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A quantum heat exchanger for nanotechnology

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Abstract: In this paper, we design a quantum heat exchanger which converts heat into light on relatively short quantum optical time scales. Our scheme takes advantage of collective cavity-mediated laser cooling of an atomic gas inside a cavitating bubble. Laser cooling routinely transfers individually trapped ions to nano-Kelvin temperatures for applications in quantum technology. The quantum heat exchanger which we propose here is expected to provide cooling rates of the order of Kelvin temperatures per millisecond and is expected to find applications in micro and nanotechnology.

Keywords: quantum thermodynamics, laser cooling, cavitation, sonoluminescence

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1. Introduction

Since its discovery in 1975 [1,2], laser cooling of individually trapped ions has become a standard technique in quantum optics laboratories world-wide [3,4]. Rapidly oscillating electric fields can be used to strongly confine charged particles, like single ions, for relatively large amounts of time [5]. Moreover, laser trapping provides unique means to control the dynamics of neutral particles, like neutral atoms [6,7]. To cool single particles, additional laser fields are applied which remove vibrational energy at high enough rates to transfer them down to near absolute-zero temperatures [8]. Nowadays, ion traps are used to perform a wide range of high-precision quantum optics experiments. For example, individually trapped ions are at the heart of devices with applications in quantum technology, like atomic and optical clocks [9,10], quantum computers [11–14], quantum simulators [15,16] and electric and magnetic field sensors [17].

For laser cooling to be at its most efficient, the confinement of individually trapped particles should be so strong that their motion becomes quantised. This means, their vibrational energy is made up of energy quanta, so-called phonons. When this applies, an externally applied laser field not only affects the electronic states of a trapped ion—it also changes its vibrational state. Ideally, laser frequencies should be chosen such that the excitation of the ion should be most likely accompanied by the loss of a phonon. If the ion returns subsequently into its ground state via the spontaneous emission of a photon, its phonon state remains the same. Overall one phonon is permanently lost form the system which implies cooling. On average, every emitted photon lowers the vibrational energy of the trapped ion by the energy of one phonon. Eventually, the cooling process stops when the ion no longer possesses any vibrational energy.

Currently, there are many different ways of designing and fabricating ion traps [18,19]. However, the main requirements for the efficient conversion of vibrational energy into light on relatively short quantum optical time scales are always the same [20,21]:

1) Individual atomic particles need to be so strongly confined that their motion becomes quantised. In the following, ν denotes the phonon frequency and $\hbar\nu$ is the energy of a single phonon.
2) A laser field with a frequency $\omega_{\rm L}$ below the atomic transition frequency ω_0 needs to be applied. As long as the laser detuning $\Delta = \omega_0 - \omega_{\rm L}$ and the phonon frequency ν are comparable in size,

$$\Delta \quad \sim \quad \nu \,, \tag{1}$$

the excitation of an ion is most likely accompanied by the annihilation of a phonon. Transitions which result in the simultaneous excitation of an ion and the creation of a phonon are possible but as long as their detuning is larger, they are less likely to occur.

3) When excited, the confined atomic particle needs to be able to emit a photon. In the following, we denote its spontaneous decay rate by Γ. This rate should not be much larger than ν,

$$\nu \;\geq\; \Gamma, \tag{2}$$

so that the cooling laser couples efficiently to atomic transitions. At the same time, Γ should not be too small so that de-excitation of the excited atomic state often happens via the spontaneous emission of a photon.

Given these three conditions, the applied laser field can result in a very efficient conversion of the vibrational energy of individually trapped ions into photons and prepares them for applications in high-precision quantum optics experiments and in quantum technology.

In this paper, we ask the question whether laser cooling could also have applications also in micro and nanoscale physics experiments. For example, nanotechnology deals with objects which have dimensions between 1 and 1000 nanometers and is well known for its applications in information and communication technology, as well as sensing and imaging. Increasing the speed at which information can be processed and the sensitivity of sensors is usually achieved by reducing system dimensions. However, smaller devices are usually more prone to heating as thermal resistances increase [22]. Sometimes, large surface to volume ratios can help to off-set this problem. Another problem for nanoscale sensors is thermal noise. As sensors are reduced in size, their signal to noise ratio usually decreases and thus the thermal energy of the system can limit device sensitivity [23]. Therefore thermal considerations have to be taken into account and large vacuums or compact heat exchangers have already become an integral part of nanotechnology devices.

Usually, heat exchangers in micro and nanotechnology rely on fluid flow [24]. In this paper, we propose an alternative approach. More concretely, we propose to use a variation of laser cooling, namely cavity-mediated collective laser cooling [25–28], and sympathetic cooling. As illustrated in Fig. 1, the proposed quantum heat exchanger mainly consist of a liquid which contains a large number of cavitating bubbles filled with noble gas atoms. Transducers constantly change the radius of these bubbles which should resemble optical cavities when they reach their minimum radius during bubble collapse phases. At this point, a continuously applied external laser field rapidly transfers vibrational energy of the atoms into light. If the surrounding liquid contains many cavitating bubbles, their surface area becomes relatively large and there can be a very efficient exchange of heat between the inside and the outside of the cavitating bubbles. Any removal of thermal energy from the trapped atomic gas inside bubbles should eventually result in the sympathetic cooling of the surrounding liquid and of the surface area of the device on which it is placed.

In this paper, we emphasise that cavitating bubbles with external laser driving can provide all of the above listed requirements $(1)-(3)$ for efficient laser cooling, especially a strong confinement of atomic particles [29,30]. They naturally contain a large number of noble gas atoms, like nitrogen. To initiate the cooling process, an appropriately detuned laser field needs to be applied in addition to the sound waves which confine the bubbles. In the following, we cannot provide accurate estimates, not even orders of magnitude, for the relevant system parameters of the experimental setup shown in Fig. 1. Nevertheless, we estimate that it seems feasible to achieve cooling rates of the order of Kelvin temperatures per millisecond for volumes of liquid on a cubic micrometer scale. We hope that the theoretical work which we present in this paper stimulates further research, for example, in the form of quantum optics experiments with cavitating bubbles.

Cavitating bubbles in sonoluminescence experiments are well-known for using sound to generate relatively large amounts of thermal energy, while producing light in the optical regime [31–33]. During this process, the atomic gas inside a cavitating bubbles can reach very high temperatures [34,35]. These
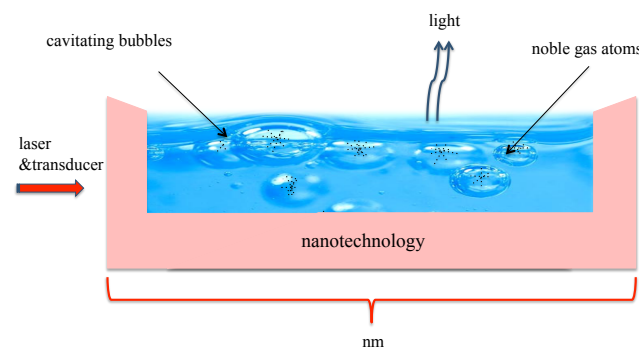
Figure 1. Schematic view of the proposed quantum heat exchanger. It consists of a liquid in close contact with the area which we want to cool. The liquid should contain cavitating bubbles which are filled with atomic particles, like Nitrogen, and should be driven by sounds waves and laser light. The purpose of the sound waves is to constantly change bubble sizes. The purpose of the laser is to convert thermal energy during bubble collapse phases into light.

hint at the presence of a very strong coupling between the electronic and the vibrational degrees of freedom of the atomic gas. Moreover, cavitating bubbles already have applications in sonochemistry, where they are used to provide energy for chemical reactions [36]. Here we propose to exploit the atom-phonon interactions in sonoluminescence experiments for laser cooling. As we shall see below, in the presence of an appropriately detuned laser field, we expect other, usually present highly-detuned heating processes to become secondary.

