Article

An Fusion of Whale and Sine Cosine Algorithms for solving optimization Functions

Gurjeet Singh Walia [1,†,‡], Narinder Singh [2,‡] and Sharandeep Singh [3,*]

[1] Department of Mathematics, Statistics & Physics, Punjab Agricultural University, Ludhiana, India; Email:drgswalia@pau.edu
[2] Department of Mathematics,Punjabi University, Patiala, Punjab, INDIA; Email: narindersinghgoria@gmail.com
[3] Department of Mathematics, Punjabi University, Patiala; Email:sssinghsharan@gmail.com
[*] Correspondence: narindersinghgoria@gmail.com; Tel.: +91-7888785729

Abstract: We developed a novel hybrid approach for solving global optimization, computer science, bio-medical and engineering real life applications that is based on the coupling of the Whale Optimizer and Sine Cosine Algorithms via a surrogate model. We relate the whale optimizer algorithm to balance between the exploitation and the exploration process in the proposed method. There exist confirmed techniques for searching approximate best optimal solutions, but our algorithm will further guarantee that such numerical and statistical solutions satisfy physical bounds of the standard and real life functions. Our experiments with the benchmark, bio-medical, computer science and engineering real life problems have illustrated the advantages of using a newly hybrid approach based on mixing Whale Optimizer and Sine Cosine algorithms. It holds considerable potential for reducing execution time for solving standard and real life problems and at the same time improving the quality of the solution.

Keywords: Function Optimization; benchmark function; Whale Optimization (WO) and Sine Cosine (SC) Algorithm

---

1. Introduction

The use of nature inspired optimization algorithm has gained popularity in a wide variety of standard, scientific, engineering and bio-medical real life applications as those techniques have some advantages over deterministic global optimization techniques. Those advantages include the capability to handle uninmodal, multi-modal and fixed dimension multi-modal objective problems without the assumptions of differentiability, continuity and the lack of the need for a good initial guess.
Several metahuristics have been proposed in the literature during last two decades but quest for an improved and efficient algorithm still continues. This manuscript proposes an efficient hybrid nature inspired optimization technique.
The proposed algorithms have been tested on several benchmark and biomedical problems. It has been observed that the proposed algorithm outperforms several other algorithms like Particle Swarm Optimization (PSO) [1], Ant Lion Optimizer (ALO) [2], Whale Optimization Algorithm (WOA) [3], Hybrid Approach GWO (HAGWO) [4], Mean GWO (MGWO) [5], Grey Wolf Optimizer (GWO) [6] and Sine Cosine Algorithm (SCA) [7] in solving several real life problems. The numerical and graphical presentation of results have been used to show the effectiveness of the proposed algorithm. The results obtained have been compared with those obtained with in terms of solution quality, solution stability, convergence speed and ability to find the global optimum and it has been concluded that our algorithm is better to other recent metaheuristics.
Recently, researchers have originated most number of population based nature inspired metaheuristics in order to search the best possible optimal solution of tested and real life problems. The first solution algorithm for the OPF problem was proposed by Dommel and Tinney [8], and since then most numbers

Preprints (www.preprints.org) | NOT PEER-REVIEWED | Posted: 15 October 2020 doi:10.20944/preprints202010.0319.v1

of other metaheuristics have been presented, some of them are: Particle Swarm Optimization (PSO) [1], Ant Colony Optimization (ACO) [9], Genetic Algorithm (GA) [10,11], Differential Evolution (DE) [12,13], fuzzy based hybrid particle swarm optimization (fuzzy HPSO) [14], Whale Optimization Algorithm (WOA) [15], Hybrid Genetic Algorithm (HGA) [15], harmony search algorithm [16], Robust Optimization (RO) [17], Grey Wolf Optimization (GWO) [6], Tabu Search (TS) [18], Gravitational Search Algorithm (GSA) [19], Artificial Neural Network (ANN) [20], Sine Cosine Algorithm (SCA) [7], Ant Lion Optimizer (ALO) [2], adaptive group search optimization (AGSO) [21], biogeography based optimization algorithm (BBO) [22], krill herd algorithm (KHA) [23], Grasshopper Optimization Algorithm (GOA) [24], Multi-Verse Optimizer (MVO) [25], Moth Flame Optimizer (MFO) [26], Dragonfly Algorithm (DA) [27], Black-Hole-Based Optimization (BHBO) [28], Cuckoo Search (CS) [29] and In addition, in case of the hybrid convergence, nature inspired algorithm hybridizations using batch modeling are combinations amid evolutionary techniques and techniques of neighbourhood or course.
A newly hybrid approach has been presented by Mafarja and Mirjalili [30] using of different feature selection techniques and Whale Optimization Algorithm (WOA). The main purpose of applying Simulated annealing here is to enhance the exploitation by finding the most promising regions located by Whale Optimization algorithm. The accuracy of the newly variants is tested on several standard functions and compared with three well-known wrapper feature selection methods in the literature.
B. Bentouati et al. [31] presents a new power system planning strategy by combining pattern search algorithm (PS) with Whale Optimization Algorithm (WOA). The existing variant has been carried out on the IEEE 30−bus test system considering several objective functions, such as voltage profile improvement, generating fuel cost, emission reduction and minimization of total power losses are also verified. The obtained numerical and statistical solutions are verified with recently published population based metaheuristic variants. Simulation solutions clearly conceal the rapidity and the effectiveness of the presented approach for solving the OPF function.

R.M.R. Allah [32] developed a new approach based on hybridizing the multi-orthogonal search strategy (MOSS) with a sine cosine algorithm (SCA), called multi-orthogonal sine cosine algorithm (MOSCA), for solving engineering design functions. The newly approach integrates the advantages of the SCA and MOSS to eliminate SCA's disadvantages, like unbalanced exploitation and the trapping in local optima. The convergence performance of the newly approach is investigated by using it on eighteen standard functions and four engineering design functions. The numerical solutions reveal that newly existing approach is a promising variant and outperforms the other recent metaheuristics in most cases.
O. E. Turgut [33] proposes a hybrid global optimization approach based on the combination of the merits of the sine–cosine algorithm (SCA) and backtracking search (BSA) to achieve the optimal design of a shell and tube evaporator. In order to verify the performance of the newly hybrid approach, ten standard optimization problems have been solved. Simulation solutions obtained from the newly hybrid variant have been verified with the literature optimizers including differential search, quantum-behaved particle swarm optimization, big bang–big crunch optimization, bat algorithm, backtracking search algorithm and intelligent tuned harmony search algorithm.

2. Whale Optimizer Algorithm (WOA)

The whale optimization algorithm is a newly population based meta-heuristics approach proposed by Mirjalili at. el. [3]. This approach simulate bubble-net attacking technique of the humpback whales when they hunting their preys.

In this variant includes three operators to simulate the find for prey, encircling prey, and bubble-net foraging behavior of humpback whales.

- **Encircling prey:** Humpback whales can recognize the location of prey and then encircle them. For the unknown position of the optimal design in the search area, the existing best agent possible

Preprints (www.preprints.org) | NOT PEER-REVIEWED | Posted: 15 October 2020 doi:10.20944/preprints202010.0319.v1

solution is the target prey or is near to the optimal in Whale Optimization Algorithm. Once the most excellent search candidate is defined, the next search candidates will thus make an effort to update their positions towards the finest search candidate. The restructured technique is represented by the following mathematical equations:

$$D = |C.X*(t) - X(t)| \tag{1}$$

$$X(t+1) = X*(t) - A.D \tag{2}$$

Where t is the current generation, $A$and $C$are coefficient vectors, $X*$is the position vector of the most excellent solution, and Xindicates the position vector of a solution, $||$is the absolute value.

The vectors $A$and $C$are mathematical calculated as below:

$$A = 2a.r.a \tag{3}$$

$$C = 2.r \tag{4}$$

Where components of $a$are linearly decreased from 2 to 0 over the course of generations and $r$is random vector in $[0;1]$

- **Bubble-net attacking method**: The humpback whales attach the prey with the bubble-net mechanism. This mechanism is mathematical respresented as follows:
- **Spiral updating position mechanism**: In this mechanism, the distance amid the whale location and the prey location is calculated then the helix-shaped movement of humpback is created as shown in the equation:

$$X(t+1) = D'.e^{bt}.\cos(2\pi l) + X*(t) \tag{5}$$

Where $D' = |X*(t) - X(t)|$is the distance amid the prey (best possible solution) and the ithwhale, $b$is a constant, $l$is a random number in $[-1;1]$.

Note: We suppose that there is fifty percent probability that whale either follow the shrinking encircling path during optimization procedure. Mathematical we modeled as belows:

$$X(t+1) = \begin{cases} X*(t) - A.D & if \ p < 0.5 \\ D'.e^{bt}.\cos(2\pi l) + X*(t) & if \ p \geq 0.5 \end{cases} \tag{6}$$

- **Search for Prey**: The vector Acan be apply for exploration to find for target and also takes the values > 1or < -1. The exploration can follows the followings mathematical equations:

$$D = |C.X_{rand} - X| \tag{7}$$

$$X(t+1) = X_{rand} - A.D \tag{8}$$

Where $p$represent random number amid $[0,1]$.

