*Review*

# Kalman Filter for Leak Diagnosis in Pipelines: Brief History and Future Research

**Lizeth Torres [1,*], Javier Jiménez-Cabas [2], Omar González [3], Lázaro Molina [4], and Francisco-Ronay López-Estrada[5*]**

1 Instituto de Ingeniería, Universidad Nacional Autónoma de México; ftorreso@iingen.unam.mx
2 Departamento de Ciencia de la Computación y Electrónica, Universidad de la Costa; jjimenez41@cuc.edu.co
3 Universidad Tecnológica de México Campus Cuitláhuac, omar180282@hotmail.com
4 Universidad Autónoma de Nuevo León, lazaro.molinasp@uanl.edu.mx
5 Tecnológico Nacional de México/Instituto Tecnológico de Tuxtla Gutiérrez, TURIX-DYNAMICS Diagnosis and Control Group
* Correspondence: frlopez@ittg.edu.mx, ftorreso@iingen.unam.mx

**Abstract:** The purpose of this paper is to provide a structural review of the progress made on detection and localization of leaks in pipelines by using approaches based on the Kalman filter. This is, to the best of our knowledge, the first review on the t opic. In particular, it is the first to try to draw the attention of the leak detection community to the important contributions that use the Kalman filter as the core of a computational pipeline monitoring system. Without being exhaustive, we try to gather the results from different research groups and present them in a unified fashion. For this reason, we propose a classification of the current approaches based on the Kalman filter. For each of the existing approaches within this classification, the basic concepts, fundamental results, and relations with the other approaches are discussed in detail. The review starts from a short summary of basic concepts about state observers. Then, a brief history of the use of the Kalman filter for diagnosing leaks is described by mentioning the most outstanding approaches. At last, we briefly discuss some emerging research problems, such as the leak detection in pipelines transporting heavy oils, and we discuss the main challenges and some open problems.

**Keywords:** leak detection; Kalman filter; pipelines

---

## 1. Introduction

Because of the operation conditions, onshore and offshore pipelines are subjected to environmental loads (wind, waves, current, seabed movements, and earthquakes) that can provoke undesirable vibrations, stress, fatigue problems and the propagation of cracks [1]. In particular, this last issue is an important source of leaks together with the aging of the pipelines, failures in the installation, illegal extractions and terrorist sabotage. For this reason, to avoid environmental disasters, the oil and gas sectors invest generous resources for the development of robust and reliable leak detection systems, which according to the API RP 1130 standard can be classified as external or internal systems [2]. External systems use local sensors (e.g., acoustic microphones or fiber optic cables) to send an alarm when a leak occurs, and they do not perform computation for diagnosing a leak. Internal systems utilize field instrumentation outputs, which monitor internal pipeline parameters (e.g., pressure, temperature, flow rate, viscosity), as well as algorithmic monitoring tools. Therefore, internal systems are also known as computational pipeline monitoring systems (CPM systems) [3].

Among the algorithmic tools that have been extensively used for dealing with the fault diagnosis of pipelines, one can find the state estimators (observers), which are powerful tools for the estimation in

Preprints (www.preprints.org) | NOT PEER-REVIEWED | Posted: 19 September 2019 doi:10.20944/preprints201909.0217.v1



real time of the internal state variables of a given system (e.g., a pipeline). These estimations are based on the knowledge of available measurements (inputs and outputs of such a system) and other known parameters. Concretely, a state estimator is a model of a system with an on-line adaptation (correction) based on available measurements for reconstructing information; see Fig. 1 [4]. Usually, the model is a state-space representation, which can in general be either of continuous-time or discrete-time, deterministic or stochastic, finite-dimensional or infinite-dimensional.

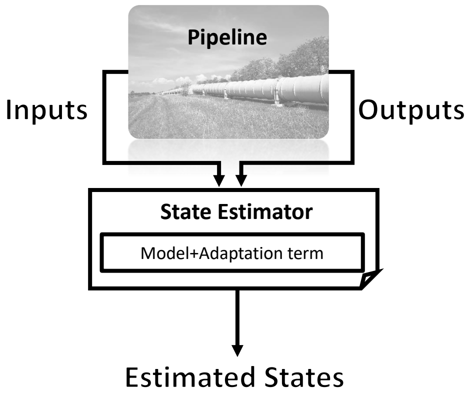


**Figure 1.** Architecture of a CPM system based on a state estimator.

Several kinds of state estimators, such as Kalman filters, Luenberger-type observers, high gain observers and sliding mode observers, have been used for leak detection and localization. In fact, three good reviews that summarize the research and development of state estimator-based leak detection systems for liquid pipelines are given in [5], [6], and [7]. The literature, however, still lacks a more in-depth review of the state of the art in leak detection using Kalman filters, which are the most commonly used estimators for detecting and localizing leaks and, according to D. Simon, are "the best linear estimators" [8]. For this reason, this work aims to fill this gap since the approaches based on these filters deserve a separate study.

There are several versions of the Kalman filter for dealing with the diversity of physical systems and their associated problems. For example, the ensemble Kalman filter (EnKF), which is suitable for problems with a large number of variables such as those described by partial differential equations [9]. For estimating the state of nonlinear systems, there are ad hoc versions such as the extended Kalman filter (EKF), the unscented Kalman filter (UKF), and the particle filter (PF). In this paper, we only focus on the versions used for leak detection, i.e., the Kalman filter, the discrete Kalman filter, and the extended Kalman filter, among others; see Appendix A. Several Kalman-based pipeline leak diagnosis methodologies have been proposed in recent years.

This paper is organized as follows. Section 2 presents a brief review of basic concepts for understanding the functioning of state observers. Section 3 introduces a tentative classification of the Kalman filter-based methods proposed to this day. Section 4 presents the history of the evolution of the Kalman filter-based methods in the area of leak detection. Concretely, this section highlights the contributions that have been a milestone in leak diagnosis. Finally, in Section 5, some recommendations for future research are given. Appendix A presents the mathematical structure of different types of Kalman filters that have been employed in leak detection tasks. Appendix B provides a didactic example of how to design a Kalman filter to detect and locate a leak.

## 2. State estimators

In many engineering applications, some variables cannot be directly measured, either because there are no sensors for them or because the cost of such sensors is expensive. An alternative for

Preprints (www.preprints.org) | NOT PEER-REVIEWED | Posted: 19 September 2019 doi:10.20944/preprints201909.0217.v1



addressing this problem is to obtain a dynamical estimation of the required variables by using state estimators.

A general definition of a state estimator is as follows: *an algorithmic tool that estimates the variables of a process using (1) a mathematical model represented in state space, (2) the available measurements of the process (inputs and outputs), and (3) a error correction (adaptation) term to ensure the convergence of the algorithm.*

To derive a general structure of a state estimator, let us consider the general structure of the continuous model of a system in a state-space representation given as follows:

$$\begin{aligned} \dot{x}(t) &= f\left(x(t), u(t)\right), \\ y(t) &= h\left(x(t)\right), \end{aligned} \tag{1}$$

where $x(t) \in \mathbb{R}^n$ is the state vector, $\dot{x}(t) \in \mathbb{R}^n$ is the state derivative vector, $u(t) \in \mathbb{R}^m$, is the external (exogenous) input vector or control signal, $y(t) \in \mathbb{R}^p$ represents the output vector, i.e., the measured states (variables) acquired by sensors, $f \in \mathbb{R}^n$ represents the vector field and $h \in \mathbb{R}^p$ is the continuous output function.

Since a state estimator is the model of the system plus a correction (adaptation term), this can be expressed as follows:

$$\begin{aligned} \dot{\hat{x}}(t) &= \underbrace{f\left(\hat{x}(t), u(t)\right)}_{\text{Model Copy}} + \underbrace{K\left(\hat{x}(t)\right)\left(y(t) - \hat{y}(t)\right)}_{\text{Correction Term}}, \\ \hat{y}(t) &= h\left(\hat{x}(t)\right), \end{aligned}$$

where $\hat{x}(t)$ and $\hat{y}(t)$ are the on-line estimations of $x(t)$ and $y(t)$, respectively, and $K(\hat{x}(t))$ is the gain of the observer. Thus, the design of the state observer consists of choosing an appropriate gain $K(\hat{x}(t))$ so that the estimation error converges to 0 when $t \to \infty$ with the desired properties of time convergence and robustness.

If the observation error $e(t)$ is defined as follow,

$$e(t) = x(t) - \hat{x}(t), \tag{2}$$

the dynamics of the error observation can be derived from (1) and (2) and expressed as

$$\dot{e}(t) = f(\hat{x}(t) + e(t), u(t)) - f(\hat{x}(t), u(t)) - K(\hat{x}(t))(h(\hat{x}(t) + e(t)) - h(\hat{x}(t))). \tag{3}$$

An observer connected to a pipeline has the structure of the block diagram shown in Fig. 2. The inputs in a pipeline can be the flow rate provided by a pump, the level of a tank, the flow rate or pressure that results from the opening or closing of a valve. These inputs, or at least a subset of them, must be registered to be injected into the state estimators. The state, which is the smallest possible subset of system variables that can represent the complete state of a system at any time, can be either the pressure or flow rate at any coordinate along the pipeline or the position of a leak. The measured outputs are the measurements provided by *in situ* sensors (flow meters, pressure transducers or thermocouples).

