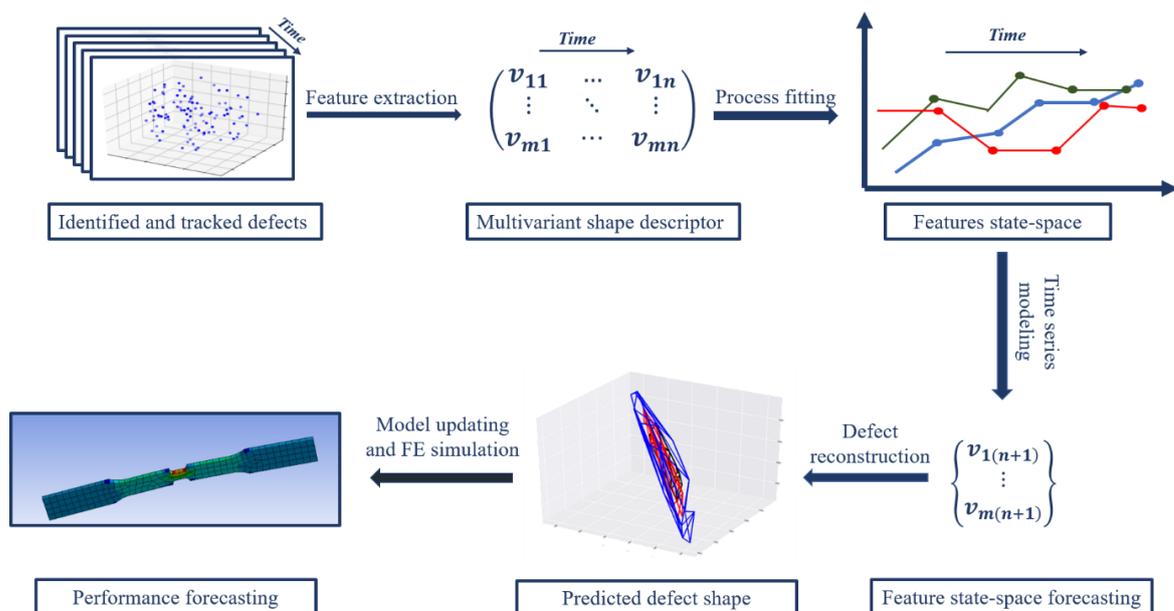
91 As stated, a critical aspect of long-term structural monitoring is the modeling of detected defects  
92 as time-dependent phenomena. Temporal modeling of defects leads to the ability to predict future  
93 defect states, and thus the future condition of the structure. While prior studies have investigated the  
94 temporal behavior of defects from an empirical perspective [44,45], efforts to quantify the dynamics  
95 of defect observations captured in remote sensing have been limited. The main objective of this study  
96 was to address these limitations for life cycle modeling of remotely sensed defects using a  
97 computational geometry approach to defect parametrization combined with time-series modeling.  
98 Presented in this paper is a novel algorithm to model the life cycle of defects manifested as either 2D

99 or 3D point clouds. Point cloud data are a geometric representation of defects that can serve as a  
 100 consistent record of the state of a structure at a given inspection interval and provide a basis for finite  
 101 element analysis, among other uses [25,34]. This algorithm extracts latent features from these defect  
 102 point clouds through computational geometry, fits a time-series process model to the evolution of  
 103 those features, and uses stochastic forecasting model to predict future state of the defect.  
 104 Consequently, a predictive analysis with regards to the future condition of the structure could be  
 105 carried out by linking the modeled evolution of defects to a numerical simulation, which ultimately  
 106 helps to provide a complete representation of structural performance and integrity over time. This  
 107 paper does not consider the predictive analysis aspect of this process, and the readers is referred to  
 108 [34–36,46] for potential applications in this domain.

109 The remainder of this paper is structured as follows. First, the complete analytical methodology  
 110 is presented. This is followed by a presentation of synthetic experimental results designed to illustrate  
 111 the key behavioral aspects of the algorithm. This is followed by experimental evaluation using fatigue  
 112 crack propagation data. The paper concludes with an overall assessment of the algorithm and  
 113 avenues for future work.