3. Sine Cosine Algorithm (SCA)

Mirjaliliet al. [7] presented newly population based meta-heuristics called Sine Cosine Algorithm (SCA) simply based on Sine and Cosine function apply for exploitation and exploration phases in global optimization functions. This variant creates singular initial random agent best possible solutions

Preprints (www.preprints.org) | NOT PEER-REVIEWED | Posted: 15 October 2020 doi:10.20944/preprints202010.0319.v1

in the search space and requires them to fluctuate outwards or towards the best possible result using following mathematical model based on sine and cosine functions.

$$\vec{x}_i^{t+1} = \vec{x}_i^t + p_1 \times \sin(p_2) \times |p_3 \times l_i^t - \vec{x}_i^t| \tag{9}$$

$$\vec{x}_i^{t+1} = \vec{x}_i^t + p_1 \times \cos(p_2) \times |p_3 \times l_i^t - \vec{x}_i^t| \tag{10}$$

Where: $\vec{x}_i^t$current position,$p_1, p_2, p_3 \in [0,1]$ are random numbers and l_i is targeted global optimal result. The above mathematical equations (9)-(10) uses $0.5 \leq p_4 < 0.5$ setting for exploitation and exploration.

$$\vec{x}_i^{t+1} = \begin{cases} \vec{x}_i^t + p_1 \times \sin(p_2) \times |p_3 \times l_i^t - \vec{x}_i^t| & , \ p_4 < 0.5 \\ \vec{x}_i^t + p_1 \times \cos(p_2) \times |p_3 \times l_i^t - \vec{x}_i^t| & , \ p_4 \geq 0.5 \end{cases} \tag{11}$$

4. Motivation of this work

Despite the Whale Optimizer and Sine Cosine Algorithm are competent to reveal an efficient performance in comparison with other population based nature inspired variants, it is not fitting for highly complex functions and is still may face the difficulty of getting trapped in local optima.To overcome these limitation and to improve its search performance, a new hybrid WOA-SCA variant is proposed to solve standard benchmark and engineering design functions. The proposed variant is called Hybrid WOA-SCA. In this variant, we improve the performance of exploitation in Whale Optimizer algorithm with the performance of exploration in Sine Cosine Algorithm (SCA) to produce both approaches' strength.

By this method, it is intended to improve the global convergence by accelerating the search seeking instead of letting the algorithm running several iterations without any improvement. The performance of newly proposed variants have been verified with several standard functions and some engineering design functions. Experimental solutions confirm that the newly proposed variant is a robust search variant for various real life and standard optimization functions.

5. The Hybrid WOA-SCA algorithm

Hybridization is an enhancement in global optimization techniques in which operators from a certain technique are combined with other operators from another technique to produce more effective and reliable synergistic entity and get superior quality of results than that of the main parent technique.

Whale Optimizer Algorithm as well as most powerful techniques have disadvantage and advantages in terms of their global optimization behavior. An advantage of generation upgrading techniques is their good exploitation quality, that is, they exactly converge to a local optimum; however, they have no method for a strong exploration of the search area. In contrast, whale optimizer showed to have superior exploration performance, but has functions with the exploitation in a promising area of the search space.

We developed a novel hybrid technique that combines whale optimizer with sine cosine algorithms. Basis of this modification, we improve the performance of exploitation in Whale Optimizer algorithm with the performance of exploration in Sine Cosine Algorithm (SCA) to produce both approaches' strength. The HWOASCA approach was mathematically modeled as follows:

In which newly hybrid variant the position of the agents has been improved by modifying the spiral updating position equation (14) using position update equation (11) of sine cosine algorithm for the purpose of extending the convergence performance of whale optimizer algorithm. The rest of

Preprints (www.preprints.org) | NOT PEER-REVIEWED | Posted: 15 October 2020 doi:10.20944/preprints202010.0319.v1

the operations of whale optimizer algorithm are same. The following position update equations are developed in this regard.

$$p_5 = 2\pi \times rand \in [0,1] \tag{12}$$

$$X(t+1) = \left(\left(D'.e^{bt}.\cos(2\pi l) + X*(t)\right) \times \sin(2\pi l p_5)\right) + X*(t) \tag{13}$$

where $p_5 \in [0,1]$is a random number, $D' = |X*(t) - X(t)|$indicates the distance of the ith whale to the pray, $b$is a constant, $l$is a random number in $[-1;1]$, $X*$is the position vector of the most excellent solution, and Xindicates the position vector of a solution.

$$X(t+1) = \begin{cases} X*(t) - A.D & if\ p < 0.5 \\ \left(\left(D'.e^{bt}.\cos(2\pi l) + X*(t)\right) \times \sin(2\pi l p_5)\right) + X*(t) & if\ p \geq 0.5 \end{cases} \tag{14}$$

6. Pseudo code of the HWOASCA algorithm

Initialize crowd
find the fitness of each solution
X^* ~ the best search member
while ($t <$ max_$generation$)
for each solution
update all constants
if 1 ($p < 0.5$)
    if 2 ($|A| < +1$)
update the direction of the solution by equation (2)
    else if 2 ($|A| > +1$)
select a random search agent ()
update the direction of the current search member by the equation (8)
end if 2
else if 1 ($p > +1$)
update the direction of the current search member by the equation (13)
end if 1
end for
check if any search agent goes beyond the search area and amend it
find the fitness of each search member
updatedX^* if there is a better solution

$$t = t + 1$$

end while
returnX^*

---

7. The steps of HWOASCA algorithm

- **Step 1**: The WOA starts by setting the all parameter values (crowd size n, coefficients $A$and C, the parameter a and maximum number of generations (max_$generation$).
- **Step 2**: Initialize the generation counter t.
- **Step 3**: The first crowd is generated randomly and all search member in the crowd is evaluated by calculating its fitness function.
- **Step 4**: Allocate the best search member.
- **Step 5**: The all following steps are repeated until the termination condition satisfied. **Step 5.1**: The generation counter is increasing $t = t + 1$.

Preprints (www.preprints.org) | NOT PEER-REVIEWED | Posted: 15 October 2020 doi:10.20944/preprints202010.0319.v1

Step 5.2: All the parameters are updated using equations (3)-(4).
Step 5.3: Update the position of current search member by using the equations (14).

- **Step 6**: The best search member is updated.
- **Step 7**: The overall process is repeated until termination condition satisfied.
- **Step 8**: Produce the best found search member (solution) so far.

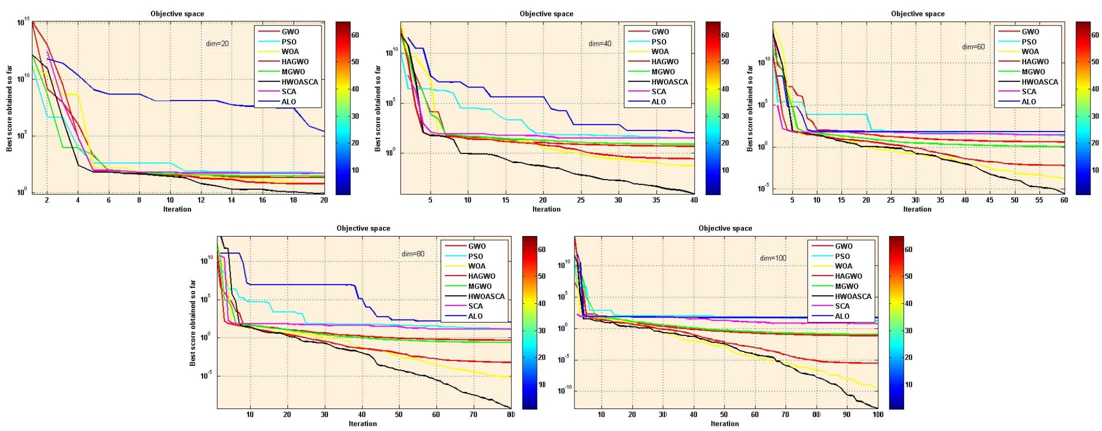
8. Standard Benchmark functions

The convergence, numerical, statistical, and time consuming performance of proposed variant have been verified with several standard benchmark and real life engineering applications and experimental results obtained are compared with recent nature inspired techniques. These standard benchmark functions have been divided into three different parts i.e. Unimodal, Multimodal and fixed dimension multimodal are listed in Appendix (Table *A*, Table *B* and Table *C*).

9. The performance of the newly proposed hybrid variant

In figures 1, we verify the general performance of the recent nature inspired algorithms with the newly proposed variant in order to test the efficiency of the proposed variant on number of generations. We set the similar parameter values for the entire algorithms to make fair comparison. We illustrate the results in figures 1 by plotting the worst optimal values of problem values against the number of iterations for simplified model of the molecule with distinct size from 20 to 100 dimension.

The figures proves that the benchmark function values quickly decrease as the number of iterations increases for newly hybrid algorithm results than those of the other nature inspired algorithms. In figures 1, PSO, ALO, WOA, HAGWO, MGWO, GWO and SCA algorithms suffers from the slow convergence, gets stuck in the partitioning procedure, nevertheless and many local minima and invoking the sine cosine algorithm in the proposed variant avoid trapping in local minima and accelerate the search.



Figures 1. The performance graph of HWOASCA

10. Experiment and Results

We test the performance of the newly proposed variant on the several standard bencharak and real life engineering functions on with different number of generations then we compared it against PSO, ALO, WOA, HAGWO, MGWO, GWO and SCA in MATLAB R2013a.

In the following subsections, we report more details the parameter settings of the newly hybrid and all other existing variants.

Preprints (www.preprints.org) | NOT PEER-REVIEWED | Posted: 15 October 2020 doi:10.20944/preprints202010.0319.v1

Table F Parameter setting

Parameter	Values
Search Agents	20
Max. number of iterations	40-100

All simulation solutions obtained from the newly hybrid variant is reported in this section. The WOA and SCA algorithms are compared to judge the effect of hybridizing Whale Optimizer Algorithm variant with the native Sine Cosine Algorithm. To find out the best variant among WOA, SCA and HWOASCA variants, all variants are verified together in Tables 1–6. The twenty two standard functions and numerious real life application have been utilized to compare the convergence and time consuming performance, efficiency and strength of the existing variant, where obtained experimental solutions by the newly hybrid variant have been verified with the PSO, ALO, WOA, HAGWO, MGWO, GWO and SCA algorithms. The standard function contains unimodal, fixed dimension multimodal and multimodal problems. The convergence performance graph and results explanation of the each benchmark function are compared in Tables 1–6 and figures 2–4, respectively.

