Positive Energy Driven CTCs In ADM 3+1 Space – Time of Unprotected Chronology

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Abstract: Chronology unprotected mechanisms are considered with a very low gravitational polarization to make the wormhole traversal with positive energy density everywhere. No need of exotic matter has been considered with the assumption of the Einstein-Dirac-Maxwell Fields, encountering above the non-zero stress-energy-momentum tensor through spacelike hypersurfaces by a hyperbolic coordinate shift.

ADM Formalism – ANEC – Einstein-Dirac-Maxwell Fields – Chronology Protection – Noether Current – Conjugate Momenta

1. Introduction

The ADM-formalism named after its inventors Arnowitt – Deser – Misner in 1959 plays a role in the Hamiltonian formalism of General Relativity and Canonical Quantum Gravity. Here, in this paper, it has been showed that, the CTC's that appears as a generic solutions of the GTR by the tipping over of light cones, for a axisymmetric formulation with rotation of a Tippler-Cylinder pervaded through a Weak-Energy-Conditions without violating them as opposed to the violation of Average-Null-Energy-Conditions with a –Ve Stress-Energy-Momentum Tensor in need of an exotic matter. The Spatial metric should be reformalized as the property with the charge conjugation with gravity (more precisely canonical momentum) in the form of a momentum-driven-current in the Noether's field that coincides with Einstein-Maxwell-Plasma field. Proceeding further, it has been shown, whether with the non negative stress-energy tensor, chronology has been protected or not.

2. Methodology
2.1 Preliminary Discussions on the Hyperbolic Shift Vector

The metric for a stationary, axisymmetric solution with rotation is given by [1,2,3],

$$ds^2 = -P(r)dt^2 + 2Q(r)d\varphi dt + R(r)d\varphi^2 + S(r)(dr^2 + dz^2)$$

Coordinate ranges could be given as, $t \in (-\infty, +\infty)$, $r \in (0, +\infty)$, $\varphi \in [0, 2\pi]$, $z \in (\mathcal{F}^{+\infty}, \mathcal{F}^{-\infty})$ where $\mathcal{F}$ is one particular foliations of the hypersurface. The determinant proved to be Lorentzian provided, $g = det(g_{\mu\nu}) = -(PR + Q^2)S^2$: where $(PR + Q^2) > 0$. The angular coordinate φ which oscillates between an interval of $[0, 2\pi]$ and $[0, -2\pi]$ gives an invariant length, closed curve (or CTC): $ds^2(\sigma) = R(r)d\varphi^2$ provided $R(r) < 0$ and $d\varphi = 2\pi$. In this interval the affine parameter for the curve $(\sigma) = \{t = const, r = const, z = const\}$.

The space-time when gets foliated in the ADM formalism to Σ_τ which are spacelike hyper surfaces, by the temporal coordinate τ and a single space slice x^i, the 3-dimensional spatial slices having a metric of $\beta_{ij}(\tau, x^i)$ and the conjugate momenta $\rho^{ij}(\tau, x^i)$ having the lapse function N and shift vector field N_i. These describes how the different foliations having a diefformisms of time, are welded together by the 'leaves' Σ_τ. Here, the values are,

$$N = \left(-^{(4)}g^{00}\right)^{-1/2}$$

$$N_i = +^{(4)}g_{0i}$$

The Hamiltonian formalism, when computed in the generalized coordinates of a 3-dimensional metric slices as $g_{ij} = +^{(4)}g_{ij}$, the conjugate (or canonical) momentum gives as,

$$\rho^{ij}(\tau, x^i) = \sqrt{+^{(4)}g} \left(+^{(4)}\Psi^0_{pq} - g_{pq}{}^{(4)}\Psi^0_{rs}g^{rs}\right)g^{ip}g^{jp}$$

Reverting back to the (3+1) ADM-Formalism [4,5,6], the line element of the space-time foliation could be given as,

$$ds^2 = -(N^2 - N^i N_i)dt^2 - 2N_i dx^i dt + h^{ij}dx^i dx^j$$

Where the contravariant form of the shift vector components N^i at time t provides the 3-velocity (purely coordinates) as to the hypersurfaces normal. h^{ij} is the intrinsic foliated metric of the hypersurface which is flat. When, the weak energy condition is projected to the hypersurfaces normal, then, this could be given in terms of Einstein equation as,

$$G^{\mu\nu}\hat{a}^{\mu}\hat{a}^{\nu} = \left(R^{\mu\nu} - \frac{1}{2}g^{\mu\nu}R\right)\hat{a}_{\mu}\hat{a}_{\nu} = 8\pi T^{\mu\nu}\hat{a}_{\mu}\hat{a}_{\nu} \qquad\qquad T^{\mu\nu}\hat{a}_{\mu}\hat{a}_{\nu} = N^2 T^{00} = E$$

