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Scaling law for the velocity of domino toppling motion in curved paths

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The arranged paths of dominoes have many shapes. The scaling law for the propagation speed of domino toppling has been extensively investigated. However, in all previous investigations, the scaling law for the velocity of domino toppling motion in curved lines was not taken into account. In the present work, the finite-element analysis (FEA) program ABAQUS was used to study the velocity of domino toppling motion in curved lines. It is shown that the domino propagation speed has a rising trend with increasing domino spacing in a straight line. It is also found that domino propagation speed is linearly proportional to the square root of domino separation. This research proved that the scaling law for the speed of domino toppling motion given by Sun (2020) is true [B-H. Sun, 2020. Scaling law for the propagation speed of domino toppling. AIP Advances, 10(9),095124.]. Moreover, the shape of domino arrangement paths has no influence on the scaling law for the propagation speed of dominoes but can affect the coefficient of the scaling law for the velocity. Therefore, the amendatory function for the propagation speed of dominoes in curved lines was formulated by the FEA data. The fitted amendatory function, φ_{revise}, provides the simple method for a domino player to quickly estimate the propagation speed of dominoes in curved lines.

I. INTRODUCTION

The game of dominoes attract many people because of its playfulness. Although dominoes is a simple game, it contains rich physical information. The mechanics of domino toppling motion have been extensively studied[1–15]. McLachlan et al.[3] presented a function for the propagation velocity, the spacing between dominoes, gravitational acceleration, and domino height when dominoes of zero thickness are spaced in a straight path. The functional relation is $v_{McLachlan} = v(h,\lambda,g) = \sqrt{gh}f(\frac{\lambda}{h})$, where g is gravitational acceleration, h the domino height, λ the spacing between dominoes, and $f(x)$ an uncertain function of x. Szirtes and Rozsa[6, 14] adopted dimensional analysis to investigate the propagation speed of dominoes. The main parameters are the domino thickness δ, domino separation λ, domino height h, gravitational acceleration g, and domino propagation speed $v = v(h,\lambda,\delta,g)$. The propagation speed of dominoes was determined by dimensional analysis to be $v_{Szirtes} = \sqrt{gh}f(\frac{\lambda}{h},\frac{\delta}{h})$. Shi et al.[12, 13] studied the law of domino toppling with different masses and the same domino spacing by using the numerical model.

In 2020, Sun[15] obtained a new domino wave speed by using directional dimensional analysis, namely

$$v_{Sun} = \lambda\sqrt{\frac{g}{h}}f(\frac{\delta}{\lambda}). \quad (1)$$

To obtain a simple explicit scaling law for the propagation speed of dominoes, the function $f(\frac{\delta}{\lambda}) \approx C(\frac{\delta}{\lambda})^{\alpha}$ was deduced by curve fitting of the experimental data. Simultaneously, the index $\alpha \approx \frac{1}{2}$ was confirmed with Stronges'[8] experimental data, and the coefficient C is affected by the friction coefficient of domino surfaces. Finally, Sun[15] indicated that domino width has little influence on the propagation speed of domino toppling and proposed the speed is $v_{Domino} \sim \lambda^{1/2}\sqrt{\frac{\delta g}{h}}$.

From the literature, it is found that all works are focused on the mechanics of domino toppling motion in straight lines. To the best of our knowledge, there is no study on domino toppling motion in curved paths. In the present work, the finite-element analysis (FEA) program AQABUS was used to explore the mechanics of domino toppling motion in curved paths. The different path shapes. i.e., straight, circular, and S-shaped paths, were investigated using FEA. The scaling law for the propagation speed of domino toppling in different curved paths was proposed by data fitting. The scaling law obtained in the present work is consistent with the result of Sun[15], when the curvature is zero. However, the scaling law proposed in this paper for Sun[15] was proved using the scaling law for the propagation speed of domino toppling.

