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Multi-Band Frequency Window for Time-Frequency Fault Diagnosis of Induction Machines

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Abstract: Induction machines drive many industrial processes, and their unexpected failure can cause heavy production losses. The analysis of the current spectrum can identify online the characteristic fault signatures at an early stage, avoiding unexpected breakdowns. Nevertheless, frequency domain analysis requires stable working conditions, which is not the case for wind generators, motors driving varying loads, etc. In these cases an analysis in the time-frequency domain -such as a spectrogram- is required for detecting faults signatures. The spectrogram is built using the short frequency Fourier transform, but its resolution depends critically on the time window used to generate it: short windows provide good time resolution, but poor frequency resolution, just the opposite than long windows. Therefore, the window must be adapted at each time to the shape of the expected fault harmonics, by highly skilled maintenance personnel. In this paper, this problem is solved with the design of a new multi-band window, which generates simultaneously many different narrow-band current spectrograms, and combines them into a single, high resolution one, without the need of manual adjustments. The proposed method is validated with the diagnosis of bar breakages during the start-up of a commercial induction motor.

Keywords: fault diagnosis; induction motors; wind energy generation; fourier transforms; spectral analysis; spectrogram; transient regime

1. Introduction

Induction machines (IMs) are a key component of many industrial processes, either as motors or as generators, such as double fed induction generators (DFIGs) used for wind energy generation. Their reliability ensures the continuity of the production processes, but they are subjected to eventual failures (broken bars, bearing faults, eccentricity, turn to turn or phase to ground short-circuits, etc.), which may cause unexpected breakdowns and high economic losses. A way to reduce these risks is the continuous monitoring of the machine's condition, in order to detect the presence of a fault at an early stage, when corrective measures can be implemented or maintenance works scheduled. Diverse quantities have been proposed in the technical literature for implementing condition based maintenance systems (CBMS) [1,2], such as the analysis of the stator currents [3–7], machine vibrations [8–10], fluxes [11,12], thermal images [13], or acoustic signals [14,15]. These techniques have been applied to detect different types of faults not only of the IM, such as stator inter-turn short circuits [11,16], broken bars [17,18], rotor asymmetries [19], eccentricity [20], bearing faults [6,21], but also of the inverter drive [3] or the mechanical coupling to the load, as gearboxes and pulleys [3,22].

Table 1. Characteristic Frequencies of Different Types of IM Faults

Type of fault	Fault Harmonics Frequency	
Shorted coils	$f_1(k \pm n \frac{1-s}{p})$	$k = 1, 3, 5, \dots$ $n = 1, 2, 3, \dots$
Rotor asymmetries	$f_1((1-s) \frac{k}{p} \pm s)$	$\frac{k}{p} = 1, 3, 5, \dots$
Mixed eccentricity	$ f_1 \pm k f_r $	$k = 1, 2, 3, \dots$
Bearing (outer race)	$\frac{N_b}{2} f_r \left[1 - \frac{D_b \cos(\beta)}{D_c} \right]$	
Bearing (inner race)	$\frac{N_b}{2} f_r \left[1 + \frac{D_b \cos(\beta)}{D_c} \right]$	
Bearing (balls)	$\frac{D_c f_r}{2 D_b} \left[1 - \left(\frac{D_b \cos(\beta)}{D_c} \right)^2 \right]$	

31 A diagnostic technique that has gained a widespread interest in recent years is the motor current
 32 signature analysis (MCSA) [23–26], which is based on the detection of the characteristic fault signatures
 33 that each type of fault impress in the current spectrum. It is a non-invasive method, which can be
 34 applied on line without disturbing the normal operation of the machine; it requires, in its more basic
 35 implementation, just a current sensor for acquiring the current signal and a fast Fourier transform
 36 (FFT) for generating its spectrum; and it can detect a wide variety of machine faults through their
 37 spectral signatures, because each type of fault generate a specific set of fault frequencies, as shown
 38 in Table 1 [27], where f_1 stands for the fundamental component frequency, p is the number of pole
 39 pairs, s is the machine slip, f_r is the rotational frequency of the IM rotor, N_b is the number of balls of
 40 the bearings, D_b is the bearing diameter, D_c is the pitch or cage diameter, and β is the contact angle.

41 Nevertheless, the use of the current spectrum as signal processing tool in MCSA limits its field of
 42 application to machines working in stationary conditions, which is not the case of industrial processes
 43 with varying load or speed conditions, or also of wind generators operating under variable wind
 44 regimes. In these cases, the fault frequencies shown in Table 1 become random time functions, and
 45 the fault harmonics do not produce isolated spectral lines in the current spectrum, which blurs their
 46 characteristic signatures.

47 To extend MCSA to the fault diagnosis of IM working in transient regime, advanced
 48 time-frequency (TF) transforms of the current are needed, so that the transient fault signatures can
 49 be identified in a joint TF domain. These transforms can be lineal, such as the short-time Fourier
 50 transform (STFT) [28–30], the short-frequency Fourier transform (SFFT) [31], and the wavelet transform
 51 (WT) [32], or quadratic, such as the Wigner-Ville distribution (WVD) [33], or the ambiguity function
 52 [34]. Quadratic TF transforms can achieve optimal resolution for mono-component chirp signals, but
 53 in case of multi-component ones, they produce cross-terms artifacts that pollute the TF representation
 54 of the current, making it difficult the correct identification of the fault harmonics. On the contrary,
 55 linear TF transforms are free from cross-terms artifacts. The STFT representation, the spectrogram, and
 56 the WVD representation, are built by multiplying the current signal with an analysing window at each
 57 time instant, and performing the FT of the resultant signal. In the case of the STFT, this window has a
 58 constant shape, which makes it difficult to obtain a good resolution both in time and in frequency: a
 59 short window gives a good time resolution, but a poor frequency resolution; on the contrary, a long
 60 window gives a good frequency resolution, but a poor time resolution. The WT solves this issue by
 61 performing a multi-resolution analysis, using different windows at different frequency bands: long
 62 windows for the lower frequency bands, and short windows for the higher frequency bands. But
 63 this approach makes the application of the WT much more complex than the FT, and distorts the TF
 64 signature of the fault harmonics.

65 Diverse solutions to this problem have been proposed recently, in order to built a current's
66 spectrogram with enough resolution for being used as an IM fault diagnosis tool. One of them is
67 to adapt the shape of the analyzing window in the TF domain to the expected shape of the fault
68 harmonics, either using a single window, as in [35], or using different window shapes for different
69 sections of the current signal, as in [36]. This approach requires a deep a priori knowledge of the
70 time evolution of the fault harmonics in the current signal, which requires highly skilled maintenance
71 personnel for implementing it, and difficult its application in automated diagnostic systems. Other
72 approach apply to the spectrogram a post-processing based on reassignment [37] or synchrosqueezing
73 [9,38] techniques, so improving its sharpness, but these techniques add a considerable computational
74 burden to the process of building the current spectrogram, departing from the simplicity of the STFT.
75 Another alternative for obtaining an improved spectrogram is the Matching Pursuit approach [39]. This
76 method is based on calculating a series of spectrograms, using a set of different windows designated
77 as "dictionary" [40], which has to be previously built. Then, combining spectrograms corresponding to
78 each window of the dictionary through a pre-defined algorithm, obtains the final spectrogram, which
79 is considered the optimum one. This method has several drawbacks, such as the need of building
80 an extensive dictionary, with a great amount of different windows for obtaining a good resolution
81 spectrogram. This implies to calculate a huge quantity of spectrograms and thus consuming a vast
82 quantity of time and computational resources

83 In this paper, a novel approach (up to the best of the authors' knowledge) is proposed to obtain
84 a high-resolution current spectrogram, useful for fault diagnostic purposes, with the simplicity of a
85 single STFT. It is based on

- 86 1. Performing the STFT with a wide range of windows with different lengths, and selecting, for
87 each point in the TF domain, the best result obtained at that point among the complete set of
88 windows.
- 89 2. Instead of running a separate STFT for each of the windows used in the analysis, a single,
90 multi-band window is built by stacking all the desired analysing windows in consecutive
91 frequency bands. This approach obtains in parallel the spectrograms corresponding to several
92 hundreds of different analysis windows with the computing cost of a single one, which makes it
93 suitable for fast, online diagnostic systems in transient regime.