Further, several real life applications have been used to verified the performance of the metaheuristics. For these experiments, the all algorithms are coded in MATLAB R2013a, running on a Laptop with an Intel HD Graphics, Pentium-Intel Core I, i5 Processor 430 M, 15.6" 16.9 HD LCD 320 GB HDD and 3GB Memory. In addition, to statistically asses the newly proposed algorithm compared with other methodologies, standard deviation and average are introduced.

The PSO, ALO, WOA, HAGWO, MGWO, GWO, SCA and HWOASCA variants were run 30 times on each standard problem. The simulation solutions (min and max objective values, average and standard deviation) are reported in Table 1 to Table 6. The all existing variants, have to be run at least more than ten times to find the best global optimial solutions. It is again a common technique that a variant is run on a standard problem several times and best solutions, min and max objective values, average and standard deviation of the superior are obtained in the last iteration.

In order to confirm the convergence performance of PSO, ALO, WOA, HAGWO, MGWO, GWO, SCA and HWOASCA variants are chosen. Here we use 40-100 iterations and 20 search members for all of the approaches. Simulated solutions in Tables 1–6 and figures 2–4 reveal that the hybrid approach is better to PSO, ALO, WOA, HAGWO, MGWO, GWO, SCA in terms of solution stability, solution quality, convergence speed and ability to find the best global optimum.

The all simulation solutions of the PSO, ALO, WOA, HAGWO, MGWO, GWO, SCA and HWOASCA variants on unimodal standard functions are shown in Table 1–4 and convergence performance represented by Figure 2. In Table 1–4, we have comparing the accuracy of proposed variant with other metaheuristics in terms of min and max objective values, average and standard deviation. On the basis of obtained results, we confirms that the proposed algorithm gives highly competitive results as compared to PSO, ALO, WOA, HAGWO, MGWO, GWO and SCA on unimodal standard problems.

Therefore, all obtained results evidence high rate of exploitation capability of the HWOASCA algorithm. Further, the experimental numerical and statistical results of the newly hybrid algorithm and other metaheuristics on multimodal problems are represented in Table 3-4 and performance plotted in Figure 3. We **examine** that the newly existing algorithm **performs superior** to other population based nature inspired techniques i.e. PSO, ALO, WOA, HAGWO, MGWO, GWO and SCA. The optimal solutions **obtained** in Table 3-4 strongly confirm that high exploration of HWOASCA algorithm is competent to explore the search area widely and give promising regions of the search space.

Preprints (www.preprints.org) | NOT PEER-REVIEWED | Posted: 15 October 2020 doi:10.20944/preprints202010.0319.v1

Table 1. 1-2. Numerical results of unimodal benchmark functions

Table 1

Problem	PSO		ALO		WOA		HAGWO	
↓	f_{min}	f_{max}	f_{min}	f_{max}	f_{min}	f_{max}	f_{min}	f_{max}
1.	11.6438	7.1632e+04	0.2136	4.4736e+04	6.2605e-10	7.4349e+04	3.2126e-08	7.5487e+04
2.	30.2562	5.1076e+12	0.0356	5.5173e+10	2.8241e-09	4.5088e+10	1.6169e-06	4.7050e+09
3.	2.2708e+03	1.9527e+05	1.2135e+05	2.0798e+04	9.1408e+04	1.1436e+05	296.2856	1.2816e+04
4.	9.9128	83.1986	0.0098	75.1142	16.0602	77.1160	40.4453	84.4233
5.	29.5918e+03	1.0804e+08	0	2.6145e+08	28.6354	3.0775e+08	28.4403	1.7418e+08
6.	13.0936	7.3806e+04	0.9655	2.8730e+04	3.7422	6.4982e+04	2.7068	7.1263e+04
7.	12.9822	82.9364	0.0066	45.9220	0.0069	156.7019	0.0186	120.2423

Table 2

Problem	MGWO		GWO		SCA		HWOASCA	
↓	f_{min}	f_{max}	f_{min}	f_{max}	f_{min}	f_{max}	f_{min}	f_{max}
1.	0.2152	6.8918e+04	0.2696	7.3865e+04	0.2125	7.4493e+04	0.2044e-22	7.8128e+04
2.	0.1118	4.1689e+10	0.0645	2.5352e+11	0.0414	1.1689e+12	0.0156e-13	7.2473e+12
3.	1.5945e+03	1.7421e+05	1.2447e+03	1.9873e+05	0	1.3256e+05	1.2031e+05	2.4550e+05
4.	1.4447	87.8777	6.4444	90.2703	0.0099	90.5999	0.0097	90.6610
5.	38.2207	2.8778e+08	661.4971	2.2601e+08	0	2.7298e+08	28.6610	3.1290e+08
6.	3.3000	6.5228e+04	4.1960	6.6630e+04	0.8856	6.6416e+04	0.7600	7.4334e+04
7.	0.0167	133.8492	0.0123	149.0505	0.0059	100.1166	0.0045	163.5383

Table 2. 3-4. Statistical results of unimodal benchmark functions

Table 3

Problem	PSO		ALO		WOA		HAGWO	
↓	μ	σ	μ	σ	μ	σ	μ	σ
1.	5.0875e+03	1.4235e+04	2.8445e+04	6.1471e+03	3.4423e+03	1.1310e+04	2.7057e+03	1.1814e+04
2.	5.1076e+10	5.1076e+11	4.9919e+09	1.6861e+10	4.5215e+08	4.5087e+09	4.7078e+07	4.7049e+08
3.	2.2581e+04	4.9495e+04	4.3428e+04	1.3934e+04	1.2557e+05	6.8692e+03	1.1446e+04	1.7216e+04
4.	19.4231	16.7300	37.5135	7.6313	43.7256	26.6360	66.3254	16.8860
5.	3.9215e+06	1.6259e+07	8.3189e+06	2.7288e+07	1.5004e+07	5.0149e+07	4.5164e+06	2.2304e+07
6.	5.1938e+03	1.4497e+04	2.1096e+04	4.7941e+03	4.7251e+03	1.3419e+04	3.9701e+03	1.3074e+04
7.	51.0305	24.0970	5.7460	5.3025	8.5687	28.6718	3.8918	16.5765

Table 4

Problem	MGWO		GWO		SCA		HWOASCA	
↓	μ	σ	μ	σ	μ	σ	μ	σ
1.	3.9306e+03	1.1535e+04	4.7094e+03	1.3687e+04	2.2878e+04	3.1900e+04	2.2007e+03	1.0711e+04
2.	4.1961e+08	4.1687e+09	2.5357e+09	2.5352e+10	1.6120e+10	1.61119e+11	1.3797e+11	1.5333e+11
3.	1.7556e+04	2.2868e+04	1.7115e+04	2.8328e+04	7.1132e+04	3.5018e+04	1.0656e+05	1.1751e+04
4.	16.7995	25.1076	31.9237	28.4636	87.4328	11.3600	21.2069	7.2216
5.	1.01908e+07	4.1003e+07	1.0820e+07	4.0393e+07	1.8684e+08	1.0227e+08	1.0811e+07	1.3229e+07
6.	3.7050e+03	1.0872e+04	3.7151e+03	1.1228e+04	2.7211e+04	2.6454e+04	1.2682e+03	1.0393e+04
7.	3.9651	16.2223	7.0891	25.6128	61.1068	41.8549	3.3752	13.4561

Table 3. 5-6. Numerical results of multimodal benchmark functions

Table 5

Problem	PSO		ALO		WOA		HAGWO	
	f_{min}	f_{max}	f_{min}	f_{max}	f_{min}	f_{max}	f_{min}	f_{max}
8.	-2.5796e+03	-1.9428e+03	-5.4177e+03	-2.2656e+03	-1.1321e+04	-2.0998e+03	-4.5349e+03	-1.8046e+03
9.	149.5336	447.2761	0	386.1930	2.1032e-12	432.1387	132.7207	441.8725
10.	3.5128	20.6666	0	19.9637	2.5507e-07	20.5739	2.5543e-04	20.0075
11.	25.2047	663.3607	0	193.2231	7.9915e-11	687.5772	0.1517	632.0270
12.	6.0648	7.1155e+08	0.1596	5.1574e+08	0.1975	5.9744e+08	0.2834	5.2363e+08
13.	2.3008	1.1074e+09	0.6025	1.2019e+09	1.4861	8.5625e+08	2.1346	1.6520e+09

Table 6

Problem	MGWO		GWO		SCA		HWOASCA	
↓	f_{min}	f_{max}	f_{min}	f_{max}	f_{min}	f_{max}	f_{min}	f_{max}
8.	-5.7333e+03	-2.1923e+03	-4.2010e+03	-1.9654e+03	-2.8348e+03	-2.9826e+03	-8.4894e+03	-2.7936e+03
9.	45.5201	448.5309	38.7564	483.4556	0	475.5980	0	490.2454
10.	0.1430	20.6511	0.0919	20.9136	0	20.5084	0	20.8137
11.	0.1849	579.3254	0.4406	524.5783	0	615.4264	0	695.6860
12.	1.1029	6.5947e+08	0.5639	5.0075e+08	0.2563	3.2515e+08	0.1319	6.8597e+08
13.	3.1407	1.0822e+09	3.4109	1.1098e+09	0.5263	1.2946e+09	0.4935	6.3958e+09

