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Celestial and Quantum Propagation, Spinning, and Interaction as 4D Relativistic Cloud-Worlds Embedded in a 4D Conformal Bulk: From String to Cloud Theory

Mohammed B. Al-Fadhli ^{1,*}

¹ College of Science, University of Lincoln, Lincoln, LN6 7TS, UK.

* Correspondence: malfadhli@lincoln.ac.uk; mo.fadhli7@gmail.com

Abstract: Considerable efforts have been devoted to modifying gravity, which aim to elucidate the possible existence or nature of dark matter and dark energy, achieve a better description of observation data, and formulate quantum gravity. In addition, despite the enormous success of the quantum field theory, the framework requires renormalization techniques and breaks down at high energies. Recently, the Planck Legacy release has confirmed the presence of an enhanced lensing amplitude in the cosmic microwave background power spectra, which prefers a positively curved early Universe with a confidence level greater than 99%. This study considers the implied positive curvature of the early Universe as the curvature of ‘the background or the 4D conformal bulk as a manifestation of vacuum energy’ and distinguishes it from the localized curvature that is induced into the bulk by the presence of celestial objects that are regarded as ‘4D relativistic cloud-worlds’. Analogously, since gravitation seems to emerge owing to spacetime curvature and does not exhibit critical characteristics shared by the other fields, it has been incorporated as the local background curvature of the bulk, due to the stress-energy of the cloud-world, affecting the embedded quantum fields that are regarded as propagating ‘4D relativistic quantum clouds’. To consider the impact of the bulk on the embedded clouds, this paper presents interaction field equations in terms of the brane-world modified gravity and the perspective of gravitization of the quantum mechanics that count for the curvature of the 4D conformal bulk as the induced gravity on the embedded 4D relativistic clouds in addition to the boundary term interactions, which could remove the singularities and satisfy a conformal invariance theory. A visualization of the evolution of 4D relativistic cloud-worlds over the conformal spacetime of the 4D bulk is presented.

Keywords: Conformal Spacetime; Brane-World Modified Gravity; Quantum Field Theory.

1. Introduction

After the formulation of Einstein’s General Relativity (GR) utilizing 4D spacetime, Kaluza discovered in 1919 a potential field unification of gravitation and electromagnetism in 5D spacetime. To deal with the 5th dimension, Klein posited that it could be compactified. Nonetheless, those attempts and their expansions to more dimensions have not culminated in testable predictions nor the competence to elucidate observations yet. As an alternative to compactification, Gogberashvili, Randall and Sundrum showed in 1999 that the weak force of gravity can be explained using a model of 4D spacetime embedded in a negatively curved and large 5th dimension; nevertheless, it required massive gravitons [1–3].

On the other hand, to achieve an effective action for quantum corrections, several theories were formulated on the modification of Lagrangian fields and curvature terms. Such modifications appear to be inevitable, which included higher-order curvature terms as well as non-minimally coupled scalar fields [4–6]. While quantum anomalies require a non-local Lagrangian, one of the major differences between GR and quantum field theory (QFT) is that GR is background independent, which allows it to require fewer inputs while the latter requires a background metric that in turn impacts its predictions [7].

Recently, the Planck Legacy 2018 (PL18) release has confirmed the existence of an enhanced lensing amplitude in the cosmic microwave background power spectra, which prefers a positively curved early Universe with a confidence level greater than 99% [8,9]. Based on this sign of early curved background and its feasible evolution over conformal time, it is obvious that the background-independent theories such as GR do not consider the evolution in the background curvature and treat celestial objects in the early Universe of a preferred curvature on equal footing with their counterparts in the present Universe of a spatially flat background; this could be the reason behind the dark matter problem. A desirable theory should consider both metrics of the celestial object and the background and it should reduce to GR in a flat background. In addition, the gravitization of quantum mechanics rather than quantizing gravity in favour of Penrose's assessment would maintain the equivalence principle and count for the impact of gravity on quantum fields [10] while the latter lacks these features. On top of that gravity seems an emergent fictitious force since it is eliminated in a free-fall as well as nothing can be shielded from it.

This study aims to formulate interaction field equations that count for the evolution of early Universe preferred curvature, signified as the curvature of the 4D conformal bulk since conformal curvature is associated with the scalar factor, and its impact on celestial objects that are regarded as '4D relativistic cloud-worlds'. In addition, it aims to incorporate the influence of the induced gravity, represented by the 4D conformal bulk curvature, on quantum fields which are regarded as propagating '4D relativistic quantum clouds'.

The paper is organised as follows. Section 2 discusses the mathematical derivations of the gravitational and electromagnetic interaction field equations. Section 3 visualizes the field equations while Section 4 discusses the gravitational, electromagnetic and Quantum interaction field equations. Section 5 reproduces quantum electrodynamics. Finally, Section 6 summarises the conclusions and suggests future works.

2. Gravitational and Electromagnetic Interaction Field Equations

The PL18 release has preferred a positively curved early Universe, that is, is a sign of a primordial background curvature or a curved conformal bulk where the evolution of the conformal curvature is associated with the expansion of the Universe's scalar factor. To incorporate the bulk curvature and its evolution over the conformal time, a modulus of spacetime deformation, E_D in terms of energy density, is introduced based on the theory of elasticity [11]. The modulus can be expressed in terms of the resistance of the bulk to the localized curvature induced by celestial objects using Einstein field equations or in terms of the field strength of the bulk using the Lagrangian formulation of the energy density existing in the bulk as a manifestation of the vacuum energy density as follows

$$E_D = \frac{T_{\mu\nu} - \frac{1}{2}Tg_{\mu\nu}}{R_{\mu\nu}/\mathcal{R}} = \frac{-\mathcal{F}_{\lambda\rho}\mathcal{F}^{\lambda\rho}}{4\mu_0} \quad (1)$$

where the stress is signified by the stress-energy tensor $T_{\mu\nu}$ of trace T while the strain is signified by the Ricci curvature tensor $R_{\mu\nu}$ as the change in the curvature divided by the scalar of the bulk curvature \mathcal{R} as the background or conformal curvature. $\mathcal{F}_{\lambda\rho}$ is the field strength tensor and μ_0 is vacuum permeability. By incorporating the bulk influence, the Einstein–Hilbert action can be extended to

$$S = E_D \int_c \left[\frac{R}{\mathcal{R}} + \frac{L}{\mathcal{L}} \right] \sqrt{-g} d^4\rho \quad (2)$$

where R is the Ricci scalar curvature representing the localized curvature induced into the bulk by a celestial object that is regarded as a 4D relativistic cloud-world of metric g_{uv} and Lagrangian density L while \mathcal{R} is the scalar curvature of the 4D bulk of metric $\tilde{g}_{\mu\nu}$ and Lagrangian density \mathcal{L} indicating its internal stresses and momenta to reflect its curvature.

