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Dirac 4×1 Wavefunction Recast into a 4×4 Type Wavefunction

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As currently understood, the Dirac theory employs a 4×1 type wavefunction. This 4×1 Dirac wavefunction is acted upon by a 4×4 Dirac Hamiltonian operator, in which process, four independent particle solutions result. Insofar as the real physical meaning and distinction of these four solutions, it is not clear what these solutions really mean. We demonstrate herein that these four independent particle solutions can be brought together under a single roof wherein the Dirac wavefunction takes a new form as a 4×4 wavefunction. In this new formation of the Dirac wavefunction, these four particle solutions precipitate into three distinct and mutuality dependent particles (ψ_L, ψ_N, ψ_R) that are eternally bound in the same region of space. Given that Quarks are readily found in a mysterious threesome cohabitation-state eternally bound inside the Proton and Neutron, we make the suggestion that these Dirac particles (ψ_L, ψ_N, ψ_R) might be Quarks. For the avoidance of speculation, we do not herein explore this idea further but merely present it as a very interesting idea worthy of further investigation. We however must say that, in the meantime, we are looking further into this very interesting idea, with the hope of making inroads in the immediate future.

Keywords: Dirac equation — modified Dirac equation — quarks — particle physics.

"*I am among those who think that Science has great beauty.*"

— **Marie Skłodowska-Curie (1867-1934)**

1 Introduction

As currently understood, the Dirac [1, 2] theory employs a 4×1 type wavefunction, ψ. This 4×1 Dirac wavefunction is acted upon by a 4×4 Dirac Hamiltonian operator, $\mathscr{H}_{\rm D}$, in which process, four independent particle solutions result, *i.e.*: $\psi[1]$, $\psi[2]$, $\psi[3]$, and, $\psi[4]$. To this day, insofar as the real physical meaning and distinction of these four solutions, it remains unclear what these solutions really mean. We demonstrate herein that these four independent particle solutions can be brought together under a single roof wherein the Dirac wavefunction takes a new form as a 4×4 wavefunction. To that end, we shall begin our journey by formally introducing the already well known Dirac equation.

That is to say: for a particle whose rest-mass and wave-function are m_0 and ψ respectively, the corresponding Dirac equation is given by:

$$\imath\hbar\gamma^{\mu}\partial_{\mu}\psi = m_0 c_0 \psi, \quad (1)$$

where: $\hbar = 1.054571817 \times 10^{-34}\,\mathrm{J\cdot s}$ (CODATA 2018) is the normalized Planck constant, $c_0 = 299792458 \times 10^8\,\mathrm{m{\cdot}s^{-1}}$ (CODATA 2018) is the speed of Light in *vacuo*, $\imath = \sqrt{-1}$, and:

$$\gamma^0 = \begin{pmatrix} \mathcal{I}_2 & \emptyset \\ \emptyset & -\mathcal{I}_2 \end{pmatrix}, \quad \gamma^i = \begin{pmatrix} \emptyset & \sigma^i \\ -\sigma^i & \emptyset \end{pmatrix}, \quad (2)$$

are the 4×4 Dirac γ-matrices where, $\mathcal{I}_2$, and, $\emptyset$, are the 2×2 identity and null matrices respectively, and, the four component Dirac wave-function, ψ, is defined as follows:

$$\psi = \begin{pmatrix} \psi_0 \\ \psi_1 \\ \psi_2 \\ \psi_3 \end{pmatrix} = \begin{pmatrix} \psi_L \\ \psi_R \end{pmatrix}, \quad (3)$$

is the 4×1 Dirac four component wavefunction and ψ_L and ψ_R are the Dirac [1, 2] bispinors that are defined such that:

$$\psi_L = \begin{pmatrix} \psi_0 \\ \psi_1 \end{pmatrix}, \quad \text{and,} \quad \psi_R = \begin{pmatrix} \psi_2 \\ \psi_3 \end{pmatrix}. \quad (4)$$

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Throughout this reading — unless otherwise specified; the Greek indices will *here-and-after* be understood to mean $(\mu, \nu, \ldots = 0, 1, 2, 3)$ and the lower case English alphabet indices $(i, j, k \ldots = 1, 2, 3)$.