## 114 2. Materials and Methods

115 The methodology presented here consists of several steps (Figure 1). Once a defect in the  
 116 structure is detected, the defect must be parameterized in a way that can support a dynamic modeling  
 117 of defect evolution over time. Parameterization is achieved with feature extraction from the point  
 118 cloud through computational geometric modeling of the convex hull of the cloud, resulting in a  
 119 combination of hull simplexes and vertices. These parameterizations are computed for multiple time  
 120 steps over the life-cycle of the structure. Once parameterized, a time-series model is fit to the sequence  
 121 of parameterizations in order to capture the underlying process of evolution. This model fitting also  
 122 enables the estimate the future state of the defect via out-sample forecasting. The defect shape is then  
 123 reconstructed by reversing the parameterizations back to a geometric point set. While not considered  
 124 in this work, this reconstructed point set can then be used to update a finite element model of the  
 125 structural component based on the predicted future defect topology, leading to predictive structural  
 126 assessment.  
 127

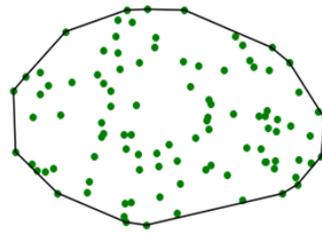


128 **Figure 1.** Schematic overview of the proposed methodology for life cycle modeling of remotely sensed  
 129 defects  
 130  
 131

### 132 2.1 Defect Parameterization

133 Once a remotely sensed defect in a structural component has been detected through computer  
 134 vision [34], the first step in the modeling process is to parameterize it so that a stochastic dynamic  
 135 model can be reliably fit the extracted parameters, or “feature vector” to track the defect evolution  
 136 over time. The complex nature of point cloud data necessitates this low dimensional  
 137 parameterization, as tracking each individual point in a cloud would lead to an intractably high  
 138 number of time-series model coefficients.

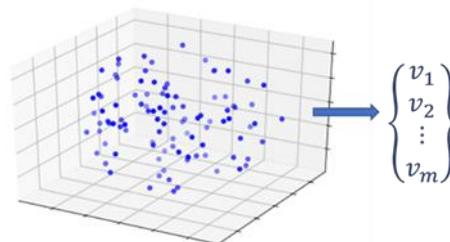
139 Here, we propose that the feature extraction can be done using the concept of a geometric convex  
 140 hull. The convex hull of a point set is a unique representation of a point set in  $R^n$ , defined as the  
 141 smallest convex polygon that surrounds all points in the point set (Figure 2) [47]. In  $R^3$  or higher  
 142 dimensional data spaces, the convex hull is similarly defined as the minimum convex polyhedron of  
 143 the point set.  
 144



145  
 146 **Figure 2.** Convex hull of a point set in  $R^2$   
 147

148 The determination of the convex hull is a geometric computation that is useful for many analyses  
 149 and has been successfully applied in domains such as image processing [48] and pattern recognition  
 150 [49]. Although a number of alternative feature extraction approaches were considered in this study  
 151 [50,51] hull parameterization is used because of its inherent advantages. The description of a defect  
 152 in terms of its parameterized hull allows for consistent temporal tracking for predictive purposes,  
 153 while also reducing the dimension and complexity of the data implicitly. In addition, the convex hull  
 154 concept can be extended to high-dimensional spaces to support the fusion of multiple sensors and  
 155 data types, a longer-term goal of this work.

156 The convex hull of point cloud  $P$  is a uniquely defined convex polygon. A natural way to  
 157 represent a generalized polygon is by listing its vertices in clockwise order, starting with an arbitrary  
 158 starting point. As such, the problem to be solved is: given a point set  $P = \{p_1, p_2, \dots, p_m\}$  in  $R^n$ , compute  
 159 a list that contains those points from  $P$  that are the vertices of the convex hull,  $CH(P)$ . To find those  
 160 vertices the algorithm sorts all points through a “divide and conquer” approach [52]. The convex hull  
 161 algorithm finds two points with maximum and minimum spatial coordinates in a single dimension  
 162 and computes a line joining these two points. This line divides the whole set into two halves. For a  
 163 given half it finds the points with a maximum distance from the dividing line, forming a triangle  
 164 defined by minimum and maximum point distances. Those points inside the triangle are determined  
 165 to not be part of convex hull. Then, these steps are iteratively repeated to search for points with  
 166 maximum distance from the separating line, until there is no point left outside of the computed  
 167 triangles. The points selected at this step constitutes the convex hull. The convex hull results in a  
 168 vector containing the Cartesian coordinates of the hull vertices. This extracted vector of vertices is the  
 169 numerical descriptor that is later fit to a stochastic process model (Figure 3).