Since the bulk modulus, E_D , is constant with regards to the cloud-world action under constant vacuum energy density condition and by considering the bulk expansion over the conformal time owing to the Universe expansion (the scale factor evolution) and its implication on the field strength of the bulk, a dual-action concerning the conservation of energy on global (bulk) and local (cloud-world) scales can be introduced as follows

$$S = \int_B \left[\frac{-\mathcal{F}_{\lambda\rho} \tilde{g}^{\lambda\gamma} \mathcal{F}_{\gamma\alpha} \tilde{g}^{\rho\alpha}}{4\mu_0} \right] \sqrt{-\tilde{g}} \int_C \left[\frac{R_{\mu\nu} g^{\mu\nu}}{\mathcal{R}_{\mu\nu} \tilde{g}^{\mu\nu}} + \frac{L_{\mu\nu} g^{\mu\nu}}{\mathcal{L}_{\mu\nu} \tilde{g}^{\mu\nu}} \right] \sqrt{-g} d^4\rho d^4\sigma \quad (3)$$

where the relationship of the conformal bulk metric $\tilde{g}_{\mu\nu}$ with the embedded cloud-world metric g_{uv} can be characterized by Weyl's conformal transformation as $\tilde{g}_{\mu\nu} = g_{\mu\nu} \Omega^2$, here Ω^2 is a conformal function [12]. The global-local action should hold for any variation as

$$\delta S = \int_B \left[\frac{-\delta(\mathcal{F}_{\lambda\rho} \tilde{g}^{\lambda\gamma} \mathcal{F}_{\gamma\alpha} \tilde{g}^{\rho\alpha}) \sqrt{-\tilde{g}}}{4\mu_0} \right] \int_C \left[\frac{\delta(R_{\mu\nu} g^{\mu\nu}) \sqrt{-g}}{\mathcal{R}_{\mu\nu} \tilde{g}^{\mu\nu}} - \frac{\delta(\mathcal{R}_{\mu\nu} \tilde{g}^{\mu\nu}) R_{\mu\nu} g^{\mu\nu} \sqrt{-g}}{(\mathcal{R}_{\mu\nu} \tilde{g}^{\mu\nu})^2} + \frac{R_{\mu\nu} g^{\mu\nu} \delta \sqrt{-g}}{\mathcal{R}_{\mu\nu} \tilde{g}^{\mu\nu}} \right] d^4\rho d^4\sigma \quad (4)$$

$$+ \left[\frac{\mathcal{F}_{\lambda\rho} \tilde{g}^{\lambda\gamma} \mathcal{F}_{\gamma\alpha} \tilde{g}^{\rho\alpha} \delta \sqrt{-\tilde{g}}}{4\mu_0} \right] \int_C \left[+ \frac{\delta(L_{\mu\nu} g^{\mu\nu}) \sqrt{-g}}{\mathcal{L}_{\mu\nu} \tilde{g}^{\mu\nu}} - \frac{\delta(\mathcal{L}_{\mu\nu} \tilde{g}^{\mu\nu}) L_{\mu\nu} g^{\mu\nu} \sqrt{-g}}{(\mathcal{L}_{\mu\nu} \tilde{g}^{\mu\nu})^2} + \frac{L_{\mu\nu} g^{\mu\nu} \delta \sqrt{-g}}{\mathcal{L}_{\mu\nu} \tilde{g}^{\mu\nu}} \right] d^4\rho d^4\sigma$$

By utilizing Jacobi's formula, $\delta \sqrt{-g} = -\sqrt{-g} g_{\mu\nu} \delta g^{\mu\nu} / 2$ [13], the variation is

$$\delta S = \int_B \left[\frac{-\mathcal{F}_{\lambda\rho} \mathcal{F}_{\gamma}^{\rho} \delta \tilde{g}^{\lambda\gamma} + \mathcal{F}_{\lambda\rho} \tilde{g}^{\lambda\gamma} \delta \mathcal{F}_{\gamma}^{\rho}}{2\mu_0} \right] \sqrt{-\tilde{g}} \int_C \left[\frac{R_{\mu\nu} \delta g^{\mu\nu} + g^{\mu\nu} \delta R_{\mu\nu}}{\mathcal{R}} - \frac{\mathcal{R}_{\mu\nu} \delta \tilde{g}^{\mu\nu} + \tilde{g}^{\mu\nu} \delta \mathcal{R}_{\mu\nu}}{\mathcal{R}^2} R - \frac{g_{\mu\nu} \delta g^{\mu\nu}}{2\mathcal{R}} R \right] \sqrt{-g} d^4\rho d^4\sigma \quad (5)$$

$$+ \left[\frac{\mathcal{F}_{\lambda\rho} \tilde{g}^{\lambda\gamma} \mathcal{F}_{\gamma\alpha} \tilde{g}^{\rho\alpha} \delta \tilde{g}_{\mu\nu} \delta \tilde{g}^{\mu\nu}}{8\mu_0} \right] \int_C \left[+ \frac{L_{\mu\nu} \delta g^{\mu\nu} + g^{\mu\nu} \delta L_{\mu\nu}}{\mathcal{L}} - \frac{\mathcal{L}_{\mu\nu} \delta \tilde{g}^{\mu\nu} + \tilde{g}^{\mu\nu} \delta \mathcal{L}_{\mu\nu}}{\mathcal{L}^2} L - \frac{g_{\mu\nu} \delta g^{\mu\nu}}{2\mathcal{L}} L \right] \sqrt{-g} d^4\rho d^4\sigma$$

By considering the cloud-world's boundary term: $\int_C g^{\mu\nu} \delta R_{\mu\nu} \sqrt{-g} d^4\rho / \mathcal{R}$, the variation in the Ricci curvature tensor δR_{uv} can be expressed in terms of the covariant derivative of the difference between two Levi-Civita connections, the Palatini identity: $\delta R_{\mu\nu} = \nabla_{\rho}(\delta \Gamma_{\nu\mu}^{\rho}) - \nabla_{\nu}(\delta \Gamma_{\rho\mu}^{\rho})$, where this variation with respect to the inverse metric $g^{\mu\nu}$ can be obtained by using the metric compatibility of the covariant derivative, $\nabla_{\rho} g^{\mu\nu} = 0$ [13], as $g^{\mu\nu} \delta R_{\mu\nu} = \nabla_{\rho}(g^{\mu\nu} \delta \Gamma_{\nu\mu}^{\rho} - g^{\mu\rho} \delta \Gamma_{\sigma\mu}^{\sigma})$. Therefore, the cloud-world's boundary term as a total derivative for any tensor density can be transformed based on Stokes' theorem as follows

$$\int_C \left[\frac{g^{\mu\nu} \delta R_{\mu\nu}}{\mathcal{R}} \right] \sqrt{-g} d^4\rho = \frac{1}{\mathcal{R}} \int_C [\nabla_{\rho}(g^{\mu\nu} \delta \Gamma_{\nu\mu}^{\rho} - g^{\mu\rho} \delta \Gamma_{\sigma\mu}^{\sigma})] \sqrt{-g} d^4\rho \quad (6)$$

$$= \frac{1}{\mathcal{R}} \int_C [\nabla_{\mu} H^{\mu}] \sqrt{-g} d^4\rho = \frac{\epsilon}{\mathcal{R}} \int_{\partial C} [K] \sqrt{|q|} d^3\varrho$$

where the bulk scalar curvature, \mathcal{R} , is left outside the integral transformation as it only acts as a scalar. The same transformation can be applied to the other boundary terms. In addition, a second approach can be applied to the bulk's evolution boundary terms as

