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Assessing Market Risk in BRICS and Oil Markets: An application of Markov Switching and Vine Copula

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Abstract: This paper investigates the dynamic tail dependence risk between BRICS economies and world energy market in the context of the COVID-19 financial crisis of 2020, to determine optimal investment decisions based on risk metrics. For this purpose, the study employs a combination of novel statistical techniques ranging from Markov Switching, GARCH and Vine copula. Using a dataset consisting of daily stock and world crude oil prices; we find high probability of transition between lower and higher volatility regimes. Furthermore, our results based on the C-Vine copula confirm the existence of two types of tail dependence: - *symmetric tail dependence* between South Africa and China; South Africa and Russia; and *lower tail dependence* between South Africa and India; South Africa and Brazil; South Africa and Oil. For the purpose of diversification in these markets, we formulate an asset allocation problem using C-vine copula-based returns and optimize it using Particle Swarm algorithm with a rebalancing strategy. The results show an inverse relationship between the risk contribution and asset allocation of South Africa and oil market supporting the existence of lower tail dependence between them. This suggests that when South African stocks are in distress, investors tend to shift their holdings in oil market. Similar results are found between China and oil. In the upper tail, South African asset allocation is found to have an inverse relationship with that of Brazil, Russia and India suggesting that these three markets might be good investment destinations when things are not good in South Africa and vice-versa.

Keywords: *BRICS, Markov Switching, Tail dependence, Vine Copula, Conditional Value-at-Risk.*

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1. Introduction

Understanding the intricate dependence and risk characteristics between BRICS economies stock and energy market (WTI Oil) is a vital question for international investors who strive to mitigate financial risks when allocation asset portfolios. By the same token, government regulatory institutions and policymakers are responsible to ensure sustainability of the financial system and macroeconomic performance in their respective economies. In financial markets, crude oil is one of the most essential global traded commodity. It functions as the underlying asset class in the pricing of other financial assets. For instance, the current global food supply chain is highly dependent on fuel and transport systems. Hence, volatility in oil price has the potential to drive higher food prices, thereby inducing inflation and increasing volatility in the market (see Nguyen and Bhatti (2012).

In wealth allocation and diversification of risk, investors' are more sensitive to a portfolio downside risk than the upside risk. Hence, they are risk averse to extreme negative returns and

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negative market sentiments, which results in overselling stocks thus, driving volatility. The current global pandemic caused by the corona virus (aka COVID-19) coupled with the oil price wars has increased market volatility impacting most asset portfolios and balance sheets gearing (leverage) ratios. This has led to mounting fears about the relationship between the recent oil price movement and its impact on stock markets performance among the top developing economies primarily due to their dependence on oil as a commodity. It is well documented, that embedded risk of a portfolio is directly connected to the dependence structure between the portfolio's risk factors (Brechmann and Czado (2013); and Kole, Koedijk, and Verbeek (2007); and Junker and May (2005) to the fact that when aggregating financial risk on a portfolio, it is important to understand the fundamental dependence structure among risk factors.

The study first contributes to the existing literature on BRICS by using novel methods to model the dependence risk properties of the portfolios under specific market situations. We attempt to examine the dynamical relationships and interdependence that exist between BRICS economies and world energy market using Markov switching GARCH and multi-dimensional copula models. Copula models have been used in portfolio optimization (see,Bekiros, Hernandez, Hammoudeh, and Nguyen (2015); Low, Alcock, Faff, and Brailsford (2013); de Melo Mendes and Marques (2012)). Furthermore, the study uses asymmetric conditional volatility specifications to relax the restrictions of symmetric standard GARCH where the variance only depends on the magnitude and not the positivity and negativity (sign) of past innovations. Nelson (1991) proposed the most popular exponential GARCH model to capture the asymmetric effects in time series data. Our combined modelling strategy is flexible to account for leverage effects, regime switching volatility as well as account for the pair wise dependence structure.

The approach provides a degree of robustness to outliers in comparison to previous studies. Hence, the type of information obtained could be used to improve the hedging strategies, risk management and portfolio rebalancing, (see Ji, Liu, Zhao, and Fan (2018); Bouri, Shahzad, Raza, and Roubaud (2018); Hernandez (2014) and Salisu and Gupta (2020)). Asset allocation for loss-averse investors is tested by minimizing conditional value at risk (CVaR) risk measure. To achieve optimal asset allocations, the study performs a Monte Carlo simulation to forecast a constrained portfolio conditional Value at Risk (CVaR) and to determine an optimal asset weight allocations. Portfolio optimization is achieved by a Particle Swam Optimization (PSO) , an evolutionary algorithm which is a nature inspired method of social swarm behaviour that offers the advantages of simplicity, for details see BoussaïD, Lepagnot, and Siarry (2013) and references therein.

There is an abundance of literature that discuss the theory on short and long run volatilities of stock market returns and the corresponding correlations, see for example the study of Sensoy, Hacihasanoglu, and Nguyen (2015). Since the establishment of BRICS grouping in 2009, several papers have been written on methodological approaches to understand the contagion and spillover effects in BRICS economies with other financial markets and in particular alternative assets classes. At the centre, are multivariate generalizations of GARCH models first introduced by Bollerslev (1990) and (Engle, 2002) and a number of other researchers in the field which try to address the ex-

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istence of conditional volatility problem in financial assets and how it helps in portfolio diversification and asset allocation. These models have undoubtedly emerged as the most popular tools that offer flexibility to capture the dynamics of conditional variance and covariance between markets and in turn aid with interpretation of dependency structure.

A few but not an exhaustive list include the most recent study in McIver and Kang (2020) who propose a multivariate dynamic equicorrelation model (DECO-GJR-GARCH) introduced by Engle and Kelly (2012) in order to overcome the curse of dimensionality of the Dynamic Conditional Correlation (DCC) GARCH. The study examines dynamics of spillovers between the BRICS and US stock markets and conclude that the U.S., Brazilian and Chinese markets are major source of volatility, whereas the Russian, Indian, and South African markets are mostly on the receiving end.

