LARGE DEVIATIONS AND INFORMATION THEORY FOR SUB-CRITICAL FOR THE SIGNAL -TO- INTERFERENCE -PLUS- NOISE RATIO RANDON NETWORK MODELS

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Abstract. The article obtains large deviation asymptotic for sub-critical communication networks modelled as signal-interference-noise-ratio(SINR) random networks. To achieve this, we define the empirical power measure and the empirical connectivity measure, as well as prove joint large deviation principles(LDPs) for the two empirical measures on two different scales. Using the joint LDPs, we prove an Asymptotic equipartition property(AEP) for wireless telecommunication Networks modelled as the subcritical SINR random networks. Further, we prove a Local Large deviation principle(LLDP) for the sub-critical SINR random network. From the LLDPs, we prove the large deviation principle, and a classical McMillan Theorem for the stochastic SINR model processes. Note that, the LDPs for the empirical measures of this stochastic SINR random network model were derived on spaces of measures equipped with the $\tau-$ topology, and the LLDPs were deduced in the space of SINR model process without any topological limitations. We motivate the study by describing a possible anomaly detection test for SINR random networks.

Keywords: Large deviation principle, Sub-critical SINR random network model, Poisson point process, Empirical power measure, Empirical connectivity measure. Relative entropy,Kullback action,Spectral potential, Anomaly detection test, Cybersecurity.

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1. Introduction

In telecommunication, Wireless networks are usually modelled by the SINR random networks. In the SINR random network model two nodes are deemed to communicate if SINR is bigger than a certain threshold as specified by some technical constant. In the process of addressing the additional requirement imposed on wireless communication networks, in particular, a higher availability of a highly accurate modeling of the SINR is required. Example, each transmission may be equipped with some battery power which may be called the mark of the node and the quantity SINR defined by the inclusion of the marks in the definition. Further study of the SINR network model has shown that an SINR model of interference is a more realistic model of interference than the protocol model of interference: a receiver node receives a packet so long as the signal to interference plus noise ratio is

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above a certain threshold. See, Bakshi et al. [2].

There are many applications of large deviation techniques to the SINR networks, which are used as models for telecommunication networks. Some of these applications include, the analysis of bi-stability in networks, such as notorious bi-stability in multiple access protocols the Aloha, and the stochastic behaviour of ATM the admission control, sizing of internal buffers, and the simulation of ATM models, see,[13]. and prevention of cyber-attacks on wireless telecommunication networks, see example [12].

Cybersecurity of the devices in a telecommunication system is a major issue when the devices become increasing dependent on computer and other local networks. And an anomaly detection in the devices networks is key to avoiding disruption in the telecommunication systems. Cybersecurity of the intelligent electronic devices in telecommunication substations has been recognized as a critical issue for smooth running of the system. One main approach to dealing with these issues is to develop new technologies to detect and disrupt any malicious activities over the networks.An Anomaly detection may be regarded as an early warning mechanism to extract relevant cybersecurity events from devices locations and correlate these events. Large deviation principles have played key role in the formulation of efficient anomaly inference algorithm for systems such as power grid, Wireless Sensor Network systems and Telecommunication systems.

In this article, we prove joint large deviation principles on the scales λ and $\lambda^2 a_\lambda$, where λ is the intensity measure of the underlining PPP of the subcritical SINR model. See, [9] or [10] or [11] for similar results fore the dense SINR random network models. From these LDPs, we prove an asymptotic equipartition property; see example [9], for the SINR models.

Further, the study shows a LLDP for the SINR models. See example, [9] and references therein. From the LLDP, we deduce asymptotic bounds on the cardinality of the set of SINR models for a given typical empirical marked measure. In addition, the study shows that from the LLDP an LDP for the SINR modelled processes.

1.1 Background

This study set a dimension $d \in \mathbb{N}$ and some measureable set $\mathcal{D} \subset \mathbb{R}^k$ with reference to the Borel-σ algebra $\mathcal{B}(\mathbb{R}^d)$. Given $\lambda\mu : \mathcal{D} \to [0,1]$, an intensity measure and probability kernel density function from $\mathcal{D}$ to $\mathbb{R}^+$, $\mathcal{K}$ and a path loss model, $\pi(\eta) = \eta^{-\alpha}$, where $\alpha \in \mathbb{R}^+$, and some technical constraint; $\iota^{(\lambda)}, \zeta^{(\lambda)} : \mathbb{R}^+ \to \mathbb{R}^+$. The study defined the SINR network model to as follows:

- We select $\sigma = (\sigma_u)_{u \in I}$, a Poisson Point Process (PPP) with rate measure $\lambda\mu : \mathcal{D} \to [0,1]$.
- Given the process σ, the locations, each σ_u is assigned a mark or power $\ell(\sigma_u) = \ell_u$ independently according to the kernel density function $\mathcal{K}(\cdot\,, \sigma_u)$.
- For any two set of marked points $((\sigma_u, \ell_u), (\sigma_v, \ell_x))$ we link an edge if and only if
$$SINR(\sigma_u, \sigma_v, \sigma) \geq \iota^{(\lambda)}(\ell_v) \text{ and } SINR(\sigma_v, \sigma_u, \sigma) \geq \iota^{(\lambda)}(\ell_u),$$
where
$$SINR(\sigma_v, \sigma_u, \sigma) = \frac{\ell_u \pi(\|\sigma_u - \sigma_v\|)}{N_0 + \zeta^{(\lambda)}(\ell_v) \sum_{u \in I \setminus \{v\}} \ell_u \pi(\|\sigma_u - \sigma_v\|)}$$