$$\int_C \left[\frac{\tilde{g}^{\mu\nu} \delta \mathcal{R}_{\mu\nu}}{\mathcal{R}^2} R \right] \sqrt{-g} d^4\rho = \int_C \left[\frac{R}{\mathcal{R}} \frac{\tilde{g}^{\mu\nu} \delta \mathcal{R}_{\mu\nu}}{\delta \tilde{g}^{\mu\nu} \mathcal{R}_{\mu\nu}} \tilde{g}_{\mu\nu} \right] \delta \tilde{g}^{\mu\nu} \sqrt{-g} d^4\rho = \int_C \left[\frac{R}{\mathcal{R}} \frac{\delta \ln \mathcal{R}_{\mu\nu}}{\delta \ln \tilde{g}^{\mu\nu}} \tilde{g}_{\mu\nu} \right] \delta \tilde{g}^{\mu\nu} \sqrt{-g} d^4\rho \quad (7)$$

$$= \int_C \left[\frac{R}{\mathcal{R}} \vartheta^2 \tilde{g}_{\mu\nu} \right] \delta \tilde{g}^{\mu\nu} \sqrt{-g} d^4\rho = \int_C \left[\frac{R}{\mathcal{R}} \frac{\vartheta^2}{\Omega^2} \tilde{g}_{\mu\nu} \right] \delta g^{\mu\nu} \sqrt{-g} d^4\rho = \int_C \left[\frac{R}{\mathcal{R}} \tilde{g}_{\mu\nu} \right] \delta g^{\mu\nu} \sqrt{-g} d^4\rho$$

where $\vartheta^2 = \delta \ln \mathcal{R}_{\mu\nu} / \delta \ln \tilde{g}^{\mu\nu}$ resembles the Ricci flow in a normalized form reflecting the conformal evolution in the extrinsic curvature of the bulk that can be expressed as a function based on Weyl's transformation as $\tilde{g}_{\mu\nu} = \tilde{g}_{\mu\nu} \vartheta^2$; $\vartheta^2 = \vartheta^2 / \Omega^2$ as $\delta g^{\mu\nu} = \Omega^2 \delta \tilde{g}^{\mu\nu}$.

By using the first approach of boundary terms' transformations given in Equation (6), the transformed boundary action, S_b , is

$$S_b = \int_{\partial B} \left[\frac{\epsilon f_\lambda}{2} \right] \sqrt{-\tilde{q}} \left(\frac{\epsilon}{\mathcal{R}} \int_{\partial C} [K] \sqrt{|q|} - \frac{R\epsilon}{\mathcal{R}^2} \int_{\partial C} [\mathcal{K}] \sqrt{|q|} + \frac{\epsilon}{\mathcal{L}} \int_{\partial C} [l] \sqrt{|q|} - \frac{L\epsilon}{\mathcal{L}^2} \int_{\partial C} [\ell] \sqrt{|q|} \right) d^3 \varrho d^3 \varsigma \quad (8)$$

where K and \mathcal{K} are the traces of the cloud-world and the bulk extrinsic curvatures, l and ℓ are the extrinsic traces of the Lagrangian density on the cloud-world and the bulk boundaries, q and \tilde{q} are the determinants of their induced metrics respectively, and ϵ equals 1 when the normal \hat{n}_u is a spacelike entity and equals -1 when it is a timelike entity. $f_\lambda = \mathcal{F}_{\lambda\rho} J^\rho$ is the 4D Lorentz force density. The boundary action should hold for any variation and by considering the transformed cloud-world's boundary term, the variation in the term yields

$$\frac{\epsilon}{\mathcal{R}} \int_{\partial C} \left[K_{\mu\nu} \delta q^{\mu\nu} + q^{\mu\nu} \delta K_{\mu\nu} + K \frac{\delta \sqrt{|q|}}{\sqrt{|q|}} \right] \sqrt{|q|} d^3 \varrho \quad (9)$$

where $K = K_{\mu\nu} q^{\mu\nu}$. By utilizing Jacobi's formula for the determinant differentiation; thus, $\delta \sqrt{|q|} = -\sqrt{|q|} q_{\mu\nu} \delta q^{\mu\nu} / 2$ and by utilizing the variation in the metric times the inverse metric, $q^{\mu\nu} q_{\mu\nu} = D$ as $q^{\mu\nu} = -q_{\mu\nu} \delta q^{\mu\nu} / \delta q_{\mu\nu}$, where D is the number of dimensions; thus, the boundary term is

$$\frac{\epsilon}{\mathcal{R}} \int_{\partial C} \left[K_{\mu\nu} \delta q^{\mu\nu} - \frac{1}{2} K \left(q_{\mu\nu} \delta q^{\mu\nu} + 2 q_{\mu\nu} \frac{\delta K_{\mu\nu}}{\delta q_{\mu\nu} K} \delta q^{\mu\nu} \right) \right] \sqrt{|q|} d^3 \varrho \quad (10)$$

here $\delta K_{\mu\nu} / \delta q_{\mu\nu} K = (\delta K_{\mu\nu} / K_{\mu\nu}) (q_{\mu\nu} / \delta q_{\mu\nu}) = \delta \ln K_{\mu\nu} / \delta \ln q_{\mu\nu}$ resembles the Ricci flow in a normalized form reflecting the conformal distortion in the boundary over conformal time, which can be expressed as a function ω^2 according to Weyl's conformal transformation as $\tilde{q}_{\mu\nu} = q_{\mu\nu} \omega^2$ [14]. Then, the term is $\epsilon / \mathcal{R} \int_{\partial C} [K_{\mu\nu} \delta q^{\mu\nu} - K \tilde{q}_{\mu\nu} \delta q^{\mu\nu} / 2] \sqrt{|q|} d^3 \varrho$ where $\tilde{q}_{\mu\nu} = q_{\mu\nu} + 2 \tilde{q}_{\mu\nu}$ is the conformally transformed induced metric on the cloud-world boundary. The same is applied to the bulk and Lagrangian boundary terms. The variation in the whole action with renaming the dummy indices is

$$\begin{aligned} \delta S = & \left(- \int_B \left[\frac{1}{2\mu_0} \left(\mathcal{F}_{\mu\lambda} \mathcal{F}_\nu^\lambda - \frac{\mathcal{F}_{\lambda\rho} \mathcal{F}^{\lambda\rho}}{4} \tilde{g}_{\mu\nu} \right) \right] \delta \tilde{g}^{\mu\nu} \sqrt{-\tilde{g}} d^4 \sigma - \int_{\partial B} \left[\frac{\epsilon}{2} \delta f_\nu / \delta \tilde{q}^{\mu\nu} \right] \delta \tilde{q}^{\mu\nu} \sqrt{|\tilde{q}|} d^3 \varsigma \right) \\ & \left(\int_C \left[\frac{R_{\mu\nu} \delta g^{\mu\nu}}{\mathcal{R}} - \frac{\mathcal{R}_{\mu\nu} \delta \tilde{g}^{\mu\nu}}{\mathcal{R}^2} \mathcal{R} - \frac{g_{\mu\nu} \delta g^{\mu\nu}}{2\mathcal{R}} \mathcal{R} + \frac{L_{\mu\nu} \delta g^{\mu\nu}}{\mathcal{L}} - \frac{\mathcal{L}_{\mu\nu} \delta \tilde{g}^{\mu\nu}}{\mathcal{L}^2} \mathcal{L} - \frac{g_{\mu\nu} \delta g^{\mu\nu}}{2\mathcal{L}} \mathcal{L} \right] \sqrt{-g} d^4 \rho \right) \\ & + \int_{\partial C} \left[\frac{\epsilon}{\mathcal{R}} \left(K_{\mu\nu} - \frac{1}{2} K \hat{q}_{\mu\nu} \right) + \frac{\epsilon}{\mathcal{L}} \left(l_{\mu\nu} - \frac{1}{2} l \hat{q}_{\mu\nu} \right) \right] \delta q^{\mu\nu} \sqrt{|q|} d^3 \varrho \\ & - \int_{\partial C} \left[\frac{R\epsilon}{\mathcal{R}^2} \left(\mathcal{K}_{\mu\nu} - \frac{1}{2} \mathcal{K} \hat{q}_{\mu\nu} \right) + \frac{L\epsilon}{\mathcal{L}^2} \left(\ell_{\mu\nu} - \frac{1}{2} \ell \hat{q}_{\mu\nu} \right) \right] \delta q^{\mu\nu} \sqrt{|q|} d^3 \varrho \end{aligned} \quad (11)$$