The Dirac equation can be recast into the Schrödinger formalism as follows:

$$\mathscr{H}_{\mathrm{D}}\psi = \mathscr{E}\psi, \tag{5}$$

where: $\mathscr{H}_{\mathrm{D}}$, is the Dirac Hamiltonian operator and, $\mathscr{E}$, is the usual quantum mechanical energy operator, and these operators are defined as follows:

$$\begin{aligned} \mathscr{H}_{\mathrm{D}} &= \imath\hbar c_0 \gamma^j \frac{\partial}{\partial x^j} - \gamma^0 m_0 c_0^2, && \text{(6a)} \\ \mathscr{E} &= -\imath\hbar \frac{\partial}{\partial t}. && \text{(6b)} \end{aligned}$$

In §(4), we shall for the purposes of efficiently making our point regarding the 4×4 wavefunction approach, use the recast Dirac Eq. (5) in the Schrödinger formalism.

Now, in-closing this introductory section, we shall give the synopsis of the present reading. In §(2), we shall for instructive, completeness and self-containment purposes, present the traditional free particle solutions of the Dirac equation. Thereafter in §(3), we shall discuss some of the major shortcomings of the Dirac equation — this we do in-order to demonstrate that there still is a lot more about the Dirac equation that still needs to be understood. Then, in §(4), we present the main task of the present reading — *i.e.*, the Dirac wavefunction is cast into a 4×4 type wavefunction. Thereafter in §(5), we proceed to make our suggestion regarding the new formulation of the Dirac wavefunction. Lastly, in §(6) a general discussion is given and no conclusion is made.

2 Free particle solutions of the Dirac equation

The free particle solutions of the Dirac equation are obtained by assuming a free particle wavefunction of the form: $\psi = ue^{\imath p_\mu x^\mu/\hbar}$, where likewise, u, is a four component object, *i.e.*:

$$u = \begin{pmatrix} u_0 \\ u_1 \\ u_2 \\ u_3 \end{pmatrix}. \tag{7}$$

This u-function is assumed to have no space and time dependence. With this in mind, one will proceed to substituting this free particle solution: $\psi = ue^{\imath p_\mu x^\mu/\hbar}$, into Eq. (1), where-after some elementary algebraic operations — they will be led to the following linear quad-set of simultaneous equations:

$$\begin{aligned} (E - m_0 c_0^2)u_0 - c_0(p_x - ip_y)u_3 - cp_z u_2 &= 0, && \text{(8a)} \\ (E - m_0 c_0^2)u_1 - c_0(p_x + ip_y)u_2 + cp_z u_3 &= 0, && \text{(8b)} \\ (E + m_0 c_0^2)u_2 - c_0(p_x - ip_y)u_1 - cp_z u_0 &= 0, && \text{(8c)} \\ (E + m_0 c_0^2)u_3 - c_0(p_x + ip_y)u_0 + cp_z u_1 &= 0. && \text{(8d)} \end{aligned}$$

An important fact to note about the above array or set of simultaneous equations is that the four solutions u_j (where: $j = 0, 1, 2, 3$) are superluminally entangled, that is to say, a change in one of the components affects every other component instantaneously *i.e.* in zero time interval. What this means is that for linearly dependent solutions of u_j, the Dirac equation — just as it predicts spin as a relativistic quantum phenomenon, it also predicts entanglement as a quantum phenomenon. If they exist as a separate reality in different regions of space, then, the particles, ψ_L, and, ψ_R, are not only entangled, but, irrevocably so.

Without further ado, we shall now present the four formal solutions of the Dirac equation, these are given by:

$$\psi[1] = \begin{pmatrix} 1 \\ 0 \\ \dfrac{c_0 p_z}{E + m_0 c_0^2} \\ \dfrac{c_0(p_x + ip_y)}{E + m_0 c_0^2} \end{pmatrix} \exp\left(\frac{\imath p_\mu x^\mu}{\hbar}\right), \tag{9}$$

$$\psi[2] = \begin{pmatrix} 0 \\ 1 \\ \dfrac{c_0(p_x - ip_y)}{E + m_0 c_0^2} \\ -\dfrac{c_0 p_z}{E + m_0 c_0^2} \end{pmatrix} \exp\left(\frac{\imath p_\mu x^\mu}{\hbar}\right), \tag{10}$$