Finally, the accuracy of the all existing metaheuristics have been tested on fixed-dimension multimodal functions and obtained results are presented in Table 5–6. The convergence performance of the metaheuristics has been plotted in Figure 4. For these standard problems we have verified the rate of convergence accuracy of the newly hybrid approach HWOASCA with PSO, ALO, WOA, HAGWO,

Preprints (www.preprints.org) | NOT PEER-REVIEWED | Posted: 15 October 2020 doi:10.20944/preprints202010.0319.v1

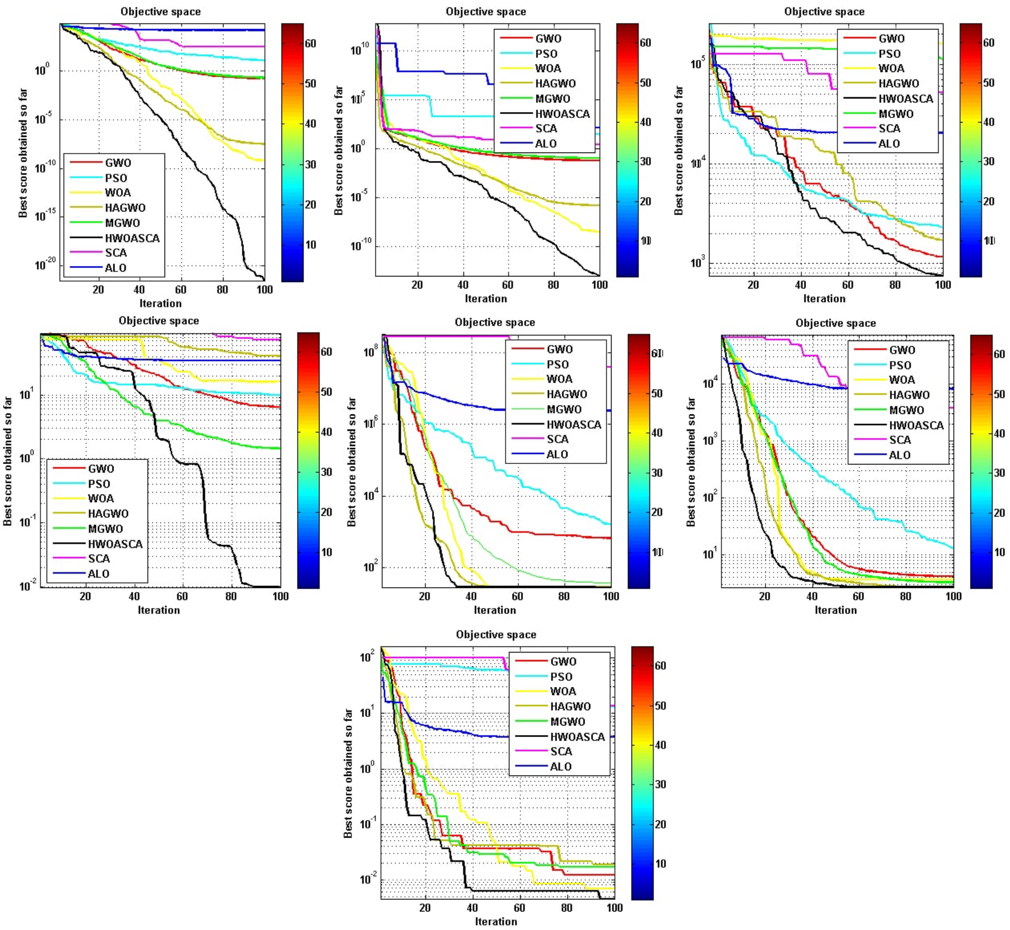


Figure 2. 1-7. Convergence graph on unimodal benchmark functions

Table 4. 7-8. Statistical results of multimodal benchmark functions

Table 7								
Problem	PSO		ALO		WOA		HAGWO	
↓	μ	σ	μ	σ	μ	σ	μ	σ
8.	-2.4472e+03	250.1510	-2.5491e+04	3.9067e+03	-1.0480e+04	1.9866e+03	-3.9504e+03	472.3243
9.	310.7920	71.4677	176.9291	69.3204	74.7250	127.8635	181.4831	55.7920
10.	8.3231	3.9980	17.3101	1.8396	2.8178	5.4858	4.4686	7.4605
11.	242.6257	194.5744	76.8344	35.8820	28.9251	110.5692	25.9122	96.1052
12.	1.3154e+07	8.1407e+07	2.6852e+07	6.5171e+07	3.1953e+07	1.2035e+08	1.1748e+07	6.5621e+07
13.	2.5692e+07	1.3810e+08	3.8775e+07	1.90454e+09	3.8292e+07	1.4759e+08	5.4586e+07	2.3120e+08

Table 8								
Proble m	MGWO		GWO		SCA		HWOASCA	
↓	μ	σ	μ	σ	μ	σ	μ	σ
8.	-3.5274e+03	1.0874e+03	-2.7570e+03	725.4048	-2.8030e+03	283.1517	-7.7324e+03	1.1640e+03
9.	149.9412	105.6169	129.0764	89.6508	238.5618	131.1048	48.4785	45.3625
10.	4.1037	6.0601	4.4015	6.4260	19.2057	2.4196	2.2206	3.3575
11.	32.8450	101.6196	27.9505	87.2026	256.6257	239.6093	21.0507	76.4345
12.	1.8186e+07	8.0264e+07	1.4943e+07	6.8810e+07	2.5745e+08	1.1081e+08	1.0124e+07	1.2145e+08
13.	3.1916e+07	1.4849e+08	4.4914e+07	1.9438e+08	5.7117e+08	6.0493e+08	3.8981e+07	1.3733e+08

MGWO, GWO and SCA in terms of min and max objective function values, standard deviation and average value. The results are consistent with those of the standard benchmark functions. On the basis of obtained results, we prove that the newly hybrid approach provides highly competitive optimal results verified with other recent nature inspired meta-heuristics, for these standard functions.

 doi:10.20944/preprints202010.0319.v1

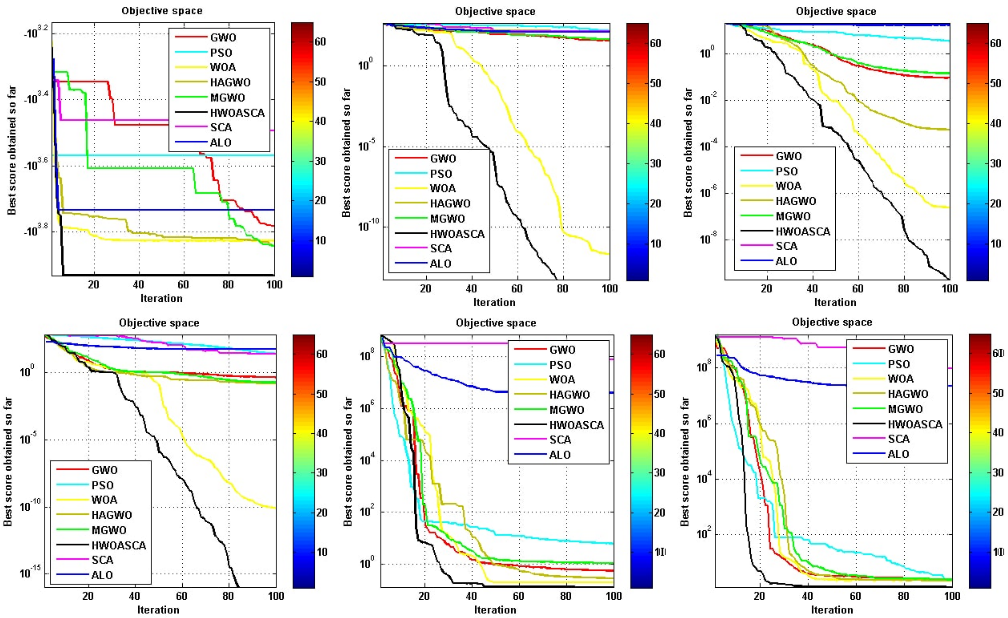


Figure 3. (8)-(13). Convergence graph on multimodal benchmark functions

Table 5. 9-10. Numercial results of fixed-dimension multimodal benchmark functions

Table 9

Problem	PSO		ALO		WOA		HAGWO	
	f_{min}	f_{max}	f_{min}	f_{max}	f_{min}	f_{min}	f_{min}	f_{max}
14.	2.9821	490.1471	1.2653	33.7293	10.7632	44.6647	10.7632	464.6207
15.	0.9980	405.1082	0.9985	448.3523	6.1079	115.0247	10.7632	460.4717
16.	-1.0316	0.9034	-1.0316	0	-1.0316	2.5343	-1.0316	1.4981
17.	0.3979	1.1174	0.3979	0.9955	0.3979	2.5717	0.3984	1.8658
18.	3.0000	51.4284	3.0000	53.5096	3.0048	18.1178	3.0367	39.7544
19.	-3.8626	-3.6348	-3.8486	0	-3.8548	-2.8722	-3.8591	-3.3905
20.	-3.2029	-0.7090	-3.1905	-0.3156	-3.1956	-1.2831	-2.4103	-1.6624
21.	-9.1532	-0.3750	-2.6305	-0.2752	-9.3912	-0.4885	-5.0546	-0.3549
22.	-10.3029	-0.4586	-2.7659	-0.5623	-4.1871	-0.3516	-5.0789	-0.5582

Table 10

Problem	MGWO		GWO		SCA		HWOASCA	
	f_{min}	f_{max}	f_{min}	f_{max}	f_{min}	f_{max}	f_{min}	f_{min}
14.	3.9683	446.4407	2.9821	7.4384	1.2536	3.0458	0.9981	498.6145
15.	13.9250	490.2823	12.9705	155.2814	0.9989	20.3814	0.9980	557.7428
16.	-1.0316	0.5289	-1.0316	3.0769	-1.0316	0	-1.0316	68.5519
17.	0.3979	0.8283	0.3982	2.8358	0.3979	1.9728	0.3979	5.3768
18.	3.0095	44.1552	3.0419	40.3638	3.0000	7.8155	3.0000	79.1330
19.	-3.8626	-2.8765	-3.8624	-3.4917	-3.8415	0	-3.8626	0
20.	-3.3215	-2.7376	-3.2615	-0.7513	-3.0064	-0.3856	-3.2793	-0.2527
21.	-5.0529	-0.6108	-2.6774	-0.3277	-0.9642	-0.2615	-9.4871	-0.2598
22.	-10.3229	-2.0957	-2.7613	-0.5292	-1.436	-0.5234	-10.3783	-0.4924

11. Clustering Problem in Wireless sensor network

In this text also we considered the accuracy of the existing variant on the clustering problem in wireless sensor network, which is most difficult and NP hard function. The all simulation results proven that the newly hybrid approach is gives most effective for these types of real life application due to fewer chances to get stuck at local minima and fast convergence. It can be concluded that the newly hybrid variant is competent to outperform the recent well known nature inspired metaheuristics in the literature.

