

Mass or Energy?

_On charge of gravity

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Abstract: The gravitational charge should be the energy instead of mass. This modification will lead to some different results and the experiments to test are also presented. Especially, we propose a scheme to achieve the negative energy and gravitational repulsive force in the lab.

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1: Introduction

There should be a conserved charge in the theory of gravitation if it is a gauge theory and the energy is a good choice because of the law of energy conservation. If the coefficient K between the energy E and mass m is always a same constant, m is equivalent to E and can also be the gravitational charge. However, the ratio can be $0 < K < c^2$ [1][2], $K > c^2$ [3], $K < 0$ [4] or other quantities[5][6]. Therefore, the mass is not conservational and cannot be the charge of any gauge theory.

2: New form

Newton's law of universal gravitation states every mass attracts any other mass by a force. It takes the form

$$G \frac{m_1 m_2}{r^2} \quad (1)$$

where $G = 6.674 \times 10^{-11} \text{Nm}^2\text{kg}^{-2}$ is the gravitational constant and r is the distance. The force is directly proportional to the product of their masses m_1 and m_2 . If the charge of the gravitational interaction is energy, the new form of the force should be

$$G' \frac{E_1 E_2}{r^2} \quad (G' = 8.262 \times 10^{-45} \text{N}^{-1} = \frac{G}{c^4}) \quad (2)$$

and the gravitational potential

$$\phi = -\frac{G' E}{r} \quad (3)$$

is a dimensionless quantity which can be linked to $g_{\mu\nu}$ directly (Eq.45). The Poisson equation is replaced by

$$\nabla^2 \phi = 4\pi G' \mathbf{N} \quad (4)$$

It can be reduced to

$$\nabla^2 \phi = \frac{4\pi G}{c^2} \rho \quad (5)$$

once the relation between the energy density \aleph and mass density ρ of the source is

$$\aleph = \rho c^2 \quad (6)$$

Eq.(2) can be written as

$$G \frac{E_1 E_2}{r^2} = G \frac{\frac{E_1}{c^2} \frac{E_2}{c^2}}{r^2} \quad (7)$$

Here, $\frac{E}{c^2}$ plays the role of the so-called gravitational mass m_G . Obviously, m_G is a redundant concept. It is unnecessary to distinguish the inertial mass m_i between gravitational mass m_G because the physical meaning of the weak principle of equivalence $m_i = m_G$ is just $m = \frac{E}{c^2}$. Einstein claimed that “The calling force of the earth depends on the gravitational mass. The answering motion of the stone depends on the inertial mass.” [7]. The sentences should be revised to “The calling force of the earth depends on the energy. The answering motion of the stone depends on the mass.”

3: Application-negative energy and repulsion

In Newton's theory, the gravitational force is always attractive. Now we use the new form to study a simple bound system. The rest energy of a deuteron is $1875.6 \times 10^6 eV$ and the force between the Earth which has a energy $Mc^2 > 0$ and positive mass $M > 0$ should be

$$G' \frac{Mc^2}{r^2} 1875.61 \times 10^6 eV = G \frac{M}{r^2} \frac{1875.61 \times 10^6 eV}{c^2} \quad (8)$$

Nevertheless, the deuteron consists of a proton and a neutron whose rest energies are $938.27 \times 10^6 eV$ and $939.57 \times 10^6 eV$, respectively. The resultant of forces

$$G' \frac{Mc^2}{r^2} 938.27 \times 10^6 eV + G' \frac{Mc^2}{r^2} 939.57 \times 10^6 eV = G' \frac{Mc^2}{r^2} 1877.84 \times 10^6 eV \quad (9)$$

is larger than (8). Thus, the gravitational force between the negative binding energy $-2.23 \times 10^6 eV$ and the positive Mc^2 of the Earth should be repulsive. Eq.(8) is equal to

$$G' \frac{Mc^2}{r^2} 938.27 \times 10^6 eV + G' \frac{Mc^2}{r^2} 939.57 \times 10^6 eV - G' \frac{Mc^2}{r^2} 2.23 \times 10^6 eV \quad (10)$$

4: Gravitational effect of the electric scalar potential

In the above example, the object is under the influence of the nuclear force. Let us consider the case that no fields or forces exist except the gravitational force. In a Faraday cage, the electrostatic field \mathbf{E} is absent although the electric scalar potential ϕ_E can be nonzero. The wave function of a free particle is

$$\psi \sim \exp \frac{i}{\hbar} (Et - \mathbf{p}\mathbf{x}) = \exp \frac{i}{\hbar} (mc^2 t - \mathbf{p}\mathbf{x}) \quad (11)$$