94 The structure of the paper is as follows. In Section 2 the generation of the spectrogram with
95 Gaussian windows shifted in the frequency domain is analyzed, and in Section 3 it is used for the
96 theoretical and practical explanation of the proposed method. In Section 4, it is validated with the
97 analysis of the start-up current of a high rated power, medium voltage squirrel cage induction motor,
98 with broken bars. Section 5 presents the conclusions of this work.

99 2. Time-Frequency Analysis of the Machine's Current via STFT with a Multi-Band Window

100 To highlight the spectral content of a time-varying current signal, which may contain the
101 characteristic fault harmonics given in Table 1, it is necessary to generate a representation of the
102 current signal in the join TF domain. Among the diverse transforms available to this end (STFT, SFFT,
103 WT, WVD, ambiguity function, etc.), the STFT has been selected in this work, because it is a linear
104 transform, without cross-terms artifacts, and is computationally very effective, which makes it suitable
105 for being implemented in low-cost, low-power embedded devices for on-line CBM systems.

106 In this Section, the traditional STFT analysis of the stator current using a Gaussian window will be
107 first reviewed, and after the proposed method using a multi-band frequency window will be presented
108 and compared with the traditional one.

109 2.1. Spectrogram of Machine's Current

110 The STFT of a current signal $i(t)$ is a linear TF transform that is able to generate a joint TF
111 representation of the current, the spectrogram, through the following steps:

112 1. For each instant τ , the current signal is multiplied element by element by the conjugate of a
 113 suitable time window centered at τ , $h(t - \tau)$

$$i_\tau(t) = i(t)h(t - \tau)^* \quad (1)$$

114

115 that emphasizes the content of the current signal at time τ , and attenuates it at other times

$$i_\tau(t) = \begin{cases} i(t), & \text{if } t \text{ is close to } \tau \\ 0, & \text{if } t \text{ is far from } \tau \end{cases} \quad (2)$$

116

2. The Fourier transform if applied to the time-windowed signal $i_\tau(t)$, which gives the frequency content of the current signal $i(t)$ around time τ

$$\begin{aligned} I_\tau(\omega) &= \frac{1}{\sqrt{2\pi}} \int e^{-j\omega t} i_\tau(t) dt \\ &= \frac{1}{\sqrt{2\pi}} \int e^{-j\omega t} i(t)h(t - \tau)^* dt \end{aligned} \quad (3)$$

117

where $\omega = 2\pi f$, and f stands for the frequency, in Hz.

3. The energy density spectrum at time τ is obtained as

$$I_{SP}(\tau, \omega) = |I_\tau(\omega)|^2 = \left| \frac{1}{\sqrt{2\pi}} \int e^{-j\omega t} i(t)h(t - \tau)^* dt \right|^2 \quad (4)$$

118

119 For each instant τ the STFT generates a different energy spectral density $I_{SP}(\tau, \omega)$, and the total
 120 set of these spectra constitutes the current spectrogram. A critical issue for obtaining a high resolution
 121 spectrogram of the current signal is the selection of the window $h(t)$ in (4). To obtain a high resolution
 122 of the energy content of the current signal in the joint TF domain, it is necessary to use a window
 123 with a high concentration of energy in the TF plane, but such energy concentration is limited by the
 124 Heisenberg's uncertainty principle : a short time window gives a good time resolution, but a poor
 125 frequency resolution, and, on the contrary, a long time window gives a good frequency resolution, but
 126 a poor time resolution. The window that can achieve the highest energy concentration in the joint TF
 127 domain is the Gaussian window [35], given in the time domain by

$$g(t) = \left(\frac{\alpha}{\pi} \right)^{1/4} e^{-\frac{\alpha}{2} t^2} \quad (5)$$

128

129 and in the frequency domain by

$$G(\omega) = \left(\frac{1}{\alpha\pi} \right)^{1/4} e^{-\frac{1}{2\alpha}\omega^2} \quad (6)$$

130

The standard deviation of the Gaussian window in the time domain (5) is $\sigma_t^2 = 1/(2\alpha)$, and in the frequency domain (6) is $\sigma_\omega^2 = \alpha/2$. Therefore, for the Gaussian window, the product of its duration σ_t and its bandwidth σ_ω gives [41]

$$\sigma_t \sigma_\omega = 1/2 \quad (7)$$

131

The Heisenberg's uncertainty principle states that one cannot construct any signal whose duration σ_t and bandwidth σ_ω are, both, arbitrarily small, because

$$\sigma_t \sigma_\omega \geq 1/2 \quad (8)$$

132

133 In this way, the Gaussian window has a duration-bandwidth product (7) that reaches the minimum
 134 value (i.e., the highest concentration in the joint TF plane) that can be achieved under the uncertainty
 135 principle (8).

136 The parameter α in (5) and in (6) is the only one that defines the shape of the Gaussian window. A
 137 low value of α gives a long window with a narrow bandwidth, while a high value of α gives a short
 138 window, with a wide bandwidth. This parameter must be tuned to the current signal to be analyzed
 139 with the STFT. As detailed in [35], the optimal Gaussian window to build the spectrogram of a given
 140 current signal is the one that has the maximum overlap with the current in the TF domain. That is, the
 141 optimal parameter α is the one whose height/width ratio $\sigma_\omega/\sigma_t = \alpha$ best approximates the slope of the
 142 current signal in the TF domain. Unfortunately, the slope of the fault harmonics in an IM in transient
 143 regime is not a constant value, because in this regime the slip and the rotational frequency in Table 1
 144 are time-varying quantities, as well as the fault frequencies that depend on them. Besides, different
 145 components of the current signal may have different slopes (such as the fundamental component and
 146 the fault harmonics). These facts preclude the use of a single, optimal Gaussian windows for building
 147 a high resolution diagnostic spectrogram of the IM current.