We let E denote the set of edges in the SINR random network and observe $Y^\lambda := Y^\lambda(\ell, \sigma, \mu) = \Big\{[(\sigma_u, \ell_u), u \in I], E\Big\}$ under the joint law of the marked PPP and the network. In this article, we call Y^{λ} an SINR Network model and $(\sigma_u,\ell_u):=\sigma_u^{\lambda}$ as the mark of site u. Recall from [9] that if $N_0=0$, then the connectivity function of the SINR random network model, T^{λ}, is defined as $T^{\lambda}((u,\ell_u),(v,\ell_v))=e^{-\lambda t_{\lambda}^{\mathcal{D}}((u,\ell_u),(v,\ell_v))}$, where

$$t_{\lambda}^{\mathcal{D}}((u,\ell_u),(v,\ell_v))=\int_D\Big[\frac{\iota^{(\lambda)}(\ell_u)\zeta^{(\lambda)}(\ell_u)}{\iota^{(\lambda)}(\ell_u)\zeta^{(\lambda)}(\ell_u)+(\|r\|^{\eta}/\|u-v\|^{\eta})}+\frac{\iota^{(\lambda)}(\ell_v)\zeta^{(\lambda)}(\ell_v)}{\iota^{(\lambda)}(\ell_v)\zeta^{(\lambda)}(\ell_v)+(\|r\|^{\eta}/\|v-u\|^{\eta})}\Big]\mu(dr).$$

This article assumes that there exists a_{λ} and a function $t:\mathcal{D}\times\mathbb{R}_+\to(0,\infty)$ such that $\lambda^2a_{\lambda}\to0$ and

$$\lim_{\lambda\uparrow\infty}a_{\lambda}^{-1}T^{\lambda}((a,\ell_a),(b,\ell_b))=t((a,\ell_a),(b,\ell_b)).$$

Sakyi-Yeboah et. al [10] and Sakyi-Yeboah et. al [11] investigates the critical SINR network model (that is $\lambda a_{\lambda}\to1$) and super-critical SINR network model (that is $\lambda a_{\lambda}\to\infty$) respectively . In this articles, we shall focus this study on sub-critical SINR Networks(that is $\lim_{\lambda\to\infty}\lambda a_{\lambda}\to0$).

For a given set $\mathcal{D}$) we define $\mathcal{S}(\mathcal{D})$ by

$$\mathcal{S}(\mathcal{D})=\cup_{x\subset\mathcal{D}}\Big\{x:\ |x\cap W|<\infty\ \text{,for any bounded } W\subset\mathcal{D}\Big\}. \tag{1.1}$$

Let $\mathcal{W}=\mathcal{S}(\mathcal{D}\times\mathbb{R}_+)$ and $\mathcal{M}(\mathcal{W})$, represent the space of positive measures on the space $\mathcal{W}$ equipped with $\tau-$ topology. Note, $\mathcal{W}$ is a locally finite subset of the set $\mathcal{D}\times\mathbb{R}_+$. See, example, [10]. Without abuse of notation we shall refer to $\mathcal{M}(\mathcal{W}\times\mathcal{W})$ as the space of symmetric measure on $\mathcal{W}\times\mathcal{W}$ endowed with the $\tau-$ topology. For any SINR random network model Y^{λ} we define a probability measure, the *empirical power measure*, $M_1^{Y^{\lambda}}\in\mathcal{M}(\mathcal{W})$, by

$$M_1^{Y^{\lambda}}\big((a,\ell_a)\big):=\frac{1}{\lambda}\sum_{u\in\mathcal{W}}\delta_{\sigma_u^{\lambda}}\big((a,\ell_a)\big)$$

and a finite measure, the *empirical connectivity measure* $M_2^{Y^{\lambda}}\in\mathcal{M}(\mathcal{W}\times\mathcal{W})$, by

$$M_2^{Y^{\lambda}}\big((a,\ell_a),(b,\ell_b)\big):=\frac{1}{\lambda^2a_{\lambda}}\sum_{(u,v)\in E}[\delta_{(\sigma_u^{\lambda},\sigma_v^{\lambda})}+\delta_{(\sigma_v^{\lambda},\sigma_u^{\lambda})}]\big((a,\ell_a),(b,\ell_b)\big).$$

It should be noted that the total mass $\|M_1^{Y^{\lambda}}\|$ of the empirical power measure is $\mathbb{1}$ and total mass of the empirical connect measure is $2|E|/\lambda^2a_{\lambda}$.

1.2 Motivation: Anomaly detection in spatial networks

Consider, SINR random network model as a model that account for the connectivity structure of the Wireless telecommunication networks (WTN). In particular, consider the subcritical SINR random networks as model for the WTNs since, in the implementation, the multihop network formed by the sensor nodes may adopt a network structure. The network will be formed randomly according to an arbitrary rule that is dependent on the distances between the device locations. Assume the device locations are marked according to their battery power, and the propagation of events is un-directed on the network. Our objective is to estimate network parameters and possible identify possible deviations form the actual values.

For instance, given a long sequence of realization $Y^{\lambda,k}$ of this sub-critical marked SINR random network, one would like to approximate parameter of the model, $\mu\times\mathcal{K}$ and t, by taking the average frequencies of the corresponding samples. In particular, if $M_1^{Y^{\lambda,k}}$ and $M_2^{Y^{\lambda,k}}$; the empirical power measure and the empirical connectivity measure of Y^{λ}, the k^{th} realization then

$$\lim_{k\to\infty}\tfrac{1}{k}\sum_{r=1}^{k}M_1^{Y^{\lambda,r}}(a,\ell_a)\to\mu\otimes\mathcal{K}(a,\,\ell_a)$$

and

$$\lim_{k\to\infty}\Big[\tfrac{1}{k}\sum_{r=1}^{k}M_2^{Y^{\lambda,r}}\Big((a,\ell_a),(b,\ell_b)\Big)/\tfrac{1}{k}\sum_{r=1}^{k}M_2^{Y^{\lambda,k}}(a,\ell_a)\otimes\tfrac{1}{k}\sum_{r=1}^{k}M_1^{Y^{\lambda,r}}(b,\ell_b)\Big]\to t\Big((a,\ell_a),(b,\ell_b)\Big),$$

with probability 1.