The design and choice of a state estimator depends on many factors: the application in which the estimates will be used, the nature of the system, the nature of the variables to be estimated, the type of information that will be available for performing the estimation, the nature of such information (e.g., discrete or continuous) or the characteristics of the required estimates. In this spirit, an abbreviated procedure for designing a state estimator for leak diagnosis purposes is proposed below. This procedure is illustrated by means of an example presented in Appendix B.

Preprints (www.preprints.org) | NOT PEER-REVIEWED | Posted: 19 September 2019 doi:10.20944/preprints201909.0217.v1



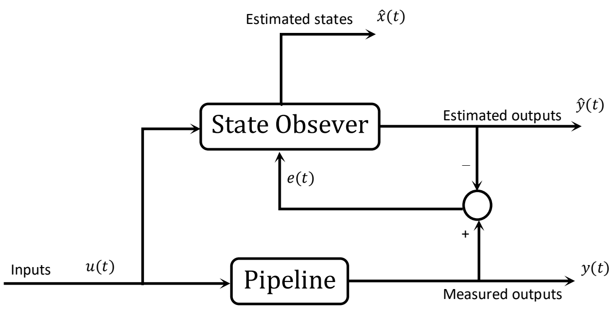


**Figure 2.** State estimation process for pipeline

**Procedure for designing a state estimator**

- **Step 1:** *Identify the available information (observations, data, measurements, records) for performing the estimation.*
- **Step 2:** *Formulate a model assuming convenient assumptions and constraints.*
- **Step 3:** *Set the model in state-space representation.*
- **Step 4:** *Set the equations of the state observer.*
- **Step 5:** *Compute the gain of the state observer.*

## 3. A proposed classification for the Kalman filter-based approaches.

The CPM systems that have used the Kalman filter as the principal algorithmic tool can be categorized into three approaches according to the architecture of the leak diagnosis algorithm: 1) the approaches based on the estimations of a bank of filters, 2) the approaches based on the estimation of variables (e.g., pressures and flow rates) at different locations along the pipeline, and 3) the approaches based on the direct estimation of the leak parameters, which are added to the pipeline flow model as if they were states.

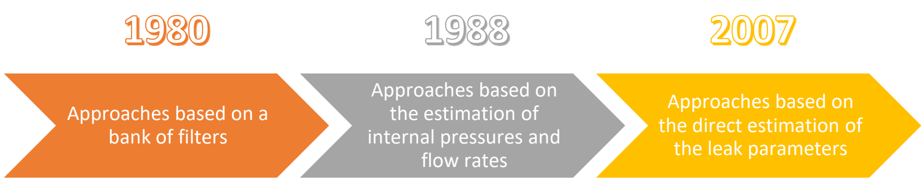


**Figure 3.** Timeline infographic of the evolution of the Kalman-filter based approaches.

This classification is inspired by three influential contributions that were presented in three different years as shown the timeline infographic in Fig. 3. In 1980, Tørris Digernes proposed the first contribution based on a bank of Kalman filters [10]. In 1988, Benkherouf and Allidina proposed the first contribution based on the estimation of the hydraulic variables at different points of the pipeline [11], and in 2007, Besançon et al. proposed the first approach based on the direct estimation of the leak parameters [12]. The following describes what each of these approaches consists of.

Preprints (www.preprints.org) | NOT PEER-REVIEWED | Posted: 19 September 2019 doi:10.20944/preprints201909.0217.v1



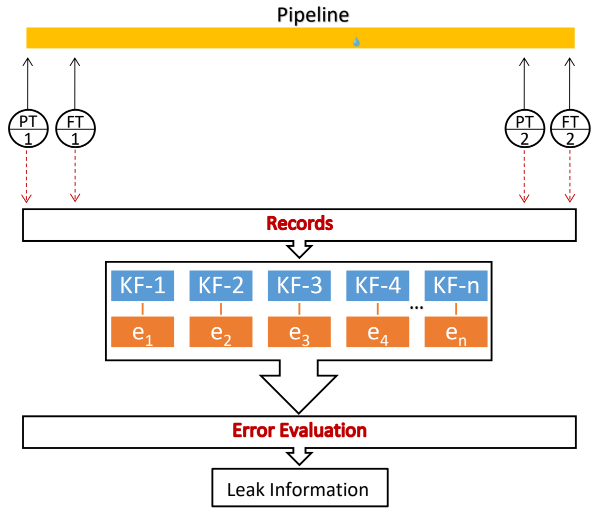


**Figure 4.** Architecture of approaches based on a bank of Kalman filters.

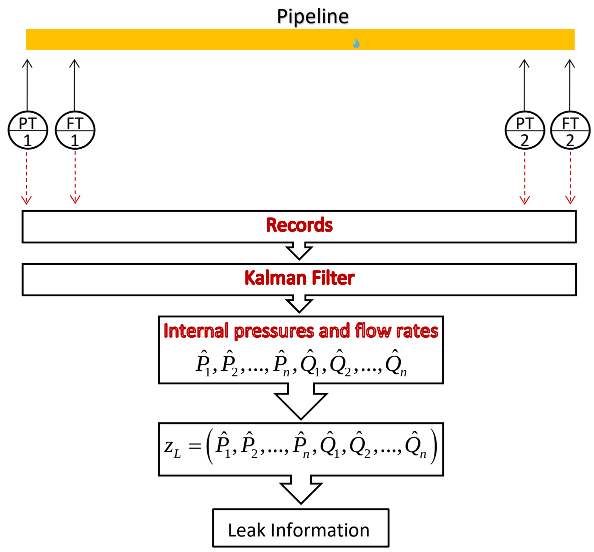


**Figure 5.** Architecture of approaches based on the estimation of internal pressures and flow rates.

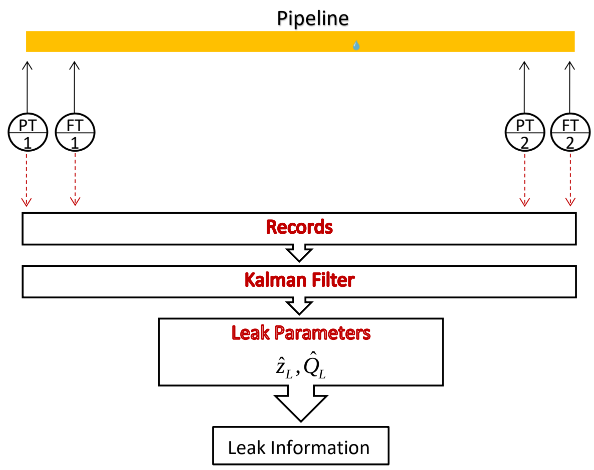


**Figure 6.** Architecture of approaches based on the direct estimation of the leak parameters.

Preprints (www.preprints.org) | NOT PEER-REVIEWED | Posted: 19 September 2019 doi:10.20944/preprints201909.0217.v1



*Approaches based on a bank of filters.* These approaches were the first proposed for detecting and localizing leaks in pipelines by using Kalman filters. The architecture of these approaches is illustrated in Fig. 3, which is basically a set of Kalman filters that act (or perform an estimate) in parallel. Each filter is different from the other because each filter is constructed from a pipeline model with a leak in a prescribed position that is different from the leak positions involved in the other models that are used to construct the other filters. For example, a leak diagnosis algorithm for a 100-meter pipe can be constructed with 10 filters. The first filter can be designed to detect a leak in the first 10 meters of the pipe, the second in the next 10 meters and so on.

On the one hand, the filters forming the bank can be independent of each other, or they can be dependent, that is, interconnected. The interconnection between filters can be cascading or peer-to-peer. On the other hand, the information that each filter receives to make the estimate can be the same (pressures and/or flow rates at the ends) or different (pressures and/or flow rates measured at certain positions of the pipe).

A bank of estimators has been successfully used by commercial leak detection systems such as PipePatrol software supplied by KROHNE Group [13]. The main disadvantage of these approaches, however, is that in order to have better accuracy regarding the leak position a bank of many filters must be designed, which implies that a high computational cost is required for finding the numerical solution of each filter. For this reason, another class of approaches was proposed for reducing the computational burden: the approaches based on the estimations of internal variables.

*Approaches based on the estimation of internal pressures and flow rates.* These approaches use a unique Kalman filter. The states of the model used for designing the Kalman filter are the internal pressures and flow rates at different (positions) coordinates distributed along the pipeline. Once the states are estimated by the Kalman filter, the leak is localized by using the estimations for solving auxiliary algebraic equations (e.g., head loss balances). The accuracy of the leak location is achieved by increasing the number of estimated internal variables. This fact implies that more states must be estimated, and therefore more computational burden is required. The architecture of these approaches is illustrated in Fig. 5.

*Approaches based on the direct estimation of the leak parameters.* These kinds of approaches were proposed to avoid using several filters and to discretize the space in many nodes. In this class of approaches, the leak localization is performed by a unique Kalman filter, which is designed from a mathematical model that involves the leak parameters as state variables in order to be estimated. Usually these approaches involve the Kalman filter described in Appendix A.3. The architecture of these approaches is illustrated in Fig. 6.

The following section details how the different Kalman-based approaches were presented over time, as well as highlights the main characteristics, advantages and disadvantages of the main contributions.