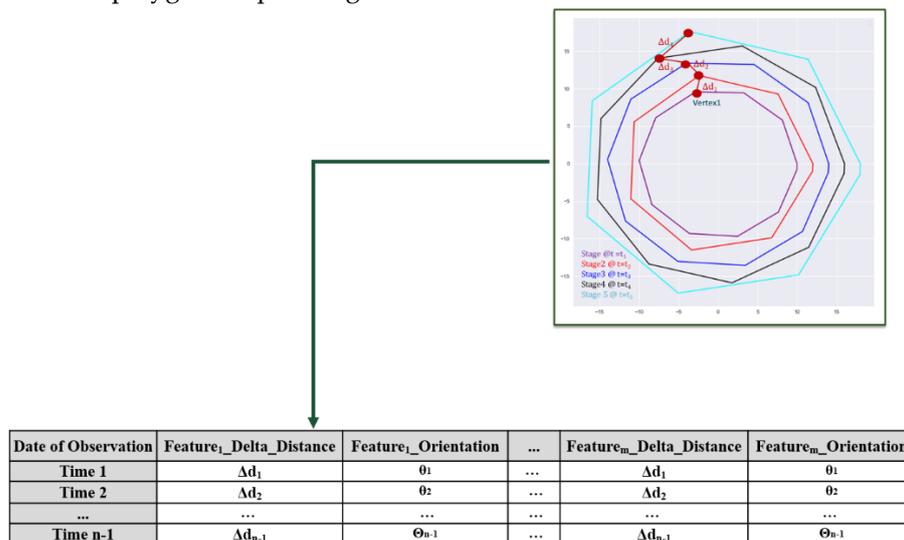


170  
 171 **Figure 3.** Feature extraction of convex hull vertices from a 3D point cloud  
 172

174 Once the hull vertices are extracted, the data must be presented in a time history context.  
 175 Therefore, it is required that multiple stages of a defect be parameterized and compiled into a matrix  
 176 of vertices representing the evolution of the defect over time. To do so, first multiple hulls computed  
 177 in the previous step are aligned and registered into a common spatial reference frame. For this work,  
 178 manual registration and transformation was used. However, registration can be done through  
 179 approaches such as the Iterative Closest Point algorithm or by determining the 2D or 3D homography  
 180 between point sets via feature-based computer vision methods [53,54]. Each vertex at  $t=t_i$  is then  
 181 spatially tracked. In this study, it is assumed that vertices of a hull in stage 1 ( $hull_1$ ) are matched to  
 182 their nearest neighbor (NN) vertices in  $hull_2$  and likewise those are equivalent to their NN vertices in  
 183  $hull_3$  and so on [55]. The nearest neighbor is found based on the smallest Euclidean distance between  
 184 vertex sets. Equation 1 shows the nearest neighbor search in  $Hull(Q): \{q_1, q_2, \dots, q_n\}$  from  $p_1 \in$   
 185  $Hull(P): \{p_1, p_2, \dots, p_m\}$  in 2D space ( $Hull(Q)$  and  $Hull(P)$  are two consecutive descriptor  
 186 vectors):  
 187

$$d(p_1, q_i) = \sqrt{(q_{ix} - p_{1x})^2 + (q_{iy} - p_{1y})^2} \quad (1)$$

188 At each time step, the NN of vertices in two time-steps are determined, and the change in  
 189 magnitude and orientation between matched vertices is computed. Once distances and orientations  
 190 between matched vertices are found, they are used for building a dataset for dynamic modeling. This  
 191 dataset consists of the distances and orientations of vertex changes between subsequent hull stages  
 192 throughout a time series. This dataset is illustrated for identified and aligned multiple stage of a  
 193 defect with a polygon shape in Figure 4.  
 194  
 195



196  
 197 **Figure 4.** Dataset representing the extracted vertices for a time-series evolution of an arbitrary polygonal  
 198 defect.  
 199

### 200 2.3 Time Series Modeling

201 Once the dataset representing the evolution of defects has been built from previous step, a  
 202 stochastic model can be fit to the dataset to model the dynamics of defect evolution. Time series  
 203 forecasting is performed in order to capture the underlying long-term life-cycle trends in inspection  
 204 data. The model can then be used to extrapolate the time series into the future. This modeling  
 205 approach is particularly useful when the temporal behavior is stochastic, as opposed to understood  
 206 deterministic evolution, and where the relationship between parameterization variables is not well  
 207 understood [56]. For example, in the problem presented later in this paper, there is no knowledge  
 208 available on the boundary conditions (i.e., applied load and support reactions) of the tested structural  
 209 component, and there is no established deterministic model for the propagation of fatigue cracks.

210 There are several different approaches to time series modeling including autoregressive, moving  
 211 average, exponential smoothing, ARIMA and multivariate time series VAR. All mentioned models  
 212 are linear, meaning that their predictions of the future values are constrained to be linear functions  
 213 of past observations. Because of their relative simplicity in understanding and implementation, linear  
 214 models have been the main research focus and applied tools during the past few decades and  
 215 similarly linear stochastic models were used in this study. Nonlinear modeling approaches such as  
 216 recurrent neural networks were not considered here due to the limited training data available for  
 217 observing defect evolution.