There are five sections in this paper. The purpose of Section 2 is to provide an introduction to cavity-mediated collective laser cooling of an atomic gas. As we shall see below, this technique is a variation of standard laser cooling techniques for individually trapped atomic particles. We provide an overview of the experimental requirements and estimate achievable cooling rates. Section 3 studies the effect of thermalisation on the thermal state of a large collection of atoms with elastic collisions. Section 4 reviews the main design principles of a quantum heat exchanger for nanotechnology. It is shown that the experimental setup shown in Fig. 1 can operate as a quantum heat exchangers for applications in micro and nanotechnology. Finally, we summarise our findings in Section 5.

2. Cavity-mediated collective laser cooling

For pedagogical reasons, let us first have a closer look at the theoretical modelling of a standard laser cooling technique for individually trapped atomic particle [20,21]. Afterwards, we review cavity-mediated laser cooling of an individually trapped atom [37–40] and cavity-mediated collective laser cooling of an atomic gas [27,28].

2.1. Laser cooling of individually trapped particles

Fig. 2(a) shows an individually confined two-level atom (or ion) with external laser driving. For simplicity, we assume that the trapping potential is approximately harmonic. Most importantly, the atom should be so strongly confined that its motion becomes quantised. In this case, the vibrational energy of the atomic particle is made up of energy-quanta, so-called phonons, of energy $\hbar\nu$. In the following, $|m\rangle$ denotes a vibrational state of the atom with exactly m phonons. Moreover, $|g\rangle$ and $|e\rangle$ denote the ground and the excited electronic state of the trapped particle with energy separation $\hbar\omega_0$. Fig. 2(b) shows the energy level of the combined atom-phonon system with the energy eigenstates $|x, m\rangle$ with x = g, e and m being an integer.

In order to cool the atom, the frequency ω_L of the applied laser needs to be smaller than the atomic transition frequency ω_0. Ideally the laser detuning $\Delta = \omega_0 - \omega_L$ should equal the phonon frequency ν, as stated in Eq. (1). In addition, efficient laser cooling requires that the phonon frequency
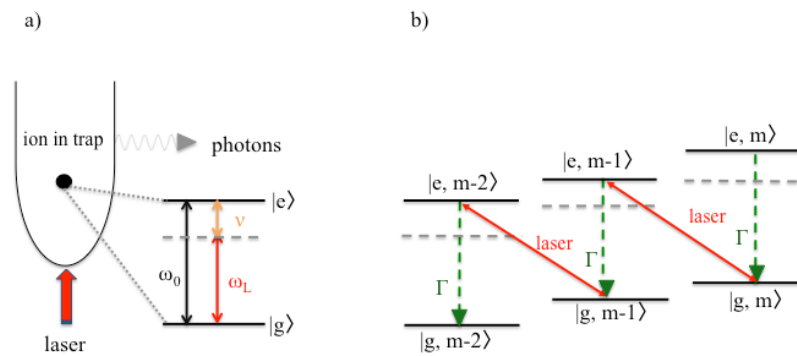
Figure 2. (a) Schematic view of the experimental setup for laser cooling of a single trapped ion. Here $|g\rangle$ and $|e\rangle$ denote the ground and the excited state of the ion, respectively, with transition frequency ω_0 and spontaneous decay rate Γ. The motion of the particle is strongly confined by an external harmonic trapping potential such that it becomes quantised. Here, ν denotes the frequency of the corresponding phonon mode and ω_L is the frequency of the applied cooling laser. (b) The purpose of the laser is to excite the ion, while annihilating a phonon, thereby causing transitions between the basis states $|x, m\rangle$ with $x = g, e$ and $m = 0, 1, \ldots$ of the atom-phonon system. If the excitation of the ion is followed by the spontaneous emission of a photon, a phonon is permanently lost which implies cooling.

ν should not be much larger than the spontaneous decay rate Γ of the excited atomic state (cf. Eq. (2)). When this applies, the laser couples resonantly to transitions which result in the excitation of the atom, while simultaneously removing one phonon from the motion of the trapped particle. If the excitation of the atom is followed by the spontaneous emission of a photon, the atom returns into its ground state without changing its phonon number. Suppose the atom was initially prepared in a state $|g, m\rangle$. Then its final state equals $|g, m - 1\rangle$. One phonon has been permanently removed from the system which implies cooling. As illustrated in Fig. 2(b), the trapped particle eventually reaches its ground state $|g, 0\rangle$ where it no longer experiences any laser driving [20,21].

Eqs. (1) and (2) ensure that the excitation of the atom is usually accompanied by the creation of a phonon. All other transitions which involve the excitation of an atom are highly detuned and only experience a very weak coupling to the applied laser field. To a very good approximation, the Hamiltonian of the atom-phonon system equals [21]

$$H_{\mathrm{I}} \;=\; \hbar g \left(\sigma^- b^\dagger + \sigma^+ b \right) \tag{3}$$

in the interaction picture with respect to its free energy of the atom-phonon system. Here g denotes the (real) atom-phonon coupling constant, while $\sigma^+ = |e\rangle\langle g|$ and $\sigma^- = |g\rangle\langle e|$ are atomic rising and lowering operators. Moreover, b and $b^\dagger$ are phonon annihilation and creation operators with $[b, b^\dagger] = 1$. To take into account the spontaneous emission of photons from the excited state of the atom with a spontaneous decay rate Γ, we describe the atom-phonon system in the following by its density matrix $\rho_{\mathrm{I}}(t)$. Its dynamics follows a master equation of Lindblad form,

$$\dot{\rho}_{\mathrm{I}} \;=\; -\frac{i}{\hbar}[H_{\mathrm{I}}, \rho_{\mathrm{I}}] + \Gamma \left(\sigma^- \rho_{\mathrm{I}} \sigma^+ - \frac{1}{2}\sigma^+ \sigma^- \rho_{\mathrm{I}} - \frac{1}{2}\rho_{\mathrm{I}} \sigma^+ \sigma^- \right). \tag{4}$$