A different approach was conducted in Kocaarslan, Sari, Gormus, and Soytas (2017) where they investigate the impact of volatility between BRIC and U.S. stock markets with a combination of quantile regressions and time varying asymmetric dynamic conditional correlation (aDCC) GARCH and find volatility asymmetries in BRIC and U.S. equity markets. A similar study was also done in Morema and Bonga-Bonga (2020) using a vector autoregressive asymmetric dynamic conditional correlation generalised autoregressive conditional heteroskedasticity (VAR-ADCC-GARCH) to assess volatility spillovers and hedge effectiveness between gold, oil and equity market. They find significant volatility spillover between the gold and stock markets, and the oil and stock markets. Whereas, Bonga-Bonga (2018) assess the extent of financial contagion between south Africa and other BRICS countries with VAR-DCC-GARCH and finds evidence of cross-transmission and dependence between South Africa and Brazil. Mensi, Hammoudeh, Nguyen, and Kang (2016) examine dynamics of spillovers between the BRICS and US stock markets multivariate Dynamic Conditional Correlation Fractionally Integrated Asymmetric Power ARCH (DCC-FIAPARCH) that captures long memory property in time series and find that Brazil and China are the major sources of spillover effects. Bhar and Nikolova (2009) find negative interdependence between the BRICS and other markets.

Since Sklar (1959) seminal paper, copula models have recently gained popularity as robust tools to quantify non-linear dependences and non-Gaussian returns in financial markets due to their flexibility to capture and model dependence structure separately from the distribution margins without the loss of information in the joint distribution. In particular, vine copula which are a class of copulas also known as pairwise copula constructions (PCCs) were first introduced in Aas, Czado, Frigessi, and Bakken (2009) as more efficient techniques built from graph theory to model high dimensional dependence structure. Perhaps, early application to financial economics using GARCH filter can be found in Brechmann and Czado (2013). A study by Kumar, Tiwari, Chauhan, and Ji (2019) examine dependence structure between the BRICS stock and foreign exchange markets with dependence-switching copula. They find symmetric tail dependence during negative correlation regimes for all countries with the exception of Russia and find asymmetric dependencies for all countries during positive correlation regimes. An application of a combination of GARCH and copula models can be found in Hou, Li, and Wen (2019) where they examine evidence on the volatility spillover between fuel oil and stock index futures markets in China with DCC-GARCH model to quantify the nonlinear interdependences.

Other recent similar studies are in BenSaïda (2018) where they investigated the contagion effect in European sovereign debt markets and find a better performance of regime-switching copula models in comparison to the single-regime copula. While, Sui and Sun (2016) use vector autoregressive model (VAR) without the volatility structure to test spillover effects. They find U.S. shocks to significantly influence stock markets in Brazil, China, and South Africa. Chkili and Nguyen (2014) compliments the study by adopting a Markov switching VAR framework with regime shifts in both the mean and variance a model. Their framework allows to not only detect potential regime shifts in the stock market returns, but also investigate the impact of crises on the stock market volatility.

More copula approaches are found in, Kenourgios, Samitas, and Paltalidis (2011) who investigated financial contagion in (BRIC) and two developed markets (U.S. and U.K.) with a regime switching copula and combined with a GARCH model. The study used a multivariate time-varying asymmetric regime-switching copula model, with marginals assumed to follow a GJR-GARCH framework. In another setting Mba and Mwambi (2020) employ Markov switching GARCH to quantify risk among cryptocurrency portfolio selection and optimization problem.

Clearly, there is a vast literature on methodological approaches which contains mixed findings on the direction and existence of co-movements among BRICS and Oil markets. However, few studies have taken into account the impact of switching volatility markets and joint dependency structure on BRICS and Oil markets to assess diversification benefits. Hence, this presents an opportunity to contribute to the existing literature by exploring alternative model construction to capture complex dependence structures that correctly account for downside risk in a portfolio. The reminder of this paper is organized as follows: Section 2, provides the adopted methodological background, Section 3 outlines the data descriptive statistics of BRICS and oil market indices together with a detailed summary of the empirical findings and finally, Section 4 concludes the paper.

2. Econometric modeling framework

2.1 Specification of VAR(2) model

In the first step of the estimation process we consider that a d-dimensional random vector of stock log returns. We filter the logarithmic returns through a VAR(2) conditional mean and then apply the two-state Markov-switching skewed Student-t GJR-GARCH model. The vector autoregressive VAR(2) model satifies the following equation,

$$\begin{aligned} r_{1,t} &= \alpha_1 + \beta_{11,1} r_{1,t-1} + \cdots + \beta_{1d,1} r_{d,t-1} + \beta_{11,2} r_{1,t-2} + \cdots + \beta_{1d,2} r_{d,t-2} + \varepsilon_{1,t} \\ r_{2,t} &= \alpha_2 + \beta_{21,1} r_{1,t-1} + \cdots + \beta_{2d,1} r_{d,t-1} + \beta_{21,2} r_{1,t-2} + \cdots + \beta_{2d,2} r_{d,t-2} + \varepsilon_{2,t} \\ \vdots \quad & \vdots \\ r_{d,t} &= \alpha_d + \beta_{d1,1} r_{1,t-1} + \cdots + \beta_{dd,1} r_{d,t-1} + \beta_{d1,2} r_{1,t-2} + \cdots + \beta_{dd,2} r_{d,t-2} + \varepsilon_{d,t} \end{aligned} \tag{1}$$

where $t \in T$ is the number of observations, T, is called the sample size or time series length. The parameter $\beta_{ij,k}$ $(i = 1, \cdots, d)$, $j = (1, \cdots, d)$, $i = 1,2)$ are fixed model coefficients and $\alpha_i = (1, \cdots, d)'$ form a fixed $(d \times 1)$ vector of intercepts. $\varepsilon_t = (\varepsilon_{1t}, \cdots, \varepsilon_{dt})'$ is a d-dimensional vector of white noise or innovation process, that is, $E(t) = 0$, the covariance $E(\varepsilon_t, \varepsilon'_t) = \Sigma_{\varepsilon_t}$ and $E(\varepsilon_s, \varepsilon'_t) = 0,\ t \neq s$. The covariance matrix Σ_{ε_t} is assumed to be non-singular if not otherwise stated. All variables in the system are simultaneously estimated and each variable $r_{i,t}$ is a linear function of the lag 1 values and lag 2 values of all variables considered in the set.