148 *2.2. Frequency Shifting of the Gaussian Analysing Window*

149 If the Gaussian window (5) is used as the analysing window to build the energy density spectrum,
 150 then (4) becomes

$$I_{SP}(\tau, \omega) = |I_\tau(\omega)|^2 = \left| \frac{1}{\sqrt{2\pi}} \int e^{-j\omega t} i(t) g(t-\tau)^* dt \right|^2 \quad (9)$$

151

152 The Gaussian window (5) is a real valued function. If it shifted in the frequency domain by a
 153 frequency f_k , corresponding to an angular frequency $\omega_k = 2\pi f_k$, then (5) becomes a complex-valued
 154 function

$$g_k(t) = g(t)e^{j\omega_k} = \left(\frac{\alpha}{\pi} \right)^{1/4} e^{-\frac{\alpha}{2}t^2} e^{j\omega_k} \quad (10)$$

155

156 Replacing (10) in (9) gives

$$I_{SP}(\tau, \omega) = |I_\tau(\omega)|^2 = \left| \frac{1}{\sqrt{2\pi}} \int e^{-j\omega t} i(t) g(t-\tau)^* e^{-j\omega_k t} dt \right|^2 \quad (11)$$

157

that is,

$$I_{SP}(\tau, \omega) = |I_\tau(\omega)|^2 = \left| \frac{1}{\sqrt{2\pi}} \int e^{-j(\omega+\omega_k)t} i(t) g(t-\tau)^* dt \right|^2 \quad (12)$$

158

159 and, making the change of variable $\omega' = \omega + \omega_k$, (12) can be expressed as

$$I_{SP}(\tau, \omega' - \omega_k) = |I_\tau(\omega' - \omega_k)|^2 = \left| \frac{1}{\sqrt{2\pi}} \int e^{-j\omega' t} i(t) g(t-\tau)^* dt \right|^2 \quad (13)$$

160

161 Therefore, as the shift ω_k of the Gaussian analysis window changes, the frequencies of the entire
 162 corresponding spectrogram (13) suffer the same ω_k shift, at each time τ .

163 *3. Proposed Multi-Band Analysing Window*

164 A possible approach for obtaining a high-resolution spectrogram of a current signals with
 165 components of different slopes in the TF domain could be a variant of the Matching Pursuit approach,
 166 using as dictionary a set of Gaussian functions with different values of α . In this way, a batch of

167 spectrograms, each one with a different value α for the Gaussian window (5) would be generated.
 168 In a second stage, for each point in the TF domain, the best value obtained among the whole set of
 169 spectrograms would be selected, giving the best approximation to the ideal TF representation of the
 170 current signal.

171 The drawback of this technique is the high amount of resources that it requires. For each possible
 172 value of α , in a given range, a full spectrogram must be built, which is a time-consuming operation,
 173 and, also, it must be stored, which implies high memory resources. Afterwards, a processing algorithm
 174 must be applied to each point of all the spectrograms to combine them. These requirements make this
 175 technique unsuitable for being deployed with on-line, low power embedded devices.

176 On the contrary, the novel technique presented in this paper can achieve the same results at
 177 roughly the cost of a single STFT, in terms of speed and storage requirements, even with the use of
 178 several hundreds of Gaussian windows with different values of α . It is based on a particular feature
 179 of the IM faults presented in Table 1: in most industrial IMs, the fault harmonics with the highest
 180 amplitudes are those with an index $k = 1$, and they are located in a narrow, low frequency band with a
 181 bandwidth f_b (normally of one or two hundreds of Hz). Nevertheless, the current signal is acquired
 182 normally using high frequencies rates, from 5 or 10 kHz up to 100 kHz and more. This implies that,
 183 when performing the FT of the windowed current signal, at each stage of the STFT process, only the
 184 values within the narrow band $[0 - f_b]$ of diagnostic interest are kept, and the rest of the spectrogram
 185 values are discarded, what represents a waste of computing resources.

186 The proposed method addresses this problem, filling the whole spectrogram with useful
 187 diagnostic contents. It relies on frequency shifting the Gaussian analysing window, as in sub-section
 188 2.2, (13). If the current signal is low-pass filtered with a cut-off frequency f_b (using, for example, a
 189 frequency filter as in [42]), then its spectrogram, built with a Gaussian window (5), will be non-zero
 190 only in the frequency band of diagnostic interest $[0 - f_b]$. But, if the Gaussian window is shifted to a
 191 frequency $f_k = f_b$ in (10), then the spectrogram of filtered current signal will appear in the frequency
 192 band $[f_b - 2f_b]$ (and will be zero outside this band). If the α parameters of both Gaussian windows (the
 193 shifted and the non-shifted one) are equal, the same spectral information will appear in both frequency
 194 bands. But, if the shifted Gaussian window has a different value of α , then the spectrograms in the
 195 frequency bands $[0 - f_b]$ and $[f_b - 2f_b]$ will be different, each one corresponding to a different value of
 196 the parameter α .

197 What is proposed in this paper is to extend this feature, using, instead of two frequency bands,
 198 a partition of the whole spectrogram in N_g adjacent bands, each one with a frequency width equal
 199 to f_b ; and to use a set of N_g Gaussian windows, with different values of α (α_k , $k = 0 \dots N_g - 1$), and
 200 with increasing shifting frequencies ($f_k = kf_b$, $k = 0 \dots N_g - 1$), for filling each of these frequency
 201 bands. This can be achieved in a single run of the FFT if, using the superposition principle, all these N_g ,
 202 frequency shifted, Gaussian windows are summed in the time domain, giving a single time window.
 203 The use of this new, multi-band window, in (4) gives a single spectrogram with N_g adjacent bands,
 204 each one corresponding to the analysis of the current signal with a different parameter α_k for the
 205 Gaussian window.

206 For a given sampling frequency f_s , and a frequency band of interest f_b , the total number N_g of
 207 adjacent frequency bands that can be used is equal to

$$N_g = f_s / f_b \quad (14)$$

208

The proposed multi-band window is built, using (10), as

$$gm(t) = \sum_{k=0}^{N_g-1} \left(\frac{\alpha_k}{\pi} \right)^{1/4} e^{-\frac{\alpha_k}{2} t^2} e^{jk2\pi f_b t} \quad (15)$$

209 As an example, a multi-band window (15) has been built using $N_g = 10$ different Gaussian windows
 210 with values of α in (5) spanning linearly the range [$\alpha_{min} = 1 - \alpha_{max} = 700$], with a step equal to
 211 $\Delta\alpha = (\alpha_{max} - \alpha_{min}) / (N_g - 1)$, as

$$gm(t) = \sum_{k=0}^{N_g-1} \left(\frac{\alpha_{min} + k\Delta\alpha}{\pi} \right)^{1/4} e^{-\frac{\alpha_{min} + k\Delta\alpha}{2} t^2} e^{jk2\pi f_b t}. \quad (16)$$

212 The multi-band window (16) is displayed in Fig. 1, which shows the real (Fig. 1.top) and the imaginary
 213 part (Fig. 1.middle) of the window, as well as its spectrum (Fig. 1.bottom). Fig. 2 shows the spectrogram
 214 of the window in the joint TF domain, with all the individual Gaussian atoms stacked in adjacent
 215 frequency bands.