## 4. Brief history

To the best of our knowledge, the first work in presenting an approach based on the Kalman filter to detect and localize faults in pipelines was written by Tørris Digernes [10]. This work, entitled *Real-time failure detection and identification applied to supervision of oil transport in pipelines*, presents a methodology based on multiple parallel filters that are independent of each other: a bank of filters. Each filter is designed from a dynamical model representing a prescribed fault situation. In particular, two fault types were treated by Digernes: single leaks and sensor faults. By applying this methodology, the fault recognition is performed by identifying the filter having the highest probability of representing the plant in the fault situation. The probability is calculated by using the multiple-model hypothesis probability test, which to be performed requires the error estimation between the available measurements from the pipeline and their estimates provided by the filter. To show the potentiality of his methodology, Digernes presented some simulations' results. In such simulations, the features of the oil pipeline between Ekofisk in the North Sea and the terminal in

Preprints (www.preprints.org) | NOT PEER-REVIEWED | Posted: 19 September 2019 doi:10.20944/preprints201909.0217.v1



Teesside, UK, were used. The filters were designed from finite models expressed by space-discrete equations that represent the mass and the momentum balance of the fluid in a pipeline. To compute the error estimation, pressures and flow rates at the inlet and outlet of the pipeline were used, as well as pressures at two points along the pipeline. The principal disadvantage of this approach is that in case of a leak a large number of filters are needed to obtain an acceptable resolution of the leak position. This pioneering work has inspired another important contributions. For example, in [14,15], the same approach was tested in a simulation environment for a long-distance pipeline of water, and only the following aspects were different: the use of a backward time-central space discretization scheme for lumping the continuity and momentum equations together, the use of a modified version of the Kalman filter and the introduction of a feedforward law for computing the leak magnitude.

Years later, Benkherouf and Allidina (B&A) presented the work entitled *Leak detection and location in gas pipelines*, which proposes a Kalman filter for detecting and finding the position of a single leak [11]. The filter is based on a lumped version of a distributed one-dimensional isothermal model (two partial differential equations describing the continuity and momentum conservation) that describes the gas flow through a single pipeline with fictitious leaks distributed along it. To obtain the distributed model, both viscous and turbulent effects of the flow were neglected, and it was assumed that both the temperature changes within the gas and the heat exchanges with the surroundings of the pipeline are small. The lumped model was formulated using the method of characteristics (MOC). By using this approach, the position of the leak is determined through the following algebraic equations that relate the fictitious leaks estimated with the Kalman filter to the real one:

$$Q_L(t) = \sum_{i=1}^{N} Q_{L_i}(t), \tag{4}$$

$$Q_L(t)z_L(t) = \sum_{i=1}^{N} Q_{L_i}(t)z_{L_i}(t), \tag{5}$$

where $z_L$ is the real position leak, $Q_{L_i}$ and $z_{L_i}$ are the flows and positions of the fictitious leaks, $i$ is the fictitious leak index and $N$ is the total number of fictitious leaks.

The methodology of B&A surpasses the Digernes approach in the sense that only one filter has to be designed. For this reason, it has been also the inspiration for a great number of works. For example, in [16], the authors used the same approach with a slight modification in the covariance formula to locate a leak in a pipeline of water. Moreover, they tested the approach in simulations as well as in the laboratory. In [17], the same approach was tested together with a technique called the extended boundary approach. In [18], a comparison between B&A's approach and the algorithm proposed in [19] (which does not have the Kalman filter as a core of the diagnosis system) was presented. According to the authors' conclusions, the cycle time of B&A's method is longer, but it is more accurate.

In 2001, Verde presented the work entitled *Multi-leak detection and isolation in fluid pipelines* [20], which proposes a bank of Kalman filters for localizing leaks in a hydraulic pipeline. Each Kalman filter is designed to diagnose a section of the pipeline, which in fact is divided into $N$ sections. Concretely, each KF is designed to estimate the states (pressures and flow rates) at prescribed points (locations) of the pipeline by considering that a leak is occurring in a pipeline section delimited by two prescribed points. If the pipeline is divided into $N$ sections, as small as desired, $N$ Kalman filters must be designed: each one by considering a leak in a different section. If there are many sections, the computational cost is higher. The estimation error of each KF is used to localize the leak. If a leak develops in a given section, the error associated with the section remains around 0, and the rest do not.

 doi:10.20944/preprints201909.0217.v1



Because the methodology was proposed for a hydraulic pipeline, the Kalman filters were designed from a space-discrete version of the water hammer (WH) equations given as follows:

$$\frac{\partial Q(z,t)}{\partial t} = -gA_r \frac{\partial H(z,t)}{\partial z} - J_s(Q(z,t)), \tag{6}$$

$$\frac{\partial H(z,t)}{\partial t} = -\frac{b^2}{gA_r}\frac{\partial Q(z,t)}{\partial z}, \tag{7}$$

which were proposed by Chaudhry in his prestigious work *Applied Hydraulic Transients* [21] by taking into account the following assumptions.

(A1) The flow is one-dimensional.
(A2) Convective changes in velocity are negligible.
(A3) The cross-sectional area is constant.
(A4) The velocity of the flow is smaller than the velocity pressure wave.
(A5) The energy loss for a given flow velocity during quasi-steady state is the same as that for steady flows at that velocity.
(A6) The fluid is slightly compressible.
(A7) The pipeline walls are linearly elastic and slightly deformable.

For WH equations, $(z,t) \in [0,L] \times [0,\infty)$ gathers the space [m] and time [s] coordinates, respectively, $L$ is the length of the pipe, $H(z,t)$ is the pressure head [m], $Q(z,t)$ is the flow rate [m$^3$/s], $b$ is the wave speed in the fluid [m/s], $g$ is the gravitational acceleration [m/s$^2$], $A_r$ is the cross-sectional area of the pipe [m$^2$], $\phi$ is the inside diameter of the pipe [m] and $Js$ is the quasi-steady friction term, which may be expressed by the Darcy-Weisbach relation as

$$J_s(Q(z,t)) = \frac{f(Q(z,t))}{2\phi A_r} Q(z,t)|Q(z,t)|, \tag{8}$$

where $f$ is the Darcy-Weisbach friction factor.

The method used to numerically solve the WH equation was the first-order finite difference method (FDM). By assuming pressures at the ends of the pipeline as the boundary conditions and after applying the FDM, the fluid model can be represented as $n$ sets of coupled nonlinear dynamic equations given in a state-space representation as follows:

$$\dot{Q}_i(t) = \frac{gA_r}{\Delta z_i}\left(H_i(t) - H_{i+1}(t)\right) - \frac{f(Q_i(t))}{2\phi A_r} Q_i(t)|Q_i(t)|, \tag{9}$$

$$\dot{H}_{i+1}(t) = \frac{b^2}{gA_r \Delta z_i}\left(Q_i(t) - Q_{i+1}(t)\right), \quad \forall i = 1, ..., n. \tag{10}$$

To involve a leak in the above model, Eq. (10) must be modified as follows:

$$\dot{H}_{i+1}(t) = \frac{b^2}{gA_r \Delta z_i}\left(Q_i(t) - Q_{i+1}(t) - Q_L(z_L, t_L)\right), \tag{11}$$

where $Q_L(z_L, t_L)$ is the leak discharge, which is a function of the pressure at the leak position $z_L$ at the leak time $t_L$. In addition, it can be expressed by the orifice equation

$$Q_L(z_L, t_L) = C_d A_L \sqrt{2gH_L(z_L, t)} = \lambda (H_L(z_L, t_L))^{1/2}, \tag{12}$$

where $\lambda = C_d A_L \sqrt{2g}$, $A_L$ is the sectional area of the leak, $C_d$ is the discharge coefficient and $H_L(z_L, t_L)$ is the pressure at the leak point.

In 2003, Verde et al. showed that the isolation (localization) of two simultaneous leaks is not feasible only with steady-state data of the fluid in a pipeline [22]. For this reason, Besançon et al.

presented an approach based on a single extended Kalman filter and suitable inputs to obtain unsteady data from the pipeline [12]. The filter was constructed from a model deduced from Eq. (9) and Eq. (10) for $n = 3$. The order of this model is the minimal to represent two leaks, so we can say that this model is a minimal-order model for two leaks. In order to estimate two leaks, four states with a constant dynamic –that represent both positions and both leak coefficients- were joined to the minimal-order model:

$$\begin{pmatrix} \dot{z}_{L_1}(t) \\ \dot{z}_{L_2}(t) \\ \dot{\lambda}_1(t) \\ \dot{\lambda}_2(t) \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \\ 0 \\ 0 \end{pmatrix}. \tag{13}$$

Since the pressures at the ends of the pipeline were considered as inputs, in order to excite the pipeline they were manipulated to be triangular. The estimation of the positions, as well as the estimation of the coefficients, was achieved. The estimation results have shown, however, that the estimation is sensitive to the initial conditions. Moreover, experimental results were not presented to validate the approach. Torres et al. presented similar methodologies in [23] and [24], but a lumped model obtained via the orthogonal collocation method was used. There are two main reasons why this algorithm could not work with experimental data. The first reason: the reduced order of the finite model that resulted from the discretization of the spatial domain into three sections; this would not be a problem if auxiliary inputs were not needed to ensure the convergence of the estimation. Usually, however, these inputs are periodical with fixed or variable frequency. If the frequency of the required input is too high, the finite model is no longer representative of a real pipeline. A solution to this concern may be the increasing of the order such that the model becomes representative to high frequencies. The second reason may be that the values of the leak positions may take values between 0 and $L$. A solution to this concern could be a reduction of this interval.