2.2 Markov Switching-GJR-GARCH models

In the second step, using the filtered residuals obtained from the VAR(2) model in section 2.1 we adopt a GJR-Garch model of Glosten, Jagannathan, and Runkle (1993) with skewed Student t innovations, see Hansen (1994). The MS-GARCH(1,1) is a two-state Markov-switching GARCH(1,1) model proposed by Haas, Mittnik, and Paolella (2004). Our formulation follows a two state Markov-switching skewed Student-*t* GJR-GARCH model of Ardia, Bluteau, Boudt, Catania, and Trottier (2019) to account for volatility switching regime. The model specification with constant unconditional variance satifies the following equation,

$$r_{it} | s_{it} = k, I_{ki,t-1} \sim sstd(0, \sigma^2_{k;it} v_{ki}) \quad (2)$$

$$\sigma^2_{k;it} = \omega_{ki} + \left(\alpha_{k;i1} + \alpha_{k;i2} \mathbb{I}_{r_{k;t-1}}\right) r^2_{k;i,t-1} + \beta_i \sigma^2_{k;i,t-1}, \quad i = 1, \cdots, d,\ k=1,2$$

$$\text{where } \mathbb{I}_{s_{t-1}} = \begin{cases} 0 & \text{if } r_{t-1} \geq 0 \\ 1 & \text{if } r_{t-1} < 0 \end{cases}$$

where r_{it} are filtered residuals (marginals) $\hat{\varepsilon}_{i,t}$ obtained from the VAR(2) mean equation (1), the parameters are such that, $\omega_i > 0$, $\alpha_i > 0$ and $\beta_i > 0$ to guarantee positive variance. Covariance-stationarity in each regime is obtained by requiring that $\alpha_{i,k} + \beta_k < 1$. $s = k, (k = 1, \text{or } 2)$. $\mathbb{I}_{t-1}$ is an indicator function which controls the leverage effect and take a value of one if the conditions hold, and zero otherwise. The coefficient α_{i2} is a state dependent variable which captures the degree of asymmetric in the conditional volatility due to the impact of positive and negative shocks. Hence, the GJR model allows good news ($r_{k;t-1}$ > 0) and bad news ($r_{k;t-1}$ < 0) to have differential effects on the conditional variance. The good news has an impact of $\alpha_{k;i1}$, while bad news has an impact sum of $\alpha_{k;i1}$+$\alpha_{k;i2}$ and when $\alpha_{k;i2} > 0$, leverage effect exists hence, negative shocks will increase volatility more than positive shocks and the leverage (gearing ratio) will increase, when $\alpha_{i2,k} < 0$ positive shocks will increase volatility more than negative shocks. The distribution of the residuals is assumed to be follow a skewed student-t (abbreviated sstd) which is suitable in capturing fat-tails and skewness. For a complete specification refer to Trottier and Ardia (2016).

Using a two state regime, the Markov probability of switching regimes at time t can be formulated in mathematical notation as follows:

$$p_{ij} = Pr(s_t = j | s_{t-1} = i), \text{ for } i,j \in s, \; t = 0,1,2,\cdots \quad (3)$$

$$P = \begin{bmatrix} p_{11} & p_{12} \\ p_{21} & p_{22} \end{bmatrix} = \begin{bmatrix} p & 1-q \\ 1-p & q \end{bmatrix}$$

where the distribution of s_t depends on the distribution of s_{t-1}, and $P = (p_{ij})$ denotes a square matrix of transition probabilities where the each sum to 1. The entry (i,j) is a conditional probability of switching to state j at time t given that the system was in state i at time $(t-1)$.

2.3 Copula models

In the last step, we use the standardized innovations filtered using a Markov switching GJR-GARCH(1,1) model with skewed Student t innovations from section 2.2 to construct a d -dimensional random vector $\mathbf{x} = (\boldsymbol{x}_1, \cdots, \boldsymbol{x}_d)$ each with n independent samples $\boldsymbol{x}_1 = (x_{11}, \cdots, x_{1d}), \ldots, \boldsymbol{x}_n = (x_{n1}, \cdots, x_{nd})$. In the first part we use Sklar(1959) theorem that states that any multivariate joint distribution function, can be decomposed in terms of cumulative univariate marginal distribution functions and a copula as:

$$F(x) := F(x_1, \cdots, x_d) = C\big(F_1(x_1), \cdots, F_d(x_d)\big), \quad \mathbf{x} \in \mathbb{R}^d \quad (4)$$

where a function $C: [0,1]^d \mapsto [0,1]$ defines the copula of $F(.)$ which connects (couples) the marginals $F_1, \cdots, F_d$ and, $F_i(x_i) = F(\infty, \cdots, x_i, \cdots, \infty)$, $x_i \in \mathbb{R}$. Hence, a copula model contains all the necessary information about the dependency structure of a set of variables. This copula representations allows us to model the marginal distribution and the dependency strucure in separate ways. The corresponding density $f(.)$ with univariate densities $f_1(x_1), f_2(x_2), \cdots, f_d(x_d)$ can be represented as follows:

$$f(\mathbf{x}) := f(x_1, \cdots, x_d) = c(F_1(x_1), \cdots, F_d(x_d)) \prod_{i=1}^{d} f_i(x_i), \; \mathbf{x} \in \mathbb{R}^d \quad (5)$$

where $c(u_1, \cdots, u_d) = \frac{\partial^d C(u_1, \cdots, u_d)}{\partial u_1, \partial u_2 \cdots \partial u_d)}$ is the density of the d-dimensional copula $C(u_1, \cdots, u_d)$. In the second part based on Sklar theorem provides a converse and states that, given any d-dimensional copula $C(.)$ and the corresponding univariate distribution functions $F_1(.),\cdots,F_d(.)$, a multivariate distribution $F(x)$ can be constructed using equation (4) having univariate margins $F_1(x_1),\cdots,F_d(x_d)$ and dependence structure $C(.)$ as follows:

$$C(\mathbf{u}) := C(u_1,\cdots,u_d) = F(F_1^{-1}(u_1),\cdots,F_d^{-1}(u_d)), \mathbf{u} \in [0,1]^d \quad (6)$$

It is the second part of the theorem that is more attractive in many applications of financial economics that quantifies the dependency structure. For a complete theoretical treatment see references in, Nelsen (2007), Cherubini, Luciano, and Vecchiato (2004)

2.4 Vine Copula models

In this section, we study dependency structure of BRICS and Oil markets by constructing multivariate distributions using bivariate building blocks also known as pair-construction copulae (PCC) see, Joe (1996); Aas et al. (2009); Bedford and Cooke (2001),(2002) The corresponding standardized residuals for each marginal distribution from equation (2) are now considered as an independent and identically distributed (i.i.d) samples generated over time. Following, Bedford and Cooke (2002), let $\boldsymbol{X} = (X_1,\cdots,X_n) \sim F(.)$ with joint density function