This equation can be used to analyse the dynamics of the expectation value $\langle A_{\mathrm{I}} \rangle = \mathrm{Tr}(A_{\mathrm{I}}\rho_{\mathrm{I}})$ of any observable A_{I}, since it implies that

$$\langle \dot{A}_{\mathrm{I}} \rangle \;=\; -\frac{i}{\hbar}[A_{\mathrm{I}}, H_{\mathrm{I}}] + \Gamma \left\langle \sigma^+ A_{\mathrm{I}} \sigma^- - \frac{1}{2}A_{\mathrm{I}} \sigma^+ \sigma^- - \frac{1}{2}\sigma^+ \sigma^- A_{\mathrm{I}} \right\rangle. \tag{5}$$

Here we are especially interested in the dynamics of the mean phonon number $m = \langle b^\dagger b \rangle$. In order to obtain a closed set of rate equations, we also need to study the dynamics of the population of the

excited atomic state $s = \langle \sigma^+ \sigma^- \rangle$ and the dynamics of the atom-phonon coherence $k_1 = i \langle \sigma^- b^\dagger - \sigma^+ b \rangle$. Using Eq. (5), we find that the time derivatives of these variables are given by

$$
\begin{aligned}
\dot{m} &= -g k_1, \\
\dot{s} &= g k_1 - \Gamma s, \\
\dot{k}_1 &= 2g(m - s) - 4g\, ms - \frac{1}{2}\Gamma k_1
\end{aligned}
\tag{6}
$$

after applying the semiclassical approximation $\langle \sigma^+ \sigma^- b^\dagger b \rangle = \langle \sigma^+ \sigma^- \rangle \langle b^\dagger b \rangle = ms$. Having a closer look at the above equations, we see that the system rapidly reaches its stationary state, where $s = k_1 = 0$ while

$$
m = 0. \tag{7}
$$

This equation shows that the atom eventually reaches a very low temperature. More detailed calculations shows that the actual stationary state phonon m number of the trapped atom depends on the system parameters but tends to zero, when the ratio Γ/ν becomes sufficiently small [21]. The above cooling equations (6) also show that the corresponding cooling rate equals

$$
\gamma_{1\,\text{atom}}^{\text{standard}} = g^2/\Gamma \tag{8}
$$

to a very good approximation and that the cooling process takes place not on mechanical but on relatively short quantum optical time scales.
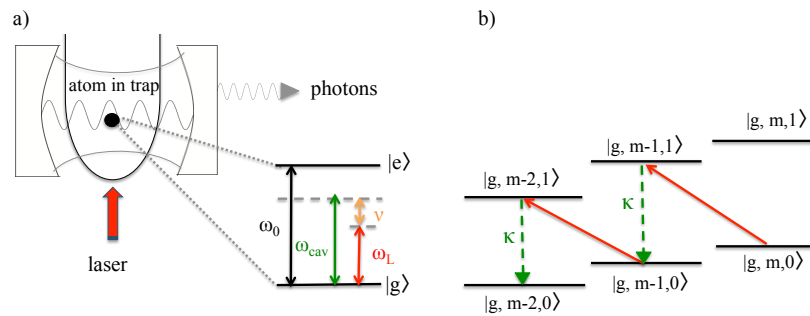
2.2. Cavity-mediated laser cooling of a single atom

Suppose we want to cool a single ion (or atom) whose transition frequency ω_0 is well above the optical regime, i.e. much larger than typical laser frequencies ω_L. In this case, it is impossible to realise the condition $\Delta = \nu$ in Eq. (1), even approximately. Hence it seems impossible to lower the temperature of the trapped particle via laser cooling. One way of overcoming this problem is to confine the particle inside an optical cavity, as illustrated in Fig. 3(a). In the following, we denote the state of the cavity with exactly n photons by $|n\rangle$. Using this notation, the energy eigenstates of the atom-phonon-photon systems can be written as $|x, m, n\rangle$ with $x = g, e$ and $m, n = 0, 1, 2, ...$ Moreover, κ denotes the spontaneous cavity decay rate, ν denotes again the phonon frequency and ω_L and ω_{cav} denote the laser and the cavity frequency, respectively. Fig. 3(b) shows the relevant energy level diagram in the presence of an externally applied cooling laser.

In the experimental setup in Fig. 3, all transitions which result in the excitation of the atom are naturally strongly detuned and can be neglected. However, it has been shown that one can nevertheless transfer the atom to a relatively low temperature by exploiting indirect couplings and converting phonons directly into cavity photons [39,40]. Suppose the cavity detuning $\Delta_{\text{cav}} = \omega_{\text{cav}} - \omega_L$ and the phonon frequency ν are comparable in size and the phonon frequency ν exceeds the spontaneous cavity decay rate κ such that

$$
\Delta_{\text{cav}} \sim \nu \quad \text{and} \quad \nu \geq \kappa, \tag{9}
$$

in analogy to Eqs. (1) and (2). Then two-step transitions which excite the atom while removing one phonon immediately followed by the de-excitation of the atom while creating a cavity photon become resonant and dominate the dynamics of the atom-phonon-photon system. The overall effect of these two-step transitions is the conversion of a phonon into a cavity photon, while the atom remains essentially in its ground state, as illustrated in Fig. 3(b). When cavity photons subsequently leak into the environment, the phonons are permanently lost. The vibrational energy of the atom is reduced which implies cooling.

Figure 3. (a) Schematic view of the experimental setup for cavity-mediated laser cooling of a single atom. The main difference between this setup and the setup shown in Fig. 2 is that the atom now couples in addition to an optical cavity with frequency ω_{cav} and the spontaneous decay rate κ. Here both the cavity field and the laser are highly detuned from the atomic transition and the direct excitation of the atom remains negligible. However the cavity detuning $\Delta_{\text{cav}} = \omega_{\text{cav}} - \omega_{\text{L}}$ should equal the phonon frequency of the trapped particle. (b) As a result, only the annihilation of a phonon accompanied by the simultaneous creation of a cavity photon are in resonance. In cavity-mediated laser cooling, the purpose of the laser is to convert phonons into cavity photons. The subsequent loss of this photon via spontaneous emission results the permanent loss of a phonon and therefore in the cooling of the trapped particle.