After a scalar potential ϕ_E is applied, the velocity v or momentum p of a particle electrically charged q in the cage does not change while the total energy is

$$mc^2 + q\phi_E \quad (m = \frac{m_0}{\sqrt{1 - \frac{v^2}{c^2}}}) \quad (12)$$

and it gains an extra phase [8]

$$\frac{q}{\hbar} \phi_E t \quad (13)$$

Under the circumstances, the gravitational force between the Earth is

$$G \frac{Mc^2}{r^2} (mc^2 + q\phi_E) = \frac{GM}{r^2} \left(m + \frac{q\phi_E}{c^2} \right) \quad (14)$$

For instance, the electric charge of an electron is $q = -e$ and the force

$$\frac{GM}{r^2} \left(m - \frac{e\phi_E}{c^2} \right) \quad (15)$$

can be zero if the electric scalar potential is equal to

$$\phi_E = \frac{mc^2}{e} \approx 0.51 \times 10^6 \text{ Volts} \quad (16)$$

It is easy to implement. Furthermore, the total energy will be negative and the gravitational force is repulsive on the condition of

$$\phi_E > \frac{mc^2}{e} \quad (17)$$

We can use the cold ion matter-wave interferometer to measure the following gravitational acceleration

$$\frac{G \frac{Mc^2}{r^2} (mc^2 + q\phi_E)}{m} = \frac{\frac{GM}{r^2} \left(m + \frac{q\phi_E}{c^2} \right)}{m} = g \left(1 + \frac{q\phi_E}{mc^2} \right) \quad (18)$$

The value is not

$$g = \frac{GM}{r^2} = 9.8 m \cdot s^{-2} \quad (19)$$

unless the mass is changed to

$$m + \frac{q\phi_E}{c^2} \quad (20)$$

simultaneously. It is impossible and some experiments can be performed to exclude Eq(20). In thermodynamics, the heat capacity C_v of electrons is in proportion to the temperature T , i.e.

$$C_v = \gamma T \propto T \quad (21)$$

$$\gamma \propto m \quad (22)$$

Suppose the mass of the electron is affected by ϕ_E , the heat capacity of a sample placed in a cage should be

$$C_v \propto \left(m - \frac{e\phi_E}{c^2} \right) T \quad (23)$$

The spectra emitted by hydrogen atoms in a Faraday cage can be observed too. The electric potential energy of an electron in this atom can be written as

$$-\frac{e^2}{4\pi\epsilon_0 r} - e\phi_E \quad (24)$$

The electric force between the proton is still

$$\frac{e^2}{4\pi\epsilon_0 r^2} \quad (25)$$

because it depends on the gradient, not the magnitude. Hence,

$$\frac{e^2}{4\pi\epsilon_0 r^2} = m \frac{v^2}{r} \quad (26)$$

Due to the Bohr's quantization condition,

$$mv\tau = n\hbar \quad (n=1,2,3,\dots) \quad (27)$$

the total energy is

$$mc^2 - \frac{e^2}{4\pi\epsilon_0 r} - e\phi_E \approx m_0c^2 - \frac{e^4}{32\pi^2\epsilon_0^2\hbar^2 n^2} m - e\phi_E \quad (28)$$

and the frequency of the spectrum should be

$$hf = E_1 - E_2 = \frac{e^4}{32\pi^2\epsilon_0^2\hbar^2} m \left(\frac{1}{n_2^2} - \frac{1}{n_1^2} \right) = 13.6eV \left(\frac{1}{n_2^2} - \frac{1}{n_1^2} \right) \quad (29)$$

It seems unlikely that

$$m \rightarrow m \left(1 - \frac{e\phi_E}{c^2} \right) \quad (30)$$

$$hf \rightarrow \frac{e^4}{32\pi^2\epsilon_0^2\hbar^2} m \left(1 - \frac{e\phi_E}{c^2} \right) \left(\frac{1}{n_2^2} - \frac{1}{n_1^2} \right) = 13.6eV \left(1 - \frac{e\phi_E}{c^2} \right) \left(\frac{1}{n_2^2} - \frac{1}{n_1^2} \right) \quad (31)$$

5: Superconducting interferometry gravimeter

The superconducting carrier is another ideal object to study the gravitational effect like the electrically charged particle in a Faraday cage because the electrostatic field also vanishes [9] inside a superconductor. In fact, it had been utilized to measure the Sagnac effect [10] of matter waves [11] in a non-inertial reference frame which can be explained in general relativity [12]. We can study the superconducting current in the following circuit (Fig.1) like the COW experiment [13] to detect the phase shift of the neutron caused by the gravitational interaction with the Earth.

