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Maxwell Equations for Material Systems

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Abstract

The Maxwell equations of electrodynamics describe electrical and magnetic forces with great accuracy in the vacuum of space. But the equations of electrodynamics applied to material systems are usually written in a way that obscures their accuracy. The usual formulation of the Maxwell equations includes a dielectric constant as a single real number that grossly over-approximates the actual properties of matter, particularly liquid matter so important in applications of electrodynamics to biology and chemistry.

We rewrite two Maxwell equations here to make clear the precision of the Maxwell equations in the presence or absence of matter. We discuss and derive two well known corollaries that are as universal and precise as the Maxwell equations themselves: (1) a continuity equation that relates charge and flux and (2) a conservation equation in which total current never accumulates at all. The total current is the right hand side of the Ampere-Maxwell law. Total current is perfectly incompressible. It is conserved exactly, everywhere and at every time, independent of the microphysics of charge conduction. The total current combines the flux of charges and the ethereal current $\varepsilon_o \, \partial \mathbf{E}/\partial t$. The ethereal current $\varepsilon_o \, \partial \mathbf{E}/\partial t$ exists everywhere, including the interior of atoms, because it is a property of space, not matter. The ethereal current exists, and thus flows, in a vacuum devoid of mass. The ethereal current is a consequence of the relativistic (Lorentz) invariance of charge. Charge does not change even if it moves at speeds approaching the velocity of light.

Total current has properties quite distinct from flux of mass because of the ethereal current. Most strikingly, in the one dimensional unbranched systems of electronic circuits and biological ion channels, the total current is independent of location, even if the flux of charges varies a great deal. Indeed, the Maxwell equations—and thus conservation of total current—act as a perfect low pass spatial filter, converting the infinite variation of (the Brownian model of) thermal motion of charges to the zero variation of the total current.

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Introduction

The Maxwell equations of electrodynamics describe electrical and magnetic forces with great accuracy in the vacuum between stars. But they are usually written in a way that obscures their accuracy when applied to matter, particularly liquid matter so important in applications of electrodynamics to biology and chemistry. The usual material formulation seems to make them immaterial and imprecise because it depends on a dielectric constant that is a gross over approximation to the actual properties of matter. We rewrite two Maxwell equations here to show at first glance that they are material and universal, in the presence or absence of matter.

Theory

The first Maxwell equation shows how charge creates the electric field. It is classically written in the presence of matter using an over approximated dielectric constant ε_r

$$\varepsilon_r \varepsilon_o \mathbf{div}\, \mathbf{E} = \boldsymbol{\rho}_{\boldsymbol{free}} \tag{1}$$

$\mathbf{E}$ is the electric field. ε_o is the electrical constant, the permittivity of free space. Equation (1) helps define an ideal dielectric, in which ε_r is the dielectric constant, a single real number. Generalizations of the dielectric constant (to a complex or time dependent function(al)) require a rewriting of the Maxwell equations that describes how $\boldsymbol{\rho}_{\boldsymbol{free}}$ varies as $\mathbf{E}$ changes. $\boldsymbol{\rho}_{\boldsymbol{Q}}$ is the charge density of all types of charge, no matter how brief or transient are its movements. $\boldsymbol{\rho}_{\boldsymbol{free}}$ is the density of charge, after the ideal dielectric term $(1-\varepsilon_r)\boldsymbol{\rho}_{\boldsymbol{Q}}$ is subtracted from the total charge

$$\boldsymbol{\rho}_{\boldsymbol{free}} = \varepsilon_r \boldsymbol{\rho}_{\boldsymbol{Q}} \tag{2}$$

$\boldsymbol{\rho}_{\boldsymbol{Q}}$ is much less than $\boldsymbol{\rho}_{\boldsymbol{free}}$ in water solutions, for example, where $\varepsilon_r(\mathrm{H_2O}) \cong 80$ because most of the free charge is screened by the ideal dielectric constant ε_r. The total current $\mathbf{J}_{\boldsymbol{Q}}$ of all charge $\boldsymbol{\rho}_{\boldsymbol{Q}}$ is related to the current $\mathbf{J}_{\boldsymbol{free}}$ produced by the movement of free charge $\boldsymbol{\rho}_{\boldsymbol{free}}$

$$\mathbf{J}_{\boldsymbol{free}} = \varepsilon_r \mathbf{J}_{\boldsymbol{Q}} \tag{3}$$

The electric field has strikingly different properties in the vacuum and in matter but they are lumped into one variable in the classical treatment, the dielectric constant ε_r. This economy of notation obscures physical reality. Lumping joins (1) polarization of material charge that has properties as complex as the motions of matter itself (2) the polarization of space that is exactly simple. Polarization of space is described by a single time derivative and an unchanging constant ε_o within the accuracy of the Maxwell equations themselves.