To model the above described dynamics, we describe the experimental setup in Fig. 3 in the following by the interaction Hamiltonian [39,40]

$$H_{\text{I}} = \hbar g_{\text{eff}}(bc^\dagger + b^\dagger c), \tag{10}$$

where g_{eff} is an effective atom-cavity coupling constant and where c with $[c, c^\dagger] = 1$ denotes the annihilation operator of a single photon inside the resonator. Since the atom remains essentially in its ground state, the spontaneous emission of photons with the atomic decay rate Γ remains negligible. To model the possible leakage photons through the cavity mirrors, we now use the master equation

$$\dot{\rho}_{\text{I}} = -\frac{i}{\hbar}[H_{\text{I}}, \rho_{\text{I}}] + \kappa \left(c\rho c^\dagger - \frac{1}{2}c^\dagger c\rho - \frac{1}{2}\rho c^\dagger c \right). \tag{11}$$

The density matrix $\rho_{\text{I}}(t)$ in this equation is the density matrix of the phonon-photon system in the interaction picture. Atomic degrees of freedome no longer have to be taken into account, since the atoms remains constantly in its ground state $|g\rangle$. Hence, the time derivative of the expectation value $\langle A_{\text{I}} \rangle = \text{Tr}(A_{\text{I}}\rho_{\text{I}})$ of an observable A_{I} now equals

$$\langle \dot{A}_{\text{I}} \rangle = -\frac{i}{\hbar}[A_{\text{I}}, H_{\text{I}}] + \kappa \left\langle c^\dagger A_{\text{I}} c - \frac{1}{2}A_{\text{I}} c^\dagger c - \frac{1}{2}c^\dagger c A_{\text{I}} \right\rangle \tag{12}$$

in analogy to Eq. (5). In the following, we use this equation to study the dynamics of the mean phonon number $m = \langle b^\dagger b \rangle$, of the mean cavity photon number $n = \langle c^\dagger c \rangle$ and of the phonon-photon coherence $k_1 = i\langle bc^\dagger - b^\dagger c \rangle$. Doing so and proceeding as in the previous subsection, we now obtain the rate equations

$$
\begin{aligned}
\dot{m} &= g_{\text{eff}}k_1, \\
\dot{n} &= -g_{\text{eff}}k_1 - \kappa n, \\
\dot{k}_1 &= 2g_{\text{eff}}(n - m) - \frac{1}{2}\kappa k_1.
\end{aligned}
\tag{13}
$$

These equations describe the continuous conversion of phonons into cavity photons. Subsequently, cavity photons leak into the environment via spontaneous emission. Hence it is not surprising to find that the stationary state of the atom-phonon-photon system corresponds to $n = k_1 = 0$ while

$$m \;=\; 0 \tag{14}$$

in analogy to Eq. (7). Independent of its initial state, the atom is again cooled to a very low temperature. In analogy to Eq. (8), the effective cooling rate for cavity-mediated laser cooling equals [39,40]

$$\gamma_{1\,\text{atom}} \;=\; g_{\text{eff}}^2 / \kappa . \tag{15}$$

Due to the resonant coupling being indirect, g_{eff} is in general a few order of magnitude smaller than atom-phonon coupling constant g in Eq. (8), if the spontaneous decay rates κ and Γ are of similar size. Cooling a single atom inside an optical resonator might therefore take a relatively large amount of time. However, as we shall see below, this reduction in cooling rate can be compensated for by collective enhancements [27].

2.3. Cavity-mediated collective laser cooling of an atomic gas

Finally we have a closer look at cavity-mediated collective laser cooling of an atomic gas inside an optical resonator [27,28]. To do so, we replace the single atom in the experimental setup in Fig. 3 by a collection of N atoms. In analogy to Eq. (10), the interaction Hamiltonian H_I between phonons and cavity photons now equals

$$H_\text{I} \;=\; \sum_{i=1}^{N} \hbar g_{\text{eff}}^{(i)} b_i c^\dagger + \text{H.c.}, \tag{16}$$

where $g_{\text{eff}}^{(i)}$ denotes the effective atom-cavity coupling constant of atom i. This coupling constant is essentially the same as g_{eff} in Eq. (15) and depends in general on the position of atom i. Moreover b_i denotes the phonon annihilation operator of atom i with $[b_i, b_j^\dagger] = \delta_{ij}$. In order to simplify the above Hamiltonian, we introduce a collective phonon annihilation operator B with

$$B \;=\; \frac{\sum_{i=1}^{N} g_{\text{eff}}^{(i)} b_i}{\left(\sum_{i=1}^{N} |g_{\text{eff}}^{(i)}|^2 \right)^{1/2}} \tag{17}$$

and $[B, B^\dagger] = 1$. In addition, we introduce an effective phonon-photon coupling constant $\tilde{g}_{\text{eff}}$,

$$\tilde{g}_{\text{eff}} \;=\; \left(\sum_{i=1}^{N} |g_{\text{eff}}^{(i)}|^2 \right)^{1/2} . \tag{18}$$

Using the above notation, the interaction Hamiltonian H_I in Eq. (16) becomes

$$H_\text{I} \;=\; \hbar \tilde{g}_{\text{eff}} B c^\dagger + \text{H.c.} \tag{19}$$

Notice that the coupling constant $\tilde{g}_{\text{eff}}$ essentially scales as the square root of the number of atoms N inside the cavity. For example, if all atomic particles couple equally to the cavity field and $g_{\text{eff}}^{(i)} \equiv g_{\text{eff}}$, then $\tilde{g}_{\text{eff}} = \sqrt{N}\, g_{\text{eff}}$. This means, in case of many atoms, the effective phonon-photon coupling is collectively enhanced [27].

Comparing the many atom interaction Hamiltonian H_I in Eq. (16) with the one atom interaction Hamiltonian H_I in Eq. (10), we see that both of them are essentially the same. Moreover the density matrix ρ_I obeys the same master equation (cf. Eq. (11)) in both cases. Hence we expect cooling in both
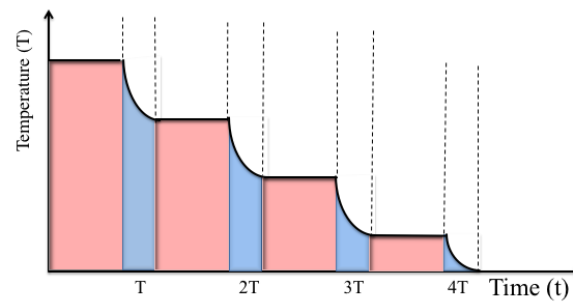
Figure 4. Schematic view of the expected dynamics of the temperature of the atomic gas during cavity-mediated collective laser cooling which involves a sequence of cooling stages (blue) and thermalisation stages (pink). During thermalisation stages, heat is transferred from the different vibrational degrees of freedoms of the atoms into a certain collective vibrational mode B, while the mean temperature of the atoms remains the same. During cooling stages, energy from the B mode into light. Eventually, the atomic gas becomes very cold.

cases as long as the phonon frequency ν of the common vibrational mode B is sufficiently large and comparable to the cavity detuning Δ_{cav} of the applied cooling laser (cf. Eq. (9)). If this applies, one can show that, if all atoms couple equally to the cavity field, the effective cooling rate of the common vibrational mode B equals

$$\gamma_{N\,\text{atoms}} \quad = \quad N g_{\text{eff}}^2 / \kappa \tag{20}$$

to a very good approximation, in analogy to Eq. (15). This cooling rate is N times larger than the cooling rate which we predicted in the previous subsection for cavity-mediated laser cooling of a single atom. In case of very large atom numbers N, the collective enhancement can easily compensate the previously described reduction in cooling rate due to indirect instead of direct phonon-photon couplings. At least in theory, it is possible to realise cooling rates $\gamma_{N\,\text{atoms}}$ with

$$\gamma_{N\,\text{atoms}} \quad \gg \quad \gamma_{1\,\text{atom}}^{\text{standard}} . \tag{21}$$

Standard laser cooling and cavity-mediated laser cooling of an atomic gas can exhibit similarly high cooling rates.