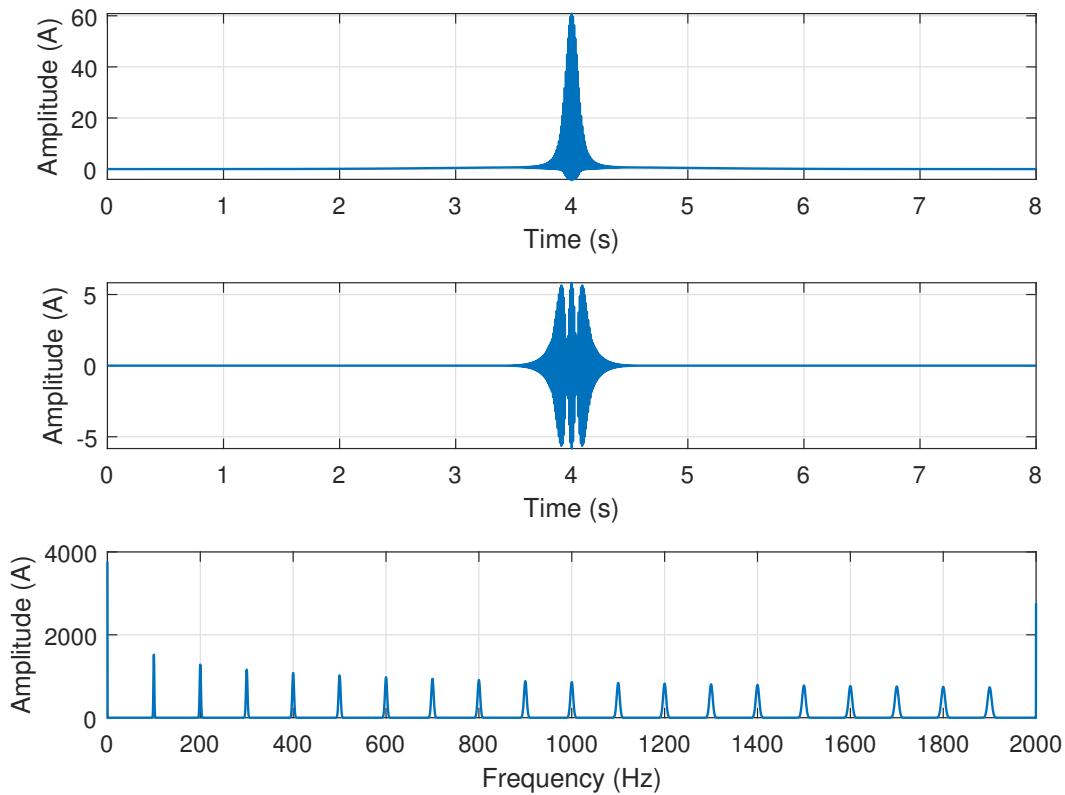


Figure 1. Multi-band window designed in Section 3 in the time domain (real part, top, imaginary part, middle), and in the frequency domain (bottom). This windows contains 20 Gaussian windows, located in 20 adjacent frequency bands of 100 Hz, spanning linearly the range [$\alpha = 1 - \alpha = 700$].

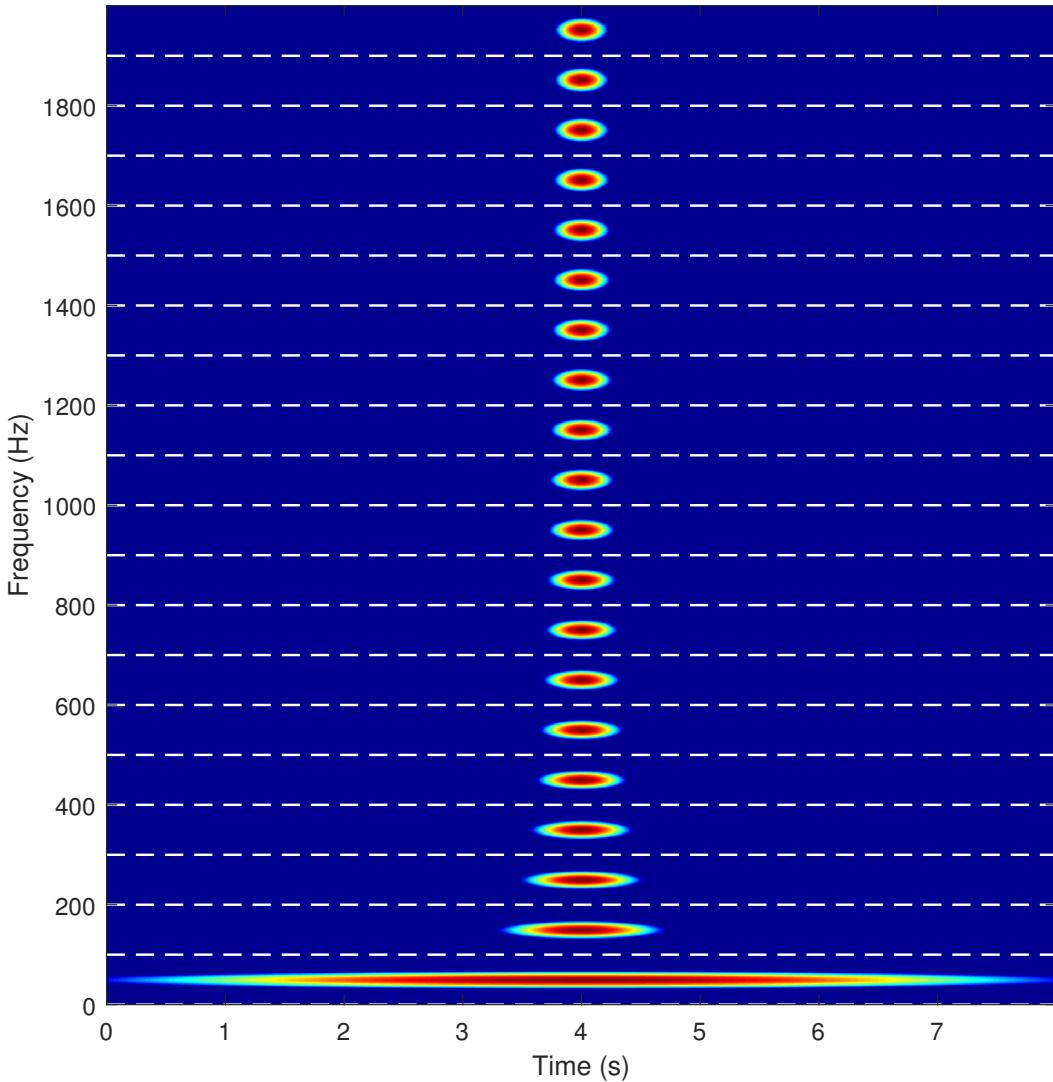


Figure 2. Atoms of the multi-band window designed in Section 3 in the joint time-frequency domain. This windows contains 20 Gaussian windows, located in 20 adjacent frequency bands of 100 Hz, spanning linearly the range from $\alpha = 1$ (bottom) up to $\alpha = 700$ (top).

²¹⁶ 3.1. Steps for Applying the Proposed Multi-Band Window

In this sub-section the process for applying the concept of multi-band window to a given current signal $i(t)$ is explained, using a synthetic current signal $i(t)$ with a sinusoidal component of 50 Hz, and a linear chirp with a frequency slope of 1 Hz/s, starting at -50 Hz. It has been built with a duration of 50 s, using a frequency sampling rate of 200 Hz (Fig. 3),

$$i(t) = \cos(2\pi 50t) + 0.05 \cos(2\pi(-50t + t^2)). \quad (17)$$

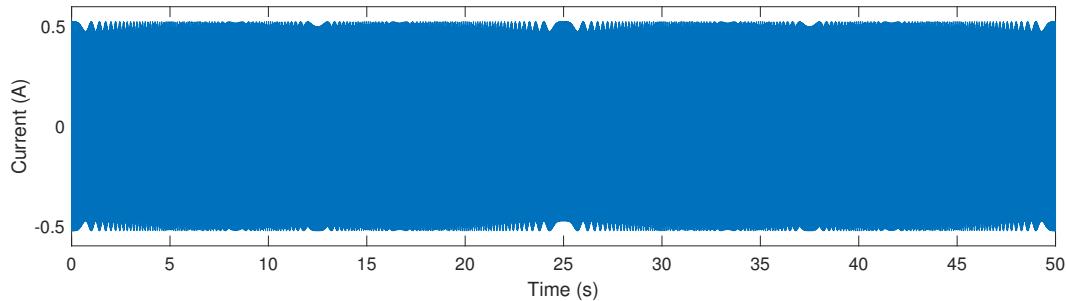


Figure 3. Synthetic current signal (17) used for illustrating the application of the proposed method.

