

Article

Chiral Perturbation Theory vs. Linear Sigma Model in A Chiral Imbalance Medium

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Abstract: We compare Chiral Perturbation Theory (ChPT) and the Linear Sigma Model (LSM) as realizations of low energy QCD for light mesons in a chirally imbalanced medium. The relations between the low-energy constants of the Chiral Lagrangian and the corresponding constants of the Linear Sigma Model are established as well as the expressions for the decay constant of π -meson in the medium and for the mass of the a_0 . In the large N_c count taken from QCD the correspondence of ChPT and LSM is remarkably good and give a solid ground for search of chiral imbalance manifestation in pion physics. A possible experimental detection of chiral imbalance (and therefore a phase with Local Parity Breaking) is outlined in the charged pion decays inside the fireball.

Keywords: chiral imbalance; Chiral Perturbation Theory; Linear Sigma Model; charged pion decay in chiral medium; local parity breaking

1. Introduction

The possible generation of a phase with Local Parity Breaking (LPB) in nuclear matter at extreme conditions such as those reached in heavy ion collisions (HIC) at RHIC and LHC [1] has been examined recently [2–4]. It has been suggested in [2] that at increasing temperatures an isosinglet pseudoscalar background could arise due to large-scale topological charge fluctuations (studied recently in lattice QCD simulations [5]).

These considerations led eventually to the observation of the so-called Chiral Magnetic Effect (CME)[2] in the STAR and PHENIX experiments at RHIC [6]. The effect should be most visible for non-central HIC where large angular momenta induce large magnetic fields contributing to the chiral charge separation. For central collisions it was proposed in [3] that the presence of a phase where parity was spontaneously broken could be a rather generic feature of QCD at finite baryon density. Local parity breaking can be induced by difference between the densities of the right- and left-handed chiral fermion fields (Chiral Imbalance).

In the hadron phase we shall assume that as a consequence of topological charge fluctuations, the environment in central HIC generates a pseudoscalar background growing approximately linearly in time. This background is associated with a constant axial vector whose zero component is identified with a chiral chemical potential. In such an environment one could search for possible manifestation of LPB in electromagnetic and lepton probes. In particular, in [7] it was shown that a good part of the excess of dileptons produced in central heavy-ion collisions [8] might be a consequence of LPB due to the generation of a pseudoscalar isosinglet condensate whose precise magnitude and time variation depends on the dynamics of the HIC.

32 The complete description of a medium with chiral imbalance should also take into account
 33 thermal fluctuations of the medium. But in the paper the description in zero temperature limit is
 34 considered and to understand the changes for non-zero temperatures we rely on the results of lattice
 35 computations of quark matter with chiral imbalance and a temperature of order 150 MeV undertaken
 36 [9]. Thus our calculations keep the tendency of increase of chiral condensate and decrease of pion
 37 masses when temperature grows.

38 This paper is mostly concerned with the possibility of identifying LPB in the hadron phase of
 39 QCD in HIC. Such a medium would be simulated by a chiral chemical potential μ_5 . Adding to the QCD
 40 Lagrangian the term $\Delta\mathcal{L}_q = \mu_5 q^\dagger \gamma_5 q \equiv \mu_5 \rho_5$, we allow for non-trivial topological fluctuations [7] in
 41 the nuclear (quark) fireball, which are ultimately related to fluctuations of gluon fields. The transition
 42 of the quark-gluon medium characteristics to a hadron matter reckons on the quark-hadron continuity
 43 [10] after hadronization of quark-gluon plasma. The behavior of various spectral characteristics for
 44 light scalar and pseudoscalar (σ, π^a, a_0^a) - mesons by means of a QCD-motivated σ -model Lagrangian
 45 was recently derived for $SU_L(2) \times SU_R(2)$ flavor symmetry including an isosinglet chiral chemical
 46 potential [11]. The structural constants of the σ -model Lagrangian were taken as input parameters
 47 suitable to describe the light meson properties in vacuum and then they are extrapolated to a chiral
 48 medium. In this way *ad hoc* there is no reliable predictability in the determination of the hadron system
 49 response on chiral imbalance, and reaching quantitative predictions requires a phenomenologically
 50 justified hadron dynamics. To increase predictability, we extend the vacuum Chiral Lagrangians [12,13]
 51 with phenomenological low-energy structural constants taking into account the chiral medium in the
 52 fireball with a chiral imbalance. It is shown that σ -model parametrization of [11] fits well the pion
 53 phenomenology at low energies as derived from ChPT.