If the intrinsic curvature term of the foliated space is $+^{(3)}R$ and the quadratic hypersurface scalar being $K_n^m K_m^n$ then this can be represented in terms of the energy constraint as,

$$8\pi E = \frac{1}{2}\left(+^{(3)}R - K_n^m K_m^n + K^2\right)$$

Where K is the extrinsic curvatures trace. Then, the energy condition term could be expanded in terms of the shift vector components as,

$$K^2 - K_n^m K_m^n = 2\partial_x N_x \partial_y N_y + 2\partial_x N_x \partial_z N_z + 2\partial_z N_z \partial_y N_y - \frac{1}{2}\left(\partial_x N_y + \partial_y N_x\right)^2 - \frac{1}{2}(\partial_x N_z + \partial_z N_x)^2 - \frac{1}{2}\left(\partial_z N_y + \partial_y N_z\right)^2$$

Negative definite terms arises in the last three terms of the above equation, while the first three terms satisfies the weak energy conditions, when we have to compute the shift vector that shifts from the elliptic to hyperbolic relations with an affine real valued function parameter γ having a spatial gradient representation as $N_i = \partial_i \gamma$, if we consider the velocity in the z direction, setting $V(x) = V(y) = 0$, the rewritten form of energy condition is,

$$K^2 - K_n^m K_m^n = 2\partial_x^2 \gamma \partial_y^2 \gamma + 2\partial_z^2 \gamma \left(\frac{2}{V_h^2}\partial_z^2 \gamma + \theta\right) - 2\left(\partial_y \partial_x \gamma\right)^2 - 2(\partial_z \partial_x \gamma)^2 - 2\left(\partial_y \partial_z \gamma\right)^2$$

Where θ is the source function, and if the potentials initial condition at $z = -\infty$ is null, the shift vector over the sources 'past wave' cone and assuming γ & θ both parameterized in the (x, y) coordinates by the norm $\tilde{s} = |x| + |y|$, the simplified energy in the 2-coordinates form (z, x), formulates to the relation,

$$E = \frac{1}{16\pi}\left(2\partial_z^2 \gamma \left(\theta + \frac{2}{V_h^2}\partial_z^2 \gamma\right) - 4(\partial_z \partial_x \gamma)^2\right) \equiv 4(\partial_z \partial_x \gamma)^2 V_h^2 = \frac{2\partial_z^2 \gamma \theta}{V_h^2} + 4(\partial_z^2 \gamma)^2 \dots (1)$$

Considering the source function $\theta \sim 0$, the above equation simplifies to,

$$|\partial_z^2 \gamma| = V_h |\partial_z \partial_x \gamma| \dots (2)$$

Now the relation only when the linear wave equation of the hyperbolic source function is considered would be examined as,

$$\partial_x^2 \gamma + \partial_y^2 \gamma - \frac{2}{V_h^2}\partial_z^2 \gamma = \theta$$

Considering the source function $\theta \sim 0$, the wave being propagated only in the (z) direction, the (x, y) direction being 0, the equation (1) & (2) simplifies to an approximate result as,

$$E \equiv \frac{1}{16\pi}\frac{2}{V_h^2}\partial_z^2 \gamma = \ell \dots (3)$$

We have already assumed $z = -\infty$ at null where in the above relation, if $\partial_z^2 \gamma \sim \ell$, an affine-valued parameter which is non-negative, then $E = \ell$ which proves the non-negative energy density everywhere. Furthersome, to note, that, if $\frac{1}{16\pi}\frac{2}{V_h^2} \cong 0$ and $\partial_z^2 \gamma = \ell$, then, as in equation (3), $E \equiv 0$, then, neglecting the terms $\frac{1}{16\pi}\frac{2}{V_h^2}$, $E = \ell$ without the intrinsic gravity conditions, which shows that $E \gtrsim 0$ and therefore, its definite to conclude the relation as,

$$E \geq 0$$

$$\therefore E \geq 2\theta \times \partial_z^2 \gamma \text{ as } \theta \sim 0$$

$$\therefore E \geq \theta \times \frac{1}{2V_h}\int dx' \partial_r \theta(r, |x'| + |y|)|_{r=z-|\Delta x|/V_h}$$

Which shows that energy could be positive as an integral of the radial norm of the derivatives of the waves trajectory in z directions.