In 2010, Dos Santos et al. introduced a new approach for detecting gas leaks in high pressure distribution networks [25]. Each pipeline of the network was modeled as a linear parameter-varying (LPV) system driven by the source node mass flow together with the pressure as the scheduling parameter. The mass flow at the offtake was considered as the system output. The leak position was added as a state of the LPV system, from which a Kalman filter was designed. The effectiveness of the CPM system was illustrated with real and simulated data.

In 2011, Navarro et al. proposed an extended Kalman filter for locating leaks in a plastic pipeline, which was constructed from a discretized model both in time and in space. For the design of the filter, the space discretization was nonuniform and was a function of the unknown leak location; furthermore, the time domain was discretized by using Heun's method. The method was validated in real time in a laboratory [26].

In 2015, Verde and Rojas presented an iterative scheme for locating sequential leaks, namely one leak after another. The core of the method is a continuous extended Kalman filter with a prescribed degree of stability, which is constructed from the model of the flow in a pipeline with an equivalent leak; check Appendix A.4. The equivalent leak is a fictitious leak with a position in which a single leak would have to produce certain values of pressure and flow rate along the pipeline at steady state, but the values are actually caused by two or more leaks [22]. The equivalent leak must satisfy two criteria: (1) water loss equivalence and (2) energy equivalence [27]. In the case of a pipeline with a single branch and a leak, the equivalent leak is caused by the branch and leak outflows; in addition, it has always a position between both extractions.

In order to address the same concern, in 2016 Delgado et al. presented an approach for localizing sequential leaks by using an extended Kalman filter (Appendix A.2) for estimating parameters (Appendix A.3): the parameters of each sequential leak such as position and size. The filter was designed from a discrete time-space model derived from the WH equations and was modified at each new leak occurrence via an adaptation strategy to augment the size of the model's state vector. The

Preprints (www.preprints.org) | NOT PEER-REVIEWED | Posted: 19 September 2019 doi:10.20944/preprints201909.0217.v1



augmentation of the state is done for including the parameters of the actual sequential leak. The approach was validated by using experimental data [28].

In 2016, Verde et al. presented a Kalman-based approach for detecting and localizing single leaks in a pipeline with a branch junction [29]. The approach requires the following information for producing a diagnosis: the flow rate together with the pressure head at the pipeline ends, the position (the spatial coordinate) of the branch junction and the flow rate through the branch. The approach involves a selector algorithm (a simple algebraic equation that can be deduced from a head loss balance) and two localization algorithms, which are two Kalman filters designed from two different mathematical models; each one representing the flow dynamics of the pipeline before and after the branch, respectively. The goal of the selector algorithm is to indicate whether the leak is to the left (upstream) or to the right (downstream) of the branch. Depending on the indication of the selector, one of the two Kalman filters can be used to estimate the position of the leak. The approach was numerically tested with synthetic and experimental data from a hydraulic test apparatus.

In 2017, Delgado et al. described the successful localization of a leak in a pipeline of the water distribution network in Guadalajara, Mexico [30]. The localization was achieved by using a discrete-time extended Kalman filter (Appendix A.3), which was constructed by a lumped version of the WH equations. Additionally, Navarro et al. presented a two-stage leak isolation methodology based on a fitting loss coefficient calibration. In the first stage, an extended Kalman filter is used to fix the equivalent straight length (ESL) of the pipeline. Once the leak is detected, an algebraic observer allows estimating the leak position from the ESL fixed by the extended Kalman filter. Since leak isolation is performed in equivalent length coordinates, a transformation to the original coordinates is necessary [31].

In 2018, Santos-Ruiz et al. introduced a methodology for leak detection and localization based on data fusion from two approaches: a steady-state estimation and an extended Kalman filter [32]. The proposed method considers only pressure head and flow rate measurements at the pipeline ends. The approach was tested in real time by using a USB device for the data acquisition. The novelty of this approach is that the steady-state solution for a nonlinear pipeline model of the pipeline is merged with the dynamic state estimation, obtained from the EKF observer, by using Bayesian data fusion in order to refine the leak diagnosis. In the same year, Delgado et al. proposed a method based on two steps for solving the leak diagnosis problem in pipeline networks [33]. In a first step, a faulty system and a nominal model are used to generate residuals with an analysis that allows identifying the region where the leak occurs. As a second step and by using the information generated in the first step, the leak position and magnitude are estimated through extended Kalman filters. The proposed two-step methodology minimizes the problem of observer design since it avoids the design of an observer for each section of the network. On the other hand, Liu et al. suggested handling multi-leak detection problems in oil pipelines by using unscented Kalman filters [34]. Leaks are detected one at a time with an observer; therefore, the number of observers must be increased when a new leak occurs.

In the next table, all these approaches are classified according to the year of their presentation. Additionally, this table contains some works that propose methodologies based on the Kalman filter for addressing different problems associated with the pipeline operation monitoring, which do not exactly concern leak or fault detection.

| Period | Contributions |
|---|---|
| 1980-1990 | [10],[11] |
| 1990-2000 | - |
| 2000-2010 | [20],[14],[35],[36],[16],[15],[12],[37],[18],[23],[38],[39],[24] |
| 2010-Up to the present | [40],[25],[26],[41],[42],[43],[44],[45],[46],[47],[48],[49],[50],[51],[46],[52][28],[53],[54],[55],[29],[30],[31],[32],[33],[34] |

Preprints (www.preprints.org) | NOT PEER-REVIEWED | Posted: 19 September 2019 doi:10.20944/preprints201909.0217.v1



In Table 1, we present a taxonomy of CPM systems according mainly to the type of Kalman filter employed in the solution formulation of the leak detection and location. In addition, other parameters are highlighted, such as the fluid involved in the study, the type of leak, either single or multiple leaks, and also the type of validation (in a simulation, in a laboratory or in the field).

**Table 1.** Taxonomy of CPM systems

| Year | Reference | Country | Fluid | Fault | Testing | Filter Type |
|---|---|---|---|---|---|---|
| 1980 | Real-time failure-detection and identification applied to supervision of oil transport in pipelines | Norway | Oil | Single leak | Simulation | Kalman Filter |
| 1988 | Leak detection and location in gas pipelines | UK | Gas | Single leak | Simulation | Extended Kalman filter |
| 1988 | Robust observer design for a fluid pipeline | China | Water | NA | Simulation, Laboratory | Kalman Filter |
| 1988 | State estimation of output-decoupled complex systems with application to fluid pipeline | China | Water | NA | Simulation, Laboratory | Extended Kalman filter |
| 1990 | An application of Kalman filter to leak diagnosis of long-distance transport pipelines | China | | | | Generalized Kalman filter |
| 2000 | Multi-leak detection and isolation in fluid pipelines | Mexico | Water | Simultaneous leak | Simulation, Laboratory | Kalman filter |
| 2002 | A non-linear multiple-model state estimation scheme for pipeline leak detection and isolation | Saudi Arabia | Water | Single leak | Simulation | Modified Extended Kalman filter |
| 2004 | Minimal order nonlinear observer for leak detection | Mexico | Water | Simultaneous leak | Simulation, Laboratory | Nonlinear Kalman filter |
| 2004 | Identificability of multi-leaks in a pipeline | Mexico | Water | Simultaneous leak | Simulation, Laboratory | Extended Kalman filter |
| 2004 | Sub-sea pipelines leak detection and location based on fluid transient and FDI | | Oil, Gas | | Industrial pipeline | Extended Kalman filter |
| 2005 | Application of Kalman filter in pipeline leak detection | | | | Laboratory | Kalman filter |
| 2007 | Leak detection in pipelines using the extended Kalman filter and the extended boundary approach | Canada | Water | Simultaneous leak | Simulation, Laboratory | Extended Kalman filter |
| 2007 | Research on state estimation of oil pipeline considering adaptive extended Kalman filtering | China | Oil | NA | Industrial pipeline | Robust Adaptative Kalman filter |
| 2007 | Direct observer design for leak detection and estimation in pipelines | Mexico, France | Water | Simultaneous leak | Simulation | Extended Kalman Filter |
| 2007 | Comparison of two detection algorithms for pipeline leaks | Mexico, France | Water | Single leak | Laboratory | Extended Kalman filter |