Assuming that we have estimated $\mu\otimes\mathcal{K}$ and t. We are interested in a test that determines whether a particular realization Y^λ is typical or not. Thus, we want to differentiate between $\mu\times\mathcal{K}$ and t (Hypothesis H_0) and any other unknown law (Hypothesis H_1). Theorem 2.1 will be the bases of providing generalized Neyman-Pearson criterion,See [7, pp.96-100],and hence an anomaly detection test for the sub-critical marked SINR random networks.

This article is structured as follows: Section 2 presents the main results; Theorem 2.1, Theorem 2.2, Theorem 2.3, Corollary 2.4 and Corollary 2.5. In Section 3 we prove the main results of the article, Theorem 2.1. Section 4 provides the proof of the AEP, see Theorem 2.2 and Section 5; Proof of Theorem 2.3, Corollary 2.4 and Corollary 2.5. Lastly, Section 6 presents the conclusion to the article.

2. Main Results

Theorem 2.1, is a joint large deviation principle for the empirical measures of the SINR network models.With reference from Subsection 1.1, we recall the definition of $t_\lambda^{\mathcal{D}}$ as

$$t_\lambda^{\mathcal{D}}((a,\ell_a),(b,\ell_b))=\int_D\Big[\tfrac{\iota^{(\lambda)}(\ell_u)\zeta^{(\lambda)}(\ell_u)}{\iota(\ell_u)\zeta(\ell_u)+(\|r\|^\eta/\|i-y\|^\eta)}+\tfrac{\tau^{(\lambda)}(\ell_v)\gamma^{(\lambda)}(\ell_v)}{\tau^{(\lambda)}(\ell_v)\gamma^{(\lambda)}(\ell_v)+(\|r\|^\eta/\|y-x\|^\eta)}\Big]\mu(dr)$$

and note that

$$t\beta\otimes\beta((a,\ell_a),(b,\ell_b))):=t((a,\ell_a),(b,\ell_b))\mu((a,\ell_a))\mu((b,\ell_b)).$$

Theorem 2.1. *Let Y^λ is a sub-critical marked SINR network model with rate measure $\lambda\mu:\mathcal{D}\to[0,1]$ and a power transition kernel function $\mathcal{K}(\cdot,y)=ce^{-cy}, y>0$ and path loss function $\pi(\eta)=\eta^{-\alpha}$, for $\alpha>0$. Thus, the link kernel function T^λ of Y^λ satisfies $a_\lambda^{-1}T^\lambda\to t$ and $\lambda a_\lambda\to 0$. Then, as $\lambda\to\infty$, the pair of measures $(M_1^{Y^\lambda},M_2^{Y^\lambda})$ satisfies a large deviation principle in the space $\mathcal{M}(\mathcal{W})\times\mathcal{M}(\mathcal{W}\times\mathcal{W})$*

(i) *with speed λ and a good rate function*

$$I^1(\beta,\phi)=\begin{cases}H\Big(\beta\Big|\mu\otimes\mathcal{K}\Big) & \text{if } \phi=t\beta\otimes\beta\\ \infty & \text{elsewhere.}\end{cases}\tag{2.1}$$

(ii) *with speed λ^2a_λ and good rate function*

$$I^2(\beta,\phi)=\begin{cases}\mathcal{H}(\phi\|t\beta\otimes\beta), & \text{if } \beta=\mu\otimes\mathcal{K}\\ \infty & \text{elsewhere.}\end{cases}\tag{2.2}$$

where

$$\mathcal{H}(\phi\|t\beta\otimes\beta):=\begin{cases} H(\phi\,\|\,t\beta\otimes\beta)+\Big(\|t\beta\otimes\beta\|-\|\phi\|\Big), & \textit{if } \|\phi\|>0.\\ \infty & \textit{elsewhere.}\end{cases} \tag{2.3}$$

Theorem 2.2. *Suppose Y^λ be a sub-critical marked SINR network model with rate measure $\lambda\beta : D \to [0,1]$ and a power probability function $\mathcal{K}(\cdot,y)=ce^{-cy}, y>0$ and path loss function $\pi(\eta)=\eta^{-\alpha}$, for $\alpha>0$. Thus, the connectivity probability T^λ of Y^λ satisfies $a_\lambda^{-1}T^\lambda\to t$ and $\lambda a_\lambda\to 0$. Suppose the sequence a_λ of Y^λ is such that $\lambda a_\lambda\log\lambda\to 0$ and $a_\lambda/\log\lambda\to -1$. Then, we have*

$$\lim_{\lambda\to\infty}\mathbb{P}\Big\{\Big|-\frac{1}{a_\lambda\lambda^2\log\lambda}\log P(Y^\lambda)-\mathbb{E}_f\Big[t((\cdot,\cdot),(\cdot,\cdot))\Big]\Big|\geq\varepsilon\Big\}=0,$$

where the expectation was taken with respect to the distribution function

$$f((x,\ell_x),(y,\ell_y))=c^2e^{-c(x+y)}\mu(d\ell_x)\mu(d\ell_y)dxdy,\quad x>0,y>0,\ell_x>0,\ell_y>0.$$

Note that the $H(f):=\mathbb{E}_f\Big[t((\cdot,\cdot),(\cdot,\cdot))\Big]$ is an entropy.