12. Bio-Medical Real life Applications

In this section two dataset biomedical applications: (i) Breast Cancer and (ii) Heart are employed (Mirjalili, S. [14]). These dataset problems have been solved by HWOASCAvariant and compared with PSO, ALO, WOA, HAGWO, MGWO, GWO and SCA approaches. Distinct parameter settings have been applied for running code of all metaheuristics and these parameter settings are given in

Table 6. 11-12. Statistical results of fixed-dimension multimodal benchmark functions

Table 11								
Problem	PSO		ALO		WOA		HAGWO	
	μ	σ	μ	σ	μ	σ	μ	σ
14.	7.1663	37.1474	3.4943	3.7517	11.5550	2.9851	14.8218	36.4007
15.	5.8346	40.4429	16.8047	61.9692	10.3995	15.8564	15.5749	45.0393
16.	-9893	0.2158	-0.9633	0.4040	-0.9741	0.3635	-1.0015	0.2540
17.	0.4397	0.1514	0.4221	0.1369	0.5045	0.4185	0.5112	0.2956
18.	7.6383	23.3996	30.0126	6.1474	4.0585	2.6770	4.8511	5.9635
19.	-3.8349	0.0504	-3.7725	0.3881	-3.8306	0.1101	-3.8233	0.0721
20.	-2.9918	0.4060	-2.8820	0.6041	-2.8764	0.4850	-2.3037	0.1127
21.	-6.0626	4.2714	-2.4310	0.5391	-6.9909	3.4712	-4.6606	0.9365
22.	-5.8551	4.0298	-2.6262	0.4101	-3.5831	1.0727	-4.7096	0.9032

Table 12								
Problem	MGWO		GWO		SCA		HWOASCA	
	μ	σ	μ	σ	μ	σ	μ	σ
14.	9.2797	42.6210	3.0774	0.6261	3.9844	0.2139	2.3740	0.0829
15.	22.6305	51.4572	16.7241	20.1200	4.7706	3.6239	1.6236	5.6982
16.	-1.0120	0.1566	-0.9789	0.4152	-0.9968	0.1141	0.2962	7.6678
17.	0.4658	0.1426	0.6212	0.5685	0.5643	0.3268	0.6272	0.8646
18.	5.2550	8.1283	5.4930	7.5195	3.9192	1.4036	4.5897	5.3702
19.	-3.7851	0.1427	-3.8437	0.0636	-3.7550	0.3994	-3.8566	0.0450
20.	-3.1139	0.2268	-2.9812	0.5275	-2.5691	0.7842	-3.1652	0.3281
21.	-2.3893	1.2945	-1.7161	0.6715	-0.9085	0.1534	-8.6276	0.8129
22.	-5.9206	2.9124	-2.1733	0.5794	-0.7874	0.3612	-8.8468	0.4306

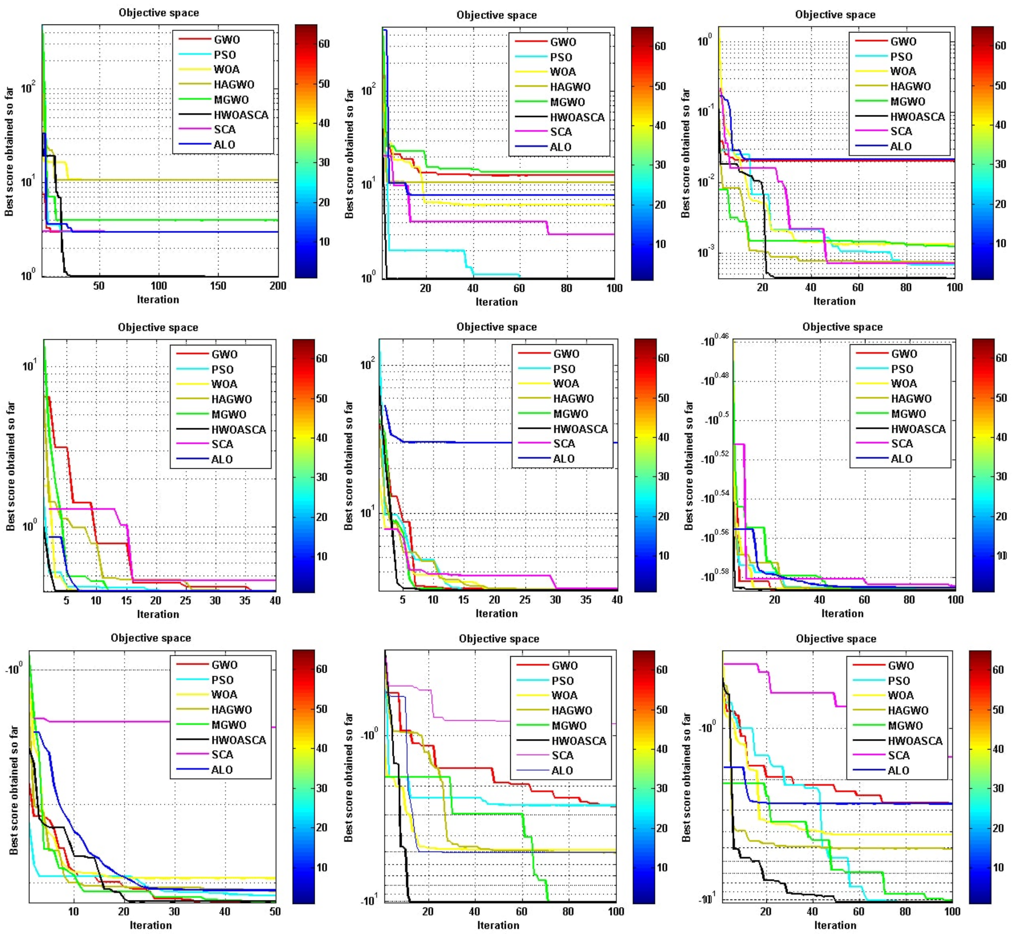


Figure 4. (14)-(22). Convergence graph on multimodal benchmark functions

Appendix [Table D] [15]. The convergence performance of the all techniques have been verified in terms of min and max objective values, standard deviation, average and classification rate[Table 14].

The all results optained in Table 8, prove that the proposed variant as comparison to others gives the best quality of optimal solutions on the biomedical problems.The simulation results of proposed

Preprints (www.preprints.org) | NOT PEER-REVIEWED | Posted: 15 October 2020 doi:10.20944/preprints202010.0319.v1

Table 7. Comparison best solutions of Clustering problem in Wireless sensor network.

Variants	Best optimal cost of FF	FND	HND	LND
PSO	38.548 mJ	2085.4	2686.4	3355.9
ALO	38.956 mJ	2020.9	2699.7	3318.1
WOA	38.412 mJ	2032.5	2718.2	3376.7
HAGWO	37.212 mJ	2043.7	2791.8	3396.2
GWO	37.421 mJ	2024.5	2745.2	3375.5
MGWO	37.102 mJ	2060.6	2785.1	3411.5
SCA	37.252 mJ	2032.6	2726.8	3387.9
HWOASCA	**37.056 mJ**	**2096.1**	**2810.2**	**3496.1**

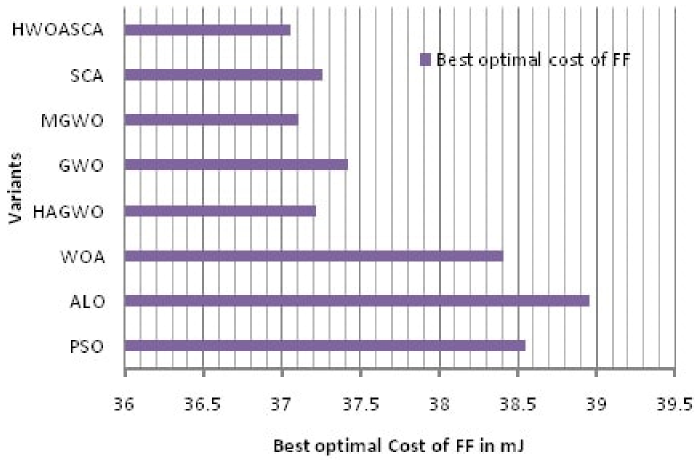


Figure 5. Comparison of variants on the best optimal cost of FF

variant prove that it has the highest capability to avoid the local optima and is considerably better than other approaches i.e. PSO, ALO, WOA, HAGWO, MGWO, GWO and SCA.