Preprints (www.preprints.org) | NOT PEER-REVIEWED | Posted: 19 September 2019 doi:10.20944/preprints201909.0217.v1



| | | | | | | |
|---|---|---|---|---|---|---|
| 2008 | A collocation model for water-hammer dynamics with application to leak detection | Mexico | Water | Single, sequential, simultaneous | Simulation | Extended Kalman filter |
| 2009 | A combined Kalman filter-discrete wavelet transform method for leakage detection of crude oil pipelines | China | Oil | Sequential leak | Industrial pipeline | Kalman Filter |
| 2009 | Estimation of the temperature field in pipelines by using the Kalman filter | Brazil, USA | Oil-gas-water mixture | NA | Industrial pipeline | Kalman filter |
| 2009 | Collocation modeling with experimental validation for pipeline dynamics and application to transient data estimations | France | Water | Single, sequential | Laboratory | Extended Kalman Filter |
| 2010 | Kalman filtering of hydraulic measurements for burst detection in water distribution systems | UK | Water | Single leak | Laboratory, Industrial pipeline | Adaptative Kalman filter |
| 2010 | Gas pipelines LPV modelling and identification for leakage detection | Portugal, USA, Germany | Gas | Single leak | Industrial pipeline | Kalman filter |
| 2011 | Real-time leak isolation based on state estimation in a plastic pipeline | Mexico, France, Venezuela | Water | Single leak | Laboratory | Extended Kalman filter |
| 2011 | Leakage detection and location in gas pipelines through an LPV identification approach | Portugal, US, Germany | Gas | Single leak | Industrial pipeline | Kalman filter |
| 2011 | Calibration of fitting loss coefficients for modelling purpose of a plastic pipeline | Mexico, France | Water | NA | Laboratory | Extended Kalman Filter |
| 2012 | Leak isolation with temperature compensation in pipelines | Mexico | Water | Single leak | Laboratory | Extended Kalman filter |
| 2012 | Real-time leak isolation based on a fault model approach algorithm in a water pipeline prototype | Mexico | Water | Single leak | Laboratory | Extended Kalman filter |
| 2013 | State estimation of pipeline models using the ensemble kalman filter | US | Gas | Single leak | Simulation | Ensemble Kalman filter |
| 2014 | Leak detection and location based on improved pipe model and nonlinear observer | Venezuela, France | Water | Single leak | Simulation, Laboratory | Extended Kalman filter |
| 2014 | Design and realization of the Kalman filter based on LabVIEW | China | Water | NA | Simulation | Kalman filter |
| 2014 | Online burst detection in a water distribution system using the Kalman filter and hydraulic modelling | UK | Water | Single leak | Industrial pipeline | Kalman filter |
| 2015 | Modeling and state estimation for gas transmission networks | Iran | Gas | NA | Simulation | Extended Kalman filter |
| 2015 | Dynamic model of a new above-ground pipeline using a Kalman estimator-based system | United Arab Emirates | | NA | Laboratory | Kalman filter |
| 2016 | Research on natural gas pipeline leak detection algorithm and simulation | Mexico, France | Water | Single leak | Laboratory | Adaptative Kalman filter |

Preprints (www.preprints.org) | NOT PEER-REVIEWED | Posted: 19 September 2019 doi:10.20944/preprints201909.0217.v1



| | | | | | | |
|---|---|---|---|---|---|---|
| 2017 | Water Leak diagnosis in pressurized pipelines: a real case study | Mexico | Water | Single leak | Laboratory | Extended Kalman filter |
| 2017 | Real-Time Leak Isolation Based on State Estimation with Fitting Loss Coefficient Calibration in a Plastic Pipeline | Mexico, France | Water | Single leak | Laboratory | Extended Kalman filter |
| 2018 | Online leak diagnosis in pipelines using an EKF-based and steady-state mixed approach | Mexico, Spain | Water | Single leak | Laboratory | Extended Kalman filter |
| 2018 | EKF-based leak diagnosis schemes for pipeline networks | Mexico, France | Water | Single leak | Laboratory | Extended Kalman filter |
| 2018 | Multi-leak diagnosis and isolation in oil pipelines based on Unscented Kalman filter | China | Water | Single leak | Simulation | Unscented Kalman filter |

According to API 1155, there are four metrics for evaluating leak detection systems: reliability, sensitivity, accuracy and robustness. Despite the large amount of academic work based on the Kalman filter, however, only some of them provide a report of such metrics within a field context. In Table 2, these works are listed.

**Table 2.** Contributions with declared performance metrics

| Paper | Results | Application |
|---|---|---|
| A combined Kalman filter – discrete wavelet transform method for leakage detection of crude oil pipelines. | Reliability: 5% of false alarms. Accuracy: 0.26% of error | Main pipelines |
| Gas pipelines LPV modelling and identication for leakage detection. | Sensitivity: A leak about 10% of the nominal flow rate was detected 24 minutes after its occurrence | Main pipelines |
| Identificability of multi-leaks in a pipeline | Accuracy: 12% of error | Main pipelines |
| Minimal order nonlinear observer for leak detection. | Accuracy: 1.36% of error with respect to the pipeline length in a noisy data scenario. | Main pipelines |
| Online burst detection in a water distribution system using the Kalman filter and hydraulic modelling. | Reliability: 85% of detected burst. | Pipeline networks |
| Real-time leak isolation based on a fault model approach algorithm in a water pipeline prototype | Accuracy: 3.6% of error with respect to the ESL. | Main pipelines |
| Research on natural gas pipeline leak detection algorithm and simulation. | Accuracy: 0.11% of locating error. | Main pipelines |

Note that the accuracy of any methodology is strongly determined by the instruments in the physical installation but also by the availability of a proper system model.

## 5. Future research

Hitherto three main approaches for estimating leak locations using the Kalman filter have been reported in the literature. Such approaches are based on banks of filters, on pressure and flow rate estimations, and on the direct estimation of leak parameters. Table 1 lists our current state of knowledge

*Preprints* (www.preprints.org) | NOT PEER-REVIEWED | Posted: 19 September 2019 doi:10.20944/preprints201909.0217.v1



regarding leak location studies. It is clear from this table that some studies still do not involve field testing. In addition, one can realize that most of the research has focused on detecting and locating leaks in water pipelines. Very few studies have addressed the problem of leaks in pipelines that transport other types of fluids, such as oil, gas or heavy oils.

In particular, proposing a leak detection system for heavy oils is important in the petroleum industry because of the enormous increase in oil demand and the progressive exhaustion of low-viscosity oil reservoirs. Moreover, the leak localization in multiphase flow pipelines (typical in oil production) is a pending issue that has not even been deeply addressed with other algorithmic tools. Therefore, there is a clear need for laboratory investigation of leak localization in multiphase flow pipelines .

The detection of individual leaks remains an important concern that requires algorithms with better performance metrics; therefore, it is necessary to continue investigating to improve the Kalman filters used for it. In addition, it is necessary that part of the research efforts focus on the development of new tools for the localization of multiple leaks since this is an important issue that has not yet been approached meritoriously. It is worth mentioning that the presence of multiple leaks is a very common problem in countries plagued by vandalism and theft of hydrocarbons. Usually, in these places the pipelines are infested by simultaneously activated illegal taps.

Finally, the location of leaks using Kalman filters is a challenging area that will require rigorous experimental validations, as well as addressing some concerns, such as the extension of existing algorithms for complex pipeline configurations, including branched pipelines or pipeline networks.

**Funding:** This research was funded by CONACyT-Convocatoria Atención a Problemas Nacionales 2017-Proyecto 4730 and CONACyT-Secretaría de Energía (Hidrocarburos)-Proyecto 280170.

**Conflicts of Interest:** The authors declare no conflict of interest. The funders had no role in the design of the study; in the collection, analyses, or interpretation of data; in the writing of the manuscript, or in the decision to publish the results.

## Appendix A Reminders about Kalman filter

The Kalman filter was first described and partially developed in technical papers by (among others) the Hungarian émigré Rudolf E. Kálmán [56–58]. It is an algorithm used to solve the so-called *linear quadratic problem*, which consists of estimating the instantaneous state of a linear dynamic system affected by white noise; therefore, it is also known as the linear quadratic estimator (LQE). In fact, the Kalman filter becomes an estimator that is statistically optimal with respect to any quadratic function of the estimation error. The following presentation seeks to briefly summarize relevant concepts presented in several prior works [56,59–62] related to the discrete Kalman filter.

### *Appendix A.1 The discrete Kalman filter*

Let us start with the state-space representation (without a direct feedthrough) of a linear dynamic system such as

$$\begin{aligned} x\left(k+1\right) &= Ax\left(k\right) + Bu\left(k\right) + w\left(k\right), \\ y\left(k\right) &= Cx\left(k\right) + v\left(k\right), \end{aligned} \tag{A1}$$

where $w\left(k\right)$ and $v\left(k\right)$ denote uncorrelated white noise processes with zero mean and covariances $Q\left(k\right)$ and $R\left(k\right)$, respectively. Notice that these noises perturb both the system states and the system outputs.

Since the objective is to find the optimal linear filter, the cost function to be minimized is the expected value of the squared prediction error as follows

$$\begin{aligned} J &= E\left\{\| \hat{x}\left(k+1\right) - x\left(k+1\right) \|_2^2\right\}, \\ &= E\left\{\left(\hat{x}\left(k+1\right) - x\left(k+1\right)\right)^T \left(\hat{x}\left(k+1\right) - x\left(k+1\right)\right)\right\}. \end{aligned} \tag{A2}$$

Preprints (www.preprints.org) | NOT PEER-REVIEWED | Posted: 19 September 2019 doi:10.20944/preprints201909.0217.v1



The Kalman filter can be conceptualized as two distinct phases: "prediction" and "correction". The prediction phase uses the state estimate from the previous time step to produce an estimate of the state one time step ahead into the future at $k+1$. This predicted state estimate, denoted by $\hat{x}(k+1|k)$, is known as the *a priori* state estimate. Thus, in the correction phase, the *a priori* state estimate is corrected based on the available measurements of the output $y(k+1)$. This improved estimate, denoted by $\hat{x}(k+1|k+1)$, is termed the *a posteriori* state estimate. The covariance matrix of the states, which provide a measure of the estimated accuracy of the state estimate, is

$$P(k) = E\left\{(\hat{x}(k) - x(k))^T (\hat{x}(k) - x(k))\right\}. \tag{A3}$$

327 Prediction Phase

The estimates of the states (since the noise $w(k)$ is assumed to be zero mean) are updated as

$$\hat{x}(k+1|k) = A\hat{x}(k) + Bu(k), \tag{A4}$$

based on the measurements up to time step $k$. By taking into account the *a priori* state estimation in (A4), the covariance matrix can be written (after some algebra) as

$$P(k+1|k) = AP(k)A^T + Q(k), \tag{A5}$$

328 where the fact has been exploited that $\hat{x}(k)$ and $x(k)$ are uncorrelated with $w(k)$.