54 Next it is described how pions modify their dynamics in decays in a chiral medium, in particular,
 55 charged pions stop decaying into muons and neutrinos for a large enough chiral chemical potential. A
 56 possible experimental detection of chiral imbalance (and therefore a phase with Local Parity Breaking)
 57 is outlined in the charged pion decays inside the fireball.

58 2. Chiral Lagrangian with chiral chemical potential

The Chiral Lagrangian for pions describing their mass spectra and decays in the fireball with
 a chiral imbalance can be implemented with the help of softly broken chiral symmetry in QCD
 transmitted to hadron media, properly constructed covariant derivative

$$D_\nu \implies \bar{D}_\nu - i\{\mathbf{I}_q \mu_5 \delta_{0\nu}, \star\} = \mathbf{I}_q \partial_\nu - 2i\mathbf{I}_q \mu_5 \delta_{0\nu}, \quad (1)$$

59 where we skipped the electromagnetic field. The axial chemical potential is introduced as a constant
 60 time component of an isosinglet axial-vector field.

In the framework of large number of colors N_c [13] the SU(3) Chiral Lagrangian in the strong
 interaction sector contains the following dim=2 operators [13],

$$\mathcal{L}_2 = \frac{F_0^2}{4} \langle -j_\mu j^\mu + \chi^\dagger U + \chi U^\dagger \rangle, \quad (2)$$

where $\langle \dots \rangle$ denotes the trace in flavor space, $j_\mu \equiv U^\dagger \partial_\mu U$, the chiral field $U = \exp(i\hat{\pi}/F_0)$, the bare
 pion decay constant $F_0 \simeq 92 \text{ MeV}$, $\chi(x) = 2B_0 s(x)$ and $M_\pi^2 = 2B_0 \hat{m}_{u,d}$, the tree-level neutral pion mass.
 The constant B_0 is related to the chiral quark condensate $\langle \bar{q}q \rangle$ as $F_0^2 B_0 = -\langle \bar{q}q \rangle$. Taking now the
 covariant derivative in (1) it yields

$$\mathcal{L}_2(\mu_5) = \mathcal{L}_2(\mu_5 = 0) + \mu_5^2 N_f F_0^2. \quad (3)$$

61 Herein we have used the identity for $U \in SU(n)$, $\langle j_\mu \rangle = 0$. In the large N_c approach the dim=4
62 operators [13] in the Chiral Lagrangian are given by

$$\mathcal{L}_4 = \bar{L}_3 \langle j_\mu j^\mu j_\nu j^\nu \rangle + L_0 \langle j_\mu j_\nu j^\mu j^\nu \rangle - L_5 \langle j_\mu j^\mu (\chi^\dagger U + \chi U^\dagger) \rangle, \quad (4)$$

63 where L_0, \bar{L}_3, L_5 are bare low energy constants. For SU(3) and SU(2) $\langle j_\mu \rangle = 0$ and there is the identity

$$\langle j_\mu j_\nu j^\mu j^\nu \rangle = -2 \langle j_\mu j^\mu j_\nu j^\nu \rangle + \frac{1}{2} \langle j_\mu j^\mu \rangle \langle j_\nu j^\nu \rangle + \langle j_\mu j_\nu \rangle \langle j^\mu j^\nu \rangle, \quad (5)$$

whereas for SU(2) there is one more identity

$$2 \langle j_\mu j^\mu j_\nu j^\nu \rangle = \langle j_\mu j^\mu \rangle \langle j_\nu j^\nu \rangle. \quad (6)$$

64 Applying these identities one finds the four-derivative Gasser-Leutwyler (GL) operators for the SU(3)
65 chiral Lagrangian

$$\begin{aligned} \mathcal{L}_4 = & L_1 \langle j_\mu j^\mu \rangle \langle j_\nu j^\nu \rangle + L_2 \langle j_\mu j_\nu \rangle \langle j^\mu j^\nu \rangle + L_3 \langle j_\mu j^\mu j_\nu j^\nu \rangle \\ & - L_5 \langle j_\mu j^\mu (\chi^\dagger U + \chi U^\dagger) \rangle \end{aligned} \quad (7)$$

with

$$L_1 = \frac{1}{2} L_0; \quad L_2 = L_0; \quad L_3 = \bar{L}_3 - 2L_0. \quad (8)$$