However, the above discussion also shows that cavity-mediated collective laser cooling only affects a single common vibrational mode B, while all other vibrational modes of the atoms do not see the cooling laser. Once this mode reaches its stationary state, the conversion of heat into light stops and there is no longer any cooling. To nevertheless take advantage of the relatively high cooling rates of cavity-mediated collective laser cooling, a mechanism is needed which rapidly exchanges energy between different vibrational degrees of freedom [28,41]. As we shall see below, one way of achieving this is to intersperse cooling stages with thermalisation stages, as illustrated in Fig. 5. The purpose of the cooling stages is to rapidly remove energy from the common vibrational mode B, thereby lowering the average temperature T of the atomic gas. The purpose of subsequent thermalisation stages is to rapidly exchange energy between all the different vibrational modes of the atoms. Repeating thermalisation and cooling stages is expected to result in the cooling of all the motional degrees of freedom of the confined atomic gas.

3. Thermalisation of an atomic gas with elastic collisions

In this section, we have a closer look at a possible realisation of the above described thermalisation stages, More concretely, we assume in the following that elastic collisions transfer the atomic gas into its thermal state. It is shown that this process re-distributes energy between all the different vibrational modes of the atoms. For simplicity, we assume here that the atoms do not see the cooling laser during

thermalisation stages. As we shall see in Section 4, this assumption is well justified for the experimental setup in Fig. 1 which provides ideal conditions for the implementation of cavity-mediated collective laser cooling of an atomic gas.

3.1. The thermal state of a single harmonic oscillator

As in the previous section, we first consider a single trapped atom inside a harmonic trapping potential. Its thermal state equals [42]

$$\rho = \frac{1}{Z} e^{-\beta H}, \tag{22}$$

where H is the relevant harmonic oscillator Hamiltonian, $\beta = 1/k_B T$ is the thermal Lagrange parameter for a given temperature T, k_B is Boltzmann's constant and Z denotes the partition function

$$Z = \mathrm{Tr}(e^{-\beta H}) \tag{23}$$

which normalises the density matrix ρ of the atom. For sufficiently large atomic transition frequencies ω_0, the thermal state of the atom is to a very good approximation given by its ground state $|g\rangle$, unless the atom becomes very hot. In the following, we therefore neglect its electronic degrees of freedom. Taking this into account the Hamiltonian H in Eq. (22) can be written as

$$H = \hbar \nu \left(b^\dagger b + \tfrac{1}{2} \right), \tag{24}$$

where ν and b are again the frequency and the annihilation operator of a single phonon. In the following, we use the density matrix ρ of the vibrational state of the atom to calculate the expectation value of its vibrational energy in the thermal state.

Using the above Hamiltonian and combining Eqs. (23) and (24), we find that

$$Z = \mathrm{Tr} \left(e^{-\lambda \left(b^\dagger b + \frac{1}{2} \right)} \right) \tag{25}$$

with $\lambda = \beta \hbar \nu$. The number operator $b^\dagger b$ can be written as $b^\dagger b = \sum_{m=0}^{\infty} m |m\rangle\langle m|$ with $|m\rangle$ denoting a number state with exactly m phonons. Taking this into account yields

$$Z = e^{-\frac{1}{2}\lambda} \sum_{m=0}^{\infty} e^{-\lambda m}. \tag{26}$$

Using geometric series, the partition function Z simplifies to

$$Z = \frac{e^{-\frac{1}{2}\lambda}}{1 - e^{-\lambda}}. \tag{27}$$

Here we are especially interested in the expectation value of the thermal energy of the vibrational mode of the trapped atom which equals $\langle H \rangle = \mathrm{Tr}(H\rho)$. Hence using Eqs. (22) and (23), one can show that

$$\langle H \rangle = \frac{1}{Z} \mathrm{Tr} \left(H e^{-\beta H} \right) = -\frac{1}{Z} \frac{\partial}{\partial \beta} Z = -\frac{\partial}{\partial \beta} \ln Z. \tag{28}$$

Finally, combining this result with Eq. (27), we find that

$$\langle H \rangle = \hbar \nu \left(\frac{e^{-\lambda}}{e^{-\lambda} - 1} + \frac{1}{2} \right) \tag{29}$$

which is Planck's expression for the average energy of a single quantum harmonic oscillator. Moreover,

$$m \;\; = \;\; \frac{e^{-\lambda}}{e^{-\lambda} - 1} \, , \tag{30}$$

since the mean phonon number $m = \langle b^\dagger b \rangle$ relates to $\langle H \rangle$ via $m = \langle H \rangle / \hbar \nu - \frac{1}{2}$.

3.2. The thermal state of many atoms with collisions

Next we calculate the thermal state of an atomic gas with strong elastic collisions inside a harmonic trapping potential. This situation has many similarities with the situation considered in the previous subsection. All the atoms constantly collide with their respective neighbours. The overall effect of these collisions is to increase the confinement of each individual particle. In the following, this is taken into account assuming that the atoms no longer experience the phonon frequency ν but an increased phonon frequency ν_{eff}. Assuming in addition that all atoms experience the same interaction, we find that their Hamiltonian H equals

$$H \;\; = \;\; \sum_{i=1}^{N} \hbar \left(\nu_{\text{eff}} + \tfrac{1}{2} \right) b_i^\dagger b_i \tag{31}$$

to a very good approximation, where b_i denotes again the phonon annihilation operator of atom i. Comparing this Hamiltonian with the harmonic oscillator Hamiltonian in Eq. (24) and substituting H in Eq. (31) into Eq. (22) to obtain the thermal state of many atoms, we find that this thermal state is simply the product of the thermal states of the individual atoms. All atoms have the same thermal state, their mean phonon number $m_i = \langle b_i^\dagger b_i \rangle$ equals

$$m_i \;\; = \;\; \frac{e^{-\lambda_{\text{eff}}}}{e^{-\lambda_{\text{eff}}} - 1} \tag{32}$$

with $\lambda_{\text{eff}} = \hbar \nu_{\text{eff}} / k_{\text{B}} T$, in analogy to Eq. (30). This equation shows that any previously depleted collective vibrational mode of the atoms becomes re-populated during thermalisation stages. To a very good approximation, the initial phonon number m of the collective phonon mode B at the beginning of a cooling stage only depends on the initial temperature T of the atoms.

4. A quantum heat exchanger with cavitating bubbles

The purpose of this paper is to introduce a quantum heat exchangers which converts heat into light on very short quantum optical time scales. A schematic view of the proposed experimental setup, which relies on a combination of cavity-mediated collective laser cooling and sympathetic cooling, is shown in Fig. 1. It consists of a liquid which sits on top of a small-scale device and is driven by a transducer and by a cooling laser. The transducer generates cavitating bubbles which should contain atomic particles and whose diameters change very rapidly in time. The purpose of the cooling laser is to encourage the conversion of the vibrational energy of the atoms into light. The cooling of the atomic particles inside the bubbles eventually aids the sympathetic cooling of the liquid which surrounds the bubbles and its environment.