329 Correction Phase

Once a new measurement $y(k+1)$ is available, it can be used to correct the estimates as follows:

$$\begin{aligned}\hat{x}(k+1|k+1) &= \hat{x}(k+1|k) + \\ &+ K(k+1)\left(y(k+1) - C\hat{x}(k+1|k)\right),\end{aligned} \tag{A6}$$

where clearly the estimates are based on the measurements up to time step $k+1$ and the optimal feedback gain $K(k+1)$ is calculated as

$$K(k+1) = P(k+1|k)C^T\left(CP(k+1|k)C^T + Q\right)^{-1}. \tag{A7}$$

According to Eq. (A6), $K(k+1)$ determines which one has more weight in updating the estimated states: the observation error $y(k+1) - C\hat{x}(k+1|k)$ or the prediction of the states based on the internal model $\hat{x}(k+1|k)$. Finally, by taking into account the *a posteriori* state estimation in (A6), the covariance matrix can be updated as

$$P(k+1|k+1) = (I - K(k+1)C)P(k+1|k). \tag{A8}$$

330 The corresponding block diagram of the Kalman filter is shown in Fig. A1.

331 *Appendix A.2 Extended Kalman filter*

In the theory of nonlinear state estimation, the *de facto* standard is the extended Kalman filter (EKF), which is the nonlinear version of the Kalman. In many situations, one is confronted with nonlinear system models of the form

$$\begin{aligned}x(k+1) &= f_k(x(k), u(k)) + w(k), \\ y(k) &= g_k(x(k)) + v(k),\end{aligned} \tag{A9}$$

 doi:10.20944/preprints201909.0217.v1



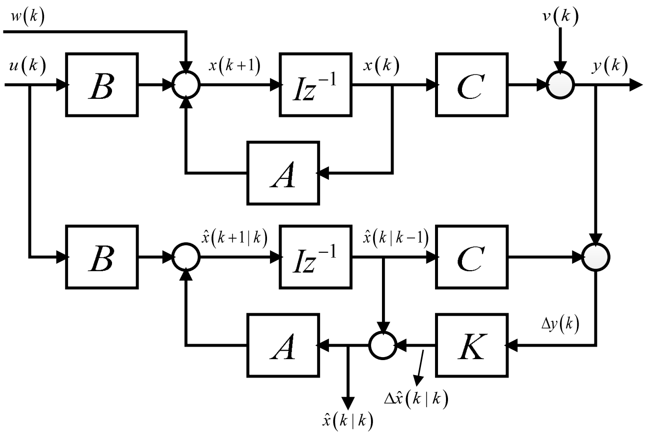


**Figure A1.** Kalman filter block diagram

where $w(k)$ and $v(k)$ denote uncorrelated white noise processes with zero mean and covariances $Q(k)$ and $R(k)$, respectively.

In the EKF, the functions $f_k$ and $g_k$ are used to compute the predicted state (from the previous estimate) and the predicted measurement (from the predicted state), respectively. To update the covariance matrix $P(k)$, however, a first-order Taylor series expansion of (A9) is used. The idea is essentially to linearize the nonlinear system around the current estimate. Thus, at each time step, the Jacobian is evaluated by considering the current predicted states. These matrices are used in the Kalman filter equations.

The extended Kalman filter is then given as follows.

Prediction Phase

$$\hat{x}(k+1|k) = f_k(x(k), u(k)), \tag{A10}$$

$$F(k) = \left.\frac{\partial f_k(x,u)}{\partial x}\right|_{x=\hat{x}(k),u=u(k)}, \tag{A11}$$

$$P(k+1|k) = F(k)P(k)F^T(k) + Q(k). \tag{A12}$$

Correction Phase

$$G(k+1) = \left.\frac{\partial g_{k+1}(x)}{\partial x}\right| x = \hat{x}(k+1|k), \tag{A13}$$

$$K(k+1) = P(k+1|k)G^T(k+1) \times \left(G(k+1)P(k+1|k)G^T(k+1) + Q(k+1)\right)^{-1}, \tag{A14}$$

$$\hat{x}(k+1|k+1) = \hat{x}(k+1|k) + K(k+1)(y(k+1) - g_{k+1}(\hat{x}(k+1|k))), \tag{A15}$$

$$P(k+1|k+1) = (I - K(k+1)G(k+1))P(k+1|k). \tag{A16}$$

*Appendix A.3 Extended Kalman filter for parameter estimation*

By augmenting the state vector $x(k)$ with a parameter vector $\theta$, the EKF can be used for state and parameter estimations. The augmented system can be written as follows:

$$\begin{bmatrix} \hat{x}(k+1) \\ \hat{\theta}(k+1) \end{bmatrix} = \begin{bmatrix} f(x(k), \theta(k), u(k)) \\ \theta(k) \end{bmatrix} + \begin{bmatrix} w(k) \\ \xi(k) \end{bmatrix}$$
$$y(k) = g(\hat{x}(k)). \tag{A17}$$

Notice that the parameters are modeled as constant quantities disturbed by a white noise.

*Appendix A.4 Continuous extended Kalman filter with a prescribed degree of stability*

Let us consider a continuous nonlinear system that can be represented by the following equations:

$$\begin{aligned} \dot{x}(t) &= f(x(t), u(t)), \\ y(t) &= h(x(t)), \end{aligned} \tag{A18}$$

where $x(t) \in \mathbb{R}^q$ is the state, $u(t) \in \mathbb{R}^p$ the input and $y(t) \in \mathbb{R}^m$ the output. An observer (A18) can then be designed as follows:

$$\dot{\hat{x}}(t) = f(\hat{x}(t), u(t)) + K(t)[y(t) - h(\hat{x}(t))], \tag{A19}$$

where the state estimate is denoted by $\hat{x}(t)$ and the observer gain $K(t)$ is a time-varying $q \times m$ calculated as

$$K(t) = \mathbf{P}(t)\mathbf{C}^{\mathrm{T}}(t)\mathbf{W}^{-1}, \tag{A20}$$

where $\mathbf{P}(t)$ is a matrix calculated by using the next differential Riccati equation

$$\dot{\mathbf{P}}(t) = (\mathbf{A}(t) + \alpha\mathbf{I})\mathbf{P}(t) + \mathbf{P}(t)(\mathbf{A}^{\mathrm{T}}(t) + \alpha\mathbf{I}) - \mathbf{P}(t)\mathbf{C}^{\mathrm{T}}(t)\mathbf{W}^{-1}\mathbf{C}(t)\mathbf{P}(t) + \mathbf{Q}, \tag{A21}$$

which involves the following Jacobians

$$\mathbf{A}(t) = \frac{\partial f}{\partial x}(\hat{x}(t), u(t)), \ \mathbf{C}(t) = \frac{\partial h}{\partial x}(\hat{x}(t)),$$

$$\mathbf{P}(0) = \mathbf{P}(0)^{\mathrm{T}} > \mathbf{0}, \mathbf{Q} = \mathbf{Q}^{\mathrm{T}} \geq \mathbf{0}, \mathbf{W} = \mathbf{W}^{\mathrm{T}} > \mathbf{0}.$$

In the Riccati equation (A21), $\alpha > 0$ is a parameter that provides a stability degree to the estimation. Furthermore, by manipulating this parameter, the estimation rate (convergence time) can be tuned.

## Appendix B Design of a Kalman filter: An example

The main purpose of the following example is to show how a Kalman filter can be designed to detect and localize a leak in a straight pipe without branches. This example involves an extended Kalman filter with a prescribed degree of stability (which is described in Appendix A.4).

**Step 1:** *Identify the available information (observations, data, measurements, records) for performing the estimation.*

In this step, the available information for performing the estimation must be analyzed and characterized. In addition, it must be defined how the information will be processed prior to being injected into the state observer. Some data features that should be considered during the analysis are the sample time, the existence of delays, and the synchronization of the information. In this step, it is important to define which measured variables will be used to calculate the estimation error.

For this example, the following information (time series data) is required: the inlet flow rate ($Q_{in}$), the upstream pressure head ($H_{in}$), the outlet flow rate ($Q_{out}$) and the downstream pressure head ($H_{out}$).

**Step 2:** *Formulate a model assuming convenient assumptions and constraints.*

In this step, the assumptions and restrictions for formulating a model are defined according to the applications and considering the applicability as well as the limitations of these equations.

In the present example, the model used for the design of the Kalman filter is the so-called rigid water column model (RWC model) [63], which describes the flow in a pipeline by taking into account the assumptions (A1)-(A5) given in Section 4 together with the following assumptions:

(A8) the pipeline wall is rigid, and
(A9) the liquid fluid is incompressible.