Therefore, as the generators g of the CTC satisfies the stress-energy-momentum tensor to be the integrand as expressed by the integral of the 'upper bound of the "good cone tip" U' as,

$$\oint_{g}^{U} T_{\mu\nu} dx^{\mu} dx^{\nu} \geq 0$$

As, in principle, the canonical momentum is the 'quantity that is conserved' in the electromagnetic interactions, therefore, the 'conserved current' can be described in terms of the Noether's theorem and Canonical stress-energy-momentum relations in terms of the shift vectors N_i, current J_i, electric field 3-vector E^i and magnetic field pseudo 3-vector B^i as,

$$\left(\sqrt{-1}\left(\frac{\partial\psi}{\partial x^\mu}\psi^* - \frac{\partial\psi^*}{\partial x^\mu}\psi\right)\eta^{\mu\nu}\right)^{-1} = (\theta_m + \theta)u_i u^0 - \theta N_i + \epsilon_{ijk}E^j B^k - E_i\left(E_j N^j\right) - \frac{1}{2}N_i\left((1-N_kN^k)E_lE^l + (N_kE^k)^2 - B^kB_k + \epsilon_{ljk}N^lE^jB^k\right)$$

Where,

$$J_i = \left(\sqrt{-1}\left(\frac{\partial\psi}{\partial x^\mu}\psi^* - \frac{\partial\psi^*}{\partial x^\mu}\psi\right)\eta^{\mu\nu}\right)^{-1}$$

And the conserved current,

$$\partial^i J_i = 0$$

This not only shows the conserved current in canonical momenta, but also showed that the non-negative stress-energy tensor would act as a means of unification of an electromagnetic Einstein-Maxwell Plasma field everywhere around the stress-energy tensor point.

2.2 Detailed Discussion on the Chronology Protection Mechanism

Theoretical physics have become as such that, there lies little hope in the advanced domain of physics to probe the theory experimentally as the conditions [7,8,9], in which the physics operate are too extreme to be tested. To give proper explainable logic to those data's, thought experiments have co me handy. The same thing has been true for time travel and its associated philosophies. If temporal displacement is truly achievable then *on what conditions do the arbitrary advanced civilization travels through time? Is there any restrictions imposed by the nature for such time travelling? Such displacement could be possible or nature has some ways to check them?* Stephen Hawking conjectured the *chronology protection* mechanism which forbids the creation of CTC's and makes time travel impossible. The EFE provides several solutions through which it can be ascertained that time travel is possible and so, as the formation of CTC's, however, the GR when combined with QR develops the new domain of physics called *quantum gravity*, where its impossible to tell whether they will suffice or restricts the formation of CTC's.

A space-time that contains abundant CTC's are the chronal regions and those where there are no CTC's are the nonchronal regions. The difference between these two regions are the chronology horizons, where the chronal regions end and CTC's are created at the future chronology horizons. These particular type of horizons are called the Cauchy horizons, where the null geodesics if pointed upwards then they leaves through the future, however they do not have past end points. Those regions are the regions where even if ANEC forms, the NEC could be violated (however, we will come to this issue later on). If the generators however enters to the past then, they can behave in two different ways! Either, they can assemble through a compact regions by erodically moving around the regions or they can asymptote to the null geodesics', where in the former case the chronology regions are compactly generated. While in the later case, the non-compact chronology regions have generators that sprouts to the future horizons like a fountains and are called the 'fountain of the generators'. The past chronology horizons are just the reversal of the future horizons where the null geodesics have past endpoints but they have no endpoints if followed to the future. If such a compact region is Ω then Hawking proved that, all the horizon generators will originate from the fountains while erodically wandering in Ω is probably not.

$$\Omega \xrightarrow{fountains} \mathscr{g}$$

It has been conjectured that, any rapidly rotating body being in axially symmetric fashion might contains the CTC's everywhere where those CTC's passes through the both past and future events thereby making a proposition of time travel. However, to be in an infinite length, it can be unphysical, therefore, proper foliations through 'leaves' has been done in this paper where in each foliations, there exists the cylinders that rotates about the boundaries of that 'leaves' making the infinity curbing down to a finite limit with positive energy density everywhere. Due to the defocusing bundle on the mouth of the cylinder, what it requires is that,

$$\oint_{FTN} T_{\mu\nu}\ell^\mu\ell^\nu d\xi < 0$$

Where, the integral is centered around the fountain FTN with $T_{\mu\nu}$ the total stress-energy tensor, ξ is the affine valued parameter with $\ell^\mu = dx^\mu/d\xi$ tangent to the fountain FTN around Ω where it has been shown that ANEC must be violated around the fountains. any null geodesics bundle that enters through the wormhole got defocused at the mouth through infinite blue shift, and the divergences is so much that it violates the ANEC with a pile up of radiation upon each transit from mouth to mouth making the collapsing of CTC's easier than that one can imagine due to the radiation pressure, such blue shift could be given by the equation,

$$\kappa \equiv \sqrt{(1+\alpha)(1-\alpha)}$$

Where κ is the infinite blue shifted parameter and α is the radiation parameter.