$$\psi[3] = \begin{pmatrix} \dfrac{c_0 p_z}{E - m_0 c_0^2} \\ \dfrac{c_0(p_x + ip_y)}{E - m_0 c_0^2} \\ 1 \\ 0 \end{pmatrix} \exp\left(\frac{\imath p_\mu x^\mu}{\hbar}\right), \tag{11}$$

$$\psi[4]=\left(\begin{array}{c}\dfrac{c_0(p_x-ip_y)}{E-m_0c_0^2}\\ \\ -\dfrac{c_0p_z}{E-m_0c_0^2}\\ \\ 0\\ \\ 1\end{array}\right)\exp\left(\frac{ip_\mu x^\mu}{\hbar}\right). \tag{12}$$

These solutions [*i.e.*, Eq. (9)-(12)] are obtained as follows:

1. From Eq. (8), u_0, and u_1 are fixed so that: $u_0=1$, and, $u_1=0$, and the resultant set of equations is solved for u_2, and, u_3.
2. Similarly, from Eq. (8), u_0, and u_1 are fixed so that: $u_0=0$, and, $u_1=1$, and the resultant set of equations is solved for u_2, and, u_3.
3. Again, from Eq. (8), u_2, and u_3 are fixed so that: $u_2=1$, and, $u_3=0$, and the resultant set of equations is solved for u_0, and, u_1.
4. Similarly, from Eq. (8), u_2, and u_3 are fixed so that: $u_2=0$, and, $u_3=1$, and the resultant set of equations is solved for u_0, and, u_1.

Now, having presented the solutions of the Dirac equation, we shall proceed to present what we feel are some of the important major shortcoming of the Dirac equation. This exercise — *of presenting these shortcomings* — is being carried out in-order to bring some kind of appreciation to the fact that, while the Dirac equation is a highly successful equation and that its success is nearly comparable to none, there still exists ample room to further improve on this noble and beautiful equation.

3 Major shortcomings of the Dirac equation

While the Dirac equation is not only one of the most *Beautiful Equations of Physics*, it is also amongst the most successful equations in *All the History of Physics*. There surely is no doubt about this. Be that as it may, despite these admirable accolades, decorations and esoteric successes, it is not without its own shortcomings, amongst them are the following:

3.1 Anomalous gyromagnetic ratios

It is a well known fact that the Dirac equation — in its bare and natural form — it predicts a gyromagnetic ratio ($\mathrm{g_D}$) that is equal to two (*i.e.*, $\mathrm{g_D}=2$) and this prediction is very close to the gyromagnetic ratio of the Electron [$\mathrm{g_E}=2+0.002319304362(2)$], hence, the Dirac equation is said to give a good description of the Electron. On the contrary, the spin-1/2 Proton ($\mathrm{g_E}$) and Neutron ($\mathrm{g_N}$), which — *like the Electron* — are thought to be fundamental particles and thus are naturally expected to readily submit to a successful description by the Dirac equation — these particles have gyromagnetic ratios that are '*disappointingly*' at variance [*i.e.*, $\mathrm{g_P}=2+3.5856947(5); \mathrm{g_N}=2-5.8260855(9)$] with the Dirac prediction. In its bare and natural form, the Dirac equation lacks in its nature infrastructure the devices to correctly predict the g-ratio of any arbitrary spin-1/2 particle. To them that seek pristine and serene beauty in a theory, this state of affairs and aspect of the Dirac equation is very disappointing. Be that as it may, with the right and appropriate amendments, the Dirac theory has been brought into accord with experience.

3.2 Negative energy solutions

As is well known, one of Dirac's foremost hopes prior to the successful construction of the esoteric equation that would latter bare his name is that — he hoped that the future equation that he would construct (discover) would, once and for all time, do away with the negative energy solutions that had so tormented and bedevilled the Klein-Gordon equation [3, 4] — an equation on the Dirac equation is itself built. These negative energy solutions reared — *again* — their '*ugly head*' in Dirac's new equation.