The RWC model for a pipeline section $\Delta z$ is expressed by the following equation [64,65]:

$$\dot{Q}(t) = \theta \frac{\Delta H(t)}{\Delta z} - \alpha(Q)|Q(t)|Q(t), \tag{A22}$$

where $Q$ is the flow rate in the pipeline section, $\Delta H$ is the pressure head loss along $\Delta z$, $\theta = gA_r$, $g$ is the gravity acceleration, $A_r$ is the cross-sectional area and $\alpha(Q) = f(Q)/2\phi A_r$, where $f(Q)$ is the friction factor, which can be computed with any of the equations presented in [66]. By assuming that the flow rate is unidirectional, (A22) becomes

$$\dot{Q}(t) = \theta \frac{\Delta H(t)}{\Delta z} - \alpha(Q)Q^2(t), \tag{A23}$$

which is the model used in this article for the conception of the Kalman filter.

**Step 3:** *Set the model in state-space representation.*

The starting point for executing this step is the formulation of the two following equations that describe the flow in two different sections of the pipeline as in Fig. A2: the section to the left of the leak (namely $\Delta z = z_L - z_0$) where the head loss is $\Delta H = H_{in} - H_L$ and the section to the right of the leak (namely $\Delta z = L - z_L$) where the head loss is $\Delta H = H_L - H_{out}$. Both equations were obtained from the RWC model (A23) by substituting $\Delta z$ and $\Delta H$ by the corresponding values:

$$\dot{Q}_{in}(t) = \frac{\theta}{z_L - z_0}(H_{in}(t) - H_L(t)) - \alpha(Q_{in}(t))Q_{in}^2(t), \tag{A24}$$

$$\dot{Q}_{out}(t) = \frac{\theta}{L - z_L}(H_L(t) - H_{out}(t)) - \alpha(Q_{out}(t))Q_{out}^2(t), \tag{A25}$$

where $z_0 = 0$ is the origin coordinate (the upstream end) of the pipeline, $z_L$ is the leak coordinate (position) and $H_L$ is the pressure head at the branch junction.

If $H_L$ is replaced by any of the equations

$$\frac{H_{in}(t) - H_L(t)}{z_L} = \frac{\alpha(Q_{in}(t))}{\theta}Q_{in}^2(t), \quad \frac{H_L(t) - H_{out}(t)}{L - z_L} = \frac{\alpha(Q_{out}(t))}{\theta}Q_{out}^2(t), \tag{A26}$$

and if the new state variables are defined as $x_1(t) = Q_{in}(t) - Q_{out}(t)$ and $x_2(t) = 1/z_L(t)$, then the following second-order system results from (A24) and (A25):

$$\begin{aligned} \dot{x}_1(t) &= x_2(t)\underbrace{\left[\theta(H_{in}(t) - H_{out}(t)) - L\left(\alpha(Q_{out}(t))Q_{out}^2(t)\right)\right]}_{u(t)} - \alpha(Q_{in}(t))Q_{in}^2(t) + \alpha(Q_{out}(t))Q_{out}^2(t) \\ \dot{x}_2(t) &= 0. \end{aligned} \tag{A27}$$

Preprints (www.preprints.org) | NOT PEER-REVIEWED | Posted: 19 September 2019 doi:10.20944/preprints201909.0217.v1



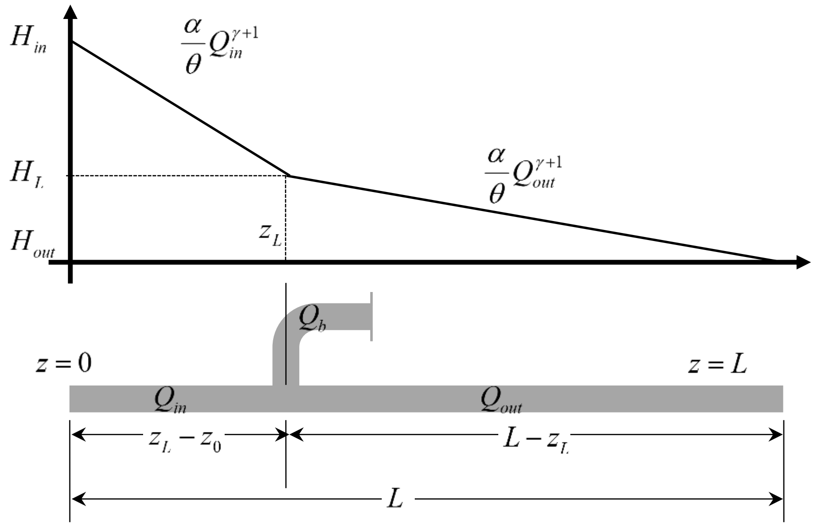


**Figure A2.** Hydraulic gradient in a pipeline with a leak.

**Step 4:** *Set the equations of the Kalman filter.*

$$\begin{aligned}\dot{\hat{x}}_1(t) &= \hat{x}_2(t)u(t) - \alpha(Q_{in}(t))Q_{in}^2(t) + \alpha(Q_{out}(t))Q_{out}^2(t) + K_1(t)e(t) \\ \dot{\hat{x}}_2(t) &= K_2(t)e(t)\end{aligned} \tag{A28}$$

where

$$e(t) = Q_{in}(t) - Q_{out}(t) - \hat{x}_1(t). \tag{A29}$$

**Step 5:** *Compute the gain of the state observer.* For computing $K_1$ and $K_2$, Eq. (A20) can be used together with Eq. (A21). Notice that for this example $\mathbf{P}(t), \mathbf{Q}, \mathbf{W} \in \mathbb{R}^{2\times 2}$.

## References


1. Drumond, G.P.; Pasqualino, I.P.; Pinheiro, B.C.; Estefen, S.F. Pipelines, risers and umbilicals failures: A literature review. *Ocean Engineering* **2018**, *148*, 412–425.
2. API RP 1130 (2007): Computational Pipeline Monitoring for Liquids. (September 2007).
3. Geiger, G.; Werner, T.; Matko, D. Leak detection and locating-a survey. PSIG annual meeting. Pipeline Simulation Interest Group, 2003.
4. Besançon, G. *Nonlinear Observers and Applications*; Springer: Berlin, Germany, 2007; p. 224.
5. Besançon, G. Observer tools for pipeline monitoring. In *Modeling and Monitoring of Pipelines and Networks*; Springer, 2017; pp. 83–97.
6. Jiménez Cabas, J.A. Liquid Transport Pipeline Monitoring Architecture Based on State Estimators for Leak Detection and Location. Master's thesis, Universidad del Norte, 2018.
7. Recent Advances in Pipeline Monitoring and Oil Leakage Detection Technologies: Principles and Approaches. *Sensors*, *19*.
8. Simon, D. Kalman filtering with state constraints: a survey of linear and nonlinear algorithms. *IET Control Theory & Applications* **2010**, *4*, 1303–1318.
9. Evensen, G. *Data assimilation: the ensemble Kalman filter*; Springer Science & Business Media, 2009.
10. Digernes, T. Real-Time Failure-Detection and Identification Applied to Supervision of Oil Transport in Pipelines. *Modeling, Identification and Control* **1980**, *1*, 39–49.
11. Benkherouf, A.; Allidina, A. Leak detection and location in gas pipelines. IEE Proceedings D-Control Theory and Applications. IET, 1988, Vol. 135, pp. 142–148.