$f(x_1,\cdots,x_d)$. The decomposition of a multivariate distribution in equation (5) into products of conditional densities can be represented as follows;

$$f(x_1,\cdots,x_d) = f_1(x_1)\cdot f_{2|1}(x_2|x_1)\cdot f_{3|2,1}(x_3|x_1,x_2)\cdots f_{d|1:(d-1)}(x_d|x_1,\cdots x_{d-1}) \quad (7)$$

Joe (1997) shows that the conditional marginal distribution of the form $F(x|\boldsymbol{v})$ for the pair-copula can be written as follows,

$$F(x|\boldsymbol{v}) = \frac{\partial C_{x,v_j|v_{-j}}((F(x|\boldsymbol{v}_{-j}),F(v_j|\boldsymbol{v}_{-j}))}{\partial F(v_j|\boldsymbol{v}_{-j}))} \quad (8)$$

where for every j; $C_{x,v_j|v_{-j}}$ is a bivariate copula distribution function; **v** is a d-dimensional vector; v_j is an arbitrarily chosen element of **v** and **v**_j denotes the **v** vector except v_j.

A regular vine copula, R vine is a regular vine distribution, where all margins are uniformly distributed on $[0,1]$. An d-dimensional regular vine tree structure $V = (T_1\cdots T_d)$ is a tree sequence of $d-1$ linked trees which satisfy the following conditions:

1. T_1 is a connected tree with nodes $N_1 = (1,\ldots,d)$ and a set of edges denoted by E_1;
2. For $i = 2,\ldots,d-1$, T_i is a connected tree with nodes $N_i = E_{i-1}$ and edge E_i;
3. Edges in tree T_i become nodes in tree T_{i+1}. That is, if two edges in tree T_i are to be joined as nodes

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in tree T_{i+1} by an edge, they must share a common node in T_i (Proximity condition).

A regular vine is called a canonical vine, C-vine, if each tree T_i has a unique node of degree $d-1$ and therefore, has the maximum degree. While, a regular vine is called a drawable vine, D-vine if all the nodes in T_1 have degrees (The number of neighbors of a node $v \in N$) no greater than 2. Note that the construction of R vine is not unique. There are $n(\mathrm{n}-1)/2$ number of all possibilities of choosing edges, R-vine tree sequence in n dimensions.

3 Empirical Analysis

3.1 Data

The dataset consists of daily stock prices and world crude oil for the BRICS countries obtained from Yahoo finance during the period January 01, 2014 and July 17, 2020 giving a total of 1300 daily observations for each asset. This sample period includes the 2020 period of the current coronavirus stock market and oil market crashes. Prices have been converted to continuous compounded returns using this relationship. $r_{i,t} = \ln\left(\frac{P_{i,t}}{P_{i,t-1}}\right) \times 100,\ t = 1, \cdots, T = 1300;$ where $r_{i,t}$ is return of market i, and $P_{i,t}$ is closing price of stock market or world crude oil.

Table 1 provides descriptive statistics of continuous compounded returns and the unconditional sample correlations. The mean of stock market returns are all positive with relative annualised standard deviation of around 24% suggesting the presence of common risk factors in stock market as a whole. However, the Oil exhibits a negative mean return with significantly higher annualised standard deviation as a measure of risk. In addition to that, stock market returns are all negatively skewed with excess kurtosis which indicates the lack of symmetry in the underlying data distribution and a higher probability for investors experiencing extreme losses on the left tail during bear markets regime (downturn). This is supported by Jarque-Bera statistic which leads to the rejection of the normal distribution and the presence of leptokurtic behaviour.

Our modelling strategy aims to account for this asymmetry behaviour by using a skewed t distribution in our Markov Switching GARCH framework.