II. PROPAGATION SPEED OF DOMINO TOPPLING IN STRAIGHT PATH

To study the law between the propagation speed of dominoes and domino spacing in the straight paths accurately, ABAQUS was used to simulate the numerical analysis for the propagation speed of 17 domino models with various domino separations. The details of the dominoes are shown in Table I. In the FEA, discrete rigid bodies were used to simulate the ground and three-dimensional (3D) deformable bodies to simulate the dominoes. The gravitational acceleration is $g = 9800m/s^2$ and the friction coefficient is 0.3. The reduced integration linear shell elements (S4Rs) and integration linear solid elements (C3D8Rs) were used for their accuracy in predicting reliable and speedy results. The details of the FE

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model are shown in Table I and Fig. 1.

Table I: Domino parameters [Length unit: mm]

Height	Width	Thickness	Number	Separation
h	w	δ	N	λ
50	20	8	60	5
50	20	8	60	7
50	20	8	60	9
50	20	8	60	11
50	20	8	60	13
50	20	8	60	15
50	20	8	60	17
50	20	8	60	19
50	20	8	60	21
50	20	8	60	23
50	20	8	60	25
50	20	8	60	27
50	20	8	60	29
50	20	8	60	31
50	20	8	60	33
50	20	8	60	35
50	20	8	60	37

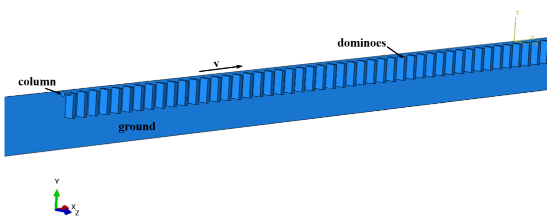

Figure 1: FE model.

The FEA results are shown in Fig. 2. It can be seen from the curve, when the domino spacing is 0∼19 mm, that the domino propagation speed exhibits a rising trend with increasing domino spacing in the straight path. However, the propagation speed of domino toppling decreases with increasing spacing as the domino separation exceeds 20 mm. In this paper, the focus is only on the part in which the propagation velocity of dominoes increases with increasing domino spacing. Thus, the data within the range of 0∼19 mm will be redrawn to specifically investigate the law for the propagation speed of dominoes.

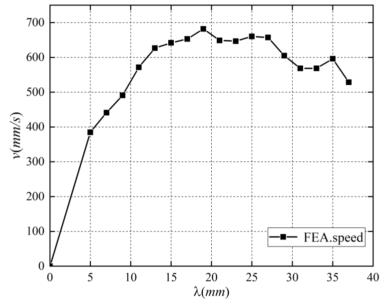

Figure 2: Domino propagation speed for FEA.

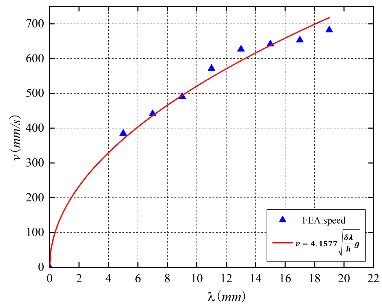

Figure 3: Dominoes propagation speed for $\lambda = 0 \sim 19mm$.

Using the function from Ref. [15] to fit the curves shown in Fig. 3, the explicit speed of domino toppling is obtained as follows:

$$v_{straight} = C_{straight}\lambda^{1/2}\sqrt{\frac{\delta g}{h}}, \tag{2}$$

where $C_{straight} = 4.1755$, δ denotes the domino thickness, λ the domino spacing, h the domino height, and g the gravitational acceleration.

It is surprising to see that the function $v_{Sun} = C_{sun}\lambda^{1/2}\sqrt{\frac{\delta g}{h}}$ from Ref. [15] can fit the curves perfectly. There is little distinction, except for the constant coefficient C. The data fitting gives $C_{straight} = 4.1577$, but Ref. [15] reports $C_{Sun} = 3.488$. As is pointed out in Ref. [15], the coefficient C difference is due to different friction coefficients. When the domino spacing range is 0∼19 mm, in general the law for propagation velocity and domino spacing satisfy the scaling law for the propagation speed of dominoes given in the literature[15]. In addition, verifying the FE models can accurately simulate the scaling law for the propagation speed of domino toppling.