A slight reformulation separates the material properties—more precisely the properties of charge with mass—from the properties of space. The latter are entirely independent of matter and arise from the relativistic properties of charge. Charge (unlike time, distance, and mass) is independent of velocity even when charge moves at velocities approaching the speed of light. Textbooks on electrodynamics and relativity discuss this issue.

$$\varepsilon_r \varepsilon_o = \overbrace{(\varepsilon_r - 1)\varepsilon_o}^{Material} + \overbrace{\varepsilon_o}^{Ethereal} \tag{4}$$

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The properties of space are named 'ethereal' because they seem to include a flux of charge in a place—that Maxwell called an 'ether'—where no mass or charge exists. See the ethereal term in eq. (6)). Maxwell's eq. (1) becomes

$$\overbrace{\mathbf{div}\,(\varepsilon_r - 1)\varepsilon_o\mathbf{E}}^{Material} + \overbrace{\mathbf{div}\,\varepsilon_o\mathbf{E}}^{Ethereal} = \boldsymbol{\rho_{free}} \tag{5}$$

The second Maxwell equation is the Ampere-Maxwell law showing how the flux $\mathbf{J_{free}}$ of free charge $\boldsymbol{\rho_{free}}$ sums with the current of an ideal dielectric $(\varepsilon_r - 1)\varepsilon_o\ \partial\mathbf{E}/\partial t$ and the ethereal current $\varepsilon_o\ \partial\mathbf{E}/\partial t$ to create a magnetic field $\mathbf{B}$.

$$\tfrac{1}{\mu_0}\mathbf{curl}\,\mathbf{B} = \mathbf{J_{free}} + \varepsilon_r\varepsilon_o\ \partial\mathbf{E}/\partial t = \overbrace{\mathbf{J_{free}}}^{Material} + \overbrace{(\varepsilon_r - 1)\varepsilon_o\ \partial\mathbf{E}/\partial t}^{Material} + \overbrace{\varepsilon_o\ \partial\mathbf{E}/\partial t}^{Ethereal} \tag{6}$$

μ_0 is the magnetic constant, the magnetic 'permeability'.

The two Maxwell equations can be combined by taking the divergence of eq. (6). The divergence of the curl of any function is always zero, as proven in every textbook of vector algebra and most textbooks of electrodynamics, so we have

$$\mathbf{div}\left(\tfrac{1}{\mu_0}\mathbf{curl}\,\mathbf{B}\right) = 0 = \mathbf{div}\left(\overbrace{\mathbf{J_{free}}}^{Material} + \overbrace{(\varepsilon_r - 1)\varepsilon_o\ \partial\mathbf{E}/\partial t}^{Material} + \overbrace{\varepsilon_o\ \partial\mathbf{E}/\partial t}^{Ethereal}\right) \tag{7}$$

The corollary called the continuity equation relates flux and charge and is derived by combining eq (5) and (7), then interchanging the operations of divergence and time differentiation.

$$\mathbf{div}\left(\tfrac{1}{\mu_0}\mathbf{curl}\,\mathbf{B}\right) = \mathbf{div}\left(\mathbf{J_{free}} + \varepsilon_r\varepsilon_o\ \partial\mathbf{E}/\partial t\right) = \mathbf{div}\,\mathbf{J_{free}} + \partial\boldsymbol{\rho_{free}}/\partial t = 0 \tag{8}$$

giving

$$\mathbf{div}\,\mathbf{J_{free}} + \partial\boldsymbol{\rho_{free}}/\partial t = 0 \tag{9}$$

The corollary called conservation of total current $\mathbf{J_{total}}$ is a restatement of eq. (7).

$$\mathbf{div}\,\mathbf{J_{total}} = 0 \tag{10}$$

$$\mathbf{J_{total}} = \overbrace{\left(\overbrace{\mathbf{J_{free}}}^{Material} + \overbrace{(\varepsilon_r - 1)\varepsilon_o\ \partial\mathbf{E}/\partial t}^{Material} + \overbrace{\varepsilon_o\ \partial\mathbf{E}/\partial t}^{Ethereal}\right)}^{\mathbf{J_{total}}} \tag{11}$$