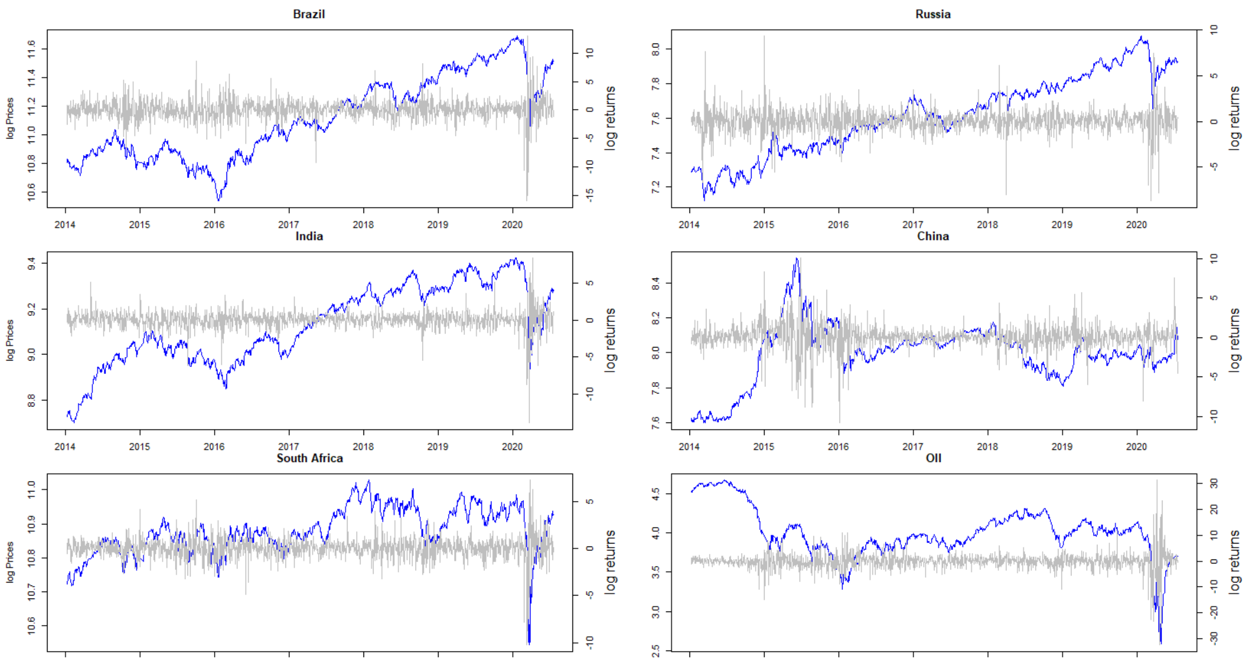
Table 1: Descriptive Statistics

	BVSP	IMOEX	NSEI	SSEC	JSE	WTI
mean%	0.051	0.048	0.043	0.035	0.017	-0.045
min	-16.000	-8.700	-13.900	-10.800	-10.200	-266.000
max	13.000	9.370	8.400	10.000	7.260	284.000
std%	1.930	1.350	1.230	1.620	1.220	11.300
AnnSD	30.500	21.300	19.500	25.700	19.300	179.000
Kurtosis	13.000	9.330	20.300	7.970	10.700	539.000
Skewness	-0.887	-0.406	-1.580	-0.684	-0.866	2.290
JB	9,306	4,751	22,954	3,543	6,353	157,764
Q(10)	45.7	34.6	59	10	12.9	339
$Q^2(10)$	1,254	313	812	297	1,366	324
acf	0.552	0.223	0.204	0.213	0.273	0.500
Pearson Correlation Matrix						
BVSP	1	0.222	0.125	0.072	0.213	0.218
IMOEX	0.222	1	0.202	0.117	0.318	0.175
NSEI	0.125	0.202	1	0.139	0.285	0.041
SSEC	0.072	0.117	0.139	1	0.195	0.058
JSE	0.213	0.318	0.285	0.195	1	0.121
WTI	0.218	0.175	0.041	0.058	0.121	1

Notes: Both the Jarque-Bera (JB) test for normality and Box-Ljung(LJ) test for autocorrelation at lag 10, (Q(10) and squared returns $Q^2(10)$ test statistics are rejected at 5% significance level. SD stands for standard deviation, AnnSD stands for annualized standard deviation. acf represents the autocorrelation function at lag 1.

Figure 1 provide basic insight of the BRICS and the world oil markets. They all provide further evidence of asymmetric behaviour and dependence structure that need further investigation using our proposed framework.

Figure 1: BRICS and Oil closing log prices and log returns



3.2 Results for marginals of VAR(2) model

Table 3 reports the initial estimation from a vector autoregressive VAR(2) model with lag length based on the BIC and AIC information criteria. The first and second lag 2 coefficient of Oil has low significance impact on the BRICS. This is also supported by the VAR(2) correlation matrix of residuals between oil and BRICS which exhibits very low values and perhaps suggests a further analysis on the co-movement.

Table 3: Parameter Estimates of the VAR(2)

	Dependent Variable					
	Brazil	Russia	India	China	SA	WTI OIL
$Brazil_{t-1}$	−0.086***	0.091***	0.100***	0.071***	0.078***	0.050
	(0.009)	(0.0001)	(0.00000)	(0.012)	(0.0002)	(0.760)
$Russia_{t-1}$	0.041	−0.022	0.083***	−0.015	0.055	0.280
	(0.380)	(0.500)	(0.005)	(0.710)	(0.066)	(0.230)
$India_{t-1}$	−0.120***	−0.140***	−0.110***	−0.063	−0.160***	−0.620***
	(0.024)	(0.0002)	(0.001)	(0.160)	(0.00000)	(0.019)
$China_{t-1}$	−0.031	−0.023	−0.028	0.012	−0.035	−0.062
	(0.370)	(0.340)	(0.190)	(0.680)	(0.110)	(0.720)
$S.Africa_{t-1}$	0.074	0.059	0.043	0.100**	0.032	0.640***
	(0.210)	(0.150)	(0.240)	(0.046)	(0.390)	(0.031)
Oil_{t-1}	0.032***	0.010	0.012***	0.006	0.010**	−0.620***
	(0.000)	(0.009)	(0.0005)	(0.220)	(0.004)	(0.000)
$Brazil_{t-2}$	−0.001	−0.008	0.035	0.007	0.037	0.0004
	(0.980)	(0.740)	(0.093)	(0.800)	(0.075)	(1.000)
$Russia_{t-2}$	−0.020	−0.007	−0.120***	−0.007	−0.022	0.067
	(0.670)	(0.820)	(0.0001)	(0.860)	(0.450)	(0.780)
$India_{t-2}$	0.005	−0.017	0.028	0.030	0.058	−0.035
	(0.920)	(0.640)	(0.400)	(0.510)	(0.080)	(0.900)
$China_{t-2}$	−0.051	−0.015	−0.021	−0.009	−0.014	−0.120
	(0.140)	(0.540)	(0.340)	(0.760)	(0.510)	(0.470)
$S.Africa_{t-2}$	0.081	0.110***	0.083***	0.030	0.008	0.730***
	(0.170)	(0.005)	(0.024)	(0.540)	(0.820)	(0.013)
Oil_{t-2}	0.012	0.002	0.001	−0.0004	0.005	−0.240***
	(0.023)	(0.570)	(0.720)	(0.920)	(0.120)	(0.000)
Intercept	0.064	0.051	0.042	0.032	0.016	−0.090
	(0.220)	(0.170)	(0.210)	(0.480)	(0.630)	(0.740)

Notes: p-Values are reported in brackets. *significant at 10% levels. **significant at 5% levels. ***significant at the 1%.
The Portmanteau test: The null hypothesis of no autocorrelation is rejected since the p-value of 0.000<0.05 is lower than the significance level.

The coefficients for India are statistically significant and highlight a strong negative influence across all other markets

3.3 Results of marginal models using MS-GJR-GARCH

Table 4 provides a summary of in-sample parameter estimates for the MS-GJR-GARCH model with a two-state asymmetric Student t specification. The table highlights interesting findings where all parameter estimates are statistically significant and indicates that the evolution of volatility process is not homogeneous across the two regimes.