To gain a better understanding of the experimental setup in Fig. 1, Section 4.1 describes the main characteristics of single bubble sonoluminescence experiments [31–35]. Section 4.2 emphasises that there are many similarities between sonoluminescence and quantum optics experiments [29,30]. From this we conclude that sonoluminescence experiments naturally provide the main ingredients for the implementation of cavity-mediated collective laser cooling of an atomic gas. Finally, in Sections 4.3 and 4.4, we have a closer look at the physics of the proposed quantum heat exchanger and estimate potentially achievable cooling rates for practical applications. Although we are unable to make exact quantitative predictions—we cannot even predict the relevant orders of magnitude—our analysis
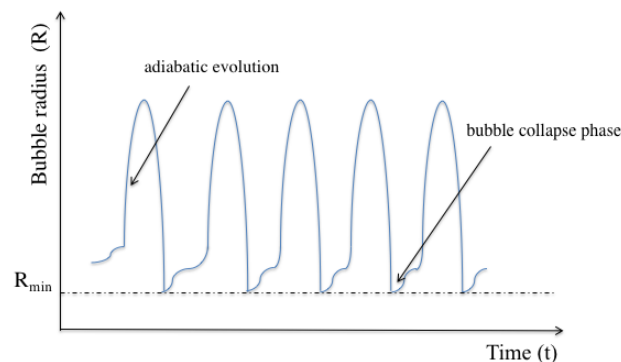
Figure 5. Schematic view of the time dependence of the bubble radius in a typical single-bubble sonoluminesence experiment. Most of the time, the bubble evolves adiabatically and exchanges thermal energy with its surroundings. However, at regular time intervals, the bubble radius suddenly collapses. At this point, the bubble becomes thermally isolated. When it reaches its minimum radius, the system usually emits a strong flash of light in the optical regime.

hopes to encourage and to provide at least some guidance for quantum optics experiments with cavitating bubbles.
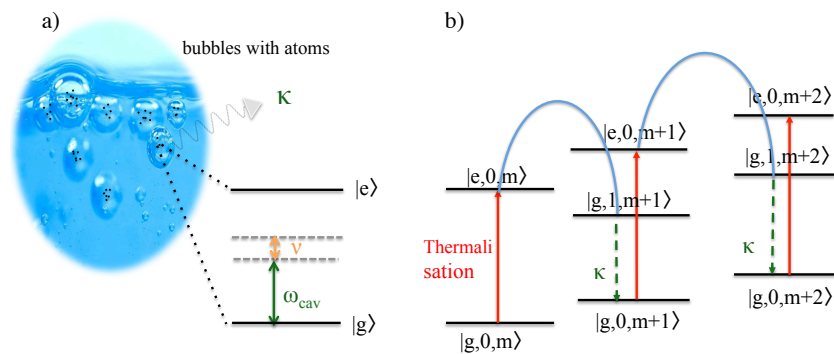
4.1. Single bubble sonoluminescence experiments

Sonoluminescence can be defined as a phenomenon of strong light emission from collapsing bubbles in a liquid, like water [31–33]. These bubbles need to be filled with noble gas atoms, like nitrogen atoms, which occur naturally in air. Alternatively, the bubbles can be filled with ions from ionic liquids, molten salts, and concentrated electrolyte solutions [43]. Moreover, the bubbles need to be acoustically confined and periodically driven by ultrasonic frequencies. As a result, the bubble radius changes periodically in time, as illustrated in Fig. 5. The oscillation of the bubble radius regenerates itself with unusual precision.

At the beginning of every expansion phase, the bubble oscillates about its equilibrium radius until it returns to its fastness. During this process, the bubble temperature changes adiabatically and there is an exchange of thermal energy between the atoms inside the bubble and the surrounding liquid. During the collapse phase of a typical single-bubble sonoluminescence, i.e. when the bubble reaches its minimum radius, its inside becomes thermally isolated from the surrounding environment and the atomic gas inside the bubble becomes strongly confined. Usually, a strong light flash occurs at this point which is accompanied by a sharp increase of the temperature of the particles. Experiments have shown that increasing the concentration of atoms inside the bubble increases the intensity of the emitted light [34,35].

4.2. A quantum optics perspective on sonoluminescence

The above observations suggest many similarities between sonoluminescence and quantum optics experiments [29,30]. For example, when the bubble reaches its minimum radius, there is a strong confinement of a large collection of atomic particles. This confinement can cause the motion of the atoms to become quantised and to contain phonons with different trapping frequencies ν which are similar to the phonons in ion trap experiments (cf. Section 2.1). Moreover, when the bubble reaches its minimum radius, its surface usually becomes opaque. When this happens, the bubble surface traps light on the inside and closely resembles an optical cavity. This cavity can be characterised by a frequency ω_{cav} and a spontaneous decay rate κ. Since the confined particles have atomic dipole moments, they naturally couple to the quantised electromagnetic field inside the cavity. The result can be an exchange of energy between atomic dipoles and the cavity mode. The creation of photons inside

Figure 6. (a) From a quantum optics point of view, one of the main characteristics of sonoluminescence experiments is that cavitating bubbles provide a very strong confinement for atomic particles. This means, the motion of atoms becomes quantised. As in ion trap experiments, we denote the corresponding phonon frequency in this paper by ν. Moreover, during its collapse phase, the surface of the bubble becomes opaque and confines light, thereby forming an optical cavity with frequency ω_{cav} and a spontaneous decay rate κ. (b) Even in the absence of external laser driving, some of the atoms are initially in their excited state $|e\rangle$ due to being prepared in a thermal equilibrium state at a finite temperature T. When returning into their ground state via the creation of a cavity photon which is only possible during the bubble collapse phase, most likely a phonon is created. This creation of phonons implies heating. Indeed, sonoluminescence experiments often reach relatively high temperatures [34,35].

the cavity is usually accompanied by the annihilation or creation of phonons in the vibrational modes of the atoms and the subsequent spontaneous emission of light in the optical regime, thereby changing the temperature of the atomic particles inside the bubble.