218 From the available time series models, ARIMA and VAR models were selected for this study  
 219 and their performances were compared against each other. The autoregressive integrated moving  
 220 average model (ARIMA) methodology developed by Box and Jenkins [57] is able to handle non-  
 221 stationary time series, in other words scenarios where the statistical properties of the time series  
 222 measurements do not remain constant over time. As such, it relaxes the requirement that time-series  
 223 data be covariance-stationary prior to modeling, and is well-suited to the challenging variations in  
 224 field conditions that impact remote sensing-based inspection practices [58]. With ARIMA, the future  
 225 value of a variable is assumed to be a linear function of several past observations and random errors.  
 226 ARIMA model contain two processes (autoregressive, moving average process) and explicitly  
 227 includes differencing in the formulation to account for the non-stationarity of the data. ARIMA has  
 228 gained enormous popularity in many areas and research practice confirms its power and flexibility  
 229 [59–61]. A generalized form of the ARIMA time series model is shown in Equation 2:  
 230

$$231 \quad Y_{(t)} = \phi_0 + \phi_1 \times Y_{(t-1)} + \phi_2 \times Y_{(t-2)} + \dots + \phi_p \times Y_{(t-p)} + \varepsilon_t + \theta_1 \times \varepsilon_{(t-1)} + \theta_2 \times \varepsilon_{(t-2)} + \dots + \theta_q \times \varepsilon_{(t-q)} \quad (2)$$

232 Where  $Y_{(t)}$  and  $\varepsilon_t$  are the actual value and random error at time  $t$ , respectively.  $\phi_i$  ( $i=1, 2,$   
 233  $\dots, p$ ) and  $\theta_j$  ( $j=0,1,2, \dots, q$ ) are the vectorized model coefficients.  $p$  and  $q$  are integers and often  
 234 referred to as orders of the model.

235 As an alternative, vector autoregressive (VAR) models are effective for multivariate time series  
 236 data where there is a potential dependency between model variables. Therefore, unlike an ARIMA  
 237 model that estimates the present value of a variable based only on its past values, VAR models  
 238 consider past values of other variables as well. This model was used here to study the dependency  
 239 and effect of the movement of convex hull vertices on other vertices. VAR model [62] has been used  
 240 for damage detection studies previously [63,64] but it has not been any effort to model life-cycle of  
 241 defect to date. The general form of autoregressive model is shown in Equation 3. Similar to ARIMA,  
 242 the VAR model relates the current value of a variable to its past values.

$$243 \quad Y_{(t)} = \phi_0 + \phi_1 \times Y_{(t-1)} + \phi_2 \times Y_{(t-2)} + \dots + \varepsilon_t \quad (3)$$

244  
 245  
 246  
 247 Where  $\varepsilon_t$  is random error (random shock) and  $\phi_i$  are constants. And similarly, VAR model  
 248 relates current value of a vector to its past values and each variable depends not only on its own past  
 249 values but on these of other variables as well in which is  $K \times 1$  random vector,  $\phi_i$  are fixed ( $K \times K$ )  
 250 coefficient matrices and  $\varepsilon_t = (\varepsilon_{t1}, \dots, \varepsilon_{tK})$  is  $K$ -dimensional random error [62]. To summarize, the  
 251 power of ARIMA modeling stems from its ability to handle non-stationary time series with non-  
 252 constant statistical properties over time. On the other hand, VAR is suitable for analyzing stationary  
 253 multivariate time series with constant statistical properties and dependency among variables .  
 254

### 255 2.3.1 Model Parameter Identification

256 Prior to fitting the coefficients of the time-series model, the model order must be optimized. The  
 257 ARIMA model contains three components for which an order must be determined: Autoregressive  
 258 (AR), Integrated (I) and Moving Average (MA) (Equation 2). The AR component uses the dependent

259 relationship between an observation and some number of lagged observations. The order of the AR  
 260 component ( $p$ ) is the number of lag observation included in the model. The Integrated component (I)  
 261 employs differencing of raw observation data in order to make the time series stationary, and its  
 262 order ( $d$ ) is the number of times that the raw observation is differenced. The MA component uses the  
 263 dependency between an observation and a residual error, and its order ( $q$ ) is the size of the moving  
 264 average window. In this study, ARIMA model orders ( $p$ ,  $d$  and  $q$ ) were evaluated based on a mean  
 265 squared error. A prototyping dataset was divided into train and test sets and, upon the optimized  
 266 combination of model orders were chosen such that they produced the least mean squared error in  
 267 the test set. For the VAR model, the model order  $p$ , the number of past observations included in the  
 268 model, was selected based on Hannan-Quinn information criterion (HQIC) [65]. This criterion was  
 269 applied because this is used to consistently estimate the order under fairly general conditions [66].  
 270 The criterion is shown in Equation 4:

$$271 \quad 272 \quad 273 \quad 274 \quad 275 \quad 276 \quad 277 \quad 278 \quad 279 \quad 280 \quad 281 \quad 282 \quad 283 \quad 284 \quad 285 \quad 286 \quad 287 \quad 288 \quad 289 \quad 290 \quad 291 \quad 292 \quad 293 \quad 294 \quad 295 \quad 296 \quad 297 \quad 298 \quad 299$$

$$HQIC = -2 \ln(L_{max}) + 2k \ln(n), \quad (4)$$

where  $n$  is the number of observations,  $k$  is the number of parameters to be estimated (e.g. the Normal distribution has  $\mu$  and  $\sigma$ ) and  $L_{max}$  is the maximized value of the log-Likelihood for the estimated model. The coefficients for  $k$  indicate the level to which the number of model parameters is being penalized. The objective is to find the model order of the selected information criterion with the lowest value HQIC value.