218 The steps for analysing the current signal (17) using the proposed multi-band windows are the
 219 following ones:

220 1. The analysis window (15) is built:

221 • first, the bandwidth of diagnostic interest $[0-f_b]$ is established ($[0-100]$ Hz in this case),
 222 which gives the maximum number of elementary Gaussian windows from (14) ($N_g =$
 223 $200/100 = 2$). The current signal is low-pass filtered with a cut-off frequency equal to f_b . In
 224 this work, a spectral filter which zeroes all the spectrum bins with a frequency greater than
 225 f_b has been used, as in [42].
 226 • second, the parameters α_k of each of these windows in (15) must be selected. For the simple
 227 signal (17), only two values of α are used, $\alpha_0 = 1$ (a long window), and $\alpha_1 = 12.6$, tailored
 228 to the chirp component according to [35].

The resulting multi-band window expression, applying (15), is

$$gm(t) = \left(\frac{1}{\pi}\right)^{1/4} e^{-t^2/2} + \left(\frac{12.6}{\pi}\right)^{1/4} e^{-12.6t^2/2} e^{j2\pi 100t} \quad (18)$$

229 This window is plotted in Fig. 4, in the time and frequency domains.

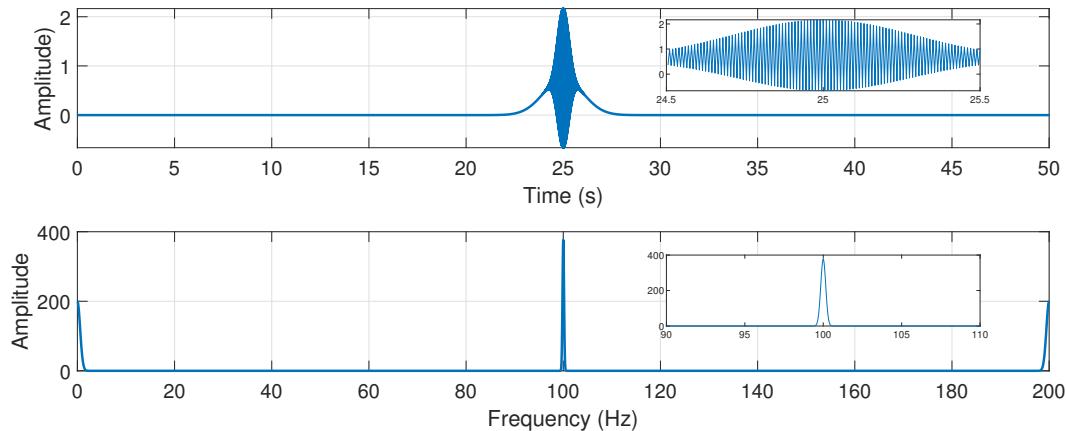


Figure 4. Multi-band window (15) in the time domain (top), and in the frequency domain (bottom). This window contains 2 Gaussian windows, in 2 adjacent frequency bands of 100 Hz, one with $\alpha = 1$ and the other one with $\alpha = 12.6$.

230 2. The spectrogram of the current signal (17) is built, using the multi-band window (18) as sliding
 231 window (Fig. 5). It shows two elementary spectrograms in adjacent TF regions, obtained with
 232 a single run of the STFT algorithm. The bottom one ($\alpha = 1$) locates the sinusoidal component
 233 at 50 Hz, but blurs the chirp component. On the contrary, the top one ($\alpha = 12.6$) locates the
 234 chirp component, but widens the sinusoidal component. This second Gaussian window, and the
 235 spectrogram that it generates, have been shifted to the frequency band [100 Hz-200 Hz].

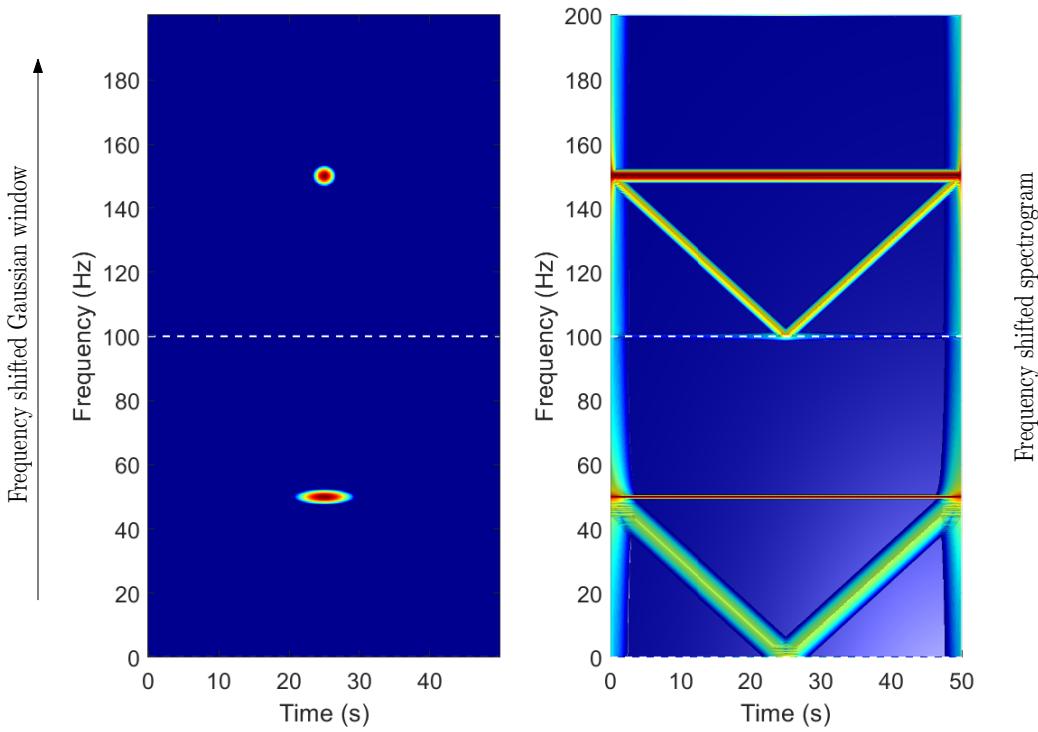


Figure 5. Spectrogram of the current signal (17) obtained in step 2 with the multi-band window (right). The Gaussian component of the window with $\alpha = 1$ (bottom, left) locates the sinusoidal component at 50 Hz (bottom, right), but fails to resolve the chirp component. On the contrary, the Gaussian component with $\alpha = 12.6$ (top, left), locates the chirp component, but widens the sinusoidal component (top, right). The Gaussian window that is frequency shifted (left) generates a spectrogram that is also frequency shift (right).

236 3. All the stacked, elementary spectrograms obtained in step 2 are shifted back to the frequency band
 237 $[0-f_b]$, as shown in Fig. 6. This process has a negligible computational cost, just the renumbering
 238 of the frequency axis of each elementary spectrogram.