A main difference between sonoluminescence and cavity-mediated collective laser cooling is the absence and presence of external laser driving (cf. Section 2.3). But even in the absence of external laser driving, there can be a non-negligible amount of population in the excited electronic states $|e\rangle$ of the atoms inside the cavitating bubble. This applies, for example, if the atoms are initially prepared in the thermal equilibrium state of a finite temperature T. Once surrounded by an optical cavity, as it occurs during bubble collapse phases, excited atoms can return into their ground state via the creation of a cavity photon. This process can be accompanied by the creation or by the annihilation of a phonon (cf. Fig. 6). Suddenly, an additional de-excitation channel has become available to them. As pointed out in Refs. [29,30], the creation a phonon is more likely to occur than by the annihilation of a phonon since

$$
\begin{aligned}
B^{\dagger} &= \sum_{m=0}^{\infty} \sqrt{m+1}\,|m+1\rangle\langle m|\,, \\
B &= \sum_{m=0}^{\infty} \sqrt{m}\,|m-1\rangle\langle m|\,.
\end{aligned}
\tag{33}
$$

Here B and $B^{\dagger}$ denote the relevant phonon annihilation and creation operators, while $|m\rangle$ denotes a state with exactly m phonons. As one can see from Eq. (33), the normalisation factor of $B^{\dagger}|m\rangle$ is slightly larger than the normalisation factor of the state $B|m\rangle$. When the cavity photon is subsequently lost via spontaneous photon emission, the newly-created phonon remains inside the bubble. Hence the light emission during bubble collapse phases is usually accompanied by heating of the atoms inside the bubble, until the sonoluminescing bubble reaches an equilibrium.
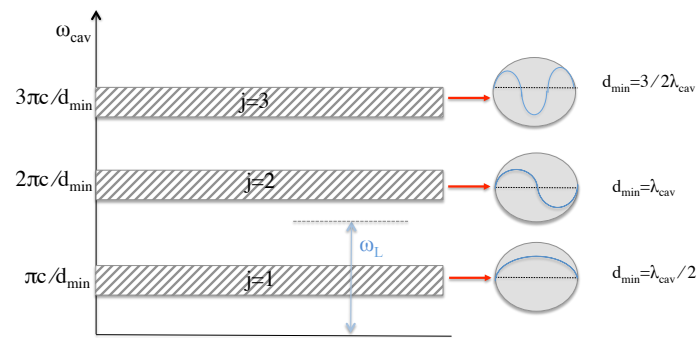
Figure 7. When the cavitating bubbles inside the liquid reach their minimum diameters d_{min}, their walls become opaque and trap light on the inside. To a very good approximation, they form cavities which can be described by spontaneous decay rates κ and by cavity frequencies ω_{cav} (cf. Eq. (34)). Suppose the diameters of the bubbles inside the liquid occupy a relatively small range of values. Then every integer number j in Eq. (34) corresponds to a relatively narrow range of cavity frequencies ω_{cav}. Here we are especially interested in the parameter j for which the ω_{cav}'s lie in the optical regime. When this applies, we can apply a cooling laser with an optical frequency ω_{L} which can cool the atoms in all bubbles. Some bubbles will be cooled more efficiently than others. But as long as the relevant frequency bands are relatively narrow, none of the bubbles will be heated.

During bubble collapse phases, cavitating bubbles are thermally isolated from their surroundings. However, during subsequent expansion phases, system parameters usually change adiabatically and there is a constant exchange of thermal energy between atomic gas inside the bubble and the surrounding liquid (cf. Fig. 5). Eventually, the atoms reach an equilibrium between heating during bubble collapse phases and loss of energy during subsequent expansion phases. Experiments have shown that the atomic gas in side the cavitating bubble can reaches temperature of the order of 10^4 K which shows that there is indeed a very strong coupling between the vibrational and the electronic states of the confined particles [34,35].

4.3. Cavity-mediated collective laser cooling of cavitating bubbles

As illustrated in Fig. 1, in this paper, we propose a quantum heat exchanger which is based on the external laser driving of cavitating bubbles. The previous subsection has shown that, during their collapse phase, the cavitating bubbles become equivalent to the experimental setup in Fig. 3 but with the single atom replaced by a strongly confined atomic gas. When the bubble reaches its minimum diameter d_{min}, its surface forms an optical cavity with a discrete set of cavity frequencies ω_{cav} which are given by

$$\omega_{\mathrm{cav}} \;=\; j \times \frac{\pi c}{d_{\mathrm{min}}} \,. \tag{34}$$

Here c denotes the speed of light in air and $j = 1, 2, \ldots$ is an integer. As illustrated in Fig. 7, the case $j = 1$ corresponds to cavity photons, whose wavelength λ_{cav} is twice the size of the bubble diameter d_{min}. The case the case $j = 2$ corresponds to cavity photons, whose wavelength λ_{cav} coincides with the bubble diameter d_{min} during the bubble collapse phase and so on. In general, not all bubbles inside the liquid have exactly the same size which is why every j is usually associated with a range of frequencies ω_{cav}. Here we are especially interested in the parameter j, where the corresponding ω_{cav}'s lie in the optical regime. All other parameters j can be neglected, once a laser field with an optical frequency ω_{L} is applied.

In addition we know that the phonon frequency ν of the relevant collective phonon mode B assumes its maximum value during the bubble collapse phase. In the following, we denote this

frequency by ν_{max}. Suppose the cavity detuning $\Delta_{cav} = \omega_L - \omega_{cav}$ of the applied laser field is chosen such that

$$\Delta_{cav} \sim \nu_{max} \quad \text{and} \quad \nu_{max} \geq \kappa \tag{35}$$

to a very good approximation, in analogy to Eq. (2). As we have seen in Section 2.3, in this case, two-step processes which result in the simultaneous annihilation of a phonon and the creation of a cavity photon become resonant and dominate the system dynamics. If the creation of a cavity photon subsequently results in the spontaneous emission of light, a phonon is permanently lost which implies cooling. Overall, we expect this cooling process to be very efficient, since the atoms experience a very strong confinement. Moreover, cooling rates are collectively enhanced and the higher the larger the number of atoms inside the bubble (c.f. Eq. (20)).

In order to cool not only tiny but larger volumes of a liquid, the experimental setup in Fig. 1 should contain a relatively large number of cavitating bubbles. Depending on the quality of the applied transducer, the minimum diameters d_{min} of the cavitating bubbles in the liquid might vary in size. Consequently, the collection of bubbles support a finite range of cavity frequencies ω_{cav} (cf. Fig. 7). In this case, it becomes impossible to realise the ideal cooling condition $\Delta_{cav} \sim \nu_{max}$ in Eq. (35) for all bubbles. However, this is not a problem. As long as the frequency ω_L of the applied cooling laser field is smaller than all cavity frequencies ω_{cav} in the optical regime and well detuned from other cavity frequencies, the system dynamics will be dominated by cooling on not by heating. When this applies, none of the bubbles should experiences any laser heating. In general, it is important that the diameters of the bubbles does not vary by too much.

Section 2.3 also shows that cavity-mediated collective laser cooling only removes thermal energy from a single collective vibrational mode B of the atoms. Once this mode is depleted, the cooling process stops. To efficiently cool an entire atomic gas, a mechanism is needed which rapidly re-distributes energy between its different vibrational degrees of freedom, for example, via thermalisation based on elastic collisions (cf. Section 3). As we have seen above, between cooling stages, cavitating bubbles evolve essentially adiabatically and the atoms experience strong collisions. In other words, the expansion phase of cavitating bubbles automatically implements the intermittent thermalisation stages of cavity-mediated collective laser cooling.