Table 4: Parameter estimates for a two state MS-GARCH with skewed t innovations

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	Brazil	Russia	India	China	South Africa	Oil
α_{01}	0.124***	0.016	0.007	0.013	0.026***	0.022
	(0.002)	(0.078)	(0.202)	(0.025)	(0.001)	(0.091)
α_{11}	0.022	0.046	0.010	0.031	0.000	0.000
	(0.144)	(0.085)	(0.238)	(0.076)	(0.263)	(0.497)
α_{21}	0.047	0.000	0.004	0.020	0.104***	0.050***
	(0.080)	(0.479)	(0.442)	(0.263)	(0.000)	(0.007)
β_1	0.913***	0.930***	0.978***	0.949***	0.913***	0.966***
	0.000	0.000	0.000	0.000	0.000	0.000
v_1	8.810***	99.900***	98.900***	3.670***	20.000***	5.570***
	(0.001)	0.000	0.000	0.000	(0.037)	0.000
ξ_1	1.120***	0.431***	0.709***	1.010***	0.846***	0.873***
	0.000	(0.001)	0.000	0.000	0.000	0.000
α_{02}	0.282***	0.112***	0.060	3.520***	3.720***	0.441***
	(0.002)	(0.040)	(0.084)	(0.002)	0.000	(0.010)
α_{12}	0.000	0.000	0.001	0.000	0.000	0.000
	(0.491)	(0.495)	(0.497)	(0.497)	(0.261)	(0.492)
α_{22}	0.243***	0.153**	0.459***	0.729***	0.037	0.379*
	(0.013)	(0.065)	(0.065)	(0.044)	(0.049)	(0.205)
β_2	0.770***	0.869***	0.752***	0.315***	0.770***	0.806***
	0.000	0.000	0.000	(0.009)	0.000	0.000
v_2	4.380***	3.720***	4.420***	5.220***	14.600***	99.800***
	0.000	0.000	0.000	(0.013)	(0.021)	0.000
ξ_2	0.865***	1.110***	0.980***	0.868***	0.044	0.971***
	0.000	0.000	0.000	0.000	(0.401)	0.000
P_{11}	0.997***	0.418***	0.464***	0.996***	0.991***	0.992***
	0.000	(0.020)	(0.031)	0.000	0.000	0.000
P_{21}	0.005	0.401***	0.441***	0.021***	1.000***	0.027***
	(0.021)	(0.001)	(0.004)	0.000	0.000	0.000
			Unconditional Volatility			
Regime 1	26.9	13.1	13.4	17.9	14.8	30.0
Regime 2	26.8	20.7	21.3	53.4	67.1	103.6
			Volatility Persistence			
Regime 1	0.96	0.98	0.99	0.99	0.97	0.99
Regime 2	0.89	0.95	0.98	0.68	0.79	0.996
Note:				*p<0	.1; **p<0.05 ***p<0.01	

The results show that the unconditional volatility are different between the regimes and leverage effects coefficients α_{2k}, $k = 1, \text{or } 2$ are positive, statistically significant at 5% level and heterogeneous. This suggests that negative shocks in a country will tend to increase volatility more than positive shocks. The results also highlights heterogeneous volatility persistence between the two regimes where regime 1 is characterized by high volatility persistence and regime 2 by low volatility persistence.

3.4 Measuring Dependence with Vine copula

We analytically construct pair-copula (PCC) with regular R-vine and canonical C-vine models by first using the independence test based on Kendall's tau for each pair. The test results rejects the null hypothesis of independence. Based on the AIC and BIC information criteria; we also find the C-Vine to be the best model in describing the dependence behaviour although R-vines are mainly preferred models which account for tail dependence of different pairs of random variables.

Table 5 and the corresponding first and second trees with pair copula families and Kendall's Tau in Figure 2 below show the results of the pair-constructions of the dependence structure with a C-vine copula where in the first tree shows South Africa is selected as the centre node. These results highlight key measurements statistics which include: the selected tree structure, the selected pair copula families describing the type of dependence, the induced Kendall's Tau (τ) values and the upper-lower tail dependence coefficients. We find two types of tail dependence structure:- *symmetric tail dependence* between South Africa and China; South Africa and Russia; and *lower tail dependence* between South Africa and India; South Africa and Brazil; South Africa and Oil.

However, the dependence in Tree 3, Tree 4 and Tree 5 are relatively negligible. The lower tail dependence found between South African stock market and India, Brazil and Oil market is vital as it might help investors diversify their portfolio during times of financial distress.

Table 5: Results of C-vine Copulas with Kendall's tau, upper-lower tail dependence

Tree	Copula	Edge	Parameter1	Parameter2	Tau	lower Tail	upper Tail
T_i	$C(.)$	E_i	θ_1	θ_2	τ	λ_l	λ_u
Tree 1	t	5,4	0.294***	17.800*	0.19	0.005	0.005
			(0.026)	(9.820)			
	SBB1	5,3	0.101**	1.320***	0.282	0.313	0.006
			(0.050)	(0.038)			
	t	5,2	0.460***	7.670***	0.304	0.108	0.108
			(0.023)	(1.790)			
	BB1	5,1	0.361***	1.090***	0.224	0.172	0.114
			(0.056)	(0.029)			
	SBB7	6,5	1.180***	0.123***	0.139	0.197	0.004
			(0.034)	(0.039)			
Tree 2	I	1,4|5	0.000	0.000	0	0	0
	C	1,3|5	0.137***	0.000	0.064	0.006	0
			(0.033)				
	t	1,2|5	0.240***	8.190***	0.154	0.041	0.041
			(0.028)	(2.250)			
	F	6,1|5	1.550***	0.000	0.164	0	0
			(0.171)				
Tree 3	I	2,4|1,5	0.000	0.000	0	0	0
	t	2,3|1,5	0.094***	13.100***	0.06	0.004	0.004
			(0.029)	(4.580)			
	BB8	6,2|1,5	1.470***	0.791***	0.096	0	0
			(0.314)	(0.184)			
Tree 4	t	3,4|2,1,5	0.086***	10.400***	0.055	0.01	0.01
			(0.030)	(3.290)			
	BB7	6,3|2,1,5	-1.040***	-0.068**	-0.053	0	0
			(0.022)	(0.033)			
Tree 5	I	6,4|3,2,1,5	0.000	0.000	0	0	0

*p<0.1; **p<0.05; ***p<0.01

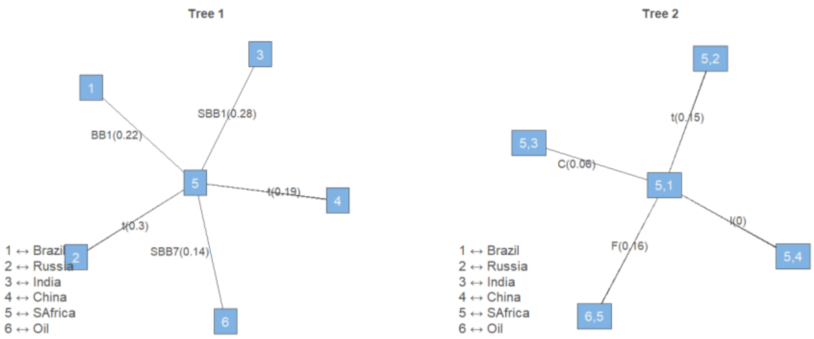

Figure 2: Results of C-vine copulas dependency trees

The R-vine copula results reported in Table 6 below also exihibit *lower tail market dependence* between Brazil and Oil, South Africa and India; symmetric tail dependence between Brazil and Russia, South Africa and Russia, South Africa and China. The dependence in Tree and Tree 5 are small to draw any meaningful conclusions.