The outcome of the global part of the action has resembled an extended electromagnetic stress-energy tensor as $\mathcal{T}_{\mu\nu} := (\mathcal{F}_{\mu\lambda} \mathcal{F}_\nu^\lambda - \mathcal{F}_{\lambda\rho} \mathcal{F}^{\lambda\rho} \tilde{g}_{\mu\nu} / 4) / \mu_0 + \delta f_\nu / \delta \tilde{q}^{\mu\nu}$ denoting energy density exists in the bulk as the vacuum energy density in addition to the 4D Lorentz force density on the bulk boundary. From Equations (1), (2) and (11), $\mathcal{T}_{\mu\nu} := E_D = \mathcal{L} = \mathcal{R} c^4 / 8\pi G_t$ is proportional to the fourth power of the speed of light that in turn is directly proportional to the frequency, which can be in accordance with frequency cut-off predictions of vacuum energy density in QFT [15,16].

By applying the principle of stationary action while choosing ϵ as a timelike entity, the general form of the field equations can be obtained based on the first approach of boundary term transformations as follows

$$\frac{R_{\mu\nu}}{\mathcal{R}} - \frac{1}{2} \frac{R}{\mathcal{R}} g_{\mu\nu} - \frac{\mathcal{R}_{\mu\nu}}{\mathcal{R}^2} R + \frac{R \left(\mathcal{K}_{\mu\nu} - \frac{1}{2} \mathcal{K} \hat{q}_{\mu\nu} \right) - \mathcal{R} \left(K_{\mu\nu} - \frac{1}{2} K \hat{q}_{\mu\nu} \right)}{\mathcal{R}^2} = \frac{\hat{T}_{\mu\nu}}{\mathcal{T}_{\mu\nu}} \quad (12)$$

The interaction field equations can be interpreted as indicating that the cloud-world's induced curvature, R , over the bulk's existing (conformal) curvature, \mathcal{R} , equals the ratio of the cloud-world's imposed energy density, $\hat{T}_{\mu\nu}$, and its flux to the bulk's vacuum energy density, $\mathcal{T}_{\mu\nu}$, and its flux through the expanding/contracting Universe. The new boundary term given by the extrinsic curvatures of the cloud-world, K , and the bulk, \mathcal{K} , is only significant at high energies when the difference between the induced and existing curvatures is significant. The boundary term could remove singularities where the field equations could satisfy a conformal invariance theory.

The field equations include four contributions that come from the cloud-world's intrinsic and extrinsic curvatures and the bulk's intrinsic and extrinsic curvatures. The field equations can be expressed in different forms depending on which contribution is required to be implicit or explicit. By applying the second approach in Equation (7) on the bulk boundary terms, the variation in the action with renaming the dummy indices is

$$\begin{aligned} \delta S = & \left(- \int_B \left[\frac{1}{2\mu_0} \left(\mathcal{F}_{\mu\lambda} \mathcal{F}_\nu^\lambda - \frac{\mathcal{F}_{\lambda\rho} \mathcal{F}^{\lambda\rho}}{4} \tilde{g}_{\mu\nu} \right) \right] \delta \tilde{g}^{\mu\nu} \sqrt{-\tilde{g}} d^4\sigma - \int_{\partial B} \left[\frac{\epsilon}{2} \delta f_\nu / \delta \tilde{q}^{\mu\nu} \right] \delta \tilde{q}^{\mu\nu} \sqrt{|\tilde{q}|} d^3\varsigma \right) \\ & \left(\int_C \left[\frac{R_{\mu\nu}}{\mathcal{R}} - \frac{R}{\mathcal{R}} \tilde{g}_{\mu\nu} - \frac{R}{2\mathcal{R}} g_{\mu\nu} - \frac{R}{\mathcal{R}} \bar{\bar{g}}_{\mu\nu} + \frac{L_{\mu\nu}}{\mathcal{L}} - \frac{L}{\mathcal{L}} \tilde{g}_{\mu\nu} - \frac{L}{2\mathcal{L}} g_{\mu\nu} - \frac{L}{\mathcal{L}} \bar{\bar{g}}_{\mu\nu} \right] \delta g^{\mu\nu} \sqrt{-g} d^4\rho \right) \\ & + \int_{\partial C} \left[\frac{\epsilon}{\mathcal{R}} \left(K_{\mu\nu} - \frac{1}{2} K \hat{q}_{\mu\nu} \right) + \frac{\epsilon}{\mathcal{L}} \left(l_{\mu\nu} - \frac{1}{2} l \hat{q}_{\mu\nu} \right) \right] \delta q^{\mu\nu} \sqrt{|q|} d^3\rho \end{aligned} \quad (13)$$

where $\tilde{g}_{\mu\nu} = \mathcal{R}_{\mu\nu}/\mathcal{R} = \mathcal{R}_{\mu\nu}/\mathcal{R}_{\mu\nu} \tilde{g}^{\mu\nu}$ represents the bulk metric regarding its intrinsic curvature contribution. By applying the principle of stationary action as

$$\frac{R_{\mu\nu}}{\mathcal{R}} - \frac{1}{2} \frac{R}{\mathcal{R}} \hat{g}_{\mu\nu} - \frac{K_{\mu\nu} - \frac{1}{2} K \hat{q}_{\mu\nu}}{\mathcal{R}} = \frac{\hat{T}_{\mu\nu}}{\mathcal{T}_{\mu\nu}} \quad (14)$$

By utilizing Equations (1), (2) and (14) that state $\mathcal{T}_{\mu\nu} := E_D = \mathcal{R}c^4/8\pi G_t$, the interaction field equations can be simplified to

$$R_{\mu\nu} - \frac{1}{2} R \hat{g}_{\mu\nu} - \left(K_{\mu\nu} - \frac{1}{2} K \hat{q}_{\mu\nu} \right) = \frac{8\pi G_t}{c^4} \hat{T}_{\mu\nu} \quad (15)$$

where $\hat{g}_{\mu\nu} = g_{\mu\nu} + 2\tilde{g}_{\mu\nu} + 2\bar{\bar{g}}_{\mu\nu}$ counting for the contribution of the cloud-world metric, $g_{\mu\nu}$, in addition to the contribution from the intrinsic, $\tilde{g}_{\mu\nu}$, and extrinsic, $\bar{\bar{g}}_{\mu\nu}$, curvatures of the bulk whereas Einstein spaces are a subclass of the conformal space [12]. The evolution in G_t reflects the field strength evolution with the Universe expansion and can accommodate the bulk curvature evolution over the conformal time against constant G for a special flat spacetime case. Based on Equation (13), an extended stress-energy tensor can be defined by including the Lagrangian on boundaries as $\hat{T}_{\mu\nu} := (2L_{\mu\nu} - L\hat{g}_{\mu\nu}) - (2l_{\mu\nu} - l\hat{q}_{\mu\nu})$ that counts for the energy density and flux of the cloud-world, $L_{\mu\nu}$, and the electromagnetic energy flux from its boundary, $l_{\mu\nu}$, over the conformal time.