 doi:10.20944/preprints201909.0217.v1




12. Besançon, G.; Georges, D.; Begovich, O.; Verde, C.; Aldana, C. Direct observer design for leak detection and estimation in pipelines. Control Conference (ECC), 2007 European. IEEE, 2007, pp. 5666–5670.
13. Geiger, I.G. Principles of leak detection. *Fundamentals of leak detection. KROHNE oil and gas* **2005**.
14. Emara-Shabaik, H.; Khulief, Y.; Hussaini, I. A non-linear multiple-model state estimation scheme for pipeline leak detection and isolation. *Proceedings of the Institution of Mechanical Engineers, Part I: Journal of Systems and Control Engineering* **2002**, *216*, 497–512.
15. Khulief, Y.; Emara-Shabaik, H. Laboratory investigation of a multiple-model state estimation scheme for detection and isolation of leaks in pipelines. *Proceedings of the Institution of Mechanical Engineers, Part I: Journal of Systems and Control Engineering* **2006**, *220*, 1–13.
16. Bai, L.; Yue, Q.; Li, H.; others. Sub-sea Pipelines Leak Detection and Location Based on Fluid Transient and FDI. The Fourteenth International Offshore and Polar Engineering Conference. International Society of Offshore and Polar Engineers, 2004.
17. Doney, K. Leak detection in pipelines using the extended Kalman filter and the extended boundary approach. PhD thesis, 2007.
18. Begovich, O.; Navarro, A.; Sanchez, E.N.; Besançon, G. Comparison of two detection algorithms for pipeline leaks. Control Applications, 2007. CCA 2007. IEEE International Conference on. IEEE, 2007, pp. 777–782.
19. Billmann, L.; Isermann, R. Leak detection methods for pipelines. *Automatica* **1987**, *23*, 381–385.
20. Verde, C. Multi-leak detection and isolation in fluid pipelines. *Control Engineering Practice* **2001**, *9*, 673–682.
21. Chaudhry, M.H. *Applied Hydraulic Transients*; New York: Van Nostrand Reinhold, 1979.
22. Verde, C.; Bornard, G.; Gentil, S. Isolability of multi-leaks in a pipeline. Proceedinss 4th MATHMOD; , 2003.
23. Torres, L.; Besançon, G.; Georges, D. A collocation model for water-hammer dynamics with application to leak detection. Decision and Control, 2008. CDC 2008. 47th IEEE Conference on. IEEE, 2008, pp. 3890–3894.
24. Torres, L.; Besançon, G.; Georges, D. Collocation modeling with experimental validation for pipeline dynamics and application to transient data estimations. European Control Conference, ECC'09, 2009.
25. Dos Santos, P.L.; Azevedo-Perdicoúlis, T.; Ramos, J.; Jank, G.; de Carvalho, J.M.; Milhinhos, J. Gas pipelines LPV modelling and identification for leakage detection. American Control Conference (ACC), 2010. IEEE, 2010, pp. 1211–1216.
26. Navarro, A.; Begovich, O.; Besançon, G.; Dulhoste, J. Real-time leak isolation based on state estimation in a plastic pipeline. Control Applications (CCA), 2011 IEEE International Conference on. IEEE, 2011, pp. 953–957.
27. Colombo, A.F.; Karney, B.W. Energy and costs of leaky pipes: toward comprehensive picture. *Journal of Water Resources Planning and Management* **2002**, *128*, 441–450.
28. Delgado-Aguiñaga, J.; Begovich, O.; Besançon, G. Varying-parameter modeling and extended Kalman filtering for reliable leak diagnosis under temperature variations. System Theory, Control and Computing (ICSTCC), 2016 20th International Conference on. IEEE, 2016, pp. 632–637.
29. Verde, C.; Torres, L.; González, O. Decentralized scheme for leaks' location in a branched pipeline. *Journal of Loss Prevention in the Process Industries* **2016**, *43*, 18–28.
30. Delgado-Aguiñaga, J.A.; Begovich, O. Water Leak Diagnosis in Pressurized Pipelines: A Real Case Study. In *Modeling and Monitoring of Pipelines and Networks*; Springer, 2017; pp. 235–262.
31. Navarro, A.; Begovich, O.; Sánchez, J.; Besançon, G. Real-Time Leak Isolation Based on State Estimation with Fitting Loss Coefficient Calibration in a Plastic Pipeline. *Asian Journal of Control* **2017**, *19*, 255–265.
32. Santos-Ruiz, I.; Bermúdez, J.; López-Estrada, F.; Puig, V.; Torres, L.; Delgado-Aguiñaga, J. Online leak diagnosis in pipelines using an EKF-based and steady-state mixed approach. *Control Engineering Practice* **2018**, *81*, 55 – 64.
33. Delgado-Aguiñaga, J.; Besançon, G. EKF-based leak diagnosis schemes for pipeline networks. *IFAC-PapersOnLine* **2018**, *51*, 723–729.
34. Liu, P.; Li, S.; Wang, Z. Multi-leak diagnosis and isolation in oil pipelines based on Unscented Kalman filter. 2018 Chinese Control And Decision Conference (CCDC). IEEE, 2018, pp. 2222–2227.
35. Verde, C. Minimal order nonlinear observer for leak detection. *Journal of Dynamic Systems, Measurement, and Control* **2004**, *126*, 467–472.

 doi:10.20944/preprints201909.0217.v1



36. Verde, C.; Visairo, N. Identificability of multi-leaks in a pipeline. American Control Conference (ACC), 2004. Proceedings of the 2004. IEEE, 2004, Vol. 5, pp. 4378–4383.
37. Gong, J.; Cai, J.; Li, X.; Song, S. Research on state estimation of oil pipeline considering adaptive extended Kalman filtering. Mechatronics and Automation, 2007. ICMA 2007. International Conference on. IEEE, 2007, pp. 1294–1298.
38. Yu, Z.; Jian, L.; Zhoumo, Z.; Jin, S. A combined Kalman filter-Discrete wavelet transform method for leakage detection of crude oil pipelines. Electronic Measurement & Instruments, 2009. ICEMI'09. 9th International Conference on. IEEE, 2009, pp. 3–1086.
39. Vianna, F.; Orlande, H.; Dulikravich, G. Estimation of the temperature field in pipelines by using the Kalman filter. 2nd International Congress of Serbian Society of Mechanics (IConSSM 2009), 2009, pp. 1–5.
40. Ye, G.; Fenner, R.A. Kalman filtering of hydraulic measurements for burst detection in water distribution systems. *Journal of pipeline systems engineering and practice* **2010**, *2*, 14–22.
41. dos Santos, P.L.; Azevedo-Perdicoúlis, T.; Jank, G.; Ramos, J.; de Carvalho, J.M. Leakage detection and location in gas pipelines through an LPV identification approach. *Communications in Nonlinear Science and Numerical Simulation* **2011**, *16*, 4657–4665.
42. Navarro, A.; Begovich, O.; Besançon, G. Calibration of fitting loss coefficients for modelling purpose of a plastic pipeline. Emerging Technologies & Factory Automation (ETFA), 2011 IEEE 16th Conference on. IEEE, 2011, pp. 1–6.
43. Pizano-Moreno, A.; Begovich, O. Leak isolation with temperature compensation in pipelines. Electrical Engineering, Computing Science and Automatic Control (CCE), 2012 9th International Conference on. IEEE, 2012, pp. 1–5.
44. Padilla, E.A.; Begovich, O. Real-time leak isolation based on a fault model approach algorithm in a water pipeline prototype. *IFAC Proceedings Volumes* **2012**, *45*, 916–921.
45. Modisette, J.P.; others. State estimation of pipeline models using the ensemble Kalman filter. PSIG Annual Meeting. Pipeline Simulation Interest Group, 2013.
46. Guillen, M.; Dulhoste, J.F.; Besançon, G.; Scola, I.R.; Santos, R.; Georges, D. Leak detection and location based on improved pipe model and nonlinear observer. Control Conference (ECC), 2014 European. IEEE, 2014, pp. 958–963.
47. Tian, J.; Wang, S.Q. Design and realization of the Kalman filter based on LabVIEW. Applied Mechanics and Materials. Trans Tech Publ, 2014, Vol. 519, pp. 1276–1280.
48. Okeya, I.; Kapelan, Z.; Hutton, C.; Naga, D. Online burst detection in a water distribution system using the Kalman filter and hydraulic modelling. *Procedia Engineering* **2014**, *89*, 418–427.
49. Behrooz, H.A.; Boozarjomehry, R.B. Modeling and state estimation for gas transmission networks. *Journal of Natural Gas Science and Engineering* **2015**, *22*, 551–570.
50. Al Ghailani, L.; El-Sinawi, A. Dynamic model of a new above-ground pipeline using a Kalman estimator-based system. IEEE Conference on Systems, Process and Control (ICSPC). IEEE, 2015, pp. 12–15.
51. Zhou, D. Research on Natural Gas Pipeline Leak Detection Algorithm and Simulation. Proceedings of the 2015 Chinese Intelligent Automation Conference. Springer, 2015, pp. 355–361.
52. Verde, C.; Rojas, J. Iterative Scheme for Sequential Leaks Location. *IFAC-PapersOnLine* **2015**, *48*, 726–731.
53. Choi, D.Y.; Kim, S.W.; Choi, M.A.; Geem, Z.W. Adaptive Kalman filter based on adjustable sampling interval in burst detection for water distribution system. *Water* **2016**, *8*, 142.
54. Durgut, İ.; Leblebicioğlu, M.K. State estimation of transient flow in gas pipelines by a Kalman filter-based estimator. *Journal of Natural Gas Science and Engineering* **2016**, *35*, 189–196.
55. Delgado-Aguiñaga, J.; Besançon, G.; Begovich, O.; Carvajal, J. Multi-leak diagnosis in pipelines based on Extended Kalman Filter. *Control Engineering Practice* **2016**, *49*, 139–148.
56. Kalman, R.E. A new approach to linear filtering and prediction Problems. *Journal of basic Engineering* **1960**, *82*, 35–45.
57. Lauritzen, S.L. Time series analysis in 1880: A discussion of contributions made by TN Thiele. *International Statistical Review/Revue Internationale de Statistique* **1981**, pp. 319–331.
58. Lauritzen, S.L. *Thiele: pioneer in statistics*; Clarendon Press, 2002.
59. Ljung, L. Asymptotic behavior of the extended Kalman filter as a parameter estimator for linear systems. *IEEE Transactions on Automatic Control* **1979**, *24*, 36–50.

60. Song, Y.; Grizzle, J.W. The extended Kalman filter as a local asymptotic observer for nonlinear discrete-time systems. American Control Conference, 1992. IEEE, 1992, pp. 3365–3369.
61. Andrews, A. Kalman filtering: theory and practice using MATLAB, 2001.
62. Isermann, R.; Münchhof, M. *Identification of Dynamic Systems: An Introduction with Applications*; Springer, 2010.
63. Cabrera, E.; Garcia-Serra, J.; Iglesias, P.L. Modelling water distribution networks: From steady flow to water hammer. In *Improving efficiency and reliability in water distribution systems*; Springer, 1995; pp. 3–32.
64. Islam, M.R.; Chaudhry, M.H. Modeling of constituent transport in unsteady flows in pipe networks. *Journal of Hydraulic Engineering* **1998**, *124*, 1115–1124.
65. Nault, J.; Karney, B. Improved rigid water column formulation for simulating slow transients and controlled operations. *Journal of Hydraulic Engineering* **2016**, *142*, 04016025.
66. Brkić, D. Review of explicit approximations to the Colebrook relation for flow friction. *Journal of Petroleum Science and Engineering* **2011**, *77*, 34–48.