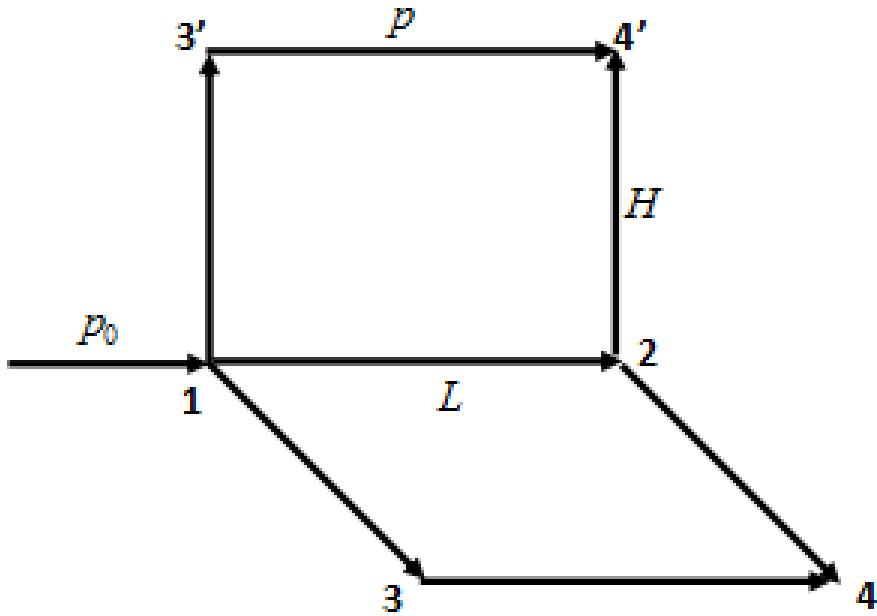


Figure.1 Schematic diagram

First, the circuit is placed in the horizontal plane $\overline{1234}$. Then it is rotating around the axis $\overline{12}$ whose length is L . The relation between the momentum p in the path $\overline{3'4'}$ and incident momentum $p_0 = m_i v_0$ (m_i is the inertial mass and v_0 is the initial velocity) should be

$$\frac{p^2}{2m_i} = \frac{p_0^2}{2m_i} - m_G g H \quad (32)$$

H is the height of $\overline{3'4'}$ relative to $\overline{1234}$. In the COW experiment,

$$\frac{p_0^2}{2m_i} \gg m_G g H \quad (33)$$

$$p \approx p_0 - \frac{2m_i m_G g H}{p_0} = p_0 - \frac{2m_G g H}{v_0} \quad (34)$$

The phase shift ϑ between $\overline{13'4'}$ and $\overline{124'}$ is proportional to the gravitational force $m_G g$.

$$\vartheta = \frac{pL}{\hbar} - \frac{p_0 L}{\hbar} = \frac{-2m_G g H}{\hbar v_0} L \propto m_G g \quad (35)$$

When an electric scalar potential ϕ_E is applied, the gravitational force should now be

$$G \frac{Mc^2}{r^2} (mc^2 + q\phi_E) = \frac{GMm}{r^2} \left(1 + \frac{q\phi_E}{mc^2}\right) = mg \left(1 + \frac{q\phi_E}{mc^2}\right) \quad (36)$$

and the gravitational phase shift is

$$\vartheta \propto \left(1 + \frac{q\phi_E}{mc^2}\right) \quad (37)$$

6: Physical significance

The gravitational field generated by the above electrically charged particle in a Faraday cage can be expressed in general relativity,

$$g_{\mu\nu} = \begin{pmatrix} -\left(1 - \frac{2G\left(mc^2 + q\phi_E - \frac{q^2}{16\pi\epsilon_0 r}\right)}{r}\right) & 0 & 0 & 0 \\ 0 & \frac{1}{1 - \frac{2G\left(mc^2 + q\phi_E - \frac{q^2}{16\pi\epsilon_0 r}\right)}{r}} & 0 & 0 \\ 0 & 0 & r^2 & 0 \\ 0 & 0 & 0 & r^2 \sin^2 \theta \end{pmatrix} \quad (38)$$