Instead of trying to '*exorcise and curse*' them away, Dirac's esoteric mind realised he had no choice but to agree to a truce by making a lasting peace agreement with these negative energy solutions. He did this by wholeheartedly embracing them as not only legitimate solutions of his beautiful equation, but, of physical and natural reality as-well. By so doing, he went on to make one of the greatest, deepest, most enduring and esoteric predictions of all times, namely that — there aught, *hitherto* exist a new form of matter in the Universe. This new form of matter came to be known as antimatter, were for every electrically charged particle, there aught to exist a corresponding antiparticle with all the properties being identical — save for the electrical charge — it is equal in magnitude but opposite in sign [5].

No sooner had Dirac made his prediction did America's Carl Anderson [6] confirm Dirac's hypothesis and latter by Giuseppe Occhialini and Patrick Blackett [7]. The existence of antimatter is now commonplace in the scientific literature. What is not clear about this antimatter particles (antiparticles) is whether or not they have negative energy and mass. Do antimatter particles fall up or down in a gravitational field? Experiments [8] are not clear and this question have been begging for an answer ever since it was conceived (see *e.g.*, Refs. [9–18]). For according to Einstein's [19] mass energy equivalence ($E=mc_0^2$), if the energy of antiparticles is negative, their mass should be negative too. If this is the case, it follows from *Newton's Law of Gravitation* that in a gravitational

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field, antiparticles aught to fall up and not down!

3.3 Whereabouts of antimatter

Apart from the the question of whether antiparticles fall up or down, there is the yet to be an answer to the question of the whereabouts of this antimatter [20–22]. The Dirac equation not only is symmetric under electric *Charge Conjugation* (C), but, symmetric under all the known discrete symmetries of *Time* (P) and *Parity* (P) reversal including the combination of any of these these discrete symmetries — *i.e.* CP, CT, PT, and, CPT. This highly symmetric nature of the Dirac equation leads it to the *unfortunate prediction* that from the very outset of the moment of *Creation of the Universe* (supposedly the *Big Bang*), the Universe must contain in it at all times equal amounts of matter and antimatter.

This prediction of the Dirac equation is '*very unfortunate*' because it is at complete variance with physical and natural as we know and experience it. That is to say, given that matter and antimatter will annihilate to form radiation should they ever come into contact, the exist of equal positions of matter and antimatter in the Universe would mean that if the Dirac prediction on the matter-antimatter census is correct, then, the Universe aught to be no more than a radiation bath. Clearly, this is not what we see around us.

3.4 Lack of a universal character

Every Fermion particle (Electron, Proton, Neutron, Neutrinos, Quarks *etc*) in *Particle Physics* is described by the Dirac equation. This gives the superficial impression that the Dirac equation is a universal equation for all spin-1/2 particles. A closer look will reveal that, while this equation is used to describe Fermion, it needs to be supplemented in-order to match-up with experimental data. As already pointed out in §(3.1), the g-ratios of every other particle save for that of the Electron are not in conformity with the natural Dirac equation. If the Dirac equation was indeed a universal equation for all Fermions, it must contain within its natural infrastructure the necessary adjustable parameters that would make it fit with the experimental data of a given particle. These post-experimental adjustments that are made in-order that the Dirac equation fits to experimental data are of *ad hoc* nature.

Apart from the inability to explain in a smooth manner the g-ratios of different Fermions, we have the issue of the universality of spin. That is to say, the Dirac equation is an equation only capable of explaining spin-1/2 particles, an not any general spin particle. For example, in-order to explain spin-3/2, we need to find another equation for this — the Rarita-Schwinger equation [23] in this case. In general, Fermions have spin $\pm n/2$ with: $n = 1, 3, 5, 7, ..., 2r + 1, etc.$ Does this mean for every spin particle, we must find a new equation to describe the given spin particle? Surely, to them that strongly believe in the simplicity and beauty of *Nature*, this state of affairs does not feel right at all — it is '*ugly*' to say the least.

Feelings, intuition, beliefs and other non-measurable tools that have been useful for the search of knowledge, these can not be quantified as veritable for the purpose of building a scientific theory — be that as it may, it is true that most of our models stem from such — hence, we can not ignore this aspect as we search for a deeper knowledge on the Dirac equation. It would be simpler that there be one equation with the spin parameter, n, as part of its infrastructure, such that, one need only substitute this, n, into the equation in-order for them to describe a spin $\pm n/2$ particle. We strongly feel and believe that the issue of the universality of the Dirac equation is tide down to the foundational issues to do with the fundamental origins of this Dirac equation.