3. Evolution of the 4D Relativistic Cloud-World Travelling in the 4D Conformal Bulk

This section visualizes the evolution of the 4D relativistic cloud-worlds over the conformal space-time of the 4D bulk. Galaxy formation and evolution as a 4D relativistic cloud-world travelling throughout a 4D conformal curved bulk as preferred by PL18 release is considered. This scenario reveals that the galaxy forms as a forced vortex due to the curved background, which could resemble galaxy rotational curves. The simplified field equations with the implicit bulk boundary term are

$$R_{\mu\nu} - \frac{1}{2}R\hat{g}_{\mu\nu} - (K_{\mu\nu} - \frac{1}{2}K\hat{q}_{\mu\nu}) = \frac{8\pi G_t}{c^4}\hat{T}_{\mu\nu} \quad (16)$$

The entire contribution comes from the boundary term when calculating the black hole entropy using the semiclassical approach [17,18]. By applying this concept and rearranging the field equations for this setting while using the general form of the field equations as follows

$$R_{\mu\nu} - \frac{1}{2}Rg_{\mu\nu} - \frac{\mathcal{R}_{\mu\nu}}{\mathcal{R}}R = \frac{8\pi G_t}{c^4}\hat{T}_{\mu\nu} - \frac{R(\mathcal{K}_{\mu\nu} - \frac{1}{2}\mathcal{K}\hat{q}_{\mu\nu}) - \mathcal{R}(K_{\mu\nu} - \frac{1}{2}K\hat{q}_{\mu\nu})}{\mathcal{R}} = 0 \quad (17)$$

From Equation (17), the field equations yield

$$R_{\mu\nu} = \frac{1}{2}Rg_{\mu\nu} + \frac{\mathcal{R}_{\mu\nu}}{\mathcal{R}}R = \frac{1}{2}R(g_{\mu\nu} + 2\tilde{g}_{\mu\nu}) = \frac{1}{2}Rg_{\mu\nu}(1 + 2\Omega^2) = 0 \quad (18)$$

where $\hat{g}_{\mu\nu} = g_{\mu\nu} + 2\tilde{g}_{\mu\nu}$ and $\tilde{g}_{\mu\nu} = \mathcal{R}_{\mu\nu}/\mathcal{R} = \mathcal{R}_{\mu\nu}/\mathcal{R}_{\mu\nu}\tilde{g}^{\mu\nu}$ is the conformal bulk metric, which can be expressed as proportional to the cloud-world metric $g_{\mu\nu}$ as $\tilde{g}_{\mu\nu} = g_{\mu\nu}\Omega^2$ by utilizing Ω^2 , the conformal transformation function. The conformally transformed metric $\hat{g}_{\mu\nu} = g_{\mu\nu}(1 + 2\Omega^2)$ can be expressed as

$$ds^2 = -A(r)(1 + 2\Omega^2(r, \mathcal{r}))c^2dt^2 + S^2(B(r)(1 + 2\Omega^2(r, \mathcal{r}))dr^2 + r^2d\theta^2 + r^2\sin^2\theta d\phi^2) \quad (19)$$

where A and B are functions of the cloud-world curvature radius r , while the conformal function Ω^2 is a function of the bulk curvature radius \mathcal{r} and it can be influenced by the cloud-world radius of curvature. S^2 is a dimensionless conformal scale factor. By performing the coordinate transformation as follows

$$ds^2 = -(A(\lambda) + 2A(\lambda)\Omega^2(\lambda, \mathcal{r}))c^2dt^2 + (B(\lambda) + 2B(\lambda)\Omega^2(\lambda, \mathcal{r}))d\lambda^2 + \lambda^2d\theta^2 + \lambda^2\sin^2\theta d\phi^2 \quad (20)$$

The mathematical derivations of the conformally metric are presented in [19], where the derived conformally metric $\hat{g}_{\mu\nu} = g_{\mu\nu} + 2\tilde{g}_{\mu\nu} = g_{\mu\nu}(1 + 2\Omega^2)$ is

$$ds^2 = \left(1 - \frac{r_s}{r} - \frac{r_b}{\mathcal{r}}\right) \left(-c^2dt^2 + S^2 \left(\frac{dr^2}{1 + \frac{r_s^2}{r^2} - 2\frac{r_s}{r}} + \frac{r^2d\theta^2 + r^2\sin^2\theta d\phi^2}{1 - \frac{r_s}{r} - \frac{r_b}{\mathcal{r}}} \right)\right) \quad (21)$$

This metric reduces to the Schwarzschild metric in a flat background ($\mathcal{r} \rightarrow \infty$), where \mathcal{r} is the background or bulk curvature radius, r_s is Schwarzschild radius and $r_b = 2G_pM_p/c^2$ is the preferred PL18's early Universe curvature radius and M_p is the early Universe plasma mass. The bulk gravitational potential can be expressed as $\varphi_b = 2G_pM_p/\mathcal{r}$, where this potential decreases with the Universe's expansion and vanishes in the flat spacetime background ($\mathcal{r} \rightarrow \infty$).

The scenario of the galaxy formation as a forced vortex due to the curvature of the background is shown in Figure 1, the evolution of the 4D cloud-world of metric $g_{\mu\nu}$ through its travel and spin in the conformal space-time of the 4D bulk of metric $\tilde{g}_{\mu\nu}$.