### 2.3.2 Forecasting and Defect Reconstruction

Once a model is fitted to a sequence of defect observations, the future state of the convex hull parameterizations can be predicted by the forecasting model. Once predicted, the future defect shape can then be reconstructed by converting the feature vector into a hull shape. A complication is that the number of extracted features may be inconsistent at different time steps, and this discrepancy in the length of the vectors in some cases leads to an inaccurate defect prediction. This case will happen when number of features extracted by convex hull computation at early time steps is significantly smaller than those in the later time steps. To handle this issue, a statistical assumption is employed. For features that have not been fit to a model, and therefore their values have not been predicted by dynamic modeling, the arithmetic mean of other features can be used as their expected value.

The pseudocode for the complete methodology is presented algorithm is shown in Figure 5. Upon reconstruction of the predicted geometric configuration of a defect, it is then possible to update a numerical simulation to account for the predicted change in the structure's geometry due to the defect. This updating process is not tested here, but such capabilities have been developed in prior related work, including efforts by the authors [34–36].

```

start
  #input: Point cloud of defects in time order
  #outout: The shape of defect in the next time step

  Align all point clouds and register into a common spatial reference
  For each point cloud
    Compute convex hull
    Extract vertices of hull as feature descriptors
  For each feature in feature descriptor vector at t = t0
    Search for nearest neighbor from feature vector (@t=t1) based on smallest Euclidean distance
    Store distance and angle between each pair of features
  Track those found nearest neighbors over time based on their Euclidean distance from the consecutive features vectors
  Build dataset from stored magnitude and orientation of spatial movement of features
  For each column of dataset determine model orders for timeseries model based on chosen criteria
  Build time series model with determined model order and fit the model to timeseries dataset
  Forecast variable/feature values in next time step
  Check the consistency between first and last feature vector:
    If they are consistent
      Do nothing
    Else
      Apply arithmetic mean of all predicted features as expected value of those not predicted
  Reverse the predicted features to a discrete point set and reconstruct shape of defect in next time step
end

```

Figure 5. Pseudo code for the proposed methodology

## 3. Experimental Validation

300 This section presents and discusses the results of experiments designed to evaluate the  
 301 developed methodology. Two series of tests are presented. The first are a set of experiments  
 302 performed on synthetic data sets. These data sets were designed to highlight key aspects of the  
 303 modeling approach and provide insight into algorithm behaviors. The second set of tests are derived  
 304 from laboratory scale tests of fatigue crack propagation in aluminum tensile specimens, in order to  
 305 illustrate the behavior of the modeling approach in a realistic use case.

306

### 307 3.1 Synthetic Data Set

308 To initially test the accuracy and robustness of the presented methodology, synthetic 2D point  
 309 clouds analogous to data derived from remote sensing (e.g. laser scanning or photogrammetry) were  
 310 generated over simulated defect life-cycles. Synthetic point clouds with distribution characteristics  
 311 representing different flaw topologies (e.g. rectangle, circle, and generalized polygon) were  
 312 generated. Additionally, varying evolution time histories were generated synthetically,  
 313 representing a variety of stochastic processes (e.g. linear, quadratic, random Gaussian, and random  
 314 uniform). For every combination of defect shape and stochastic process, a set of twenty defect time  
 315 steps was generated. White noise was also introduced into the point clouds for each time step, in  
 316 order to simulate more realistic measurements. Finally, uniform and non-uniform feature evolution  
 317 were considered. Uniform feature evolution refers to a case where all vertices of the convex hull  
 318 (features in the extracted descriptor vector) have the same expansion magnitude regardless of the  
 319 trend of evolution. Cases where vertices were allowed to expand at varying magnitudes over a life-  
 320 cycle simulation were considered non-uniform.

321

#### 322 3.1.1 Time Series Stationarity Assessment

323 Time series are stationary if the statistics calculated on the time series (e.g. the mean or variance  
 324 of the observations) are consistent over time. Most statistical modeling methods assume or require  
 325 the time series to be stationary to be effective. There are many methods to check whether a time series  
 326 is stationary or non-stationary such as reviewing a time series plot, reviewing the summary statistics  
 327 for time series, or using statistical tests. The Augmented Dickey-Fuller test [67], one of the more  
 328 widely used, was used in this study. It uses an autoregressive model and optimizes an information  
 329 criterion across multiple different lag values. The null hypothesis ( $H_0$ ) of the test is that the time series  
 330 can be represented by a unit root that it is not stationary (i.e. it has some time-dependent structure).  
 331 The alternate hypothesis (rejecting the null hypothesis) is that the time series is stationary.