3.5 Fundamental origins of the Dirac equation

Another very important and yet largely ignored reality is that of the fundamental origins of the Dirac equation. That is to say, despite its exquisite beauty and success, it remains that Dirac guessed his equation — *albeit*, in a very educated and esoteric manner. All he sought was an equation linear in both the space and time derivatives such that when this equation is '*squared*' (*i.e.*, in the Dirac parlance of squaring) it would yeield the well known Klein-Gordon equation [3, 4].

The Klein-Gordon equation [3, 4] is itself derived from the Einstein [24] energy-momentum dispersion relation: $E^2 = p^2c_0^2 + m_0^2c_0^4$, *via* the successful method of canonical quantization that was used by Schrödinger to arrive at his successful equation that describes the atomic world. Dirac's prescription or method of arriving at his equation is not fundamental at all, and to this day, no real progress on this has been on this issue of seeking the foundational and fundamental origins of this equation. Where does the Dirac equation really come from? This is yet another question that is profusely begging for an answer.

4 4× 4 Dirac wavefunction

The very fact that the Dirac Hamiltonian, $\mathscr{H}_{\mathrm{D}}$, is a 4×4 component object acting on, ψ, this readily implies that, ψ, can be a $4 \times k$ component object where: $k = 1, 2, 3, 4, 5, etc.$ If: $1 \leq k < 4$, the resulting system of equations is overdetermined and will thus have more than one solution, and if: $k > 4$, the resultant system of equations is under-determined and is unable to yield a solution. If: $k = 4$, the system has one and only solution, and this is the case of the 4×4 Dirac wavefunction that we would like to have a look at.

In the event of a 4×4 Dirac wavefunction, ψ, this wavefunction is given by:

$$\psi = \begin{pmatrix} \psi_{00} & \psi_{01} & \psi_{02} & \psi_{03} \\ \psi_{10} & \psi_{11} & \psi_{12} & \psi_{13} \\ \psi_{20} & \psi_{21} & \psi_{22} & \psi_{23} \\ \psi_{30} & \psi_{31} & \psi_{32} & \psi_{33} \end{pmatrix} = \begin{pmatrix} \psi_a & \psi_b \\ \psi_c & \psi_d \end{pmatrix}, \quad (13)$$

where likewise:

$$\begin{matrix} \psi_a = \begin{pmatrix} \psi_{00} & \psi_{01} \\ \psi_{10} & \psi_{11} \end{pmatrix} & \psi_b = \begin{pmatrix} \psi_{02} & \psi_{03} \\ \psi_{12} & \psi_{13} \end{pmatrix} \\ \psi_c = \begin{pmatrix} \psi_{20} & \psi_{21} \\ \psi_{30} & \psi_{31} \end{pmatrix} & \psi_d = \begin{pmatrix} \psi_{22} & \psi_{23} \\ \psi_{32} & \psi_{33} \end{pmatrix} \end{matrix}, \quad (14)$$

As usual, for a free particle solution, we assume the following:

$$\psi = u \exp\left(\frac{\imath p_\mu x^\mu}{\hbar}\right), \quad (15)$$

where u is a 4×4 matrix whose terms are independent of the spacetime coordinates x^μ. That is to say:

$$u = \begin{pmatrix} u_{00} & u_{01} & u_{02} & u_{03} \\ u_{10} & u_{11} & u_{12} & u_{13} \\ u_{20} & u_{21} & u_{22} & u_{23} \\ u_{30} & u_{31} & u_{32} & u_{33} \end{pmatrix}, \quad (16)$$

where likewise:

$$\begin{matrix} u_a = \begin{pmatrix} u_{00} & u_{01} \\ u_{10} & u_{11} \end{pmatrix} & u_b = \begin{pmatrix} u_{02} & u_{03} \\ u_{12} & u_{13} \end{pmatrix} \\ u_c = \begin{pmatrix} u_{20} & u_{21} \\ u_{30} & u_{31} \end{pmatrix} & u_d = \begin{pmatrix} u_{22} & u_{23} \\ u_{32} & u_{33} \end{pmatrix} \end{matrix}. \quad (17)$$