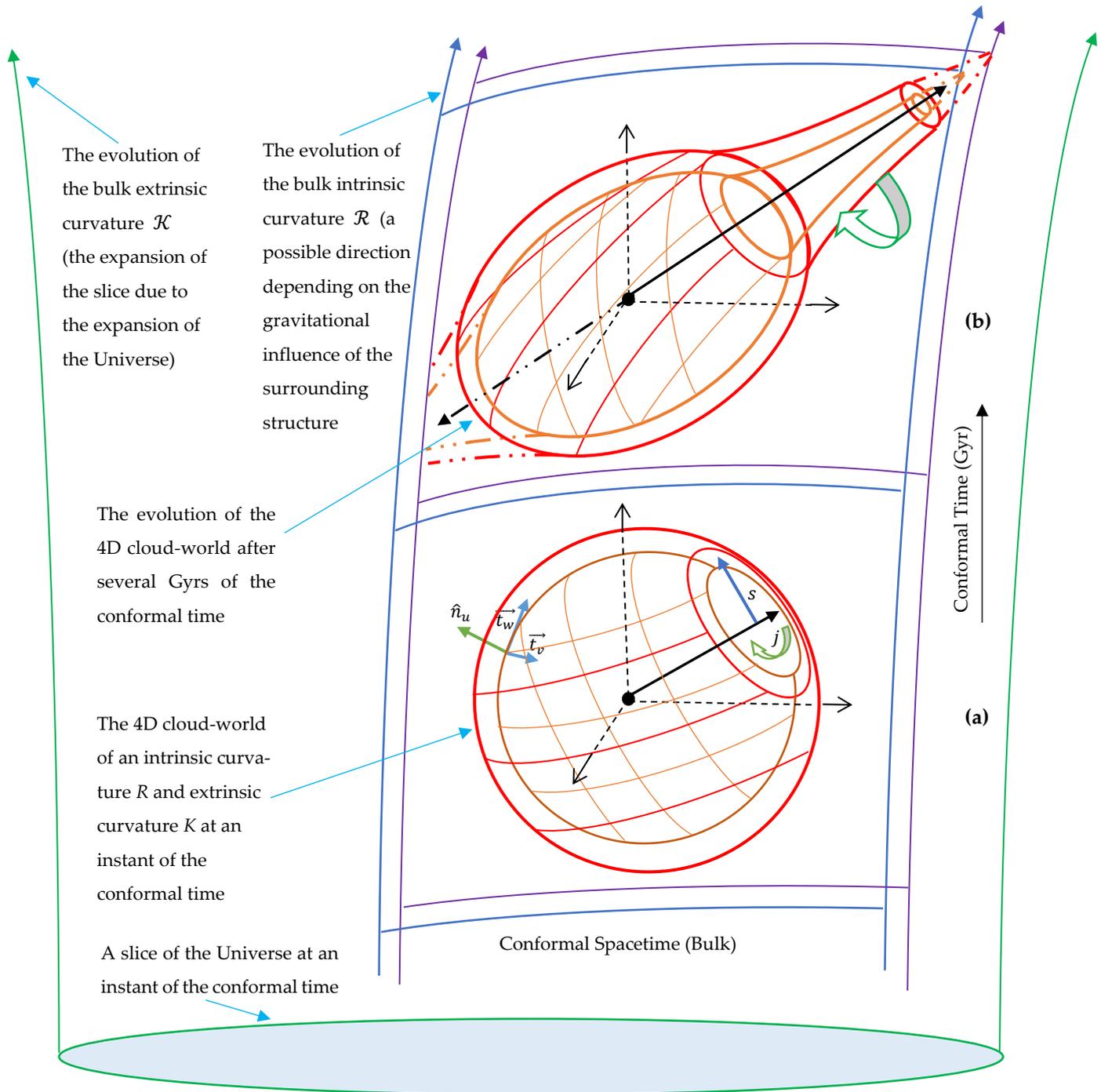


Figure 1. The hypersphere of a compact core of a galaxy (the red-orange 4D cloud-world) along with its travel and spin through the conformal spacetime (the blue-purple 4D bulk representing the bulk of distinctive curvature evolving over the conformal time).

4. Gravitational, Electromagnetic and Quantum Interaction Field Equations

The action in Equation (3) is expanded to investigate the interaction of quantum fields with the electromagnetic fields under the influence of the field strength of vacuum energy that is reliant on the curvature of the cloud-world and the bulk as follows

$$S = \int_B \left[\frac{-\mathcal{F}_{\lambda\rho} \tilde{g}^{\lambda\gamma} \mathcal{F}_{\gamma\alpha} \tilde{g}^{\rho\alpha}}{4\mu_0} \right] \sqrt{-\tilde{g}} \int_C \left[\frac{R_{\mu\nu} g^{\mu\nu}}{\mathcal{R}_{\mu\nu} \tilde{g}^{\mu\nu}} \right] \sqrt{-g} \int_Q \left[\frac{p_\mu p_\nu q^{\mu\nu}}{\pi_\mu \pi_\nu g^{\mu\nu}} + \frac{L_{\alpha\beta} q^{\alpha\lambda} L_{\lambda\gamma} q^{\beta\gamma}}{n \mathcal{L}_{\mu\nu} g^{\mu\nu}} \right] \sqrt{-q} d^4\alpha d^4\rho d^4\sigma \quad (22)$$

where $L_{\alpha\beta} L^{\alpha\beta}$ are Lagrangian densities of two entangled quantum fields of a metric tensor $q_{\mu\nu}$ and four-momentum $p_\mu p^\nu$ while $\pi_\mu \pi^\nu$ are the four-momentum of vacuum energy density and n is a proportionality constant. The action should hold for any variation as

$$\delta S = \int_B \left[\begin{array}{c} \frac{-\mathcal{F}_{\lambda\rho} \mathcal{F}_\gamma^\rho \delta \tilde{g}^{\lambda\gamma}}{2\mu_0} \\ \frac{-\mathcal{F}_{\lambda\rho} \tilde{g}^{\lambda\gamma} \delta \mathcal{F}_\gamma^\rho}{2\mu_0} \\ \frac{\mathcal{F}_{\lambda\rho} \mathcal{F}^{\lambda\rho} \tilde{g}_{\mu\nu} \delta \tilde{g}^{\mu\nu}}{8\mu_0} \end{array} \right] \sqrt{-\tilde{g}} \int_C \left[\begin{array}{c} \frac{R_{\mu\nu} \delta g^{\mu\nu} + g^{\mu\nu} \delta R_{\mu\nu}}{\mathcal{R}} \\ \frac{\mathcal{R}_{\mu\nu} \delta \tilde{g}^{\mu\nu} + \tilde{g}^{\mu\nu} \delta \mathcal{R}_{\mu\nu}}{\mathcal{R}^2} \\ \frac{-g_{\mu\nu} \delta g^{\mu\nu}}{2\mathcal{R}} R \end{array} \right] \sqrt{-g} \quad (23)$$

$$\int_Q \left[\begin{array}{c} \frac{p_\mu p_\nu \delta q^{\mu\nu} + q^{\mu\nu} \delta(p_\mu p_\nu)}{\pi_\mu \pi^\nu} - p_\mu p^\nu \frac{(\pi_\mu \pi_\nu) \delta g^{\mu\nu} + g^{\mu\nu} \delta(\pi_\mu \pi_\nu)}{(\pi_\mu \pi^\nu)^2} - \frac{p_\mu p^\nu q_{\mu\nu} \delta q^{\mu\nu}}{\pi_\mu \pi^\nu} \frac{1}{2} \\ \frac{L_{\alpha\gamma} L_\beta^\gamma \delta q^{\alpha\lambda} + q^{\alpha\lambda} L_{\alpha\beta} \delta L_\beta^\gamma}{n \mathcal{L}} - 2L_{\alpha\beta} L^{\alpha\beta} \frac{\mathcal{L}_{\mu\nu} \delta g^{\mu\nu} + g^{\mu\nu} \delta \mathcal{L}_{\mu\nu}}{n \mathcal{L}^2} - L_{\alpha\beta} L^{\alpha\beta} \frac{q_{\mu\nu} \delta q^{\mu\nu}}{2n \mathcal{L}} \end{array} \right] \sqrt{-q} d^4\alpha d^4\rho d^4\sigma$$