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III. PROPAGATION SPEED OF DOMINO TOPPLING IN CIRCULAR PATH

Using accurate FE models, domino models were designed with a circular path. The radius of the circular path is 150∼250 mm and the domino spacing range is 0∼19 mm. Owing to the fact that the speed direction of the domino with the circular path will change, only the speed value is explored. Details of the FE models and dominoes are shown in Fig. 4 and Table II.

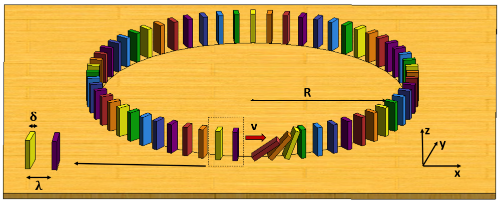

Figure 4: Details of dominoes in circular path.

Table II: Parameters of dominoes in circular path [Length unit: mm]

Height	Width	Thickness	Radius	Separation
h	w	δ	R	λ
50	20	8	150	7.7
50	20	8	170	9.7
50	20	8	190	11.7
50	20	8	210	14
50	20	8	230	16.09
50	20	8	250	18.18

Table II shows that the radius increased from 150 to 250 mm and domino spacing gradually increased from 7.7 to 18.18 mm. Using the FE results, the scatter diagram between the propagation speed of domino toppling and domino spacing is plotted. Figure 5 indicates that the domino propagation speed has a rising trend with increasing radius in the circular path when the radius is 150∼250 mm (domino spacing is 7.7∼18.18 mm). Compared to the FE model in which the dominoes are in the straight path, the propagation speed of domino toppling in the circular path is lower than in the straight model at the same domino spacing. It is found that domino propagation speed in the circular path is linearly proportional to the square root of domino separation by using the data fitting. The equation is expressed as follows:

$$v_{circle} = C_{circle}\lambda^{1/2}\sqrt{\frac{\delta g}{h}}, \quad (3)$$

where $C_{circle} = 3.289$. It can be seen that formula (3) is similar to the law provided in the literature,[15] except for the value of the coefficient C. Therefore, it is known that the shape of the domino arrangement paths has little influence on the scaling law for the propagation speed of domino toppling.

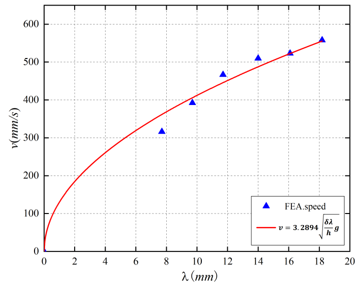

Figure 5: Dominoes propagation speed with circular path.

IV. PROPAGATION SPEED OF DOMINO TOPPLING IN S-SHAPED PATH.

The results of the domino FE models with straight and circular paths demonstrated that the shape of domino arrangement paths has little influence on the scaling law for the propagation speed of domino toppling, but it can change the value of the velocity. Therefore, domino FE models with different curvature path were designed. The details of the geometric parameters of dominoes are shown in Table III. To change the curvature, the spacing must be first fixed as 19 mm of each domino, and then the dominoes moved in the width direction and the dominoes arranged in an S shape. There are many different curvatures because of different distances (Δw). The curvature calculation formula is $K = 1/R$. The model of a domino with an S-shaped path as displayed in Fig. 6.