Of this 4×4 component wavefunction, ψ, we shall require of it to observe the following constraint:

$$\psi^\dagger \psi = \psi\psi^\dagger = \varrho \mathcal{I}_4, \quad (18)$$

where: $\mathcal{I}_4$, is the 4×4 identity matrix, and, $\varrho \in \mathbb{R}$, is a real zero-rank object — it is the quantum mechanical probability density amplitude. This constraint [*i.e.*, Eq. (18)] is required by the unified theory of gravitation and electromagnetism [25] that we are currently working on.

Now, substituting the new 4×4 component wavefunction into the Dirac Eq. (1), we will have:

$$\begin{pmatrix} (E - m_0 c_0^2)\mathcal{I}_2 & -c_0 \boldsymbol{\sigma}\cdot\boldsymbol{p} \\ c_0 \boldsymbol{\sigma}\cdot\boldsymbol{p} & -(E + m_0 c_0^2)\mathcal{I}_2 \end{pmatrix} \begin{pmatrix} u_a & u_b \\ u_c & u_d \end{pmatrix} = 0. \quad (19)$$

As we proceed, the reader must — *here and after* — take note of the fact that the object, $\boldsymbol{\sigma}\cdot\boldsymbol{p}$, is a 2×2 matrix, *i.e.*:

$$\boldsymbol{\sigma}\cdot\boldsymbol{p} = \begin{pmatrix} p_z & p_x - \imath p_y \\ p_x + \imath p_y & -p_z \end{pmatrix}. \quad (20)$$

This matrix, $\boldsymbol{\sigma}\cdot\boldsymbol{p}$, is hermitian.

Now, from Eq. (19), four equations will result and these are:

$$u_a = \left(\frac{c_0 \boldsymbol{\sigma}\cdot\boldsymbol{p}}{E - m_0 c_0^2}\right) u_c, \quad (21a)$$

$$u_b = \left(\frac{c_0 \boldsymbol{\sigma}\cdot\boldsymbol{p}}{E - m_0 c_0^2}\right) u_d, \quad (21b)$$

$$u_c = \left(\frac{c_0 \boldsymbol{\sigma}\cdot\boldsymbol{p}}{E + m_0 c_0^2}\right) u_a, \quad (21c)$$

$$u_d = \left(\frac{c_0 \boldsymbol{\sigma}\cdot\boldsymbol{p}}{E + m_0 c_0^2}\right) u_b. \quad (21d)$$

For a solution to this set of simultaneous equation, we shall set as a constraint the following:

$$u_a = u_d = \sqrt{\frac{\varrho}{2}}\mathcal{I}_2. \quad (22)$$

This naturally leads to the following for, u_b, and, u_c, *i.e.*:

$$u_b = \sqrt{\frac{\varrho}{2}}\left(\frac{c_0 \boldsymbol{\sigma}\cdot\boldsymbol{p}}{E - m_0 c_0^2}\right). \quad (23a)$$

$$u_c = \sqrt{\frac{\varrho}{2}}\left(\frac{c_0 \boldsymbol{\sigma}\cdot\boldsymbol{p}}{E + m_0 c_0^2}\right). \quad (23b)$$

Hence:

$$u = \sqrt{\frac{\varrho}{2}} \begin{pmatrix} \mathcal{I}_2 & \dfrac{c_0 \boldsymbol{\sigma}\cdot\boldsymbol{p}}{E - m_0 c_0^2} \\ \dfrac{c_0 \boldsymbol{\sigma}\cdot\boldsymbol{p}}{E + m_0 c_0^2} & \mathcal{I}_2 \end{pmatrix}. \quad (24)$$