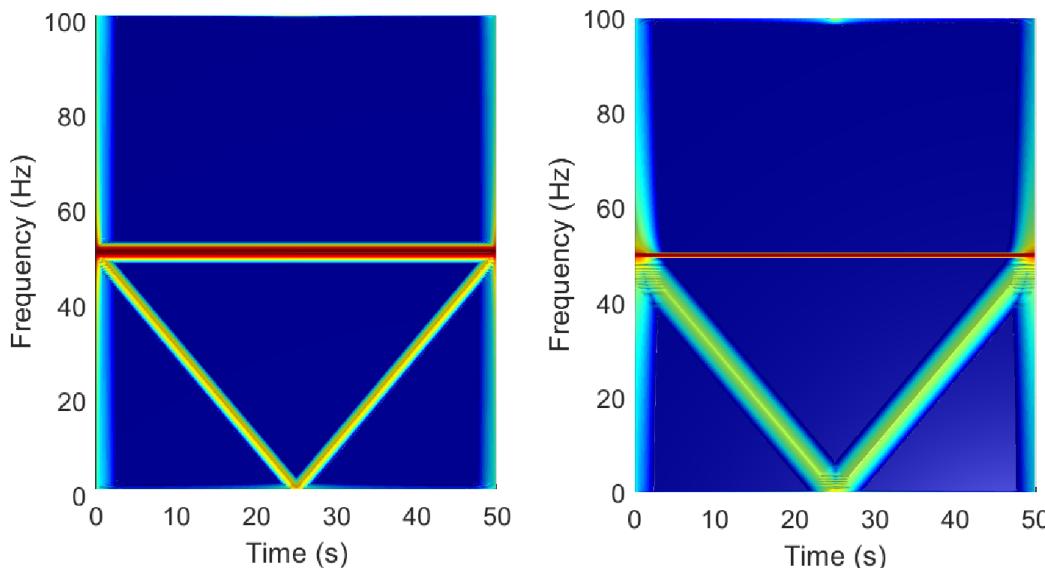


Figure 6. Relocation in the frequency axis of the elementary spectrograms, so that all of them span the same frequency band $[0-f_b]$.

239 4. The points with the same time-frequency coordinates in all the relocated spectrograms obtained
 240 in step 3 (Fig. 6, are combined to give a single high resolution spectrogram of the TF region of
 241 diagnostic interest, in the frequency band [0 - f_b] (Fig. 7). The combination process used in this
 242 work consists in selecting, for each point of this region, the minimum value obtained among all
 243 the relocated spectrograms. The final result shows with a high resolution both the sinusoidal
 244 component at 50 Hz and the chirp component.

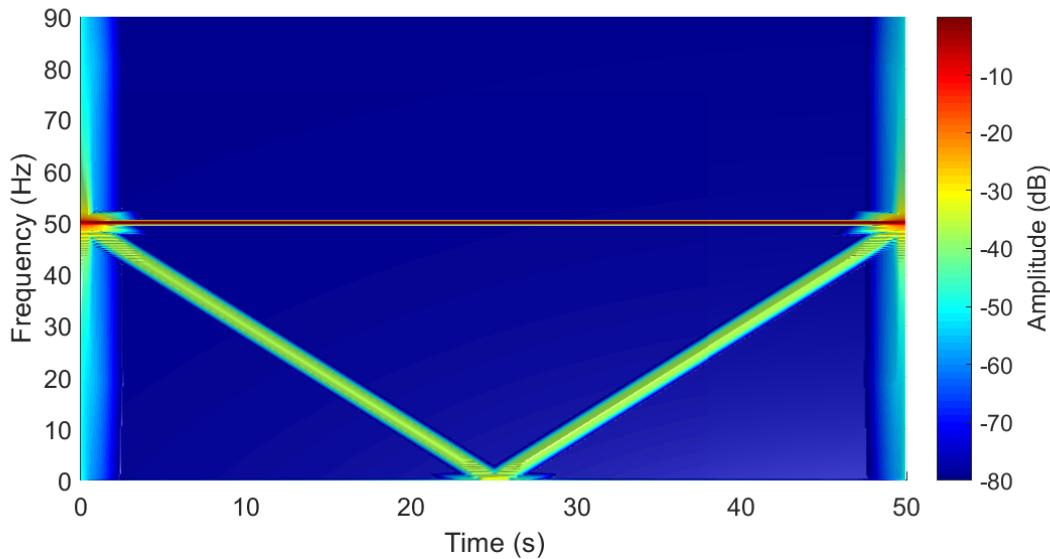


Figure 7. High resolution spectrogram of the current signal (17): for each point of the TF region of interest, the minimum value obtained among all the stacked spectrograms of Fig. 5, right, is selected. The final result shows with a high resolution both the sinusoidal component at 50 Hz and the chirp component.

245 4. Experimental Validation

246 For the experimental verification of the proposed approach, an IM whose characteristics are given
 247 in Appendix A has been prepared with a forced rotor fault, by drilling a hole in one of the rotor bars.
 248 The stator current during a startup transient has been acquired with a frequency rate $f_s = 5000$ Hz,
 249 during 12 seconds, using a current probe whose characteristics are given in Appendix B, and it is
 250 shown in Fig. 8.

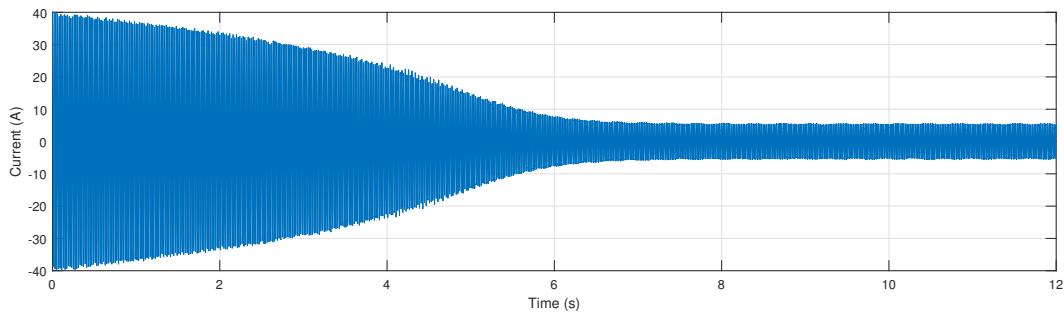


Figure 8. Start-up current of the motor of Appendix A with a broken bar fault.

From Table 1, the characteristic frequencies of the fault harmonics in an IM with a rotor asymmetry, such as a broken bar, are, for the main fault harmonics ($k/p = 1$ in Table 1),

$$f_{bb} = f_1(1 \pm 2s) \quad (19)$$

In particular, the fault harmonic with a frequency given by

$$f_{LSH} = f_1(1 - 2s), \quad (20)$$

which is known as the lower side-band harmonic (LSH), is commonly tracked for the diagnosis of rotor asymmetries. During a start-up transient, the trajectory of the LSH in the TF plane given by (20) generates a typical V-shaped fault signature [43], with a frequency that initially ($s = 1$) is equal to the fundamental frequency f_1 , decreases to 0 ($s = 0.5$), and then increases again until its steady-state regime value of $f_1(1 - 2s) \approx f_1$, according to (20). The ability to detect this fault harmonic using the proposed method is to be assessed in this section.

The three steps presented in Section 3.1 will be followed in this experimental case to obtain the spectrogram of the current signal presented in Fig. 8.

1. The analysis window (15) is built:

- first, the bandwidth of diagnostic interest is established. In this case, from (20), the frequency band of interest is the [0 - 50 Hz] band. Due to the presence of higher order harmonics in the current spectrum, apart from the LSH, a wider band [0 - 125 Hz] has been selected, in order to better assess the strength of the LSH compared with them. In this way, With a sampling frequency of 5000 Hz, the maximum number of elementary Gaussian windows from (14) is $N_g = 5000/125 = 40$ windows.
- second, the parameter α_k of each of these windows in (15) is selected. In this case, a linear range of the parameter α is used, covering the range [$\alpha_{min} = 1 - \alpha_{max} = 700$].