Interpretation: To transmit information contain in a large SINR random network modls one require with a large probability

$$-\lambda^2a_\lambda\log\lambda\Big[H(f)\Big]/\log 2 \textbf{ bits.}$$

Let $\mathcal{G}$ be the set of all SINR networks with rate measure $\lambda\mu:\mathcal{D}\to[0,1]$ and state the Local Large deviation principle as follows:

Theorem 2.3. *Suppose Y^λ is a sub-critical marked SINR network model with rate measure $\lambda\mu:\mathcal{D}\to[0,1]$ and a mark transition kernel $\mathcal{M}(y)=ce^{-cy}, y>0$ and path loss function $\pi(\eta)=\eta^{-\alpha}$, for $\eta>0$ and $\alpha>0$. Thus, the link probability T^λ of Y^λ satisfies $a_\lambda^{-1}T^\lambda\to t$ and $\lambda a_\lambda\to 0$. Then,*

- *for any functional $\phi\in\mathcal{G}$ and a number $\varepsilon>0$, there exists a weak neighbourhood B_ϕ such that*
$$\mathbb{P}_\beta\Big\{Y^\lambda\in\mathcal{G}\,\Big|\,M_2^{Y^\lambda}\in B_\phi\Big\}\leq e^{-\frac{1}{2}\lambda^2a_\lambda\mathcal{H}(\phi\|t\beta\otimes\beta)-\lambda a_\lambda\varepsilon},\ \textit{where } \beta=\mu\otimes\mathcal{K}.$$
- *for any $\phi\in\mathcal{G}_\beta$, a number $\varepsilon>o$ and a fine neighbourhood B_ϕ, we have the compute:*
$$\mathbb{P}_\beta\Big\{Y^\lambda\in\mathcal{G}\,\Big|\,M_2^{Y^\lambda}\in B_\phi\Big\}\geq e^{-\frac{1}{2}\lambda^2a_\lambda\mathcal{H}(\phi\|t\beta\otimes\beta)+\lambda_\lambda a_\lambda\varepsilon},\ \textit{where } \beta=\mu\otimes\mathcal{K}.$$

For the given telecommunication network model, we define an entropy as $h:\mathcal{M}(\mathcal{W}\times\mathcal{W})\to[0.\infty]$ by

$$h(\phi):=\Big(\|\phi\|-\|\lambda\beta\otimes\beta\|-\Big\langle\phi\,,\,\log\frac{\phi}{\|t\beta\otimes\beta\|}\Big\rangle\Big)/2,\quad\text{where } \beta=\mu\otimes\mathcal{K}. \tag{2.4}$$

Corollary 2.4 (McMillian Theorem)**.** *Let Y^λ be a sub-critical marked SINR network model with rate measure $\lambda\mu:\mathcal{D}\to[0,1]$ and a mark transition kernel $\mathcal{K}(\cdot,\,y)=ce^{-cy}, y>0$ and path loss function $\pi(\eta)=\eta^{-\alpha}$, for $\eta>0$ and where $\beta=\mu\otimes\mathcal{K}$. $\alpha>0$. Thus, the link probability T^λ of every $Y^\lambda\in\mathcal{G}$ satisfies $a_\lambda^{-1}T^\lambda\to t$ and $\lambda a_\lambda\to 0$.*

(u) *For any empirical link measure ϕ on $\mathcal{W}\times\mathcal{W}$ and $\varepsilon>0$, there exists a neighborhood B_ϕ such that*
$$Card\Big(\big\{Y^\lambda\in\mathcal{G}\,|\,M_2^{Y^\lambda}\in D_\phi\big\}\Big)\geq e^{\lambda^2a_\lambda(h(\phi)-\varepsilon\big)}.$$

(ii) *for any neighborhood B_ρ and $\varepsilon > 0$, we have*

$$Card\Big(\{Y^\lambda \in \mathcal{G} \,|\, M_2^{Y^\lambda} \in B_\phi\}\Big) \le e^{\lambda^2 a_\lambda (h(\phi)+\varepsilon)},$$

where $Card(\gamma)$ means the cardinality of γ.

remark 1 Given $\phi = t\beta\otimes\beta$, we have $Card\Big(\Big\{j \in \mathcal{G}\Big\}\Big) \approx e^{\lambda^2 a_\lambda \|t\beta\otimes\beta\| \mathcal{H}\left(t\beta\otimes\beta/\|t\beta\otimes\beta\|\right)}$, where $\beta = \mu\otimes\mathcal{K}$.

Interpretation: Note from Corollary 2.4 that, for the typical empirical connectivity measure, $t\mu^2 \otimes \mathcal{K}^2$, the cardinality of the space of SINR models is nearly equal to $e^{\lambda^2 a_\lambda \|t\mu^2\otimes\mathcal{K}^2\| H\left(t\mu^2\otimes\mathcal{K}^2/\|t\mu^2\otimes\mathcal{K}^2\|\right)}$. The next theorem is the LDP for the SINR random network processes.

Corollary 2.5. *Let Y^λ be a sub-critical marked SINR random network model with rate measure $\lambda\mu : \mathcal{D} \to [0,1]$ and a mark kernel function $\mathcal{K}(\cdot, y) = ce^{-cy}, y > 0$ and path loss function $\pi(\eta) = \eta^{-\alpha}$, for $\alpha > 0$. Thus, the link probability T^λ of Y^λ satisfies $a_\lambda^{-1}T^\lambda \to t$ and $\lambda a_\lambda \to 0$.*

- *Let U be closed subset $\mathcal{G}$. Then we have*
$$\limsup_{\lambda\to\infty} \frac{1}{\lambda^2 a_\lambda} \log \mathbb{P}_{\mu\times\mathcal{K}}\Big\{Y^\lambda \in \mathcal{G} \,\Big|\, M_2^{Y^\lambda} \in U\Big\} \le -\tfrac{1}{2} \inf_{\phi\in U} \Big\{\mathcal{H}(\phi \| t\mu\times\mathcal{K}\otimes\mu\times\mathcal{K})\Big\}$$
- *Let O be open subset $\mathcal{G}$. Then we have*
$$\liminf_{\lambda\to\infty} \frac{1}{\lambda^2 a_\lambda} \log \mathbb{P}_{\beta}\Big\{Y^\lambda \in \mathcal{G} \,\Big|\, M_2^{Y^\lambda} \in O\Big\} \ge -\tfrac{1}{2} \inf_{\phi\in O} \Big\{\mathcal{H}(\phi \| t\mu\times\mathcal{K}\otimes\mu\times\mathcal{K})\Big\}.$$