Table 6: Results of R-vine Copulas with Kendall's tau, upper-lower tail dependence

Tree T_i	Copula $C(.)$	Edge E_i	Parameter1 θ_1	Parameter2 θ_2	Tau τ	lower Tail λ_l	upper Tail λ_u
Tree 1	SBB1	1,6	0.143***	1.160***	0.197	0.184	0.016
			(0.049)	(0.031)			
	t	2,1	0.359***	5.000***	0.234	0.143	0.143
			(0.027)	(0.888)			
	t	5,2	0.460***	7.670***	0.304	0.108	0.108
			(0.023)	(1.790)			
	SBB1	5,3	0.101**	1.320***	0.282	0.313	0.006
			(0.050)	(0.038)			
	t	5,4	0.294***	17.800*	0.19	0.005	0.005
			(0.026)	(9.820)			
Tree 2	F	2,6|1	1.090***	0.000	0.119	0	0
			(0.169)				
	t	5,1|2	0.214***	11.200***	0.138	0.015	0.015
			(0.028)	(4.050)			
	t	3,2|5	0.117***	10.900***	0.075	0.01	0.01
			(0.030)	(3.400)			
	t	4,3|5	0.093***	8.560***	0.059	0.019	0.019
			(0.030)	(2.270)			
Tree 3	I	5,6|2,1	0.000	0.000	0	0	0
	t	3,1|5,2	0.085***	19.000*	0.054	0.001	0.001
			(0.029)	(9.900)			
Tree 3	I	4,2|3,5	0.000	0.000	0	0	0
Tree 4	BB7	3,6|5,2,1	-1.040***	-0.070**	-0.056	0	0
			(0.022)	(0.032)			
Tree 4	I	4,1|3,5,2	0.000	0.000	0	0	0
Tree 5	I	4,6|3,5,2,1	0.000	0.000	0	0	0
Note:					*p<0.1	; **p<0.05; ***p<0.01	

3.5 Market Risk Under Portfolio Rebalancing

In this section, we propose a multi-objective asset allocation model for a portfolio that maximizes return and minimizes the Conditional Value-at-Risk (CVaR) with a confidence level of 0.95. The transformed data obtained from R-vine copula are now used to construct a monthly rebalancing optimization strategy using a Particle Swarm Optimization (PSO) algorithm.

Let $\mathbf{w} = (w_1, w_2, w_3, w_4, w_5, w_6)$ be the portfolio weights vector for 6 risky assets which represent the decision variables. Then, the investor solves the following optimization problem;

$$
\begin{aligned}
&\underset{\mathbf{w}}{\text{maximize}} && \mu_p && \\
&\underset{\mathbf{w}}{\text{minimize}} && \text{CVaR}_\alpha && = \mathbb{E}[L|L > VaR_\alpha(L)] \\
&\text{subject to} && && \\
& && \sum_{i=1}^{6} w_i && = 1, w_i \in [0,1] \\
& && w_i && \geq 0.\ \ i = 1, \cdots, 6
\end{aligned}
\qquad (11)
$$

where $\mathbb{E}[L|L > VaR_\alpha(L)] = \frac{1}{1-\alpha}\int_{-\infty}^{-VaR} f(L)dL$ and $VaR_\alpha(L)$ is defined as the smallest number L such that the probability of a negative loss L is not higher than $(1-\alpha)$ quantile of the distribution; $\mu_p = \mathbf{w}\mathbb{E}[R]$ is the maximum portfolio mean. We assume no short selling portfolio (long portfolio) thus, w_i are non-negative portfolio weights.

First, Table 7 reports the results of a benchmark model with no regime switching and dependency structure. It is clear that no asset dominates the weight allocation and the overall portfolio CVaR is around 0.034. Table 8 takes into account the regime switching with no dependency structure with overall risk contribution averaging A similar observation 0.021. Table 9 shows the results that accounts for both regime switching and dependency structure with smallest CVaR of 0.010 . Overall, the R vine portfolio has best performance over the rebalancing period as dispicted in Figure 3.

Table 7 Risk and weight allocation without regime switching and dependence.

Period	Mean	CVaR	Brazil	Russia	India	China	SAfrica	Oil
P	μ		w_1	w_2	w_3	w_4	w_5	w_6
P_1	0.0004	0.022	0.160	0.190	0.179	0.120	0.213	0.131
P_2	0.0003	0.029	0.180	0.254	0.148	0.168	0.188	0.054
P_3	0.0002	0.045	0.112	0.188	0.158	0.342	0.140	0.054
P_4	0.0002	0.043	0.090	0.238	0.114	0.351	0.166	0.050
P_5	-0.0003	0.040	0.020	0.106	0.060	0.148	0.060	0.602
P_6	-0.0002	0.044	0.030	0.146	0.062	0.120	0.070	0.568
P_7	0.0003	0.046	0.119	0.197	0.136	0.325	0.155	0.078

Table 8 Risk and weight allocation with regime switching only.

period	mean	ES	Brazil	Russia	India	China	SAfrica	Oil
P	μ		w_1	w_2	w_3	w_4	w_5	w_6
P_1	0.0000	0.016	0.166	0.154	0.150	0.191	0.167	0.180
P_2	-0.0000	0.021	0.118	0.140	0.232	0.250	0.118	0.138
P_3	-0.0001	0.021	0.154	0.096	0.212	0.310	0.118	0.119
P_4	-0.0001	0.021	0.136	0.116	0.128	0.330	0.176	0.110
P_5	-0.0001	0.022	0.104	0.149	0.194	0.229	0.123	0.204
P_6	-0.0001	0.021	0.132	0.177	0.173	0.295	0.108	0.116
P_7	-0.0000	0.021	0.108	0.138	0.186	0.278	0.126	0.155

Table 9 Risk and weight allocation with regime switching and dependence.

period	mean	ES	Brazil	Russia	India	China	SAfrica	Oil
P	μ		w_1	w_2	w_3	w_4	w_5	w_6
P_1	-0.0000	0.010	0.136	0.138	0.182	0.209	0.142	0.194
P_2	-0.0001	0.011	0.167	0.167	0.167	0.167	0.167	0.167
P_3	-0.0001	0.011	0.167	0.167	0.167	0.167	0.167	0.167
P_4	-0.0000	0.011	0.167	0.167	0.167	0.167	0.167	0.167
P_5	-0.0000	0.010	0.160	0.156	0.150	0.264	0.116	0.150
P_6	-0.0000	0.010	0.127	0.133	0.260	0.214	0.123	0.134
P_7	-0.0000	0.010	0.098	0.134	0.228	0.252	0.134	0.156

Figure 4 displays the three equity curves highlighting the relative performance with respect to this rebalancing strategy. The R vine portfolio strategy clearly outperforms both the single MSGARCH and a portfolio with no regime switching and dependence. The sharp portfolio draw-downs witnessed during the COVID 19 financial market crisis in 2020 are adequately captured.