332

333

334

**Table 1.** The p-values from Augmented Dickey-Fuller test stationary test

Defect Shape	Triangle	Rectangle	Circle	Polygon
Defect Evolution				
<b>Uniform</b>				
Linear	0.002	0.002	0.020	0.030
Quadratic	1.000	1.000	1.000	1.000
Random Uniform	0.950	0.950	0.390	0.940
Random Gauss	0.960	0.960	0.990	0.950
<b>Non-uniform</b>				
Random Uniform	0.950	0.950	0.96	0.990

335

336 Table 1 shows the average p-values of each generated time series, which is used in the  
 337 Augmented Dickey-Fuller test to evaluate stationarity for various defect shapes and evolutions. A p-

338 value above 0.05 suggests that a test fails to reject the null hypothesis (H0) and it is concluded that  
 339 such time series models are non-stationary. This analysis shows that all of the generated time series  
 340 simulations, with the exception of the simplest linear evolution process, are non-stationary. As such,  
 341 it was anticipated that the ARIMA approach would perform better than the VAR approach for the  
 342 synthetic data sets.

343

### 344 3.1.2 Time Series Modeling

345 After the synthetic data sets were generated, ARIMA and VAR model orders were selected prior  
 346 to fitting to time series. For the VAR models, all extracted feature vectors were input as a matrix at  
 347 once and the model order (p) was set as the same for all features, whereas for the ARIMA model,  
 348 each feature vector was considered individually and model order suitable to each time series were  
 349 chosen based on least mean squared error. Models were then fit to the time series.

350

### 351 3.1.3 Metrics

352 The key metric for evaluating the time-series model behavior was defect reconstruction  
 353 accuracy. To evaluate the results, the predicted defect shape for a future time step was compared  
 354 against an established ground truth for each scenario. Two geometric metrics were computed: the  
 355 percentage difference between the area of two shapes and their overlap area percentage. These two  
 356 metrics were necessary in order to identify scenarios where the predicted and ground truth defect  
 357 sizes were similar, but where there was a divergence in the geometric topology.

358

### 359 3.1.4 Results and Discussion

360 Table 2 and 3 show the comparison of predicted defect shapes for both the ARIMA and VAR  
 361 models against the ground truth. The time series generated from linear, quadratic expansion models  
 362 are defined as deterministic time series, as their future value can be exactly computed by a  
 363 mathematical function. These mathematical functions are  $y_t = \theta t^0$  for linear expansion, and  
 364  $y_t = \theta \times d^{(t-1)}$  for quadratic expansion ( $\theta$  and  $d$  are both constants). As expected, the predicted  
 365 feature state from both models completely matched with the ground truth, regardless of the defect  
 366 shape. The ability to forecast such a simple deterministic process is inherent to both ARIMA and VAR  
 367 modeling approaches and was used to validate basic model performance.

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371 **Table 2.** Comparison of predicted defect shape using ARIMA model against ground truth

Defect Shape	Triangle		Rectangle		Circle		Polygon	
Defect Evolution	Overlap	Area_Diff	Overlap	Area_Diff	Overlap	Area_Diff	Overlap	Area_Diff
Metrics	(%)	(%)	(%)	(%)	(%)	(%)	(%)	(%)
<b>Uniform</b>								
Linear	100	0	100	0	100	0	100	0
Quadratic	100	0	100	0	100	0	100	0
Random Uniform	100	15	100	10	100	19	89	15
Random Gauss	100	4	100	5	100	8	92	11
<b>Non-uniform</b>								
Random Uniform	95	7	96	5	98	4.5	87	17

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More interesting results can be seen for the Gaussian and uniform random stochastic time series, which are more realistic representations of defect life-cycle dynamics in practical problems. Such stochastic processes are more challenging for any predictive model. As can be seen, ARIMA models provide relatively better prediction, though results show many similarities. The reason for the difference in predictive accuracy is the capability of ARIMA in handling nonstationary time series, as well as the assumptions of variable dependencies in VAR. Also, results show that both models can predict a defect with a Gaussian underlying process better than those with a uniform random evolution. The reason lies in the difference between the statistical properties of the two processes. Gaussian processes have a single most likely value in the distribution (the mean), whereas in uniform distributions every allowable value is equally likely, degrading predictive capabilities. Overall, the results of these synthetic experiments indicated that the convex hull parameterization approach and time series modeling provides reliable and accurate representations of defect evolution across a range of defect topologies and is reasonably robust to noisy measurements. As anticipated, ARIMA provided higher prediction accuracy as stationarity assumptions became increasingly unrealistic.