Note that $\mathbf{J_{total}}$ is perfectly incompressible. It cannot accumulate anywhere at any time.

$$\mathbf{div}\left(\overbrace{\left(\overbrace{\mathbf{J_{free}}}^{Material} + \overbrace{(\varepsilon_r - 1)\varepsilon_o\ \partial\mathbf{E}/\partial t}^{Material} + \overbrace{\varepsilon_o\ \partial\mathbf{E}/\partial t}^{Ethereal}\right)}^{\mathbf{J_{total}}}\right) = 0 \tag{12}$$

The flux of free charge $\mathbf{J_{free}}$ can accumulate. As it accumulates, $\boldsymbol{\rho_{free}}$ is created, according to eq. (9), the continuity equation. The flux of charge $\mathbf{J_Q}$ can also accumulate. As it accumulates, it

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creates $\boldsymbol{\rho_Q}$ according to eq. (19), the appropriate continuity equation. Only $\mathbf{J}_{\boldsymbol{total}}$ cannot accumulate.

The dielectric constant ε_r is used traditionally to define perfect dielectrics, but those traditions arose in the 1880's (if not earlier) when knowledge of dielectrics was limited. In applications involving the liquid state, including electrochemistry and biology, the material response to the electric field depends on many properties of the system and is far too complex to be described even approximately by a single real number dielectric constant ε_r. Even in crystalline solids, the response of charge to the electric field has complex dependence on time that cannot be captured by a single real dielectric constant ε_r in the time range used in electronics applications, say times < 10^{-8} sec. A wide range of optical applications of electrodynamics involve responses of matter (mostly electrons) to the electric field that are too nonlinear to be described by the traditional formulation.

Results

A revised formulation of the Maxwell equations focuses attention on the physics of charge movement in response to an electric field. The revision describes all material charge—that is to say, all charge with mass, no matter how transient its movement—in one variable $\boldsymbol{\rho_Q}$. The flux of that charge is $\mathbf{J}_{\boldsymbol{Q}}$. That is to say, $\mathbf{J}_{\boldsymbol{Q}}$ is the flux of charge with mass, no matter how small or transient its movements.

A revised version of Maxwell's first equation involves only one material property, the charge density $\boldsymbol{\rho_Q}$.

$$\overbrace{\mathbf{div}\,\varepsilon_o\mathbf{E}}^{Ethereal} = \overbrace{\boldsymbol{\rho_Q}}^{Material} \tag{13}$$

The revised version of Maxwell's second equation, the Ampere Maxwell law, is

$$\tfrac{1}{\mu_0}\mathbf{curl}\,\mathbf{B} = \mathbf{J}_{\boldsymbol{total}} \tag{14}$$

$$\mathbf{J}_{\boldsymbol{total}} = \overbrace{\mathbf{J}_{\boldsymbol{Q}}}^{Material} + \overbrace{\varepsilon_o\,\partial\mathbf{E}/\partial t}^{Ethereal} \tag{15}$$

$$\tfrac{1}{\mu_0}\mathbf{curl}\,\mathbf{B} = \overbrace{\left(\overbrace{\mathbf{J}_{\boldsymbol{Q}}}^{Material} + \overbrace{\varepsilon_o\,\partial\mathbf{E}/\partial t}^{Ethereal}\right)}^{\mathbf{J}_{\boldsymbol{total}}} \tag{16}$$

A rewriting of Maxwell's second equation anticipates its description of radiation that moves at the velocity of light c, where

$$c^2 = \frac{1}{\mu_0\varepsilon_o} \tag{17}$$

giving an illuminating formulation of the Ampere Maxwell law

$$\mathbf{curl}\,\mathbf{B} = \overbrace{\mu_0\mathbf{J}_{\boldsymbol{Q}}}^{Material} + \overbrace{\tfrac{1}{c^2}\partial\mathbf{E}/\partial t}^{Ethereal} \tag{18}$$

The revised versions of Ampere Maxwell law do not involve any material properties other than the flux of material charge. The term $\varepsilon_o\,\partial\mathbf{E}/\partial t$ creates a magnetic field as if it were a

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current, even if the term exists in a space devoid of mass. Hence the name ethereal. The ethereal term $\varepsilon_o\ \partial\mathbf{E}/\partial t$ is separated in eq. (16) and (18) so the revised Maxwell equations can be applied (as written) to a vacuum devoid of matter $\boldsymbol{\rho_Q} = 0$ where $\mathbf{J_Q}(vacuum)\ = 0$.