Not all chronology horizons that are compactly generated have the 'generically emerge from fountain situation' as in Taub-NUT space [10,11] having a Cauchy horizon and a compactly generated chronal regions, Taub-NUT space satisfies the local NEC with $T_{\mu\nu} > 0$ everywhere, where the generators are itself a fountain rather than peeling off from a fountain. Every generator itself is a smoothly closed null geodesics.

Hawking showed that the smoothly closed null geodesics that emerged from that fountains of a compactly generated chronology horizons, the ANEC can be $\oint_{FTN} T_{\mu\nu}\ell^\mu\ell^\nu < 0$ everywhere around the fountains, however in case of Taub-NUT or Misner space, or the rotating cylinders that we assumed in the paper have either $\oint_{FTN} T_{\mu\nu}\ell^\mu\ell^\nu = 0$ or $\oint_{FTN} T_{\mu\nu}\ell^\mu\ell^\nu > 0$ thus no ANEC is violated having none nonpositive local values everywhere around the fountains.

The high-energy blue shifted wave packet, on each circuit through $\kappa \equiv \sqrt{(1+\alpha)(1-\alpha)}$ piles up on the fountains, however, with each circuit, the divergence induced ANEC spreads laterally reducing its amplitude by a factor of $\delta/2D$ with each circuit, where, δ is the radius of the mouth of the wormhole and D is the distance between 2 mouths with 1 complete circuits from left to right and right to left has been represented as $2D$ here. If the distance between the 2 mouths is large enough with $(\delta/2D)\xi < 1$, then in each circuit, the energy density decreases with each pileup and at the horizons, the total energy density remains finite and chronology is not protected – not atleast in this version.

Repeated value of the stress-energy tenser through repeated circuits in the hypersurface, yields a repeatedly contributing stress-energy tensor through the Hadamard function where the renormalized $T_{\mu\nu}$ limits the point splitting relations, through each foliations where $G = c = 1$ and ℓ_{Pl} is the Plank length, such that $\ell_{Pl} = \sqrt{\hbar}$ with the spatial σ parameter that sharply varying with refocusing parameter as $(\Delta > 1)$ and defocusing parameter $(\Delta < 1)$ giving the relations,

$$T_{\mu\nu} = \frac{\sqrt{\Delta}}{6\pi^2}\frac{\ell_{Pl}^{\ 2}}{\sigma^3}\left(2k_\mu l_\nu + 2k_\nu l_\mu + k_\mu k_\nu + l_\nu l_\mu + g_{\mu\nu}l_a k^a\right)$$

Moreover, the backreaction of the pileup radiation pressure that arises through the wormhole mouth through each circuit $\frac{\delta}{2D}$ as a vacuum polarization (VP) distortion to the wormhole mouth is too weak to prevent chronology because, when the time interval $\Delta t \sim \ell_{Pl}$ through the horizon passage which is the Planks region, time at such small scales does not make classical sense, and if all this hold feasible, quantum gravity will smoothen the divergence, the metric while divergence occurs, becomes too small at $\omega g_{\mu\nu}^{VP} \sim \frac{\ell_{Pl}}{D} \sim 10^{-35}$ for $D \sim 1\ meter$ in case of $n \sim \infty$ turns, as given by the equation,

$$\omega g_{\mu\nu}^{VP} \sim \frac{\ell_{Pl}^{\ 2}}{D\Delta t}\sum_n \left(\frac{\delta}{2D}\right)^{n-1}$$

2.3 Further Extrapolation on the Traversal Dirac Fermions

The paper [12] contains a solution of Traversal Wormholes (TWs) as a solution of the Einstein-Dirac-Maxwell (EDM) fields, that encounters with two gauged relativistic fermions having opposite charges for maintaining a spherical symmetry, where working with the units, $G = c = \hbar = 1$, where to solve analytically, the limit $q = 0$ and the angular frequency $\omega = 0$, the line element ds^2, charge density Q and two real spinor functions $\Psi_f(r)$ and $\Psi_g(r)$, electrostatic potential $J(r)$, with the U(1) metric potential, described by the following relations,

$$ds^2 = -(1 - Mr^{-1})^2 dt^2 + \left((1 - r_0 r^{-1})(1 - Q(r_0 r)^{-1})\right)^{-1} dr^2 (r)^2 d\Omega^2$$