Table 8. Comparison of variants on the best possible solution of Bio Medical applications

(i)	Breast cancer dataset problem					
	Algorithm	**Best Min value**	**Best Max value**	**Average**	**S.D.**	**Classification Rate**
	GWO	0.0016	0.0471	0.0027	0.0039	99.00%
	PSO	0.0021	0.0236	0.0158	0.0251	27.99%
	WOA	0.0046	0.00876	0.0038	0.0125	63.29%
	HAGWO	0.0014	0.0502	0.0015	0.0029	99.23%
	MGWO	0.0014	0.0499	0.0019	0.0036	99.15%
	SCA	0.0016	0.0392	0.064	0.0082	89.99%
	ALO	0.0013	0.0326	0.0088	0.0090	79.56%
	HWOASCA	**0.0009**	**0.00621**	**0.0010**	**0.0019**	**99.71%**
(ii)	**Heart dataset problem**					
	Algorithm	**Best Min value**	**Best Max value**	**Average**	**S.D.**	**Classification Rate**
	GWO	0.0612	0.2891	0.0999	0.0197	76.00%
	PSO	0.0699	0.2741	0.1377	0.0271	53.26%
	WOA	0.1339	0.2786	0.1311	0.0253	58.41%
	HAGWO	0.0678	0.2811	0.0991	0.0188	76.60%
	MGWO	0.0501	0.2882	0.0984	0.0161	77.12%
	SCA	0	0.2911	0.0934	0.0176	78.48%
	ALO	0	0.2751	0.1291	0.0248	59.00%
	HWOASCA	**0**	**0.3158**	**0.0681**	**0.0101**	**78.71%**

13. HWOASCA algorithm for a tension/compression spring

In this section, the accuracy of HWOASCA algorithm was also tested with four constrained engineering design application like tension/compression spring Mirjalili, S. et al. [3] and comparison with the optimal solutions of GA, ES, PSO, SCA and WOA metaheuristics.

The main motive of this test function is to reduce or minimize the weight of the tension/compression spring. Optimum design must satisfy constraints on deflection, surge frequency and deflection. There are three design variables: number of active coils (N), mean coil diameter (D) and wire diameter (d). The mathematical optimization function is formulated as bellows:
Consider

$$Y = [y_1, y_2, y_3] = [d, D, N] = Minf(Y) = (y_3 + 1)y_2 y_1^2 \tag{15}$$

Subject to

$$l_1(Y) = 1 - \frac{y_2^3 y_3}{71785 y_1^4} \le 0 \tag{16}$$

$$l_2(Y) = \frac{4y_2^2 - y_1 y_2}{12566(y_2 y_1^3 - y_1^4)} + \frac{1}{5108 y_1^2} \le 0 \tag{17}$$

$$l_3(Y) = 1 - \frac{140.45 y_1}{y_2^2 y_3} \le 0 \tag{18}$$

$$l_4(Y) = \frac{y_1 + y_2}{1.5} \le 0 \tag{19}$$

Variable range

$$0.05 \le y_1 \le 2.00, \quad 0.25 \le y_2 \le 1.30, \quad 2.0 \le y_3 \le 15.0 \tag{20}$$

Table 9. Comparison of HWOASCA optimization solutions with literature for the tension/compression spring design problem.

Algorithm	Optimum variables			Optimum weight
	d	D	N	
HWOASCA	**0.051198**	**0.344389**	**12.078036**	**0.0126648**
WOA	0.051207	0.345215	12.004032	0.0126763
SCA	0.051203	0.345035	12.005536	0.0126735
PSO	0.051728	0.357644	11.244543	0.0126747
ES	0.051989	0.363965	10.890522	0.0126810
GA	0.051480	0.351661	11.632201	0.0127048

This function was solved using different metaheuristics like Genetic Algorithm (GA) [35], Evolution Strategy (ES) [36], Particle Swarm Optimization (PSO) [3] and Whale Optimizer algorithm (WOA) [3].

The optimal solution of newly existing variant and sine cosine algorithm are compared with literature in Table 9. A several penalty problem constraint handling technique was utilized in order to perform a reasonable comparison with literature [34]. It can be notice that HWOASCA variant outperforms all others metaheuristics.

14. Economic Dispatch Problem (EDP)

During last few years, many researchers have used different types of optimization techniques to find the best quality solutions of Economic Dispatch Problems in the literature such as General Algebraic

Modeling System (GAMS) [37], Hybrid PSO-SQP [38], Quadratic Programming (QP) [39], MPSO [40], Simulated Annealing (SA) [41], Particle Swarm Optimization (PSO) [42], PSO-LRS [43], Variable Scaling hybrid differential evolution (VSHDE) [44], qPSO [45], HGPSO, HGAPSO and HPSOM [46], Anti-predatory Particle Swarm Optimization (APSO) [47], Self-organizing Hierarchical PSO (SOH-PSO) [48], Mean PSO [49,50], Quantum PSO (QPSO) [51], Biogeography-Based optimization (BBO) [52], Simulated Annealing (SA) [53], Quadratic Approximation Particle Swarm Optimization (qPSO) [54] and Particle Swarm Optimization (PSO) [55].

In this section, the performance of the existing variant has been also tested with economic dispatch problem and comparsion with the generation cost of Mean PSO, VSHDE, SA, QP, GAMS, HGPSO, HGAPSO, MPSO, HPSOM, PSO-SQP, PSO-LRS, NPSO-LRS, APSO, SPSO, SOH-PSO, qPSO, BBO, HPSO (Park 2007), QPSO and MSPSO metaheuristics

The purpose of ED problem is to reduce the total fuel cost of power plants subject to the operating constraints of a power system. Commonly, it can be formulated with an two constraints and objective function (Park, J.B.,[42] and Deep, K. and Bansal, J.C., [54]):

$$Min\, C_T = \sum_{i=1}^{N} C_i(P_i) \tag{21}$$

where,
C_T: Total generation cost, C_i: Cost function of generator$i$,P_i: Power output of function generator i, Nnumber of generators and

$$C_i(P_i) = l_i + m_i P_i + n_i P_i^2 \quad \forall i = 1, 2, ..., N \tag{22}$$

l_i, m_i, n_i : are cost coefficients of generator$i$.

(a) Equality constraints

$$P_D + P_L - \sum_{i=1}^{N} P_i = 0 \tag{23}$$

where P_D and P_L are total system demand and transmission loss of the system.

(b) Inequality constraint

$$P_{i,\min} \leq P_i \leq P_{i,\max} \tag{24}$$

where $P_{i,\min}$and $P_{i,\max}$are minimum and maximum power output unit.
The generation cost function $C_i(P_i)$ may be written as:

$$C_i(P_i) = l_i + m_i P_i + n_i P_i^2 + |e_i \times \sin(f_i \times (P_{i,\min} - P_i))| \quad \forall i = 1, 2, ..., N \tag{25}$$

where $e_i$and$f_i$are the cost coefficient of generator$i$. The results obtained by Table 10, illustrate the performance of the newly hybrid variant and other recent metaheuristics in the terms of least generation cost, mean and standard deviation. The results are also compared with newly published economic dispatch problem solutions. From Table 10, it is clear that HMOASCA variant gives a superior quality of results and signifies HWOASCA's higher efficiency to find the solution of economic dispatch problem as compared to other meta heuristics.

Further the generation cost obtained by difference metaheuristics has been compared by Figure 6. On the basis of experimental results and performance plotted by Figure 6, it can be observed that for power system economic dispatch problem of greater size with higher non-linearities, the HWOASCA algorithm is proved to be the best approach among all the variants.

Preprints (www.preprints.org) | NOT PEER-REVIEWED | Posted: 15 October 2020 doi:10.20944/preprints202010.0319.v1

Table 10. Comparison of HWOASCA optimization solutions with literature for the Economic Dispatch Problem

Method	Unit	Total Power (MW)	Generation Cost	Mean	S.D.
Mean PSO	40	10,500	153562.45	160178.5514	3762.512976
VSHDE	40	10,500	143943.90	--	--
SA	40	10,500	143930.41	--	--
QP	40	10,500	143926.32	--	--
GAMS	40	10,500	143926.32	--	--
HGPSO	40	10,500	124797.13	126,855.70	1160.91
HGAPSO	40	10,500	122780.00	124,575.70	906.04
MPSO	40	10,500	122252.26		
HPSOM	40	10,500	122112.40	124,350.87	978.75
PSO-SQP	40	10,500	122094.67	122,245.25	
PSO-LRS	40	10,500	122035.79	122,558.45	
NPSO-LRS	40	10,500	121664.43	--	--
APSO	40	10,500	121663.52	122,153.67	
SPSO	40	10,500	121504.29	121632.3979	97.617794
SOH-PSO	40	10,500	121501.14	--	--
qPSO	40	10,500	121500.93	121565.906	39.777128
BBO	40	10,500	121479.50	121,512.06	
HPSO (Park 2007)	40	10,500	121452.67	121537.1906	--
QPSO	40	10,500	121448.21	--	--
MSPSO	40	10,500	121433.73	121588.6508	109.929025
HWOASCA	**40**	**10,500**	**121156.12**	**122951.1253**	**125.659238**