The resulting multi-band window, applying (15), is

$$gm(t) = \sum_{k=0}^{39} \left(\frac{k\alpha_k}{\pi} \right)^{1/4} e^{-\frac{k\alpha_k}{2} t^2} e^{jk2\pi 125}, \quad (21)$$

with

$$\alpha_k = 1 + k \frac{700 - 1}{39} \quad k = 0, 1 \dots 39. \quad (22)$$

The multi-band window (21) is displayed in Fig. 9, which shows the real (Fig. 9.top) and the imaginary part (Fig. 9.middle) of the window, as well as its spectrum (Fig. 9.bottom).

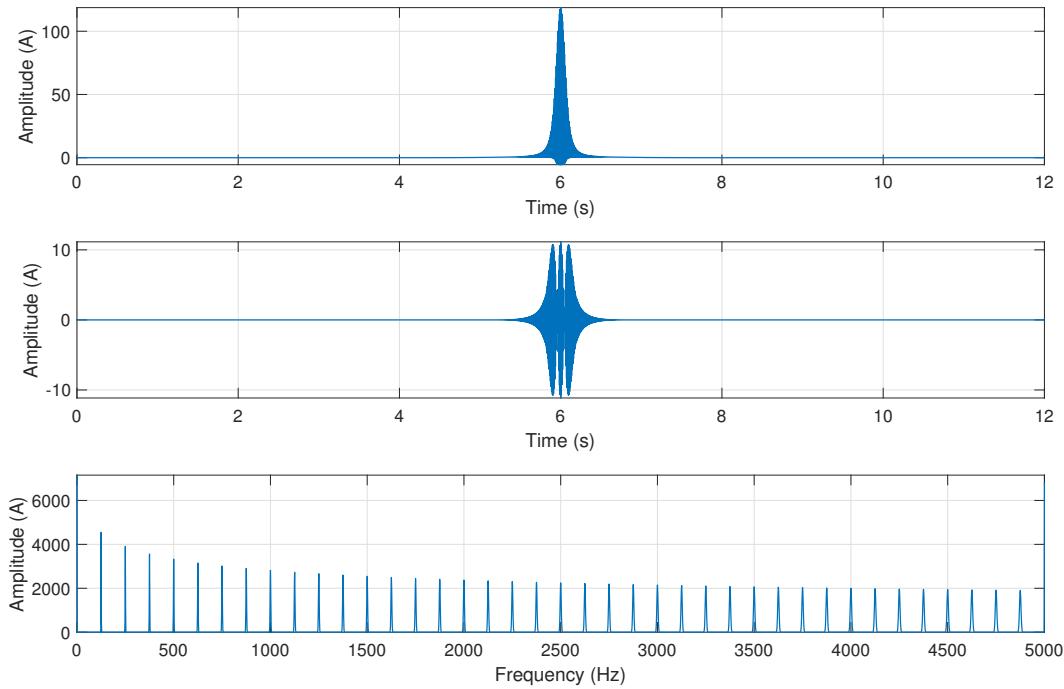


Figure 9. Multi-band window (21) in the time domain (real part, top, imaginary part, middle), and in the frequency domain (bottom). This windows contains 40 Gaussian windows, located in 40 adjacent frequency bands of 125 Hz, with values of α ranging from $\alpha_{min} = 1$ to $\alpha_{max} = 700$.

270 2. The spectrogram of the current signal of Fig. 8 is built. First, the current signal is low-pass
 271 filtered, keeping only the frequency bins of its spectrum lower than 125 Hz. After, and using the
 272 multi-band window (21) as sliding window, the STFT algorithm (4) is applied, which generates
 273 the spectrogram sown in Fig. 10. This spectrogram contains 40 elementary spectrograms in
 274 adjacent TF regions (Fig. 10, right), obtained with 40 different Gaussian windows (Fig. 10, left),
 275 at the cost of a single run of the STFT algorithm (6 seconds with the computer of Appendix C).

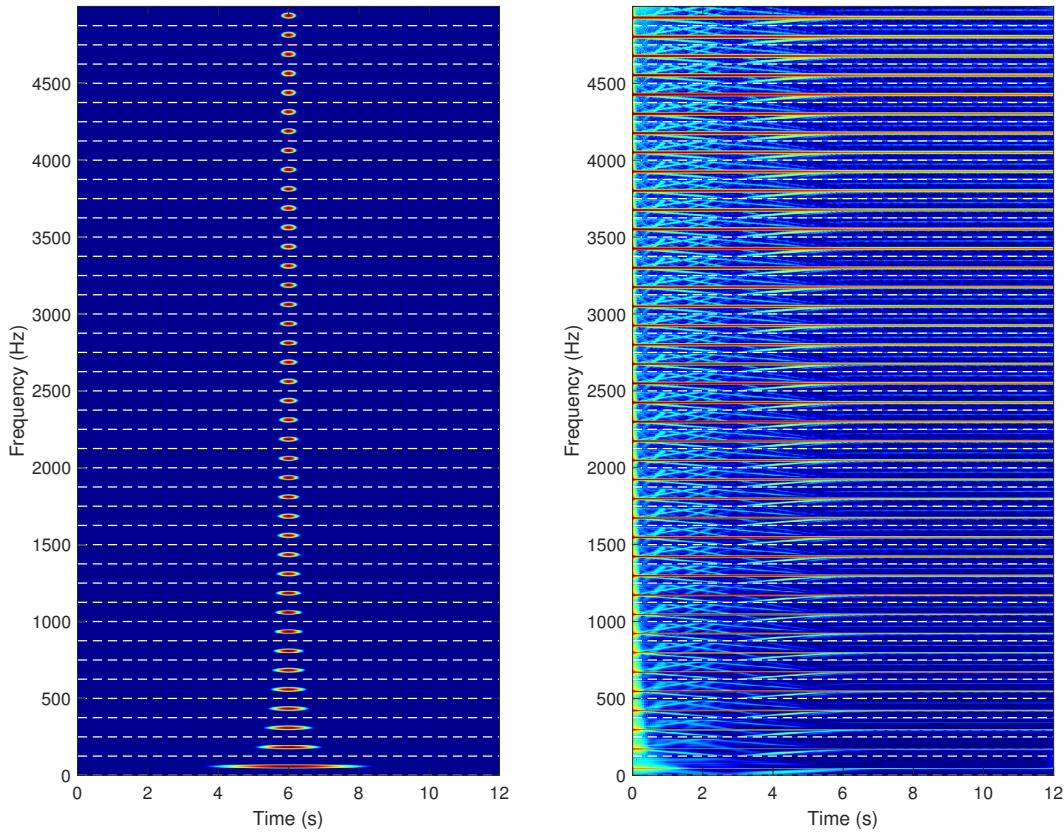


Figure 10. Spectrograms of the current of Fig. 8 (right) obtained with the multi-band window (21) (left). The 40 stacked spectrograms have been obtained with a single run of the STFT algorithm.

276 Two of the 40 individual Gaussian windows shown in Fig. 10 are displayed in Fig. 11, left, along
 277 with their corresponding current spectra (Fig. 11, right). The two zoomed bands corresponds to
 278 the spectrogram located in the base frequency band [0-125 Hz] (Fig. 11, bottom), which defines
 279 clearly the fundamental component, but blurs the fault harmonics, and to the spectrogram shifted
 280 to the frequency band [1500-1625 Hz], (Fig. 11, top) which defines clearly the fault harmonics,
 281 but widens the fundamental component.