The limit $\phi_E = 0$ is the Reissner-Nordstrom metric. However, its gravitational acceleration (18) in another source's field is associated with the mass m . It is at variance with general relativity whose equation of motion

$$\frac{d^2 x^\mu}{ds^2} + \Gamma_{\alpha\beta}^\mu \frac{dx^\alpha}{ds} \frac{dx^\beta}{ds} = 0 \quad (39)$$

is independent of the mass. Actually, a geometric theory is valid to the objects whose gravitational charge to mass ratio is a constant K . In relativity, the constant is $K = c^2$. But the ratio in above examples

$$\frac{mc^2 + q\phi_E}{m} = c^2 + \frac{q\phi_E}{m} \quad (40)$$

is not a constant.

7: Geometric theories of gravity

If K is a constant, there should be [1]~[4]

$$ds^2 = g_{\mu\nu} dx^\mu dx^\nu \quad (41)$$

$$dx^\mu dx^\nu = Kt^2 + dx^2 + dy^2 + dz^2 \quad (42)$$

$$g_{\mu\nu} = \begin{pmatrix} -1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix} \quad (43)$$

to describe an inertial reference frame. In a gravitational field, Eqs.(41)&(42) are still tenable and $g_{\mu\nu}$ is determined by Einstein's field equation

$$R_{\mu\nu} - \frac{1}{2} g_{\mu\nu} R = 8\pi G' T_{\mu\nu} \quad (G' = 8.262 \times 10^{-45} N^{-1}) \quad (44)$$

For the sake of convenience, let us consider the Earth's field

$$g_{\mu\nu} = \begin{pmatrix} -(1+2\phi) & 0 & 0 & 0 \\ 0 & \frac{1}{1+2\phi} & 0 & 0 \\ 0 & 0 & r^2 & 0 \\ 0 & 0 & 0 & r^2 \sin^2 \theta \end{pmatrix} \quad (45)$$

Since $E = Mc^2$ of the Earth, another expression of Eq.(45) familiar to us is

$$g_{\mu\nu} = \begin{pmatrix} -\left(1 - \frac{2GM}{c^2r}\right) & 0 & 0 & 0 \\ 0 & \frac{1}{1 - \frac{2GM}{c^2r}} & 0 & 0 \\ 0 & 0 & r^2 & 0 \\ 0 & 0 & 0 & r^2 \sin^2 \theta \end{pmatrix} \quad (46)$$

Therefore,

$$ds^2 = -\left(1 - \frac{2GM}{c^2r}\right)Kdt^2 + \frac{dr^2}{1 - \frac{2GM}{c^2r}} + r^2d\theta^2 + r^2 \sin^2 \theta d\varphi^2 \quad (47)$$

$K = c^2$ is the well-known

$$ds^2 = -\left(1 - \frac{2GM}{c^2r}\right)c^2dt^2 + \frac{dr^2}{1 - \frac{2GM}{c^2r}} + r^2d\theta^2 + r^2 \sin^2 \theta d\varphi^2 \quad (48)$$

The approximation of the motion equation (39) is

$$\frac{d^2x^i}{Kdt^2} + \nabla\phi = 0 \quad (49)$$

The acceleration is

$$a = \frac{d^2x^i}{dt^2} = -K\nabla\phi \quad (50)$$

On the surface of the Earth,

$$\phi = -G\frac{E}{r} = -\frac{G}{c^4} \frac{Mc^2}{r} = -\frac{GM}{c^2r} \quad (51)$$

$$a = \frac{K}{c^2} \frac{GM}{r^2} \quad (52)$$

In the age of Newton, all experimental objects can be measured effectively satisfy

$K = c^2$. Thus,

$$a = \frac{GM}{r^2} \quad (53)$$

This is just the Newton's law

$$ma = \frac{GM}{r^2} m \quad (54)$$

8: Negative mass and attraction

A negative mass is inconceivable in Newton's time. Now scientists can produce abnormal waves in metamaterials which propagate from sources to the receivers while the wave vectors are reversed. The phenomena imply that the masses of quanta of these waves are less than zero [4]. According to Newton's formula, the gravitational force between the quanta and the earth ($M > 0$) should be repulsive. But the energy hf of such a quantum is positive and the force

$$G' \frac{Mc^2}{r^2} hf \quad (55)$$

is still attractive in the new equation. The sign of gravity depends on the product of energies rather than masses.