3. Proof of Main Results

3.1 Proof of Theorem 2.1(i)

Suppose $W_1, ..., W_n$ is a decomposition of the space $\mathcal{D}\times\mathbb{R}_+$. Note that, for every $(u,v) \in A_x\times A_y$, $x, y = 1, 2, 3, ..., n$, $\lambda M_2^{Y^\lambda}(u,v)$ given $\lambda M_1^{Y^\lambda}(u) = \lambda\beta(u)$ denotes a number of bernoulli trial with parameters $\lambda^2\beta(u)\beta(v)/2$ and $T^\lambda(u,v)$. Consider $\mathcal{K}$ to represent as the gamma distribution with mean $1/c$. With reference to the function $t_\lambda^{\mathcal{D}}$ from the preceding sections, we observe that Lemma 2.3 is fundamental in the application of the Gartner-Ellis Theorem. See [7].

Lemma 3.1. *Suppose Y^λ is a sub-critical marked SINR random model with rate measure $\lambda\mu : \mathcal{D} \to [0,1]$ and a power probability function $\mathcal{K}(\cdot, y) = ce^{-cy}, y > 0$ and path loss function $\pi(\eta) = \eta^{-\alpha}$, for $\eta > 0$ and $\alpha > 0$. Thus, the link probability T^λ of Y^λ satisfies $a_\lambda^{-1}T^\lambda tot$ and $\lambda a_\lambda \to 0$. Suppose Y^λ be a sub-critical SINR network model, conditional on the event $M_1^{Y^\lambda} = \beta$. Let $q : \mathcal{W}\times\mathcal{W} \to \mathbb{R}$ be bounded function. Then,*

$$\begin{aligned}\lim_{\lambda\to\infty} \frac{1}{\lambda} \log \mathbb{E}\Big\{e^{\lambda\langle q,\, M_2^{Y^\lambda}\rangle} \Big| M_1^{Y^\lambda} = \beta\Big\} &= \frac{1}{2} \lim_{n\to\infty} \sum_{y=1}^{n}\sum_{x=1}^{n} \Big\langle q,\, t\beta\otimes\beta\Big\rangle_{A_x\times A_y} \\ &= \frac{1}{2}\Big\langle q,\, t\beta\otimes\beta\Big\rangle_{\mathcal{W}\times\mathcal{W}}.\end{aligned}$$

Proof. Now we observe that

$$\mathbb{E}\Big\{e^{\int\int \lambda q(u,v) M_2^{Y^\lambda}(du,dv)/2} \Big| M_1^{Y^\lambda} = \beta\Big\} = \mathbb{E}\Big\{\prod_{u\in\mathcal{W}}\prod_{v\in\mathcal{W}} e^{\lambda q(u,v)_2^\lambda(du,dv)/2}\Big\}$$

$$\mathbb{E}\Big\{\prod_{u\in\mathcal{W}}\prod_{v\in\mathcal{W}} e^{q(u,v)\lambda M_2^{Y^\lambda}(du,dv/2)}\Big\} = \prod_{x=1}^{n}\prod_{y=1}^{n}\prod_{u\in W_x}\prod_{v\in W_y}\mathbb{E}\Big\{e^{q(u,v)\lambda M_2^{Y^\lambda}(du,dv)/2}\Big\}$$

$$\log\Big\{e^{\lambda\langle q,M_2^{Y^\lambda}\rangle/2}\Big|M_1^{Y^\lambda}=\beta\Big\} = \sum_{y=1}^{n}\sum_{x=1}^{n}\int_{W_y}\int_{W_x}\log\Big[1-T^\lambda(u,v)+T^\lambda(u,v)e^{q(u,v)/\lambda a_\lambda}\Big]^{\lambda^2\beta\otimes\beta(du,dv)/2}+o(n)$$

Introducing the dominated convergence theorem

$$\frac{1}{\lambda}\log E\{e^{\lambda\langle q,M_2^{Y^\lambda}\rangle/2}\mid M_1^{Y^\lambda}=\beta\} = \frac{1}{\lambda}\sum_{y=1}^{n}\sum_{x=1}^{n}\int_{W_x}\int_{W_y}\log\Big[1-\big(1-e^{q(u,v)/\lambda a_\lambda}\big)T^\lambda(u,v)\Big]^{\lambda^2\beta\otimes\beta(du,dv)/2}+o(n)/\lambda$$

$$\frac{1}{\lambda}\log\mathbb{E}\{e^{\lambda\langle q,M_2^{Y^\lambda}\rangle/2}|M_1^{Y^\lambda}=\beta\} = \lim_{\lambda\to\infty}\sum_{y=1}^{n}\sum_{x=1}^{n}\int_{W_x}\int_{W_y}\log\Big[1+q(u,v)t(u,v)/\lambda+o(\lambda)/\lambda\Big]^{\lambda\beta\otimes\beta(du,dv)/2}+o(n)/\lambda$$

$$\lim_{\lambda\to\infty}\frac{1}{\lambda}\log\mathbb{E}\{e^{\lambda\langle q,M_2^{Y^\lambda}\rangle/2}\mid M_1^{Y^\lambda}=\beta\} = \frac{1}{2}\sum_{y=1}^{n}\sum_{x=1}^{n}\Big\langle g,\,t\beta\otimes\beta\Big\rangle_{W_x\times W_y}$$

$$\begin{aligned}\lim_{\lambda\to\infty}\frac{1}{\lambda}\log\mathbb{E}\{e^{\lambda\langle q,M_2^{Y^\lambda}\rangle/2}\Big|M_1^{Y^\lambda}=\beta\} &= \frac{1}{2}\lim_{n\to\infty}\sum_{y=1}^{n}\sum_{x=1}^{n}\Big\langle q,\,t\beta\otimes\beta\Big\rangle_{W_x\times W_y}\\ &= \frac{1}{2}\Big\langle q,\,t\beta\otimes\beta\Big\rangle_{\mathcal{W}\times\mathcal{W}}.\end{aligned}$$