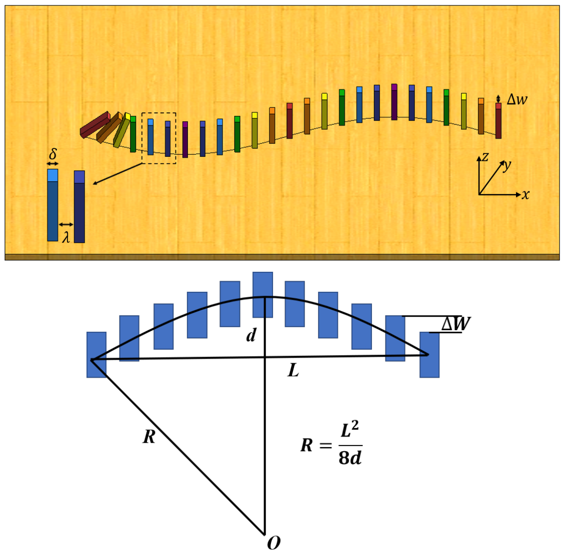

Figure 6: Parameters of dominoes in S-shaped path.

Table III shows seven FE models with various curvatures that were designed. The range of Δw is 2∼14 mm. FE analysis shows that the domino is not successively toppling when

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Table III: Parameters of dominoes in S-shaped path.

h	w	δ	λ	Δw	K
50	20	8	19	2	0.00096
50	20	8	19	4	0.00188
50	20	8	19	6	0.00274
50	20	8	19	8	0.00352
50	20	8	19	10	0.00420
50	20	8	19	12	0.00478
50	20	8	19	14	0.00525

$\Delta w = 12$ *and* $14mm$. The failure modes are shown in Figs. 7 and 8.

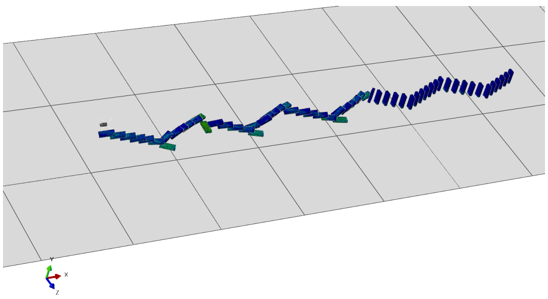

Figure 7: Failure mode for $\Delta w = 12mm$.

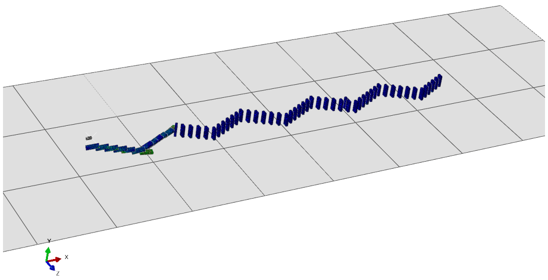

Figure 8: Failure mode for $\Delta w = 14mm$.

From Fig. 7, when $\Delta w = 12mm$, only 37 dominoes topple. With increasing Δw, the number of toppling dominoes decreases. When $\Delta w = 14mm$, just 13 dominoes topple. This is demonstrated when Δw exceeds half the width of the dominoes, and the dominoes do not successively topple completely. Owing to the fact that 60 dominoes in the two models have not been successfully toppled, the focus is on the propagation speed of dominoes toppling with $\Delta w = 2 \sim 10mm$. The domino propagation velocities are shown in Table IV.

As shown in Table IV, it can be clearly seen that the propagation speed of dominoes gradually decreases as curvature increases. In addition, with increasing path radius, the propagation speed of dominoes shows an increasing trend. This is the same law as seen in the FE models with a circular path. To study the effect of curvature on the propagation speed of domino toppling, dimensionless methods were used to deal with $v_{S-num.}$, i.e., dividing the propagation speed of domino toppling by $v_{straight}$. Using the data from FE results, the

Table IV: Propagation velocities of dominoes in S-shaped path.

h	δ	Δw	R	K	$v_{S-num.}$	φ
50	8	2	1046.17	0.0009559	618.109	0.906
50	8	4	532.08	0.0018794	589.327	0.864
50	8	6	364.72	0.0027418	574.262	0.842
50	8	8	284.04	0.0035206	485.807	0.712
50	8	10	238.03	0.0042011	393.164	0.577