Interestingly, the gravitational acceleration

$$\frac{G' \frac{Mc^2}{r^2} hf}{m} \quad (m < 0) \quad (56)$$

should be in the opposite direction of the gravitational force (55). From another perspective, it is because of $K < 0$ [4] in Eq.(52).

9: Metric tensor and non-inertial effects

In a non-inertial reference frame, $g_{\mu\nu}$ depends on not only the acceleration but also the coefficient K . For example, in a rotating frame,

$$ds^2 = -\left(1 - \frac{\Omega^2 r^2}{K}\right)K dt^2 + dr^2 + r^2 d\theta^2 + dz^2 + 2\Omega r^2 d\theta dt \quad (57)$$

where Ω is the angular frequency. The frequency relation should be

$$f_1 \sqrt{1 - \frac{\Omega^2 r_1^2}{K}} = f_2 \sqrt{1 - \frac{\Omega^2 r_2^2}{K}} \quad (58)$$

When $K = c^2$,

$$ds^2 = -\left(1 - \frac{\Omega^2 r^2}{c^2}\right)c^2 dt^2 + dr^2 + r^2 d\theta^2 + dz^2 + 2\Omega r^2 d\theta dt \quad (59)$$

$$f_1 \sqrt{1 - \frac{\Omega^2 r_1^2}{c^2}} = f_2 \sqrt{1 - \frac{\Omega^2 r_2^2}{c^2}} \quad (60)$$

It was verified long ago by using the photons in vacuum [14]. Moreover, a transverse Doppler effect of sound

$$f_1 \sqrt{1 - \frac{\Omega^2 r_1^2}{C_s^2}} = f_2 \sqrt{1 - \frac{\Omega^2 r_2^2}{C_s^2}} \quad (61)$$

was predicted in 2000 [1] and experimentally studied in 2011[15]. It cannot be explained by Eqs.(59)&(60). We have to substitute the ratio $K = C_s^2$ of a quantum of the sound wave [1] into Eq.(57) where the sonic speed C_s is a constant. In air, $C_s = 340 \text{ m/s}$. Consequently,

$$ds^2 = -\left(1 - \frac{\Omega^2 r^2}{C_s^2}\right) C_s^2 dt^2 + dr^2 + r^2 d\theta^2 + dz^2 + 2\Omega r^2 d\theta dt \quad (62)$$

and the result (61) can be interpreted. By comparison, the equation to describe sound in a gravitational field should be $K = C_s^2$ in Eq.(47)

$$ds^2 = -\left(1 - \frac{2GM}{c^2 r}\right) C_s^2 dt^2 + \frac{dr^2}{1 - \frac{2GM}{c^2 r}} + r^2 d\theta^2 + r^2 \sin^2 \theta d\varphi^2 \quad (63)$$

and the gravitational frequency shift of sound

$$f_1 \sqrt{1 - \frac{2GM}{c^2 r_1}} = f_2 \sqrt{1 - \frac{2GM}{c^2 r_2}} \quad (64)$$

is equal to that of light. There is the Mössbauer effect to measure the gravitational shift of light [16] but no technologies to detect such a tiny quantity of sound now.

Additionally, Eq(59) is also utilized to explain the Sagnac phase of light [12]

$$\frac{4\pi f}{c^2} \Omega S \quad (65)$$

in a rotating system. S denotes the area. To the acoustic wave, it should be

$$\frac{4\pi f}{K} \Omega S = \frac{4\pi f}{C_s^2} \Omega S \quad (66)$$

As a uniformly accelerated reference frame [17], the non-inertial frequency shift is consistent with the result derived from $E = mc^2$ and the relation $madx = dE = hdf$ [18].

Once the squared speed of light c^2 is replaced by K , they are applicable to others such as sound [18].

10: Conclusions

The charge of the gravitational interaction should be replaced by energy. Newton did not know the latter whose concept became mature in the 19th century, about 100 years after his death.

Another reason is all available objects in his time satisfy $\frac{E}{m} = K = c^2$ and $G' \frac{E_1 E_2}{r^2}$ is

equivalent to $G \frac{m_1 m_2}{r^2}$. A geometric theory of gravity is effective under the premise of

$K = \text{constant}$, even though it is not c^2 . But K can also be a variable quantity and we design some experiments to test.

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