Finally, let us point out that it does not matter, whether the cooling laser is turned on or off during thermalisation stages, i.e. during bubble expansion phases. As long as optical cavities only form during the bubble collapse phases, the above described conversion of heat only light only happens, when the bubble reaches its minimum diameter. During bubble expansions, the atoms do not see a cooling laser with a frequency in the optical regime. The reason for this is that noble gas atoms, like nitrogen, have very large transition frequencies ω_0. The direct laser excitation of atomic particles is therefore relatively unlikely. If we could excite the atoms directly by laser driving, we could easily cool them directly (cf. Section 2.1). In general cooling only occurs during the bubble collapse phase.

4.4. Sympathetic cooling of the surrounding

The purpose of the heat exchanger which we propose in this paper is not to cool an atomic gas inside cavitating bubbles but to constantly remove thermal energy from the surrounding liquid and the nanotechnology device on which the liquid is placed (cf. Fig. 1). As described in the previous subsection, this is achieved here by converting heat at very high rates into light. In addition, a mechanism is needed which encourages the exchange of thermal energy between the bubbles and the surrounding liquid. As we have seen above, cavitating bubbles evolve adiabatically between collapse phases and bubble expansion phases naturally realise sympathetic cooling of the liquid surrounding the bubbles. As illustrated in Fig. 8, alternating cooling and thermalisation stages (or collapse and expansion phases) is therefore expected to implement a quantum heat exchanger which does not require the actual transport of particles from one place to another. On a coarse grained time scale, there
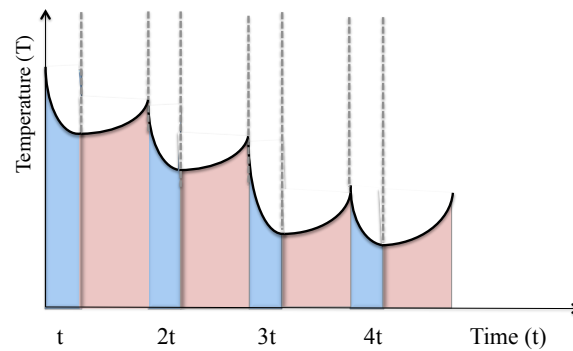
Figure 8. Schematic view of the expected dynamics of the temperature of a confined atomic gas during bubble collapse stages (blue) and expansion stages (pink). During expansion stages, heat is transferred from the outside into the inside of the bubble, thereby increasing the temperature of the atoms. During bubble collapse stages, heat is converted into light, thereby resulting in the cooling of the system in Fig. 1. Eventually, both processes balance each other out and the temperature of the system remains constant on a coarse grained time scale.

is a continuous draining of thermal energy from the liquid into collective atomic phonon modes from where it can be transformed into light.

Due to the relatively high complexity of the quantum heat exchanger in Fig. 1, it is difficult to make quantitative predictions about achievable cooling rates. Not only do the bubbles exchange heat with the surrounding liquid. Moreover, the liquid constantly exchanges energy with its environment at an unknown rate. However, any energy which is taken from an atomic gas inside a cavitating bubble comes eventually from the environment which we aim to cool. In this final subsection, we have closer look at potentially achievable cooling rates for devices with length dimensions in the nano and micrometer regime to obtain at least some guidance for quantum optics experiments with cavitating bubbles.

In the following, we assume that the relevant phonon frequencies ν_{max} are sufficiently large to ensure that every emitted photon indicates the loss of one phonon, i.e. the loss of one quantum of energy $\hbar\nu_{\mathrm{max}}$. Moreover, suppose our quantum heat exchanger contains a certain amount of liquid, let us say water, of mass m_{water} and heat capacity $c_{\mathrm{water}}(T)$ at an initial temperature T_0. Then we can ask the question, how many photons N_{photons} do we need to create in order to cool the water by a certain temperature ΔT. From thermodynamics, we know that the change in the thermal energy of the water equals

$$\Delta Q \;=\; c_{\mathrm{water}}(T_0)\, m_{\mathrm{water}} \Delta T \tag{36}$$

in this case. Moreover, we know that

$$\Delta Q \;=\; N_{\mathrm{photons}}\, \hbar\nu_{\mathrm{max}} \,. \tag{37}$$

Hence the number of photons that needs to be produced is given by

$$N_{\mathrm{photons}} \;=\; \frac{c_{\mathrm{water}}(T_0)\, m_{\mathrm{water}}\, \Delta T}{\hbar\nu_{\mathrm{max}}} \,. \tag{38}$$

The time t_{cool} it would take to create this number of photons equals

$$t_{\mathrm{cool}} \;=\; \frac{N_{\mathrm{photons}}}{N_{\mathrm{atoms}}\, I} \,, \tag{39}$$

where I denotes the average single-atom photon emission rate and N_{atoms} is the number of atoms involved in the cooling process. When combining the above equations, we find that the cooling rate $\gamma_{\mathrm{cool}} = t_{\mathrm{cool}}/\Delta T$ of the proposed cooling process equals

$$\gamma_{\mathrm{cool}} = \frac{c_{\mathrm{water}}(T)\, m_{\mathrm{water}}}{N_{\mathrm{atoms}}\, I\, \hbar \nu_{\mathrm{max}}} \tag{40}$$

to a very good approximations.

As an example, suppose we want to cool one cubic micrometer of water at room temperature. In this case, $V_{\mathrm{water}} = 1\,\mu\mathrm{m}^3$ and $T_0 = 20\,^0\mathrm{C}$. Hence $m_{\mathrm{water}} = 10^{-15}\,\mathrm{g}$ and $c_{\mathrm{water}}(T_0) = 4.18\,\mathrm{J/gK}$ to a very good approximation. Suppose $\nu = 100\,\mathrm{MHz}$ (a typical frequency in ion trap experiments is $\nu = 10\,\mathrm{MHz}$), $I = 10^6/\mathrm{s}$ and $N_{\mathrm{atoms}} = 10^8$ (a typical bubble in single bubble sonoluminescence contains about 10^8 atoms). Substituting these numbers into Eq. (39) yields a cooling rate of

$$\gamma_{\mathrm{cool}} = 3.81\,\mathrm{ms}/K. \tag{41}$$

Achieving cooling rates of the order of Kelvin temperatures per millisecond seems experimentally feasible. As one can see from Eq. (39), to reduce cooling rates further, one can either reduce the volume that requires cooling, increase the number of atoms involved in the cooling process or increase the trapping frequency ν_{max} of the atomic gas during the bubble collapse phase. All of this is, at least in principle, possible.

5. Conclusions

In this paper we point out similarities between quantum optics experiments with strongly confined atomic particles and single bubble sonoluminescence experiments [29,30]. In both cases, interactions are present which can be used to convert thermal energy very efficiently into light. When applying an external cooling laser to cavitating bubbles, as illustrated in Fig. 1, we therefore expect a rapid transfer of heat into light which can eventually result in the cooling of relatively small devices. Our estimates show that it might be possible to achieve cooling rates of the order of milliseconds per Kelvin temperature for cubic micrometers of water. The proposed quantum heat exchanger is expected to find applications in research experiments and in micro and nanotechnology. A closely related cooling technique, namely laser cooling of trapped ions, is already routinely used to cool individual ions to nanoKelvin temperatures for applications in quantum technology (cf. e.g.. Refs. [10–17]).

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