The revised continuity equation is one corollary of the Maxwell equations

$$\mathbf{div}\,\mathbf{J_Q} + \partial\boldsymbol{\rho_Q}/\partial t = 0 \tag{19}$$

The revised conservation of total current is another corollary of the Maxwell equations

$$\mathbf{div}\left(\overbrace{\overbrace{\mathbf{J_Q}}^{Material} + \overbrace{\varepsilon_o\ \partial\mathbf{E}/\partial t}^{Ethereal}}^{\mathbf{J_{total}}}\right) = 0 \tag{20}$$

$\mathbf{J_{total}}$ is perfectly incompressible. It cannot accumulate anywhere at any time. The flux of material charge $\mathbf{J_Q}$ can accumulate. As it accumulates, $\boldsymbol{\rho_Q}$ is created according to the continuity eq. (19). Only $\mathbf{J_{total}}$ does not accumulate at all. For that reason, $\mathbf{J_{total}}$ obeys Kirchhoff's current law at the technologically crucial time scales < 10^{-8} sec in the electronic circuits of our computers and digital technology (which are one dimensional branched systems), while $\mathbf{J_Q}$ and $\mathbf{J_{free}}$ do not.

A third corollary is the vector wave equation describing the propagation of light (i.e., electromagnetic signals). It is written here for completeness without the derivation easily available in textbooks. $\boldsymbol{\nabla^2}$ is the vector Laplacian defined in textbooks of vector algebra and electrodynamics.

$$\boldsymbol{\nabla^2}\mathbf{E} = \tfrac{1}{c^2}\partial^2\mathbf{E}/\partial t^2 \tag{21}$$

The revised approach requires a separate theory to describe how material charge $\boldsymbol{\rho_Q}$ changes and flows as the electric field is changed.[1] This revised formulation allows other force fields—like convection, diffusion, or temperature—to drive flux and change charge density by specifying $\boldsymbol{\rho_Q}$ and $\mathbf{J_Q}$.

It is hard to exaggerate the importance of applications in which diffusion moves charge. Semiconductor devices, electrochemical systems, and biological cells (tissues and organs) use diffusion to create and control electric fields and flows. Models must include diffusion of charges in a concentration field as much as migration in an electric field. Convection of charges is a central property of complex fluids that cannot be ignored. And temperature driven flows are important throughout physics and engineering.

Large communities of scientists work on systems in which diffusion and convection move charge and create current. Applications of great economic importance depend on the proper treatment of diffusion, convection, and electrodynamics.

It seems useful to have a form of the Maxwell equations that easily includes charge $\boldsymbol{\rho_Q}$ and flows $\mathbf{J_Q}$ created by diffusive and other fields. The same equations that describe the polarization response of matter to the electric field can be used to describe the movement of charge driven

[1] The change and flow of $\boldsymbol{\rho_Q}$ has many effects that all interact. It (1) creates $\mathbf{J_Q}$; (2) The change of $\boldsymbol{\rho_Q}$ changes the charge distribution and thus electric field. (3) The flow $\mathbf{J_Q}$ creates a magnetic field $\mathbf{B}$.

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by convection, diffusion, and temperature gradients. Charge movements of polarization, diffusion, convection and other fields can be combined with electrodynamics using models of $\boldsymbol{\rho_Q}$ and $\mathbf{J_Q}$ and combining those models with the Maxwell equations in the revised form (13), (15), and their corollaries.

Discussion

Most models start simply and (in their initial forms) do not need to specify realistically the polarization process by which matter moves when the electric field is changed. The classical formulations of the Maxwell equations is then appropriate, eq.(1) or (5) and (6) (and corollaries). It is better to use an over-simplification than nothing.

When models are refined, they need to describe the polarization process more realistically. Conversion to the revised formulation (13), (16) (and corollaries) is then needed, and fortunately quite straightforward: just set $\varepsilon_r = 1$ and thereby remove the dielectric constant ε_r from the equations eq. (1) or (5) and (6) (and corollaries). A model or lookup table is then needed to describe the polarization phenomena and the material charge $\boldsymbol{\rho_Q}$ and its flow $\mathbf{J_Q}$. Those models can easily accommodate the charges and flows created by fields other than classical electrodynamics, e.g., concentration fields that drive the diffusion of charges or pressure fields that drive the convection of charges.