By considering the boundary term of the quantum cloud: $\int_Q q^{\mu\nu} \delta(p_\mu p_\nu) \sqrt{-q} d^4\alpha / \pi_\mu \pi^\nu$, the variation in the four-momentum δp_μ , i.e., the change in the total energy of charged fields enclosed within the quantum cloud boundary Q can represent the flow of the four-current J_μ through the cloud boundary ∂Q , where multiplying this current by the four potential that is generated by the current itself, A_μ , and that externally applied, B_μ , gives a scalar as follows $\delta p_\mu \equiv \delta \sqrt{(E/c - p)} \equiv (A_\mu + B_\mu) J^\mu$. This deduction is based on the gauge theory. However, the cloud's volume and its boundary surface should be taken into consideration. Since the boundary term represents two entangled quantum clouds, thus,

$$\int_Q \left[\frac{q^{\mu\nu} \delta(p_\mu p_\nu)}{\pi_\mu \pi^\nu} \right] \sqrt{-q} d^4\alpha = \int_{\partial Q} \left[\epsilon \frac{J^\mu (A_\mu + B_\mu) J^\nu (A_\nu + B_\nu)}{2\pi_\mu \pi^\nu} \right] \sqrt{-e} d^3\zeta \quad (24)$$

where $e_{\mu\nu}$ is the induced metric on the quantum cloud boundary. On the other hand, the cloud-world's boundary term can be transformed as follows

$$\begin{aligned} \int_Q \left[\frac{g^{\mu\nu} \delta(\pi_\mu \pi_\nu)}{(\pi_\mu \pi^\nu)^2} p_\mu p^\nu \right] \sqrt{-q} d^4\alpha &= \int_Q \left[\frac{p_\mu p^\nu}{\pi_\mu \pi^\nu} \frac{g^{\mu\nu} \delta(\pi_\mu \pi_\nu)}{\delta g^{\mu\nu} \pi_\mu \pi_\nu} g_{\mu\nu} \right] \delta g^{\mu\nu} \sqrt{-q} d^4\alpha \\ &= \int_Q \left[\frac{p_\mu p^\nu}{\pi_\mu \pi^\nu} \frac{\delta \ln(\pi_\mu \pi_\nu)}{\delta \ln g^{\mu\nu}} g_{\mu\nu} \right] \delta g^{\mu\nu} \sqrt{-q} d^4\alpha = \int_Q \left[\frac{p_\mu p^\nu}{\pi_\mu \pi^\nu} \bar{q}_{\mu\nu} \right] \delta q^{\mu\nu} \sqrt{-q} d^4\alpha \end{aligned} \quad (25)$$

where $\vartheta^2 = \delta \ln(\pi_\mu \pi_\nu) / \delta \ln g^{\mu\nu}$ resembles the Ricci flow in a normalized form reflecting the conformal evolution of the extrinsic curvature of the cloud-world that can be expressed as a positive function ϑ^2 according to Weyl's conformal transformation as $\bar{q}_{\mu\nu} = \tilde{g}_{\mu\nu} \vartheta^2$; $\vartheta^2 = \varphi^2 / \Omega^2$ as $\delta q^{\mu\nu} = \Omega^2 \delta g^{\mu\nu}$ since $\delta g^{\mu\nu} = \delta(q^{\mu\nu} \Omega^2) = \delta q^{\mu\nu} \Omega^2 + q^{\mu\nu} \delta \Omega^2$ as $\delta \Omega^2 \cong 0$.

The variation in the whole action with renaming the dummy indices is

$$\begin{aligned}
\delta S = & \left(- \int_B \left[\frac{1}{2\mu_0} \left(\mathcal{F}_{\mu\lambda} \mathcal{F}_\nu^\lambda - \frac{\mathcal{F}_{\lambda\rho} \mathcal{F}^{\lambda\rho}}{4} \tilde{g}_{\mu\nu} \right) \right] \delta \tilde{g}^{\mu\nu} \sqrt{-\tilde{g}} d^4 \sigma - \int_{\partial B} \left[\frac{\epsilon}{2} \delta f_\nu / \delta \tilde{q}^{\mu\nu} \right] \delta \tilde{q}^{\mu\nu} \sqrt{|\tilde{q}|} d^3 \zeta \right) \\
& \left(\int_C \left[\frac{R_{\mu\nu}}{\mathcal{R}} - \frac{R}{\mathcal{R}} \tilde{g}_{\mu\nu} - \frac{R}{\mathcal{R}} \bar{g}_{\mu\nu} - \frac{R}{2\mathcal{R}} g_{\mu\nu} \right] \delta g^{\mu\nu} \sqrt{-g} d^4 \rho + \int_{\partial C} \left[\frac{\epsilon}{\mathcal{R}} \left(K_{\mu\nu} - \frac{1}{2} K \hat{q}_{\mu\nu} \right) \right] \delta q^{\mu\nu} \sqrt{|q|} d^3 \varrho \right) \\
& \left(\int_Q \left[\frac{p_\mu p_\nu}{\pi_\mu \pi^\nu} - \frac{\pi_\mu \pi_\nu}{(\pi_\mu \pi^\nu)^2} p_\mu p^\nu - \frac{\bar{q}_{\mu\nu}}{\pi_\mu \pi^\nu} p_\mu p^\nu - \frac{p_\mu p^\nu}{\pi_\mu \pi^\nu} \frac{q_{\mu\nu}}{2} \right] \delta q^{\mu\nu} \sqrt{-q} d^4 \alpha \right) \\
& + \int_Q \left[\frac{2L_{\mu\nu} L^\nu}{n\mathcal{L}} - \frac{2L_{\alpha\beta} L^{\alpha\beta} \mathcal{L}_{\mu\nu}}{n\mathcal{L}^2} - \frac{2L_{\alpha\beta} L^{\alpha\beta} \bar{g}_{\mu\nu}}{n\mathcal{L}} - \frac{L_{\alpha\beta} L^{\alpha\beta} q_{\mu\nu}}{2n\mathcal{L}} \right] \delta q^{\mu\nu} \sqrt{-q} d^4 \alpha \\
& + \int_{\partial Q} \left[\epsilon \frac{J^u (A_\mu + B_\mu) J^\nu (A_\nu + B_\nu)}{2\pi_\mu \pi^\nu} + \epsilon \frac{\delta f_\nu / \delta e^{\mu\nu}}{n\mathcal{L}} \right] \delta e^{\mu\nu} \sqrt{-e} d^3 \varsigma \Big)
\end{aligned} \tag{26}$$

By applying the principle of stationary action as

$$\frac{p_\mu p_\nu}{\pi_\mu \pi^\nu} - \frac{p_\mu p^\nu}{\pi_\mu \pi^\nu} \frac{\bar{q}_{\mu\nu}}{2} - \frac{1}{2} \frac{J^u (A_\mu + B_\mu) J^\nu (A_\nu + B_\nu)}{\pi_\mu \pi^\nu} = \frac{T_\mu T_\nu}{\mathcal{T}_{\mu\nu}} \tag{27}$$

where $\bar{q}_{\mu\nu} = q_{\mu\nu} + 2\tilde{q}^{\mu\nu} + 2\bar{q}_{\mu\nu}$ is the conformally transformed metric tensor counting for the contribution of the quantum cloud's metric, $q_{\mu\nu}$, in addition to the contribution from the intrinsic, $\tilde{q}^{\mu\nu}$, and extrinsic, $\bar{q}_{\mu\nu}$, curvatures of the cloud-world. $T_\mu T_\nu = (2L_{\mu\alpha} L^\alpha_\nu - L_{\alpha\beta} L^{\alpha\beta} \bar{q}_{\mu\nu} / 2) / n + \delta f_\nu / \delta e^{\mu\nu}$ denote the Cauchy stress tensors, extended into four-dimension of the deformed configuration of the two entangled quantum fields while $\mathcal{T}_{\mu\nu}$ represents the overall stress-energy tensor of the cloud-world and the bulk. Figure 2 shows the quantum cloud where T_n is the traction vector on the inner surface S_i while n is the unit vector on the inner surface.