Hence,by the Gartner-Ellis theorem, conditional on the event $\Big\{M_1^{Y^\lambda}=\beta\Big\}$, $M_2^{Y^\lambda}$ obey a large deviation principle with speed λ and variational formulation of the rate function

$$I_\beta(\phi)=\frac{1}{2}\sup_q\Big\{\Big\langle q,\,\phi\Big\rangle_{\mathcal{W}\times\mathcal{W}}-\Big\langle q,\,t\beta\otimes\beta\Big\rangle_{\mathcal{W}\times\mathcal{W}}\Big\}$$

the solution can be found, see example [4], would obviously reduces to the good rate function as such

$$I_\beta(\phi)=0. \tag{3.1}$$

■

3.2 Proof of Theorem 2.1(ii)

Analogously we consider $W_1,...,W_n$ as decomposition of the space $\mathcal{D}\times\mathbb{R}_+$. We refer to t_λ^D and observe that, Lemma 3.2 will play an important role in the application of the Gartner-Ellis Theorem. See, [7].

Lemma 3.2. *Let Y^λ be a sub critical powered SINR network with rate measure $\lambda\mu:\mathcal{D}\to[0,1]$ and a power probability function $\mathcal{K}(y)=ce^{-cy}, y>0$ and path loss function $\pi(\eta)=r^{-\alpha}$, for $\eta>0$ and $\alpha>0$. Thus, the link probability T^λ of Y^λ satisfies $a_\lambda^{-1}T^\lambda\to t$ and $\lambda a_\lambda\to\infty$. Let Y^λ be a sub-critical SINR network, conditional on the event $M_1^{Y^\lambda}=\beta$. Let $q:\mathcal{W}\times\mathcal{W}\to\mathbb{R}$ be bounded function. Then,*

$$\begin{aligned}\lim_{\lambda\to\infty}\frac{1}{\lambda^2 a_\lambda}\log\mathbb{E}\Big\{e^{\lambda^2 a_\lambda\langle q,\,M_2^{Y^\lambda}\rangle}\Big|M_1^{Y^\lambda}=\beta\Big\} &= -\frac{1}{2}\lim_{n\to\infty}\sum_{y=1}^{n}\sum_{x=1}^{n}\Big\langle 1-e^q,\,t\beta\otimes\beta\Big\rangle_{W_x\times W_y}\\ &= -\frac{1}{2}\Big\langle 1-e^q,\,t\beta\otimes\beta\Big\rangle_{\mathcal{W}\times\mathcal{W}}.\end{aligned}$$

Proof. Now we note that

$$\mathbb{E}\Big\{e^{\int\int \lambda^2 a_\lambda q(u,v) M_2^{Y^\lambda}(du,dv)/2}\Big| M_1^{Y^\lambda}=\beta\Big\}=\mathbb{E}\Big\{\prod_{i\in\mathcal{W}}\prod_{j\in\mathcal{W}} e^{\lambda^2 a_\lambda q(u,v) M_2^{Y^\lambda}(du,dv)/2}\Big\}$$

$$\mathbb{E}\Big\{\prod_{i\in\mathcal{W}}\prod_{j\in\mathcal{W}} e^{q(u,v)\lambda M_2^{Y^\lambda}(du,dv/2)}=\prod_{x=1}\prod_{y=1}\prod_{i\in W_x}\prod_{j\in W_y}\mathbb{E}\Big\{e^{\lambda^2 a_\lambda q(u,v) M_2^{Y^\lambda}(du,dv)/2}\Big\}\times e^{o(n)}$$

$$\log\Big\{e^{\lambda^2 a_\lambda\langle q, M_2^{Y^\lambda}\rangle/2}\Big| M_1^{Y^\lambda}=\beta\Big\}=\sum_{y=1}^{n}\sum_{x=1}^{n}\int_{W_y}\int_{W_x}\log\Big[1-T^\lambda(u,v))+T^\lambda(u,v)e^{q(u,v)}\Big]^{\lambda^2\beta\otimes\beta(du,dv)/2}+o(n)$$

Using the dominated convergence theorem

$$\frac{1}{\lambda^2 a_\lambda}\log E\{e^{\lambda\langle q, M_2^{Y^\lambda}\rangle/2}\mid M_1^{Y^\lambda}=\beta\}=\frac{1}{\lambda^2 a_\lambda}\sum_{y=1}\sum_{x=1}\int_{W_x}\int_{W_y}\log\Big[1-(1-e^{q(u,v)})T^\lambda(u,v)\Big]^{\lambda^2\beta\otimes\beta(du,dv)/2}+o(n)/\lambda^2 a_\lambda$$

$$\frac{1}{\lambda^2 a_\lambda}\log\mathbb{E}\{e^{\lambda\langle q, M_2^{Y^\lambda}\rangle/2}|M_1^{Y^\lambda}=\beta\}=\lim_{\lambda\to\infty}\sum_{y=1}\sum_{x=1}\int_{W_x}\int_{W_y}\log\Big[1-(1-e^{q(u,v)})T^\lambda(u,v)\Big]^{\lambda\beta\otimes\beta(du,dv)/2}+o(n)/\lambda^2 a_\lambda$$

$$\lim_{\lambda\to\infty}\frac{1}{\lambda^2 a_\lambda}\log\mathbb{E}\Big\{e^{\lambda\langle q, M_2^{Y^\lambda}\rangle/2}|M_1^{Y^\lambda}=\beta\Big\}=-\frac{1}{2}\sum_{y=1}\sum_{x=1}\int_{W_x}\int_{W_y}\Big[(1-e^{q(u,v)})t(u,v)\beta\otimes\beta(du,dv)\Big]$$