The polarization model is a description of the movement of mass in response to the electric and magnetic fields and must be solved together with the Maxwell equations in a joint field problem that so far lacks a general name. Variational methods are helpful in dealing with such joint problems because their solutions are always consistent. Their solutions automatically satisfy all field equations and boundary conditions with one set of unchanging parameters.

A simple but helpful model of polarization in many systems includes charged masses on damped springs that (perhaps) interact according to Maxwell's equations or in their approximation, the Poisson equation. It is important that these models do not simply assume the shape or strength of the electric (or magnetic) fields because the shape of those fields is not a constant (in space or time or with conditions) in most applications. Rather the models must calculate the electric and magnetic fields from the charges, currents, and boundary conditions.

Polarization in other applications will need other models. Models are most useful if they include (idealizations of) the setups used in actual experiments, like the conductivity cell of electrochemistry and biophysics. Analysis of those setups provide operational definitions of polarization and dielectric phenomena as described in experimental publications. General models of polarization are likely to be too vague to be of particular use.

The corollaries of the Maxwell equations have striking implications. The forces exerted by electric and magnetic fields $\mathbf{E}$ and $\mathbf{B}$ move atoms so total current is exactly conserved at all times and places, in a result that might seem magical if it were not the immediate consequence of the mathematical identity $\mathbf{div\ curl} = 0$.

These electrodynamic forces are the dominant forces between atoms in many, even most conditions. They are usually much more important than the steric forces (including van der Waals forces). Indeed, the classical systems of ideal gases, or ideal ionic solutions do not include steric forces at all.

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Steric forces appear when charged particles have finite size, in (nonideal) gases and ionic solutions. They appear when atoms approach each other and (nearly) collide. Steric forces tend to dominate only when atoms are crowded at very large densities, of the order of $10\ \mathrm{M} \cong 6 \times 10^{21}$ cm^{-3}, as found in some important biological applications, where steric forces balance electrical forces to create selectivity between different elements or molecules. For reference, solid Na^+Cl^- is about 37 M**,** water is 55 M.

Calculating forces between the enormous numbers of atoms in macroscopic systems is challenging and so it is difficult to implement the continuity equation (19) on an atomic scale. In simulations of proteins, for example, it is difficult to keep track of all the movements of atoms as they interact. Conservation of total current eq. (20) is much easier to implement. It does not require calculation of forces between atoms at all.

The striking generality of conservation of total current $\mathbf{J}_{total}$ is not magic. It arises because the Maxwell equations specify precisely those $\mathbf{E}$ and $\mathbf{B}$ fields needed to enforce the corollaries. $\mathbf{E}$ and $\mathbf{B}$ fields are solutions of the Maxwell equations that change (as conditions, parameters, locations, or time change) so total current $\mathbf{J}_{total}$ is conserved exactly, everywhere, at any time that the Maxwell equations apply, independent of the microphysics of conduction of $\mathbf{J}_{free}$ or $\mathbf{J}_{Q}$.

$\mathbf{J}_{total}$ can have very different properties from $\mathbf{J}_{Q}$ or $\mathbf{J}_{free}$. The total current is a special variable (surprisingly different from other flux variables) because of its ethereal component $\varepsilon_o\ \partial\mathbf{E}/\partial t$ that is a property of space, not matter. (Models of mass flow have no such term because those models describe nothing in a vacuum. Only electrical total current contains an ethereal component.)

When systems are one dimensional, as in many biological and engineering applications, conservation of current is particularly easy to understand. The electric and magnetic fields $\mathbf{E}$ and $\mathbf{B}$ assume values that enforce equality of current (in unbranched systems) or enforce Kirchhoff's current law in general. The changing $\mathbf{E}$ field creates an ethereal current $\varepsilon_o\ \partial\mathbf{E}/\partial t$. The changing $\mathbf{E}$ field also helps move charges $\boldsymbol{\rho}_{Q}$ to make a material current $\mathbf{J}_{Q}$. The sum of the ethereal current $\varepsilon_o\ \partial\mathbf{E}/\partial t$ and material current $\mathbf{J}_{Q}$ is the total current $\mathbf{J}_{total}$ that is exactly conserved.