For SU(2) one has a further reduction of the dim=4 lagrangian,

$$\mathcal{L}_4 = \frac{1}{4} l_1 \langle j_\mu j^\mu \rangle \langle j_\nu j^\nu \rangle + \frac{1}{4} l_2 \langle j_\mu j_\nu \rangle \langle j^\mu j^\nu \rangle - \frac{1}{4} l_4 \langle j_\mu j^\mu (\chi^\dagger U + \chi U^\dagger) \rangle \quad (9)$$

with normalization so that

$$l_1 = 4L_1 + 2L_3, \quad l_2 = 4L_2, \quad (l_1 + l_2) = 2L_3 + 6L_2; \quad l_4 = 4L_5. \quad (10)$$

66 We stress that this chain of transformations is valid only if $\langle j_\mu \rangle = 0$.

67 The response of the Chiral Lagrangian on chiral imbalance is derived with the help of the covariant
68 derivative (1) applied to the Lagrangian (4),

$$\Delta \mathcal{L}_4(\mu_5) = -\mu_5^2 \{ 8(l_1 + l_2) \langle j^0 j^0 \rangle - 4(l_1 + l_2) \langle j_k j_k \rangle - l_4 \langle \chi^\dagger U + \chi U^\dagger \rangle \}. \quad (11)$$

We notice that this result is drastically different from what one could obtain from the final Lagrangian (9). This is because the identities (5) and (6) are violated if $\langle j_\mu \rangle \neq 0$. The above modifications change differently the coefficients in the dispersion law in energy p^0 and three-momentum $|\vec{p}|$ for the mass shell as well as modify the mass term for pions (all together it gives the inverse propagator of pions),

$$\mathcal{D}^{-1}(\mu_5) = (F_0^2 + 32\mu_5^2(l_1 + l_2))p_0^2 - (F_0^2 + 16\mu_5^2(l_1 + l_2))|\vec{p}|^2 - (F_0^2 + 4l_4\mu_5^2)m_\pi^2 \rightarrow 0. \quad (12)$$

69 In the leading order of large N_c expansion (neglecting the RG logarithm as a contribution
70 next-to-leading in the large N_c expansion) the empirical values of the SU(2) GL constants [12] are

$$\begin{aligned} l_1^r &= (-0.4 \pm 0.6) \times 10^{-3}; \quad l_2^r = (8.6 \pm 0.2) \times 10^{-3}; \\ l_1^r + l_2^r &= (8.2 \pm 0.8) \times 10^{-3}; \quad l_4^r = (2.64 \pm 0.01) \times 10^{-2}. \end{aligned} \quad (13)$$

71 They can be obtained also if they are normalized at the Renormalization Group scale $\mu \simeq M_\pi \simeq$
72 140 MeV , $\log(m_\pi/\mu) \simeq 0$.

Thus in the pion rest frame

$$F_\pi^2(\mu_5^2) \simeq F_0^2 + 32\mu_5^2(l_1 + l_2); \quad m_\pi^2(\mu_5^2) \simeq \left(1 - 4\frac{\mu_5^2}{F_0^2}(8(l_1 + l_2) - l_4)\right)m_\pi^2(0), \quad (14)$$

i.e. the pion decay constant is growing and its mass is decreasing in the chiral media.

3. Linear Sigma Model for light pions and scalar mesons in the presence of chiral imbalance: comparison to ChPT

Let us compare these constants with those ones estimated from the Linear Sigma Model built in [14,15]. The Sigma Model was build with realization of SU(2) chiral symmetry to describe pions and isosinglet and isotriplet scalar mesons. Its Lagrangian reads

$$L = N_c \left\{ \frac{1}{4} \langle (D_\mu H (D^\mu H)^\dagger) \rangle + \frac{B_0}{2} \langle m(H + H^\dagger) \rangle + \frac{M^2}{2} \langle HH^\dagger \rangle - \frac{\lambda_1}{2} \langle (HH^\dagger)^2 \rangle - \frac{\lambda_2}{4} \langle (HH^\dagger) \rangle^2 + \frac{c}{2} (\det H + \det H^\dagger) \right\}, \quad (15)$$

where $H = \xi \Sigma \xi$ is an operator for meson fields, N_c is a number of colours, m is an average mass of current u, d quarks, M is a "tachyonic" mass generating the spontaneous breaking of chiral symmetry, $B_0, c, \lambda_1, \lambda_2$ are real constants.