Writing this 4×4 matrix [Eq. (24)] in full, we will have:

$$u=\sqrt{\frac{\varrho}{2}}\begin{pmatrix} 1 & 0 & \dfrac{c_0p_z}{E-m_0c_0^2} & \dfrac{c_0(p_x-\imath p_y)}{E-m_0c_0^2} \\ 0 & 1 & \dfrac{c_0(p_x+\imath p_y)}{E-m_0c_0^2} & -\dfrac{c_0p_z}{E-m_0c_0^2} \\ \dfrac{c_0p_z}{E+m_0c_0^2} & \dfrac{c_0(p_x-\imath p_y)}{E+m_0c_0^2} & 1 & 0 \\ \dfrac{c_0(p_x+\imath p_y)}{E+m_0c_0^2} & -\dfrac{c_0p_z}{E+m_0c_0^2} & 0 & 1 \end{pmatrix} \tag{25}$$

Immediately, one will be quick to notice that columns (1), (2), (3), and (4) of this matrix [Eq. (25)] are in-fact the traditional solutions to the Dirac equation given in Eq. (9), (10), (11), and, (12), respectively. What this means is that the 4×4 wavefunction is a coagulation of these four traditional solutions into one giant set of mutually dependent set of the four particle systems.

5 Quarks

Apart from the pristinely obvious simplification brought about by the tying together of four independent particle solutions into a neat, single and mutually dependent particle solution — *if any at all*, what difference does the recasting of the 4×1 Dirac wavefunction into a 4×4 formulation bring onto the *Esoteric Table of Enquiry*? This surely is a most natural question which every seeking mind will ask if they are to afford any ounce of worthiness to this recasting programme. Is it just an erudite and subtle mathematical exercise devoid of any physical meaning or significance? We think not! We have a suggestion to make regarding this recasting of the Dirac wavefunction into a 4×4 wavefunction.

To that end, let us start-off by writing down the full 4×4 Dirac wavefunction: $\psi = ue^{\imath p_\mu x^\mu/\hbar}$. For the 4×4 Dirac wavefunction, the u-function has been defined in Eq. (25) and from that definition, it follows that:

$$\psi = \begin{pmatrix} \psi_N & \psi_R \\ \psi_L & \psi_N \end{pmatrix}, \tag{26}$$

where — accordingly:

$$\psi_N = \sqrt{\frac{\varrho}{2}}\mathcal{I}_2 \exp\left(\frac{\imath p_\mu x^\mu}{\hbar}\right), \tag{27a}$$

$$\psi_R = \sqrt{\frac{\varrho}{2}}\left(\frac{c_0\boldsymbol{\sigma}\cdot\boldsymbol{p}}{E-m_0c_0^2}\right) \exp\left(\frac{\imath p_\mu x^\mu}{\hbar}\right), \tag{27b}$$

$$\psi_L = \sqrt{\frac{\varrho}{2}}\left(\frac{c_0\boldsymbol{\sigma}\cdot\boldsymbol{p}}{E+m_0c_0^2}\right) \exp\left(\frac{\imath p_\mu x^\mu}{\hbar}\right). \tag{27c}$$

In-comparison, *i.e.*, between ψ as defined in Eq. (3) and the resultant definition of it in Eq. (26), we see that the initially four particles: ψ_a, ψ_b, ψ_c, and, ψ_d, have been reduced to three because, ψ_a, and, ψ_d, are identical — *i.e.*: $\psi_a = \psi_d = \psi_N$. In Eq. (26), we have according to the parlance of the Dirac formalism identified ψ_b, and, ψ_c, with the right and the left-handed Dirac components. In terms of handedness, we have in the same parlance of the Dirac formalism defined a new form of handedness in the ψ_N-particle, a handedness that we shall call — *Neutral-Handedness*, hence, ψ_N, is a neutral-handed particle, this particle is neither left nor right-handed, hence our calling it neutral-handed particle and hence the subscript-N in its denotation.

Now, in the set: ψ_N, ψ_R, and, ψ_L, we have a trio of particles that are not only mutually dependent but entangled, and in addition to this, they are confined in the same region of space. Each of these particles do not exist independently of the other, they can never be free of each other far-away from the region defined by the ψ-particle system. The boundary in spacetime of the ψ-particle system is defined by the normalization of conditions of this particle system, *i.e.*, by initial conditions entering into the integral of the probability density amplitude ($\varrho\mathcal{I}_4 = \psi^\dagger\psi$), *i.e.*: $\iiint \varrho d^3x = 1$.