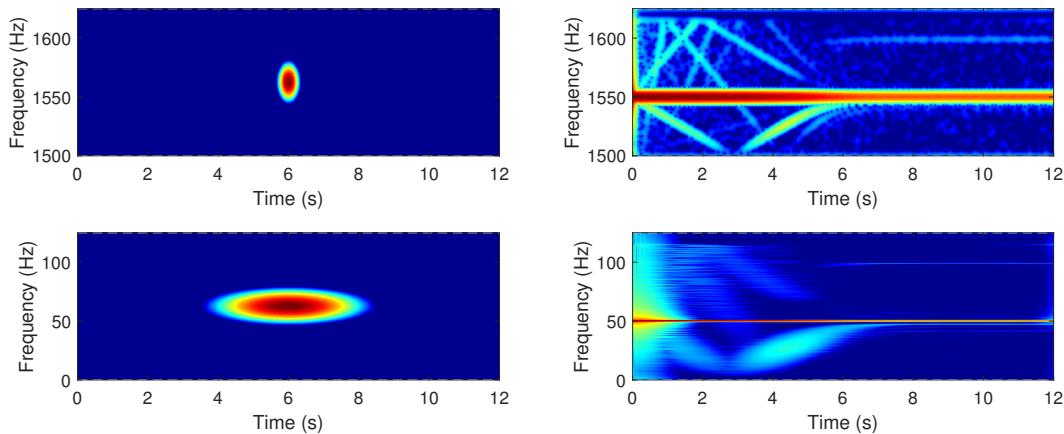
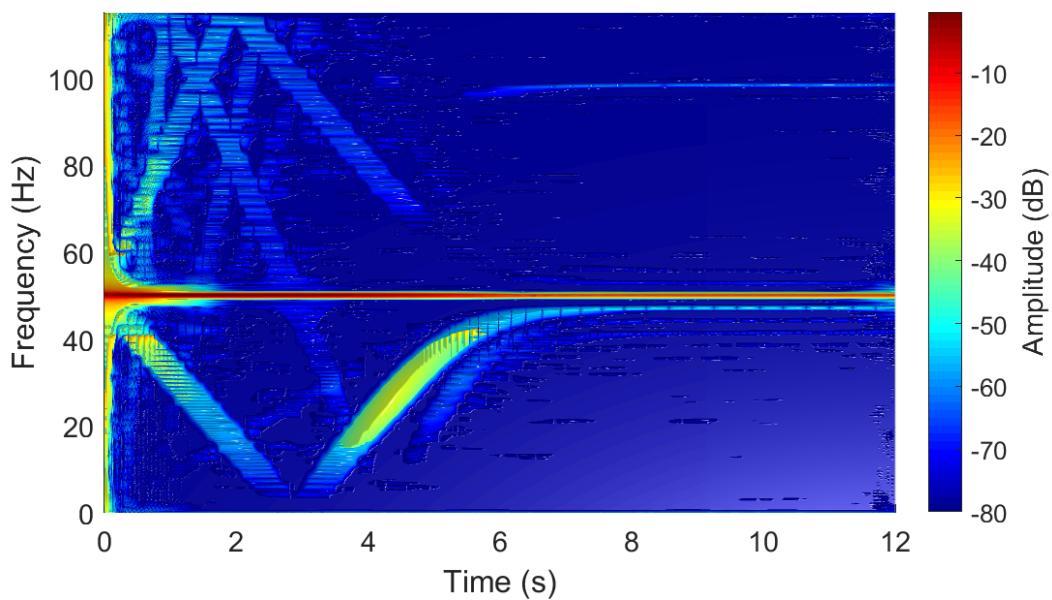


Figure 11. Zoom of Fig. 10 showing two of the individual Gaussian windows contained in the multi-band window of Fig. 10 (left), and the corresponding spectrograms generated with them (right). The two zoomed bands corresponds to the spectrogram located in base frequency band [0-125 Hz] (bottom), which defines clearly the fundamental component, but blurs the fault harmonics, and to the spectrogram shifted to the frequency band [1500-1625 Hz] (top), which defines clearly the fault harmonics, but widens the fundamental component.

282 3. All the stacked, elementary spectrograms obtained in step 2 (Fig. 10, right), are relocated in the
 283 base frequency band [0 125 Hz], by renumbering their frequency axis.
 284 4. All the relocated, elementary spectrograms obtained in step 3 (Fig. 10, right), are combined to give
 285 a high resolution spectrogram of the TF region of diagnostic interest (Fig. 12). The combination
 286 process used in this work consists in selecting, for each point of this [0 - 125 Hz] region, the
 287 minimum value obtained among all the relocated spectrograms. The final result shows with a
 288 high resolution both the sinusoidal component at 50 Hz and the LSH fault component. Unlike
 289 the individual spectrograms, the optimized spectrogram clearly shows the LSH not only during
 290 the transient periode, but also when the steady state is reached; it is also remarkable the set
 291 of fault-related second-order components that are revealed, which helps to get a more reliable
 292 diagnostic.



294 **Figure 12.** High resolution spectrogram of the current of Fig. 8: for each point of the TF region of
 295 interest, the minimum value obtained among all the stacked spectrograms of Fig. 10, right, is selected.
 296 The final result shows with a high resolution both the sinusoidal component at 50 Hz and the LSH fault
 297 component.

298 5. Conclusions

299 In this work, a novel technique for performing the fault diagnosis of IMs working in transient
 300 regime has been presented and validated experimentally. This technique consists in building a
 301 multi-band analysing window, composed by several, different Gaussian windows, stacked in the
 302 frequency domain. When this window is multiplied by the current signal in the time domain, it
 303 generates a spectrum which contains all the spectra generated by each of the Gaussian windows
 304 that form the multi-band window. In this way, a spectrogram containing even hundreds of different
 305 Gaussian windows can be obtained at the cost of a single run of the STFT algorithm. The selection
 306 of the parameters of the individual windows that compose the multi-band window can be setup by
 307 the user using different criteria, without affecting the performance of the proposed approach. In this
 308 work, a blind approach has been used, by selecting a range of individual windows that cover a wide
 309 range of, a priori, unkown fault harmonic components. This approach can be particularly useful for
 310 automated diagnostic system, which can operate without expert assistance, and for the detection of
 311 different types of faults. In this case, a single multi-band window can avoid the design of multiple
 312 analysis windows specifically designed for each single type fault.

313 **Author Contributions:** R.P. and A.S. developed the theoretical explanation of the method. J.B. and J.M. designed
 314 and carried out the experimental validations; M.P. wrote the paper and M.R. carried out the revision the paper.

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315 **Appendix A Motor Characteristics**

316 Three-phase induction machine. Rated characteristics: $P = 1.1$ kW, $f = 50$ Hz, $U = 230/400$ V,
317 $I = 2.7/4.6$ A, $n = 1410$ r/min, $\cos \varphi = 0.8$.

318 **Appendix B Current Clamp**

319 Chauvin Arnoux MN60, Nominal measuring scope: 100 mA–20A, ratio input/output:
320 1 A/100 mV, intrinsic error: $\leq 2\% + 50$ mV, frequency use: 400 Hz–10 kHz.

321 **Appendix C Computer Features**

322 CPU: Intel Core i7-2600K CPU @ 3.40 GHZ RAM memory: 16 GB, Matlab Version: 9.6.0.1072779
323 (R2019a).

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