The matrix Σ includes the singlet scalar meson σ , its vacuum average v and the isotriplet of scalar mesons a_0^0, a_0^-, a_0^+ , the details see in [14,15]. The covariant derivative of H including the chiral chemical potential μ_5 is defined in (1). The operator ξ realizes a nonlinear representation (see (2)) of the chiral group $SU(2)_L \times SU(2)_R$, namely, $\xi^2 = U$.

The diagonal masses for scalar and pseudoscalar mesons read

$$\begin{aligned} m_\sigma^2 &= -2 \left(M^2 - 6(\lambda_1 + \lambda_2)F_\pi^2 + c + 2\mu_5^2 \right) \\ m_a^2 &= -2 \left(M^2 - 2(3\lambda_1 + \lambda_2)F_\pi^2 - c + 2\mu_5^2 \right) \\ m_\pi^2(\mu_5) &= \frac{2bm}{F_\pi} \simeq m_\pi^2(0) \left(1 - \frac{\mu_5^2}{2(\lambda_1 + \lambda_2)F_0^2} \right) \\ F_\pi^2(\mu_5) &= \frac{M^2 + 2\mu_5^2 + c}{2(\lambda_1 + \lambda_2)} = F_0^2 + \frac{\mu_5^2}{\lambda_1 + \lambda_2}. \end{aligned} \quad (16)$$

From spectral characteristics of scalar mesons in vacuum one fixes the Lagrangian parameters, $\lambda_1 = 16.4850$, $\lambda_2 = -13.1313$, $c = -4.46874 \times 10^4 \text{ MeV}^2$, $B_0 = 1.61594 \times 10^5 \text{ MeV}^2$ [14].

The change of pion coupling constant F_0 is determined by potential parameters as compared to the ChPT definition,

$$\frac{\Delta F_\pi^2}{\mu_5^2} = \frac{1}{\lambda_1 + \lambda_2} \approx 0.3 \quad \text{vs} \quad 32(l_1 + l_2) \approx 0.26. \quad (17)$$

It is a quite satisfactory correspondence.

Analogously, in the rest frame using the pion mass correction, $m_\pi^2(\mu_5)F_\pi^2(\mu_5) \simeq 2m_q B_0 F_\pi(\mu_5)$ it is easy to find the estimation for

$$l_4 \approx 2.64 \times 10^{-2} \quad \text{vs} \quad \frac{1}{8(\lambda_1 + \lambda_2)} \approx 3.8 \times 10^{-2}, \quad (18)$$

wherefrom one can also guess the relation $4(l_1 + l_2) \sim l_4$ following from the LSM.

For moving mesons with $|\vec{p}| \neq 0$ and the CP breaking mixing of scalar and pseudoscalar mesons the effective masses $m_{eff\mp}^2$ take the form,

$$\begin{aligned} m_{eff-}^2 &= \frac{1}{2} \left(16\mu_5^2 + m_a^2 + m_\pi^2 - \sqrt{(16\mu_5^2 + m_a^2 + m_\pi^2)^2 - 4(m_a^2 m_\pi^2 - 16\mu_5^2 |\vec{p}|^2)} \right), \\ m_{eff+}^2 &= \frac{1}{2} \left(16\mu_5^2 + m_a^2 + m_\pi^2 + \sqrt{(16\mu_5^2 + m_a^2 + m_\pi^2)^2 - 4(m_a^2 m_\pi^2 - 16\mu_5^2 |\vec{p}|^2)} \right). \end{aligned} \quad (19)$$

For small $\mu_5^2, m_\pi^2 \ll m_a^2 \simeq 1\text{GeV}^2$ one can approximate the dependence on the wave vector \vec{p}

$$m_{eff-}^2 \simeq m_\pi^2 - 16\mu_5^2 \frac{|\vec{p}|^2}{m_a^2}. \quad (20)$$

Comparing with (12) one establishes the relationship of isotriplet scalar mass and GL constants

$$m_a = \frac{F_0}{\sqrt{l_1 + l_2}} \simeq 1\text{GeV}, \quad (21)$$

90 which reproduces the PDG value within the experimental error bars[16].