[1] Propagation speed of dominoes with S-shaped path by FEA is $v_{s-numerical}$.
[2] $\varphi = v_{s-numerical}/v_{straight}$ when $\lambda = 19mm$ *and* $v_{straight} = 681.927mm/s$.

modified formula of propagation speed is proposed, and the equation is expressed as follows:

$$\varphi_{revise} = C_{straight}\left[\alpha(\frac{h}{R})^3 + \beta(\frac{h}{R})^2 + \gamma(\frac{h}{R}) + 1\right], \quad h = 50mm, \quad R = (240 \sim 1050mm), \tag{4}$$

where $\alpha = -109.38$, $\beta = 27.54$, and $\gamma = -2.36$.

Therefore, the equation for the propagation speed of domino toppling with curved paths is expressed as follows:

$$v_s = C_{straight}\lambda^{1/2}\left[\alpha(\frac{h}{R})^3 + \beta(\frac{h}{R})^2 + \gamma(\frac{h}{R}) + 1\right]\sqrt{\frac{\delta g}{h}}, \quad \lambda = (0 \sim 19mm), \quad R = (240 \sim 1050mm), \quad h = 50mm. \tag{5}$$

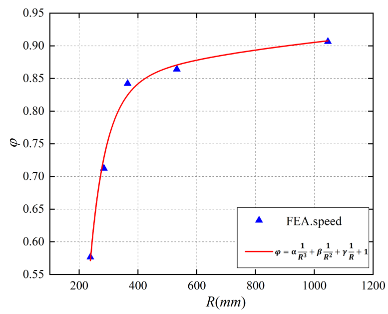

Figure 9: Modified formula of domino toppling motion.

V. EQUATION VERIFICATION

To validate formula (5), the propagation speeds of domino toppling obtained from this equation were then compared with those obtained from FE analysis. The results are shown in Table V. As illustrated by the results presented in Table 5, the propagation speeds calculated from Eqs.(5) show a strong similarity to the FE results. The errors of the three model results are within 6%. It can be shown that the propagation speed of domino toppling in the curved paths was realized using Eqs. (5).

Table V: Comparisons of FEA results with those from Eqs. (5) [Length unit: mm, time: second]

h	δ	λ	R	K	v_s	$v_{S-num.}$	Error (%)
50	8	7	314.17	0.003183	383.30	383.8	0.1
50	8	9	406.17	0.002462	455.52	438.4	3.9
50	8	18.18	250	0.004	529.46	557.76	5.1

VI. CONCLUSIONS

The propagation speed of domino toppling in differently shaped paths was studied numerically. In the numerical analysis, the domino propagation speed in the straight path is linearly proportional to the square root of domino separation. The law for domino propagation velocity and spacing satisfies the scaling law for the propagation speed of dominoes given in the literature,[15] and the FE analysis proved that the scaling law for the speed of domino-toppling motion given in the literature[15] is true. By analyzing the FE models with a circular path, the domino propagation speed in the circular path is linearly proportional to the square root of domino separation. Comparing the FE models in the straight path, the same scaling law for the propagation speed of domino toppling applies, but the value for speed in the circular path is lower than that of the straight models at the same domino spacing. It was revealed that over a specific range of domino spacing the shape of domino arrangement paths has no effect on the scaling law of the propagation speed of domino toppling, but can change the value of the domino propagation speed. To study the effect of curvature in a curved path on the propagation speed of domino toppling, domino FE models with different curvatures were considered. The modified formula φ_{revise} for propagation speed was proposed. Furthermore, the equation for the propagation speed of domino toppling with curved paths over a specific range of domino spacing was obtained; the function is

$$v_s = \varphi v_{straight} = \varphi_{revise}\lambda^{1/2}\sqrt{\frac{\delta g}{h}}$$

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