$$M = \frac{2Q^2 r_0}{Q^2 + r_0^2}$$

$$J(r) = \frac{M}{Q}\sqrt{\left(1 - \frac{r_0}{r}\right)\left(1 - \frac{Q^2}{r_0 r}\right)}$$

$$\Psi_f(r) = \frac{\kappa_0\left(\sqrt{1 - \frac{Q^2}{r_0 r}} - c\sqrt{1 - \frac{r_0}{r}}\right)^2}{\sqrt{1 - \frac{M}{r}}}$$

$$\Psi_g(r) = \frac{cr_0}{32\kappa_0(Q^2 + r_0^2)}\frac{\kappa_0\left(\sqrt{1 - \frac{Q^2}{r_0 r}} - c\sqrt{1 - \frac{r_0}{r}}\right)^2}{\sqrt{1 - \frac{M}{r}}}$$

Where $\kappa_0 \neq 0$ is an arbitrary constant, which describes a TWs solutions with r_0 being the throat radius and the electric charge $Q < r_0$ where M is the ADM mass having the parameterized value given $M > Q$. The spinors contribute the stress-energy-momentum tensor which is regular everywhere, and when $Q \to r_0$, then the Reissner-Nordstrom BH is approached with $T_{uv}^{(D)} \to 0$, thus maintaining a positive energy density everywhere.

3. Conclusion

This paper presents an affirmative approach on account of "temporal displacement" or time travelling. Not only, it has been shown that, there exists a Einstein-Maxwell Plasma fields which couples with gravity as virtue of conjugate momentum, that in account conserves both the charges and the Null-Energy-Conditions with a nonzero stress-energy-momentum tensor. The paper begins with Tipper's cylindrical approach which seems unphysical because of its infinite length, but spatial foliation has been done in (3+1) ADM space-times where, it can be conjectured that at each hypersurface or foliated "sheets" or "leaves' there lies a single cylinder which makes the infinite solutions renormalized. This on the other hand due to the Electric-Gravity coupling hints a new additional Kaluza-Klein 5th dimensions that could be described by means of the metric tensors but that is not the purpose of this paper. This particular dimension is too compactified or tiny to get observed. In the end, on the occasion of the marriage of GR and QM, it has been clarified that, not only the quantum gravity smoothen out the defocusing gravitational polarizations but also, the scale is too small to protect chronology. On this occasion, even if Null-Energy-Conditions could be violated, but the Average-Null-Energy-Conditions can not be compromised leading that to a nonzero value or even greater than zero. This leads to the idea of a very weak chronal protections with the view of Weak-Energy-Conditions to make the journey through the future chronology horizon or the Cauchy horizon stable.

4.   Reference

[1]   Wald, R.M. (1984) General Relativity, University of Chicago Press, Chicago.
[2]   Visser, M. (1995) Lorentzian Wormholes: From Einstein to Hawking, American Institute of Physics, New York.
[3]   LOBO, F., & CRAWFORD, P. (2003). TIME, CLOSED TIMELIKE CURVES AND CAUSALITY. arXiv:gr-qc/0206078v2
[4]   Misner. C. W., Thorne, K., & Wheeler. J. (1973) Gravitation (San Francisco: W. H. Freeman) ISBN 978-0-7167-0344-0, 978-0-691-17779-3
[5]   Bardeen, J. M. & Piran, T. (1983) Physics Reports 96 205 – 250 ISSN 0370-1573 URL http://www.sciencedirect.com/science/article/pii/0370157383900698
[6]   Lentz. E. W. (2021). Breaking the warp BARRIER: Hyper-fast Solitons In Einstein–Maxwell-plasma theory. *Classical and Quantum Gravity, 38*(7), 075015. doi:10.1088/1361-6382/abe692
[7]   Hawking, S. (1973). *The Large Scale Structure of Space-Time*. Cambridge University Press.
[8]   Hawking, S. W. (1992). Chronology protection conjecture. *Physical Review D, 46*(2), 603-611. doi:10.1103/physrevd.46.603
[9]   Throne, K. S. (1993). Closed timelike curves. *Caltech Goldernrod Preprint in Theoretical Astrophysics or Gravitational Physics.*
[10] Taub, A. H. (1951). *Ann. Math.* 53, 472
[11] Newman, E. T., Tamburino, L., & Unti, T. J., (1963). *J. Math. Phys.* 4. 915-923
[12] Blázquez-Salcedo, J. L., Knoll, C., & Radu, E. (2021). Traversable wormholes in einstein-dirac-maxwell theory. *Physical Review Letters, 126*(10). doi:10.1103/physrevlett.126.101102

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