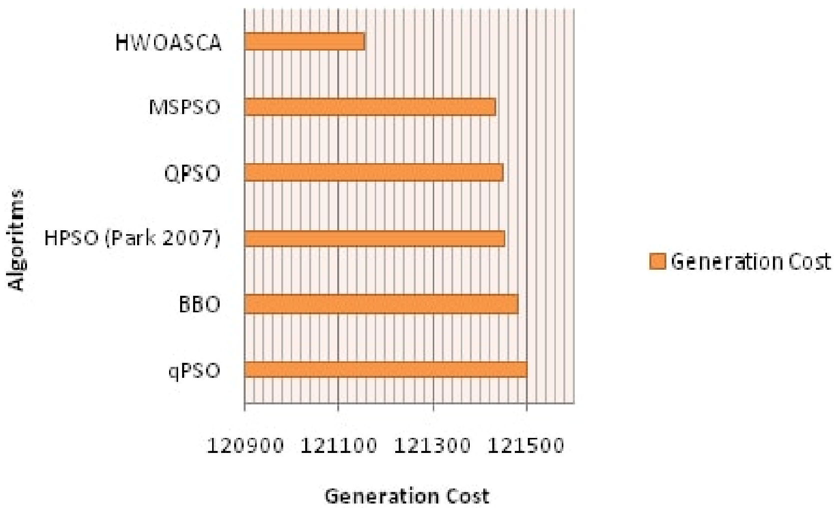


Figure 6. Comparison of HWOASCA variant optimization results with literature for the of Economic dispatch problems

15. Conclusion and future work

In this article, we propose a new hybrid whale optimizer algorithm with Sine Cosine algorithm in order to find the best possible solutions of the twenty two standard benchmark problems and real life applications. We call the newly proposed variant by Whale Optimizer algorithm and Sine Cosine algorithm (HWOASCA). We relate the whale optimizer algorithm to balance between the exploitation and the exploration process in the newly proposed variant. The obtained optimal solutions proved that the newly hybrid variant benefits form high exploration in comparison to the recent metaheuristics.

Further, we also tested the clustering problem in wireless senor network, breast cancer, heart dataset problem, tension/compression spring and economic dispatch problems is verified the performance of the existing variant with recent metaheuristics. The results show that the HWOASCA algorithms is found to be highly effective for real life applications due to fast convergence and fewer chances to get stuck at local minima. Hence the HWOASCA algorithm is able to outperform the recent well known and powerful nature inspired metaheuristics in the literature. The solutions prove the capability and advantage of HWOASCA to existing metaheuristic variants and it has an capability to become and helpful tool for solving real life optimization applications.

The future work will be concentrated on two parts: (i) composite functions, aircraft's wings, feature selection, Structural Damage Detection, the gear train design problem, Welded beam design, Cantilever beam, Pressure vessel design problem, bionic car problem, and mechanicial engineering problems (ii) Developing newly modified population based nature inspired metaheuristics for these tasks. To end with, we expectation that this work will encourage young researchers and other scientists, who are working on recent evolutionary metaheuristics concepts.

Author Contributions: Narinder Singh designed the numerical experiments, developed code and prepared the manuscript. Both the authors revised and finalized the final draft of manuscript.

Funding: This research received no external funding.

Conflicts of Interest: The authors declare that there is no conflict of interest.

16. Appendix

Table A. Unimodal benchmark functions

Function	Dimension	Range	Min. function value
$F_1(x) = \sum_{i=1}^{n} x_i^2$	30	[-100,100]	0
$F_2(x) = \sum_{i=1}^{n} \mid x_i \mid + \prod_{i=1}^{n} \mid x_i \mid$	30	[-10,10]	0
$F_3(x) = \sum_{n=1}^{n} (\sum_{j-1}^{i} x_j)^2$	30	[-100, 100]	0
$F_4(x) = max_i\{\mid x_i \mid, 1 \leq i \leq n\}$	30	[-100, 100]	0
$F_5(x) = \sum_{i=1}^{n-1} [100(x_{i+1} - x_i^2)^2 + (x_i - 1)^2]$	30	[-30, 30]	0
$F_6(x) = \sum_{i=1}^{n} ([x_i + 0.5])^2$	30	[-100, 100]	0
$F_7(x) = \sum_{i=1}^{n} ix_i^4 + rand[0,1)$	30	[-1.28, 1.28]	0

Table B. Multimodal benchmark functions

Function	Dimension	Range	Min. function value
$F_8(x) = \sum_{i=1}^{n} -x_i \sin(\sqrt{\mid x_i \mid})$	30	[-500,500]	0
$F_9(x) = \sum_{i=1}^{n} [x_i^2 - 10\cos(2\pi x_i) + 10]$	30	[-5.12,5.12]	0
$F_{10}(x) = -20\exp(-0.2\sqrt{\frac{1}{n}\sum_{i=1}^{n} x_i^2}$	30	[-32, 32]	0
$-\exp(\frac{1}{n}\sum_{i=1}^{n}\cos(2\pi x_i)) + 20 + e$			
$F_{11}(x) = \frac{1}{4000}\sum_{i=1}^{n} x_i^2 - \prod_{i=1}^{n}\cos(\frac{x_i}{\sqrt{i}}) + 1$	30	[-600, 600]	0
$F_{12}(x) = \frac{\pi}{n}\{10\sin(\pi y_i)$	30	[-50, 50]	0
$+\sum_{i=1}^{n-1}(y_i - 1)^2[1 + 10\sin^2(\pi y_{i+1}) + (y_{n-1})^2]\}$			
$+\sum_{i=1}^{n}(x_i, 10, 100, 4)$			
$y_i = 1 + \frac{x_i + 1}{4}$			
$u(x_i, a, k, m) = \{ \begin{matrix} k(x_i - a)^m & x_i > a \\ 0 & -a < x_i < a \\ k(-x_i - a)^m & x_i < -a \end{matrix}$			
$F_{13}(x) = 0.1\{\sin^2(3\pi x_i) + \sum_{i=1}^{n}(x_i - 1)^2[1 + \sin^2(3\pi x_i + 1)]$	30	[-50, 50]	0
$+\sum_{i=1}^{n} u(x_i, 5, 100, 4)$			

Preprints (www.preprints.org) | NOT PEER-REVIEWED | Posted: 15 October 2020 doi:10.20944/preprints202010.0319.v1

Table C. Fixed-dimension multimodal benchmark functions

Function	Dimension	Range	Min. function value
$F_{14}(x) = \left(\frac{1}{500} + \sum_{j=1}^{25} \frac{1}{j + \sum_{j=1}^{2}(x_2 - a_{ij})^6}\right)^{-1}$	2	[-65,65]	1
$F_{15}(x) = \sum_{i=1}^{11} \left[a_i - \frac{x_1(b_i^2 + b_i x_2)}{b_i^2 + b_i x_i + x_4}\right]^2$	4	[-5,5]	0.00030
$F_{16}(x) = 4x_1^2 - 2.1x_1^4 + \frac{1}{3}x_1^6 + x_1 x_2 - 4x_2^2 + 4x_2^4$	2	[-5, 5]	-1.0316
$F_{17}(x) = \left(x_2 - \frac{5.1}{4\pi}^2 x_1^2 \frac{5}{\pi} x_1 - 6\right)^2 + 10\left(1 - \frac{1}{8\pi}\right)\cos x_1 + 10$	2	[-5, 5]	0.398
$F_{18}(x) = \left[1 + (x_1 + x_2 1)^2(19 - 14x_1 + 3x_1^2 - 14x_2 + 6x_1 x_2 + 3x_2^2)\right] \times \left[30 + (2x_1 - 3x_2)_2 \times (18 - 32x_1 + 12x_1^2 + 48x_2 - 36x_1 x_2 + 27x_2^2)\right]$	2	[-2, 2]	3
$F_{19}(x) = -\sum_{i=1}^{4} c_i \exp\left(-\sum_{j=1}^{3} a_{ij}(x_j - p_{ij})^2\right)$	3	[1, 3]	-3.86
$F_{20}(x) = -\sum_{i=1}^{4} c_i \exp\left(-\sum_{j=1}^{6} a_{ij}(x_j - p_{ij})^2\right)$	6	[0, 1]	-3.32
$F_{21}(x) = -\sum_{i=1}^{5} \left[(X - a_i)(X - a_i)^T + c_i\right]^{-1}$	4	[0, 10]	-10.1532
$F_{22}(x) = -\sum_{i=1}^{7} \left[(X - a_i)(X - a_i)^T + c_i\right]^{-1}$	4	[0, 10]	-10.4028
$F_{23}(x) = -\sum_{i=1}^{10} \left[(X - a_i)(X - a_i)^T + c_i\right]^{-1}$	4	[0, 10]	-10.5363

Table D. Bio-Medical Classification datasets (Mirjalili et al. (2014))[56]

Classification datasets	Number of attributes	Number of training samples	Number of test samples	Number of classes
Breastcancer	9	599	100	2
Heart	22	80	187	2