Conservation of total current is not just a restatement of the Maxwell equations. Eq. (20) is enough to allow the design and understanding of important devices. For example, the circuits of our electronic technology or the ion channels of biological membranes (or their biomimetic analogs) can be designed with little else. Kirchhoff's current law applied to $\mathbf{J}_{total}$ is often all that is needed for design, without knowing atomic locations or forces at all. [2] Indeed, circuits of electronic devices designed using Kirchhoff's law function as expected at times < 10^{-9} sec, even though the original derivations of Kirchhoff's law for the material fluxes $\mathbf{J}_{Q}$ or $\mathbf{J}_{free}$ are not helpful on those time scales.

[2]This is not true for $\mathbf{J}_{Q}$ or $\mathbf{J}_{free}$. Only $\mathbf{J}_{total}$ follows Kirchhoff's law under all conditions, because only $\mathbf{J}_{total}$ does not accumulate. $\mathbf{J}_{Q}$ or $\mathbf{J}_{free}$ accumulate net charge that must be described by a circuit element added to the original circuit, often called a 'stray' capacitance. $\mathbf{J}_{Q}$ and $\mathbf{J}_{free}$ have much more complicated circuit properties than $\mathbf{J}_{total}$.

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The special properties of total current are particularly striking when thermal noise is important, as it always is in ion channels, and as it often is in semiconductor devices, where shot noise can be so important.

Consider the special case when $\mathbf{J}_{total}$ is just thermal noise. $\mathbf{J}_{total}$ (thermal) satisfies conservation of current like any other total current, because the Maxwell equations (and its corollaries) are valid on all time and space scales including those of thermal motion. The consequences are dramatic in the one dimensional systems of biological ion channels and the two terminal devices of electronic circuits. In those systems $\mathbf{J}_{total}$(thermal) is a constant in space because in those systems conservation of total current guarantees equality of total current in space.

$\mathbf{J}_{total}$(thermal) is not a constant in all respects. $\mathbf{J}_{total}$(thermal) varies in time, for example, as shot noise does in models inspired by the pioneering work of Landauer.

The properties of thermal movement of material charge $\mathbf{J}_{Q}$ (thermal) are very different from the properties of the thermal total current $\mathbf{J}_{total}$(thermal). The spatial dependence of thermal movement of charge $\mathbf{J}_{Q}$ (thermal) is nearly a Brownian stochastic process that reverses direction (in one dimensional systems) an infinite number of times in any finite distance, no matter how short. But the thermal total current $\mathbf{J}_{total}$(thermal) does not reverse itself at all. It is a constant in space in these important systems at all times, including the time/space scale of thermal motion.

The Maxwell equations act as a perfect low pass spatial filter, converting the infinite variation of material movement $\mathbf{J}_{Q}$(thermal) in a Brownian trajectory to the zero variation of total current $\mathbf{J}_{total}$(thermal). The ethereal current simplifies the system quite remarkably, suggesting that it should be included explicitly in models, not just as part of *ad hoc* stray capacitances.

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Appendix: Sources and References

References are grouped here by topic to keep the text less cluttered,.

Electrodynamics texts of great quality are widely available, including [1-16]. Ref. [15] is a modern place to start. Ref. [3, 11] explicitly recommend a revision in the classical treatment of polarization. Ref. [12] describes specific models of polarization. Many of these texts describe the relation of special relativity and the Maxwell equations because as Einstein put it "The special theory of relativity ... was simply a systematic development of the electrodynamics of Clerk Maxwell and Lorentz" (p. 57 of [17]). Special focus is found in [5, 18, 19].

Vector algebra is described to my taste in ref. [20] and [21] and there are innumerable other references and treatments in texts of electrodynamics.

Dielectric properties are described in thousands of papers which can be accessed directly or though the classical papers [22-27]. Ref. [28] shows explicitly how the descriptions of molecular spectroscopy are equivalent to descriptions of electrodynamics.

Optical applications are found in [29-32].

Theory of electronic devices and networks are described in innumerable books [33-39], including neglected classics [40]. Computational electronics and Landauer inspired models of shot noise are found in [41-51].

'Crowded Charge' as a determinant of selectivity in biological systems has received a great deal of attention starting with the early work of Nonner and Eisenberg [52, 53] reviewed in [54, 55]. Recent reviews include [56-58]. Physical systems receive emphasis in [59-62].

A number of preprints and papers describe the slow evolution of my understanding of these issues [63-72].

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