### 3.2 Experimental Data Set

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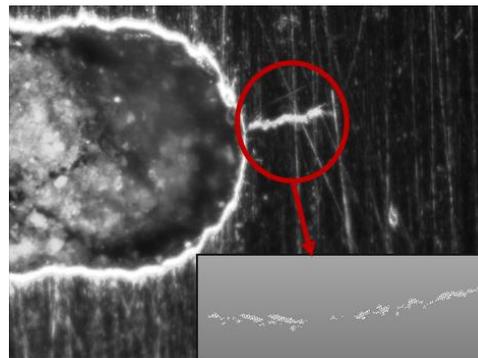
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To further evaluation of the methodology under more realistic conditions, a data set from prior experimental testing was repurposed. In these laboratory tests, aluminum tensile coupons were tested to observe fatigue crack growth under cyclic fatigue loading. Marine grade aluminum 5052-H32, with a nominal thickness of 2.29 mm was used. Specimen had machined elliptical flaw in the center and increasing load caused initiation and growth of cracks on both the right and left sides of this notch. Cycling tension loading was performed over 80k cycles and the state of crack growth was captured at 30 intermediate intervals during the test, using an inspection microscope. The captured images were then segmented to isolate the crack, and the crack patterns were transformed from pixels into point clouds (Figure 6). The convex hulls of these point clouds were computed, and features evolution were exported as time series, per the methodology delineated in Section 2. An analysis of the data sets yielded an average p-value of 0.35, indicating that the statistical uncertainty of the experimental measurements was non-stationary. Three different tests are presented in this section to evaluate the performance of the proposed methodology including single step prediction, multiple step prediction, and prediction during nonlinear system dynamical behavior.



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Figure 6. Extracted point cloud from the captured image

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#### 3.2.1 Single Step prediction

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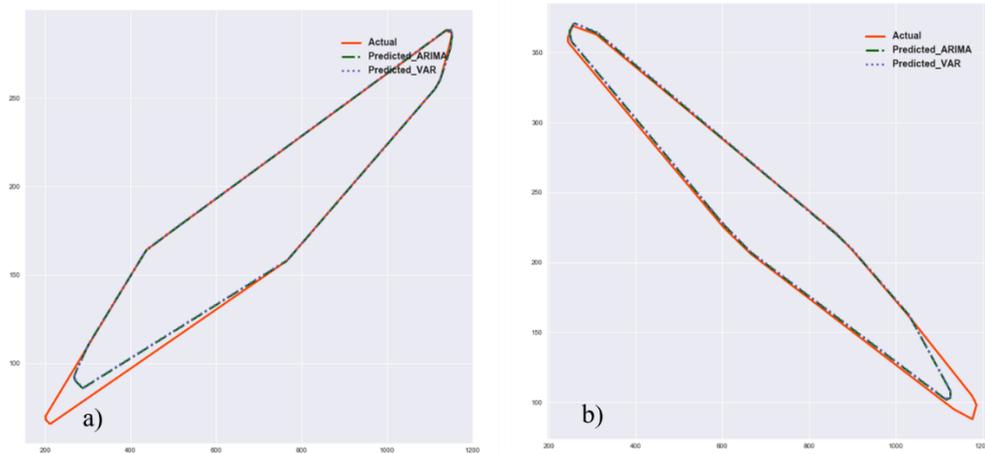
Performance of the proposed algorithm for predicting a single future step is evaluated in this section. Both ARIMA and VAR model were used to find the pattern of crack growth and predict the future state of crack. The convex hull of the right and left crack at 80k cycles load were computed and held out as the ground truth for one single step prediction. Results are shown in Table 4 and Figure 7. Since the performance of ARIMA and VAR model is almost the same, the predicted shape from both models has overlap and only ARIMA model can be seen in the Figure 7.

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Table 4 – Comparison of predicted crack shape from ARIMA and VAR against ground truth

Metric	ARIMA		VAR	
	Overlap (%)	Area_Diff (%)	Overlap (%)	Area_Diff (%)
Right Crack	100.0	7.0	100.0	7.0
Left Crack	99.0	5.0	96.0	5.0

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**Figure 7.** Comparison of the predicted crack shape against the ground truth for a) right and b) left cracks.

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### 3.2.2 Multiple Step Prediction

423 To evaluate the capability of the proposed algorithm for prediction of multiple steps, 20 steps of  
424 the right-side crack, corresponding to approximately 40 thousand loading intervals, were used to fit  
425 to the ARIMA and VAR models. The true convex hulls of the crack at time steps 21 to 30 were then  
426 computed and held out as the ground truth. Then, prediction of 1 to 10 time steps were computed  
427 and evaluated. Table 5 shows the results from the ARIMA model. Since the performance of ARIMA  
428 and VAR was very similar for this specific problem, only ARIMA is shown here.