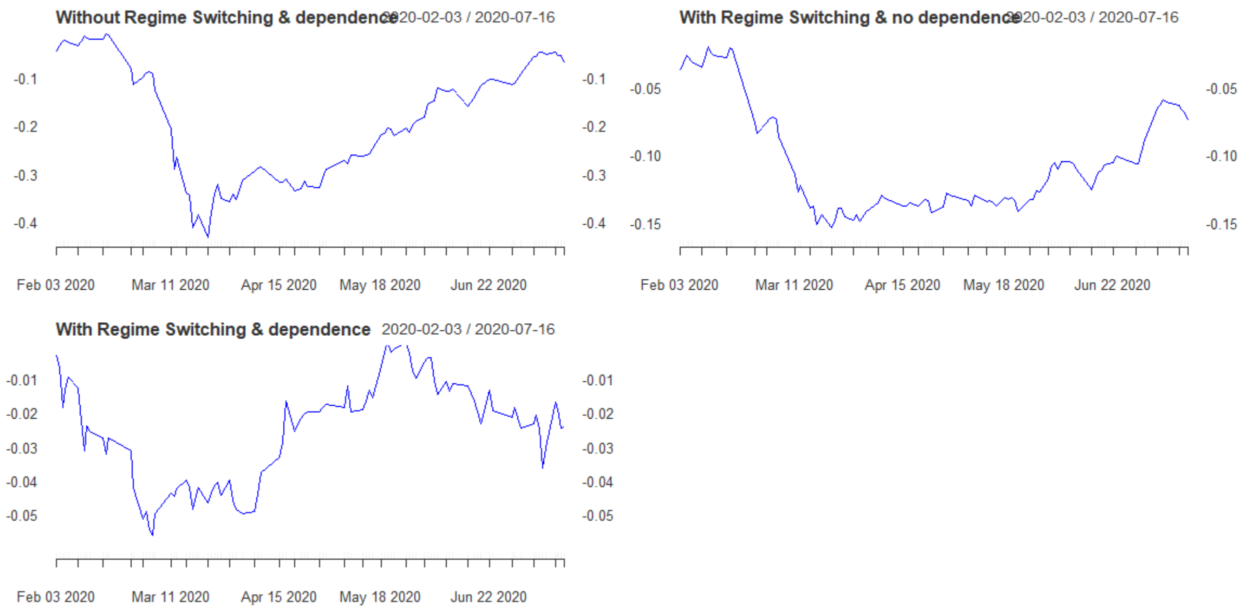

Figure 3 Trajectory of maximum return and minimum-CVaR rebalancing portfolio values.

4 Conclusion

This study attempted to analyze the tail dependence structure in BRICS and oil markets, and to determine optimal investment decisions in these markets using a combination of different statistical techniques including Markov switching GJR-GARCH, vine copula and Particle Swam optimization techniques. The returns series of the indices were first pre-filtered using VAR to model the dependence on the conditional mean and then use a MS-GJR-GARCH process to remove the effects of autocorrelation and heteroscedasticity and at the same time account for regime switching to separate lower from higher volatility regime. We found high probability of transition between lower and higher volatility regimes. Pairwise copula construction was done using vine copula. The copula was adopted to account for different dependence structure among each pair in the BRICS and oil markets. Our estimation using Akaike Information criteria showed that C-vine copula was able to best fit each pair of constructed dependence.

We found two types of tail dependence structure: - *symmetric tail dependence* between South Africa and China; South Africa and Russia; and *lower tail dependence* between South Africa and India; South Africa and Brazil; South Africa and Oil. However, the dependence in Tree 3, Trees 4 and Tree 5 were relatively negligible. The lower tail dependence found between South African stock market and India, Brazil and Oil market is vital as it might help investors diversify their portfolio during times of financial distresses. To determine optimal investment strategies in these markets; we made use of the estimated C-vine copula to simulate returns for these markets and applied the Particle Swam optimization technique to determine optimal weight allocations. Particle Swam is a nonlinear and nonparametric techniques that finds optimal solution iteratively by trying to improve the candidate solution and avoid producing sub-optimal investment solutions.

The optimization results under a rebalancing strategy confirm the existence an inverse relationship between the risk contribution and asset allocation of South Africa and oil supporting the existence of lower tail dependence between them. We argue that when South African stock market is in distress, investors tend to shift their holdings in oil market. Similar results were found between China and oil. In the upper tail, South African asset allocation was found to have an inverse relationship with that of Brazil, Russia and India suggesting that these three markets might be good investment destinations when South African stock market is in financial turmoil and vice-versa. Furthermore, we find that the rebalancing strategy with regime switching and dependence structure outperforms a portfolio strategy without regime switching and dependency and a portfolio with only regime switching. These findings are vital for international investors, policymakers, and regulators during both bull and bear markets. However, the limitation of this model is its inability to account for other hidden stylized facts such as price jumps and long-memory process in stock returns and future research with try to account for these.

Supplementary Materials:

Author Contributions: the contribution of each author is as follows: Conceptualization, J.W. Muteba Mwamba; methodology, J.W. Muteba Mwamba, and Sutene Mwambi.; software, Sutene Mwambi; validation, J.W. Muteba Mwamba; formal analysis, Sutne Mwambi; investigation, J.W. Muteba Mwamba, and Sutene Mwambi; resources, Sutene Mwambi; data curation, Sutene Mwambi.; writing—original draft preparation, Sutene Mwambi; writing—review and editing, J.W. Muteba Mwamba; visualization, Sutene Mwambi; supervision, J.W. Muteba Mwamba.; project administration, Sutene Mwambi; funding acquisition, Not Applicable. All authors have read and agreed to the published version of the manuscript.

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Conflicts of Interest: The authors declare no conflict of interest. The authors certify that they have no affiliations with or involvement in any organization or entity with any financial or non-financial interest in the subject matter or materials discussed in this manuscript. The authors declare that there is no conflict of interest.

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