References

1. Kennedy J. and Eberhart R.C., Particle Swarm Optimization, *In proceedings of IEEE International Conference on Neural Networks* **1995**,1942-1948.
2. Mirjalili S. The Ant Lion Optimizer. *Advances in Engineering Software, Elsevier* **2015**; *83*,80-98.
3. Mirjalili S. The Whale Optimization Algorithm. *Advances in Engineering Software* **2016**;*9*,51-67.
4. Singh, N. and Singh, S.B., An effective Hybrid WOA-MGWO Strategy for Classical and Bio-Medical Science Optimization Problems, *Journal of Optimization, (Under Review)* **2017**.
5. Singh, N. and Singh, S.B., A Modified Mean Gray Wolf Optimization Approach for Benchmark and Biomedical Problems, *Journal of Evolutionary Bioinformatics* **2017**; *13*, 1-28.
6. Mirjalili, S., Mirjalili, S.M. and Lewis, A., Grey Wolf Optimizer, *Advances in Engineering Software* **2014**;*69*, 46-61.
7. Mirjalili, S., SCA: A Sine Cosine Algorithm for solving optimization problems, *Knowledge-Based Systems* **2016**, 1-14.
8. Dommel H, Tinney W. Optimal Power Flow Solutions. *IEEE Tran. Power Appar. Syst.* **1968**; *87*, 1866-1876.
9. Soares J, Sousa T. Vale ZA, Morais H, Faria P. Ant colony search algorithm for the optimal power flow problem. *IEEE Power Energy Soc, Gen, Meet.* **2011**, 1-8.
10. Chung TS and Li YZ. A hybrid GA approach for OPF with consideration of FACTS devices. *IEEE Power Engineering Review* **2001**, 47-50.
11. Cai LJ. I Erlich and G Stamtsis. Optimal choice and allocation of FACTS devices in deregulated electricity market using genetic algorithms. *IEEE* **2004**
12. Kalaiselvi K, Kumar V, Chandrasekar K. Enhanced Genetic Algorithm for Optimal Electric Power Flow using TCSC and TCPS. *Proc. World* **2010(II)**
13. Bakirtzis AG, Biskas P, Zoumas CE, Petridis V. Optimal power flow by enhanced genetic algorithm. *Power Syst. IEEE Trans.* **2002**; *17*, 229-236.
14. Hsun LR, Ren TS, Tone CY. Tseng Wan-Tsun. Optimal power flow by a fuzzy based hybrid particle swarm optimization approach. *Electr Power Syst Res* **2011**; *81*, 1466-74.
15. Slimani L, Bouktir T. Optimal Power Flow Solution of the Algerian Electrical Network using Differential Evolution Algorithm. *TELKOMNIKA* **2012**; *10*, 199-210.
16. Sinsupan N, Leeton U, Kulworawanichpong T. Application of Harmony Search to Optimal Power Flow Problems.*In proceeding of International Conference on Advances in Energy Engineering* **2010**, 219-222.
17. Ben-Tal A, El haoui, L, Nemirovski, A. Robust Optimization. *Princeton Series in Applied Mathematics. Princeton University Press* **2009**, 9-16.
18. Abido MA. Optimal power flow using tabu search algorithm. *Electric Power Compon Syst.* **2002**;*30*, 469–83.
19. Duman S, Güvenç U, Sönmez Y, Yörükeren N. Optimal power flow using gravitational search algorithm. *Energy Convers, Manag.* **2012**; *59*, 86-95.
20. Chowdhury BH.Towards the concept of integrated security: optimal dispatch under static and dynamic security constraints . *Electric Power Syst. Research* **1992**; *25*, 213-225.
21. Daryani N, Hagh MT, Teimourzadeh S. Adaptive group search optimization algorithm for multi-objective optimal power flow problem. *Appl. Soft Comput.* **2016**; *38*, 1012–1024.
22. Simon D. Biogeography-based optimization. *IEEE Transaction on Evolutionary Computation* **2008**; *12*, 702-713.
23. Mukherjee A, Mukherjee V. Solution of optimal power flow using chaotic krill herd algorithm. *Chaos. Solitons Fractals.* **2015**; *78*, 10-21.
24. Mirjalili S. Grasshopper Optimisation Algorithm: Theory and application. *Advances in Engineering Software* **2016**; *105*, 30-47.
25. Mirjalili S, Mirjalili SM, Hatamlou A. Multi-Verse Optimizer: a nature-inspired algorithm for global optimization. *Neural Computing and Applications* **2016**; *2*, 495-513.
26. Mirjalili S. Moth-flame optimization algorithm: A novel nature-inspired heuristic paradigm. *Knowledge-Based System, Elsevier* **2015**; *89*, 228-249.
27. Mirjalili S. Dragonfly algorithm: a new meta-heuristic optimization technique for solving single-objective, discrete, and multi-objective problems. *Neural Computing and Applications* **2016**; *4*, 1053-1073.
28. Bouchekara HREH. Optimal power flow using black-hole-based optimization approach. *Appl. Soft Comput.* **2014**; *24*, 879-888.

29. Rao MR, Babu NVN. Optimal Power Flow Using Cuckoo Optimization Algorithm. *ijareeie* **2013**, 4213-4218.
30. M.M. Mafarja and S. Mirjalili, Hybrid Whale Optimization Algorithm with Simulated Annealing for feature selection, *Neurocomputing* **2017**; *260*, 302-312.
31. B. Bentouati, L. Chaib and C. Saliha, A hybrid whale algorithm and pattern search technique for optimal power flow problem, *In proceeding of 8th International Conference on Modeling, Identification and Control (ICMIC-2016), IEEE Xplore* **2016**
32. R.M.R. Allah, Hybridizing sine cosine algorithm with multi-orthogonal search strategy for engineering design problems, *Journal of Computational Design and Engineering* **2017** In Press, 2017.
33. O.E. Turgut , Thermal and Economical Optimization of a Shell and Tube Evaporator Using Hybrid Backtracking Search-Sine Cosine Algorithm, *Arabian Journal for Science and Engineering* **2017**; *42(5)*, 1-20.
34. Yang XS. Nature-inspired metaheuristic algorithms. *Luniver Press* **2011**
35. Coello Coello CA. Use of a self-adaptive penalty approach for engineering optimization problems. *Comput Ind* **2000**; *41*, 113-27.
36. Mezura-Montes E, Coello Coello CA. An emprirical study about the usefulness of evolution strategies to solve constrained optimization problems. *International journal Genetic System* **2008**; *37*, 443-73.
37. D. Chattopadhyay, Application of General algebraic modeling system to power system optimization, *IEEE Trans. on Power Systems* **1999**; *14(1)*, 15-22.
38. T.A.A.Victoire and A.E. Jeyakumar, Hybrid PSO-SQP for economic dispatch with valve-point effect, *Electric Power System* **2004**; *71(1)*, 51-59.
39. R.M.S. Dhanraj and F. Gajendran, Quadratic programming solution to Emission and Economic Dispatch Problem, *Journal of the Institution of engineers, pt EL* **2005**; *86*, 129-132.
40. J.B. Park, K.S. Lee, J.R. Shin and K.Y. Lee, A particle swarm optimization for economic dispatch with non smooth cost functions, *IEEE Trans. Power System* **2005**; *20(1)*, 34-42.
41. M. Basu, A simulated annealing based goal attainment method for economic emission load dispatch of fixed head hydrothermal power systems, *International Journal of Electrical Power and Energy Systems* **2005**; *27(2)*, 147-153.
42. J.B. Park, Y.W. Jeong, J.R. Shin, K.Y. Lee and J.H. Kim, A Hybrid Particle Swarm Optimization Employing Crossover Operation for economic Dispatch Problems with Valve-point Effects, *Pro. Int. Conf. in Intelligent Systems Applications to Power Systems* **2007**; 281-286.
43. A.I. Selvakumar and K. Thanushkodi, A new particle swarm optimization solution to nonconvex economic dispatch problems, *IEEE Trans. Power System* **2007**; *22(1)*, 42-51.
44. Ji. Pyng Chiou, Variable Scaling hybrid differential evolution for large scale economic dispatch problem, *Electrical power systems Research* **2007**; *77*, 212-218.
45. Deep, K., and Das, K.N., Quadratic Approximation based Hybrid Genetic Algorithm for Function Optimization, *Applied Mathematics and Computation* **2008**; *203*, 86-98.
46. S.H. Ling, H.H.C. Iu, K.Y. Chan, H.K. Lam, B.C.W. Yeung and F.H. Leung, Hybrid particle swarm optimization with wavelet mutation and its industrial applications, *IEEE Trans. Syst. Man Cybern. Part B - Cybern.* **2008**; *38(3)*, 743-763.
47. A.I. Selvakumar and K. Thanushkodi, Anti-predatory particle swarm optimization: solution to nonconvex economic dispatch problems, *Electrical Power System* **2008**; *78(1)*, 2-10.
48. K.T. Chaturvedi, M. Pandit and L. Srivastava, Self-organizing hierarchical particle swarm optimization for non convex economic dispatch, *IEEE Trans. Power System* **2008**; *23(3)*, 1079–1087.
49. K. Deep and J.C. Bansal, Mean particle swarm optimization for function optimization, *International Journal of Computational Intelligence Studies* **2009**; *1(1)*, 72-92.
50. K. Deep and J.C. Bansal, Solving Economic Dispatch Problems with Valve-point Effects using Particle Swarm Optimization, *Journal of Universal Computer Science* **2012**; *18(13)*, 1842-1852.
51. K. Meng, Z.Y. Dong, H.G. Wang and K.P. Wong, Quantum-inspired particle swarm optimization for value-point economic load dispatch, *IEEE Trans. On Power Systems* **2010**; *25*, 215-222.
52. A. Bhattacharya and P.K. Chattopadhyay, Biogeography- Based Optimization for Different Economic Load Dispatch Problems, *IEEE Transactions on Power Systems* **2010**; *25(2)*, 1064-1077.
53. M.S. Kaurav, H.M. Dubey, M. Pandit and B.K. Panigrahi, Simulated Annealing Algorithm for Combined Economic and Emission Dispatch, *Proceedings of International Conference, ICACCN* **2011**; 631-636.

54. K. Deep and J.C. Bansal, Solving Economic Dispatch Problems with Valve-point Effects using Particle Swarm Optimization, *Journal of Universal Computer Science* **2012**; *18(13)*, 1842-1852.
55. S. Tiwaril, A. Kumar, G.S. Chaurasia and G.S. Sirohi, "Economic Load Dispatch Using Particle Swarm Optimization, *International Journal of Application or invocation in Engineering and Management* **2014**; *2(4)*, 476-485.
56. S. Mirjalili, S.M. Mirjalili and A. Lewis, Let a biogeography-based optimizer train your Multi-Layer Perceptron, *Information Sciences* **2014**; *269*, 188-209.