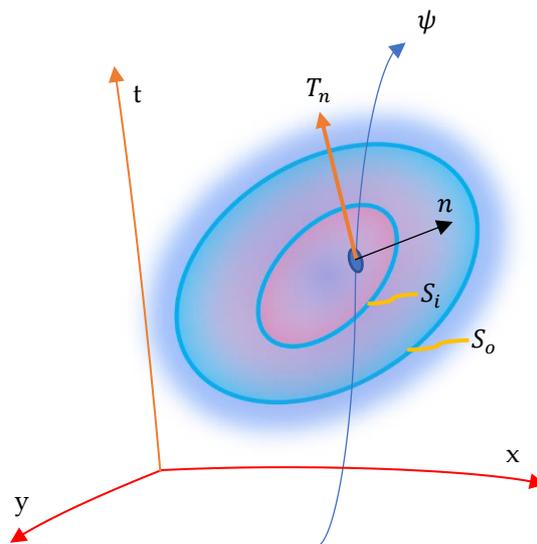


Figure 2. The deformed configuration of the 4D relativistic quantum cloud of metric $q_{\mu\nu}$ along its travel and spin, based on its wavefunction ψ . The configuration is given by, S_i , the inner surface of the cloud that separates its continuum into two portions and encloses an arbitrary inner volume while S_o is the outer surface denoting the cloud's boundary.

By separating the two entangled quantum fields with renaming the dummy indices and utilizing the dimensional analysis, the field equations in Equation (27) can be simplified to

$$p_\mu - \frac{1}{2}p^\nu \bar{q}_{\mu\nu} - \frac{1}{2}J^\mu(A_\mu + B_\mu) = \frac{\hbar G_t}{2c^2 g_R} T_\mu \quad (28)$$

where T_μ in (N/m^2) represents the 4D Lorentz force density applied on the boundary of the quantum cloud and $g_R = -M_C G_t / R^2$ in (N/Kg) is the gravitational field strength of the cloud-world of mass M_C and at radius R . From Equation (28), the expected value of the quantum cloud's volume is $V = 2\hbar R^2 / M_C c$, where c is the speed of light in vacuum. This reveals that the volume of the quantum cloud is quantized. The equations in terms of the operators are

$$\hat{p}_\mu - \frac{1}{2}\hat{p}^\nu \bar{q}_{\mu\nu} - \frac{1}{2}J^\mu(A_\mu + B_\mu) = \frac{\hbar G_t}{c^2 g_R} \hat{T}_\mu \quad (29)$$

where \hat{p}_μ is the momentum operator and \hat{T}_μ is the stress-energy (gravitational) operator. A plane wavefunction, $\psi = Ae^{i(kx - \omega t)}$, can be expressed by utilizing Equation (29) as $\psi = Ae^{\frac{-iR^2}{M_C c^2}(T_t ct - T_x x)}$, thus:

$$i\hbar\gamma^\mu \partial_\mu \psi - \frac{1}{2}i\hbar\gamma^\mu \partial^\nu \bar{q}_{\mu\nu} \psi - \frac{1}{2}J^\mu(A_\mu + B_\mu)\psi = \frac{1}{4} \frac{\hbar}{x_\mu \gamma^\mu} R \partial_R \psi \quad (30)$$

where \hbar is the Planck constant and γ^μ are the Dirac matrices.

5. Reproducing the Quantum Electrodynamics

The interaction field equations can be utilized to reproduce the quantum electrodynamics using an undeformed configuration of the quantum cloud given by Minkowski metric $\eta_{\mu\nu}$ while disregarding the curvature of the background and its gravitational field strength as follows

$$i\hbar\gamma^\mu \partial_\mu \psi - \frac{1}{2}i\hbar\gamma^\mu \partial^\nu \eta_{\mu\nu} \psi - \frac{1}{2}J^\mu(A_\mu + B_\mu) = \frac{\hbar G_t}{2c^2 g_R} T_\mu \psi \quad (32)$$

The T_μ can be defined intrinsically as the energy density while the volume of the quantum cloud is $V = 2\hbar R^2 / M_C c$ and the gravitational field strength of the cloud-world is given by $G_t M_C / R^2$, thus, $T_\mu = m M_C c^3 / 2\hbar R^2$ where m is the mass of a single electron. Accordingly, the field equations are

$$i\hbar\gamma^\mu \left(\frac{\partial_t}{c} + \vec{\nabla} \right) \psi - \frac{1}{2}i\hbar\gamma^\mu \left(\frac{\partial_t}{c} - \vec{\nabla} \right) \eta_{\mu\nu} \psi - \frac{1}{2}J^\mu(A_\mu + B_\mu)\psi = \frac{1}{2}mc\psi \quad (33)$$

For a single electron, the current $J^\mu = e$ and by choosing the quantum metric signature as $(1, -1, -1, -1)$ as

$$\frac{1}{2}i\hbar\gamma^\mu \left(\frac{\partial_t}{c} + \vec{\nabla} \right) \psi - \frac{1}{2}e\gamma^\mu(A_\mu + B_\mu)\psi = \frac{1}{2}mc\psi \quad (34)$$

This can be reformatted as

$$i\hbar\gamma^\mu \partial_\mu \psi - m\psi = e\gamma^\mu(A_\mu + B_\mu)\psi \quad (35)$$

Equation (35) resembles the Dirac equation and the interaction with the electromagnetic field.

6. Conclusions and Future Works

This study has presented interaction field equations in terms of the brane-world modified gravity and the perspective of gravitization of the quantum mechanics that count for the curvature of the 4D conformal bulk as the induced gravity on the embedded 4D relativistic clouds in addition to the boundary term interactions. The study has considered the implied positive curvature of the early Universe γ as the curvature of the background or the 4D conformal bulk and distinguished it from the localized curvature that is induced into the bulk by the presence of celestial objects that are regarded as 'relativistic 4D cloud-worlds'. Similarly, the quantum clouds are regarded as propagating '4D relativistic quantum clouds' that are embedded in vacuum energy of a field strength reliant on the background curvature as the induced gravity. The interaction field equations were utilized to reproduce the quantum electrodynamics using an undeformed configuration of the quantum cloud. The new boundary term is only significant at high-energy limits such as within black holes and it can remove the singularities and satisfy a conformal invariance theory. Finally, this theoretical work will be tested using observational data in future works.

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