$$\lim_{\lambda\to\infty}\frac{1}{\lambda^2 a_\lambda}\log\mathbb{E}\{e^{\lambda\langle q, M_2^{Y^\lambda}\rangle/2}|M_1^{Y^\lambda}=\beta\}=-\frac{1}{2}\sum_{y=1}^{n}\sum_{x=1}^{n}\Big\langle 1-e^{q},\, t\beta\otimes\beta\Big\rangle_{W_x\times W_y}$$

$$\begin{aligned}\lim_{\lambda\to\infty}\frac{1}{\lambda^2 a_\lambda}\log\mathbb{E}\{e^{\lambda\langle q, M_2^{Y^\lambda}\rangle/2}\Big|M_1^{Y^\lambda}=\beta\}&=-\frac{1}{2}\lim_{n\to\infty}\sum_{y=1}^{n}\sum_{x=1}^{n}\Big\langle 1-e^{q},\, t\beta\otimes\beta\Big\rangle_{W_x\times W_y}\\&=-\frac{1}{2}\Big\langle 1-e^{q},\, t\beta\otimes\beta\Big\rangle_{\mathcal{W}\times\mathcal{W}}\end{aligned}$$

Hence,by the Gartner-Ellis theorem, conditional on the event $\Big\{M_1^{Y^\lambda}=\beta\Big\}$, $M_2^{Y^\lambda}$ obey a large deviation principle with speed λ and variational formulation of the rate function is given by

$$I_\beta(\phi)=\frac{1}{2}\sup_q\Big\{\Big\langle q,\,\phi\Big\rangle_{\mathcal{W}\times\mathcal{W}}+\Big\langle 1-e^{q},\, t\beta\otimes\beta\Big\rangle_{\mathcal{W}\times\mathcal{W}}\Big\}$$

which when solved, see example [4], will clearly reduce to the good rate function given by

$$I_\beta(\phi)=\frac{1}{2}\mathcal{H}(\phi\|t\beta\otimes\beta). \tag{3.2}$$

■

3.3 Proof of Theorem 2.1(ii) by Method of Mixtures. For any $\lambda\in(0,\infty)$ we define

$$\mathcal{M}_\lambda(\mathcal{W}):=\Big\{\beta\in\mathcal{M}(\mathcal{W})\,:\,\lambda\beta(u)\in\mathbb{N}\text{ for all }u\in\mathcal{W}\Big\},$$

$$\tilde{\mathcal{M}}_\lambda(\mathcal{W}\times\mathcal{W}):=\Big\{\phi\in\mathcal{M}(\mathcal{W}\times\mathcal{W})\,:\,\lambda\,\phi(u,v)\in\mathbb{N},\ \text{ for all }\ u,v\in\mathcal{W}\Big\}.$$

We denote by $\Upsilon_\lambda := \mathcal{M}_\lambda(\mathcal{W})$ and $\Upsilon := \mathcal{M}(\mathcal{W})$. We write

$$P^{(\lambda)}_{\beta_\lambda}(\phi_\lambda) := \mathbb{P}\big\{M_2^{Y^\lambda} = \phi_\lambda \,\big|\, M_1^{Y^\lambda} = \beta_\lambda\big\}\,,$$
$$P^{(\lambda)}(\beta_\lambda) := \mathbb{P}\big\{M_1^{Y^\lambda} = \beta_\lambda\big\}$$

Th joint distribution of $M_1^{Y^\lambda}$ and $M_2^{Y^\lambda}$ is the mixture of $P^{(\lambda)}_{\beta_\lambda}$ with $P^{(\lambda)}(\beta_\lambda)$, as follows:

$$d\tilde{P}^\lambda(\beta_\lambda, \ell_\lambda) := dP^{(\lambda)}_{\beta_n}(\ell_\lambda)\, dP^{(\lambda)}(\beta_\lambda). \tag{3.3}$$

(Biggins, Theorem 5(b), 2004) provides condition for the validity of large deviation principles for the mixtures and for the goodness of the rate function if individual large deviation principles are known. The following three lemmas ensure validity of these conditions.

Note that the family of measures $(P^{(\lambda)}\colon \lambda \in (0,\infty))$ is exponentially tight on Υ.

Lemma 3.3. (u) *The family of measures $(\tilde{P}^\lambda\colon \lambda \in (0,\infty))$ is exponentially tight on $\Upsilon \times \tilde{\mathcal{M}}(\mathcal{W} \times \mathcal{W})$.*
(ii) *The family measures $(T^\lambda\colon \lambda \in (0,\infty))$ is exponentially tight on $\Upsilon \times \mathcal{M}(\mathcal{W} \times \mathcal{W})$.*

We refer to [9, Lemma 4.3] for similar proof for Large Deviation Principle on the scale λ^2
Define the function $I^2_{sc}, I^1_{sc}\colon \Upsilon \times \mathcal{M}(\mathcal{W} \times \mathcal{W}) \to [0,\infty]$, by

$$I^1(\beta,\phi) = \begin{cases} H\Big(\beta\Big|\mu\otimes\mathcal{K}\Big) & \text{if } \phi = t\beta\otimes\beta \\ \infty & \text{otherwise.}\end{cases} \tag{3.4}$$

$$I^2(\beta,\nu) = \frac{1}{2}\mathcal{H}\Big(\nu\|t\beta\otimes\beta\Big). \tag{3.5}$$

Lemma 3.4. (u) *I^1 is lower semi-continuous.*
(ii) *I^2 is lower semi-continuous.*

By (Biggins, Theorem 5(b), 2004) the two previous lemmas, the LDP for the empirical power measure, see, [9, Theorem 2.1] and the large deviation principles we have established Theorem 2.1 ensure that under $(\tilde{P}^\lambda)$ and T^λ the random variables $(\beta_\lambda, \ell_\lambda)$ satisfy a large deviation principle on $\mathcal{M}(\mathcal{W})\times\mathcal{M}(\mathcal{W}\times\mathcal{W})$ and $\Upsilon \times \mathcal{M}_\lambda(\mathcal{W}\times\mathcal{W})$ on the speeds λ and $\lambda^2 a_\lambda$ with good rate functions I^1 and I^2 respectively, which ends the proof of Theorem 2.1.