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Table 5 – Comparison of predicted crack shape from ARIMA against ground truth

	1-step	2-steps	3-steps	4-steps	5-steps
Overlap (%)	100	100	99	98	96
Area_Diff (%)	1	1	4	3	4
	6-steps	7-steps	8-steps	9-steps	10-steps
Overlap (%)	95	94	93	92	91
Area_Diff (%)	3	6	5	3	3

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### 3.2.3 Prediction During Nonlinear System Behavior

433 The goal of this study was to evaluate the performance of the model during a nonlinearity in the  
434 evolution of a defect over time. For the crack fatigue problem studied here, there was a sudden  
435 change in the direction of crack growth after 48k load cycles. Of course, the predictive time series  
436 models could not accurately forecast the convex hull immediately after this event. Rather, the  
437 question here was how long it would take the time series models to correct for this nonlinearity  
438 in the dynamic evolution. The results for both ARIMA and VAR models are shown in Tables 6 and 7.  
439 As can be seen, the ARIMA model quickly adapts after only two time steps (at 50,000 load cycles).  
440 The VAR model struggles to adjust for far longer, only regaining consistent predictive accuracy after  
441 60,000 load cycles, equivalent to an additional 4 model time steps.  
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Table 6. Comparison of the predicted crack shape from ARIMA model against ground truth

Load	48k	50k	52k	54k	58k
Overlap (%)	83	96	95	99	95
Area_Diff (%)	8	14	17	15	3

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Table 7. Comparison of predicted crack shape from VAR model against ground truth

Load	48k	50k	52k	54k	58k	60k	62k	64k
Overlap (%)	63	70	66	92	70	95	87	97
Area_Diff (%)	8	14	17	15	16	5	2	6

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### 3.2.2 Results and Discussion

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## 4. Conclusion and Future Work

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In this work, a methodology to parametrize and model the dynamics of defect evolution based on a convex hull parametrization and time-series modeling was introduced. Using convex hull parametrization, 2D synthetic and experimental point clouds representing various defect shapes and stochastic evolutions were parametrized and their evolution were modeled using time series forecasting models. The future state of defects was then forecasted and evaluated against ground truth. The results indicate that this convex hull approach provides consistent and accurate representations of defect evolution across a range of defect topologies and is reasonably robust to noisy measurements, however the behavior of the underlying dynamical process plays a significant role in predictive accuracy. Predictive accuracy degrades for both ARIMA and VAR models as defect evolution becomes increasingly random, though ARIMA is slightly more robust under such conditions.

The proposed methodology has a number of advantages over current practices. First, it provides engineers with an intuitive and consistent representation of remotely sensed information over a structures' life-cycle through the reduced dimension convex hull representation. Tracking the evolution of damages and their connections to structural performance also results in more reliable forecasting capabilities and a more complete understanding of structural performance, particularly compared to existing NDE techniques that often do not quantify damage evolution through time. This process also does not require extrapolation from other data sets for prediction, rather it builds up a time-series representation based solely on the observed evolution of a given defect.

This study was part of an on-going research program and various part of the presented methodology are being considered for further improvement. The parametrizations and hull modeling are being studied for temporal tracking of non-geometric changes such as color change in structures. The hull parametrization method is being extended to high-dimensional feature space analyses, supporting

489 the fusion of multiple sensors and survey information for holistic life-cycle modeling. As it was  
490 presented in the methodology description, the results of this work will be used to support finite  
491 element model updating for predictive analysis of structural capacity as well. One notable avenue for  
492 future work is to adapt the algorithm to more realistically parameterize defect shape using a  
493 combination of a convex and concave hull algorithm [68]. Such an approach would allow for more  
494 accurate depiction of complex geometric topologies similar to the fatigue cracks evaluated in this  
495 work. In addition, nonlinear time series modeling methods such as Recurrent Neural Network can  
496 be studied for more complex defect evolutions, however such machine-learning driven approaches  
497 need much larger datasets to be employed.  
498

499 **Author Contributions:** conceptualization, S.M and D.L.; methodology, S.M.; validation, S.M.; formal analysis,  
500 S.M.; resources, D.L.; writing—original draft preparation, S.M.; writing—review and editing, S.M. and D.L.;  
501 supervision, D.L.; project administration, D.L.; funding acquisition, D.L.

502 **Funding:** This work was supported in part by a grant from the Office of Naval Research (No. N00014-18-1-2014).  
503 Any opinions, findings, and conclusions or recommendations expressed in this material are those of the authors  
504 and do not necessarily reflect the views of the Office of Naval Research.

505 **Acknowledgments:** The authors would like to thank Dr. Hugh Bruck and Paul Lara (University of Maryland)  
506 for providing access to experimental data used in this work.

507 **Conflicts of Interest:** The authors declare no conflict of interest.  
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