Now, given the following there facts about the Proton, Neutron and Quarks — namely that:

1. The Proton and Neutron are each known to contain three Quarks living inside them.
2. Further, the Quarks strongly appear to be unable to exist independent of each other.
3. Furthermore, these Quarks strongly appear to be eternal prisoners inside the Proton and Neutron. They are unable to exist beyond the radius demarcating the Proton and Neutron particle systems.

From these facts — *i.e.*, the pristine similarity in the *Nature of Quarks* and the trio: ψ_N, ψ_R, ψ_L, it is natural to wonder whether or not these three particles: ψ_N, ψ_R, ψ_L, are the Quarks whose origins we have thus far elusively sought to understand? Surely, from this *viewpoint and*

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this alone, the recasting of the Dirac wavefunction finds a truly worthwhile reason for it to be considered as something tangible and necessary for our understanding of the goings-on in the particle World. With that having been said, we must at this very juncture say that — it is not our intention to explore this idea that the set (ψ_N, ψ_R, ψ_L) might explain Quarks and the reason for this is simple that we feel it is too early for us to do so, otherwise all that we would do is to speculate — *hypothesis non fingo*!

In the most immediate times ahead of us, we shall be reflecting on this interesting find and upon it, ponder much deeper than meets the mundane eye, and when all is clear, we shall present our findings. In the meantime, we strongly believe and feel that it is important for us to share this interesting idea with others who are or might be seeking the same answers on the *Nature of Quarks* and also them seeking a much deeper understanding of the *All-Beautiful, Noble and Esoteric Dirac Equation*.

6 Discussion

As currently accepted and understood, the Dirac theory [1, 2] employs a 4×1 type wavefunction. This 4×1 Dirac wavefunction is acted upon by a 4×4 Dirac Hamiltonian, in which process, four independent particle solutions result and insofar as the real physical meaning and distinction of these four solutions, it is not clear what these solutions really mean. It is this that this reading has made an endeavour to provoke a thought process were a physical meaning can be attached to these four independent particle solutions of the Dirac equation and this is *via* the recasting of the Dirac wavefunction into a 4×4 type wavefunction.

We first presented this idea of a 4×4 Dirac wavefunction in Ref. [26, 27]. Prior to the said presentation [26, 27], we had never seen or heard of it anywhere in the literature. Therein Refs. [26], this idea was presented as no more than a *mathematical curiosity*, with no physical meaning attached to it. That is to say, we felt that since this solution exists and looks interesting, mention of it in the literature is needed in case something worthwhile might spring forth when it is put under the lenses of another looker.

We had to come back to this idea now because we realised that it is necessary for the theory that we are currently working on [25], that is, a unified field theory of the gravitational and electromagnetic phenomenon. In the said theory [25] whose follow-up paper will be out soon, we realise that if — *as per our desideratum* — the quantum phenomenon is to be part and parcel of this unified theory the gravitational and electromagnetic force, then, the wavefunction must enter into the fold of this theory as a 4×4 object and not 4×1. Actually, we must say that, we did demonstrate in Ref. [27] that the origins of the Dirac equation can be sought from a theory such as that presented in Ref. [25].

We herein realise that — of Dirac's four independent particle solutions, what we have done with these particles is to demonstrate that these can be brought into some form of intimate unity under a single roof of the same particle system wherein the Dirac wavefunction takes a new form as a 4×4 wavefunction. In this new formation of the Dirac wavefunction, these four particle solutions precipitate into three distinct and mutuality dependent particles (ψ_L, ψ_N, ψ_R) that are eternally bound in the same region of space.

Realizing that Quarks are readily found in a mysterious threesome cohabitation-state where they are eternally bound inside the Proton and Neutron, we proceeded logically to make the natural suggestion to the effect that these Dirac particles (ψ_L, ψ_N, ψ_R) might be Quarks. Whether or not these particles are Quarks, this surely is something that further investigations will have to be establish.

In-closing, allow us to say that — for the avoidance of speculation, we have not explored this idea further than merely presenting it as a very interesting idea worthy of further investigation. In the meantime, we are looking further into this very interesting idea, with the hope of making inroads in the immediate future — *for* — what we have here is more of work in progress, rather than a complete theory.

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