91 4. Possible experimental detection of chiral imbalance in the charged pion decays

The predicted distortion of the mass shell condition can be detected in decays of charged pions when the effective pion mass approaches to muon mass. Let us find the threshold value for the $\pi^+ \rightarrow \mu^+ \nu$ decay. If a charged pion was generated in chiral medium its mass is lower than in the vacuum and the condition for its decay follows from (12),

$$\left(1 - 16(l_1 + l_2) \frac{\mu_5^2}{F_0^2}\right) \left(|\vec{p}|^2 + m_{0,\pi}^2\right) \geq |\vec{p}|^2 + m_\mu^2, \quad \frac{m_a^2}{16\mu_5^2} \geq \frac{|\vec{p}|^2 + m_{0,\pi}^2}{m_{0,\pi}^2 - m_\mu^2}, \quad (22)$$

92 where we have used the relations $4(l_1 + l_2) \simeq l_4$ and (21). The decay channel is closed for $|\vec{p}|^2 \simeq 0$
 93 if $\mu_5 \simeq 160\text{MeV}$. It must be detected as a substantial decrease of muon flow originated from pion
 94 decays in fireball. When considering the decay process of a charged pion into a muon + neutrino at
 95 values of the chiral chemical potential lower than $\mu_5 \simeq 160\text{MeV}$ then still the muon yield from the
 96 fireball obviously decreases at sufficiently large momenta. It gives a chance to measure the magnitude
 97 of chiral chemical potential for sufficiently high statistics.

98 5. Results

- 99 • For light mesons in the chiral imbalance medium we compared the Chiral Perturbation Theory
 100 (ChPT) and the Linear Sigma Model (LSM) as realizations of low energy QCD. The relations
 101 between the low-energy constants of the Chiral Lagrangian and the corresponding constants of
 102 the Linear Sigma Model are established as well as the expressions for the decay constant of the
 103 pion in the medium and the mass of the a_0 meson are found.
- 104 • The low energy QCD correspondence of ChPT and LSM in the large N_c limit is satisfactory and
 105 give a solid ground for search of chiral imbalance manifestation in pion physics at HIC.
- 106 • The resulting dispersion law for pions in the medium allows us reveal the threshold of decay
 107 of a charged pion into a muon and neutrino which can be suppressed with increasing of chiral
 108 chemical potential.
- 109 • [14], that at higher energies exotic decays of isoscalar mesons into three pions arise due to mixing
 110 of π and a_0 meson states in the presence of chiral imbalance. It was shown [7,11,14] that a wider
 111 class of direct parity breaking at higher energies, in the framework of linear sigma model with
 112 isotriplet scalar (a_0) and pseudoscalar (pions) mesons their mixing arises with generation of $\pi\pi$

and $\pi\pi\pi$ decays of heavier scalar state. As well the independent check of our estimates could be done by lattice computation (cf.[9]).

- A manifestation for LPB in the presence of chiral imbalance in the sector of ρ and ω vector mesons [3,4] can also happen and in this case the Chern-Simons interaction plays the main role. It turns out [3] that the spectrum of massive vector mesons splits into three components with different polarizations $\pm, long$ having different effective masses $m_{V,+} < m_{V,long} < m_{V,-}$.
- Thus a possible experimental detection of chiral imbalance in medium (and therefore a phase with LPB) in the charged pion decays and vector meson polarizations inside of the fireball can be realized.
- We draw also attention to the recent proposal to measure the photon polarization asymmetry in $\pi\gamma$ scattering [14,17,18] as a way to detect LPB due to chiral imbalance. It happens in the ChPT including electromagnetic fields due to the Wess-Zumino-Witten operators.
- One may concern for appearance of changes in the properties of muons and neutrinos in the medium, but in our opinion, this does not change the main estimates in Eq. (22), as a possible influence of chiral chemical potential on lepton properties would be controlled by extra power of Fermi coupling constant, i.e. by a next order in weak interactions with a little hope to register it.

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