4. Proof of Theorem 2.2 by Large deviations

To prove the Shannon-Mcmillian Breiman (SMB) or the AEP, we first prove a weak law of large numbers (WLLN) for the empirical marked measure and the empirical connectivity measure of the SINR network model.

Lemma 4.1. *Let Y^λ be a sub-critical marked SINR model with rate measure $\lambda\mu : \mathcal{D} \to [0,1]$ and a marked transition function $\mathcal{K}(\cdot, y) = ce^{-cy}, y > 0$ and path loss function $\pi(\eta) = \eta^{-\alpha}$, for $\alpha > 0$. Thus, the link probability T^λ of Y^λ satisfies $a_\lambda^{-1}T^\lambda \to t$ and $\lambda a_\lambda \to 0$. Then,*

$$\lim_{\lambda\to\infty} \mathbb{P}\Big\{\sup_{(a,\ell_a)\in\mathcal{W}} \Big|M_1^{Y^\lambda}(a,\ell_a) - \mu\otimes\mathcal{K}(a,\ell_a)\Big| > \varepsilon\Big\} = 0$$

and

$$\lim_{\lambda\to\infty} \mathbb{P}\Big\{\sup_{([y_u,\ell_u],[y_v,\ell_v])\in\mathcal{W}\times\mathcal{W}} \Big|M_2^{Y^\lambda}([u,\ell_u],[y_v,\ell_v]) - t\mu\otimes\mathcal{K}\times\mu\otimes\mathcal{K}([y_u,\ell_u],[y_v,\ell_v])\Big| > \varepsilon\Big\} = 0$$

Proof. Let

$$U_{1,\mathcal{W}}=\Big\{\beta:\sup_{(a,\ell_a)\in\mathcal{W}}|\beta(a,\ell_a)-\mu\otimes\mathcal{K}(a,\ell_a)|>\varepsilon\Big\},$$

$$U_{2,\mathcal{W}}=\Big\{\phi:\sup_{([u,\ell_u],[j,\ell_v])\in\mathcal{W}\times\mathcal{W}}|\phi([y_u,\ell_u],[y_v,\ell_v])-t\mu\otimes\mathcal{K}\times\mu\otimes\mathcal{K}([y_u,\ell_u],[y_v,\ell_v])|>\varepsilon\Big\}$$

and $U_{3,\mathcal{W}}=U_{1,\mathcal{W}}\cup U_{2,\mathcal{W}}$. Now, observe from Theorem 2.1 that

$$\lim_{\lambda\to\infty}\frac{1}{\lambda}\log\mathbb{P}\Big\{(M_1^{Y^\lambda},M_2^{Y^\lambda})\in U_{3,\mathcal{W}}^c\Big\}\le-\inf_{(\beta,\phi)\in F_{3,\mathcal{W}}^c}I(\beta,\phi).$$

It meets the requirement for the study to prove that I is strictly positive. For instance,there is a sequence $(\beta_n,\phi_n)\to(\beta,\phi)$ such that $I(\beta_\lambda,\phi_\lambda)\downarrow I(\beta,\phi)=0$. This means $\beta=\mu\otimes\mathcal{K}$ and $\phi=t\mu\otimes\mathcal{K}\times\mu\otimes\mathcal{K}$ which contradicts $(\beta,\phi)\in U_3^c$. This ends the proof of the Lemma. □

We write $M_\Delta^\lambda=\frac{1}{\lambda}\sum_{u\in I}\delta_{(\sigma^\lambda,\sigma^\lambda)}$ and observe that the distribution of the marked SINR random network $P(y)=\mathbb{P}\Big\{Y^\lambda=y\Big\}$ is given by

$$P_\lambda(u)=\prod_{u=1}^{I}|\mu\otimes\mathcal{K}(y_u,\ell_u)\prod_{(u,v)\in E}\frac{T^{r^\lambda}([y_u,\ell_u],[y_v,\ell_v])}{1-T^\lambda([y_u,\ell_u],[y_v,\ell_v])}\prod_{(u,v)\in\mathcal{E}}(1-T^\lambda([y_u,\ell_u],[y_v,\ell_v]))\prod_{x=1}^{I}(1-T^\lambda([y_u,\ell_u],[y_v,\ell_v]))$$

$$\begin{aligned}-\frac{1}{a_\lambda\lambda^2\log\lambda}\log P_\lambda(y)&=\frac{1}{a_\lambda\lambda\log\lambda}\Big\langle-\log\mu\otimes T\,,M_1^{Y^\lambda}\Big\rangle+\frac{1}{\log\lambda}\Big\langle-\log\Big(\tfrac{T^{r^\lambda}}{1-T^{r^\lambda}}\Big)\,,M_2^{Y^\lambda}\Big\rangle\\&+\frac{1}{a_\lambda\log\lambda}\Big\langle-\log(1-T^{r^\lambda})\,,M_1^{Y^\lambda}\otimes M_1^{Y^\lambda}\Big\rangle+\frac{1}{a_\lambda\lambda\log\lambda}\Big\langle-\log(1-T^\lambda)\,,M_\Delta^\lambda\Big\rangle\end{aligned}$$

Notice,

$$\lim_{\lambda\to\infty}\frac{1}{a_\lambda\lambda\log\lambda}\Big\langle-\log\mu\otimes\mathcal{K}\,,M_1^{Y^\lambda}\Big\rangle=\lim_{\lambda\to\infty}\frac{1}{\lambda}\Big\langle-\log(1-T^\lambda\,,M_\Delta^\lambda\Big\rangle=\lim_{\lambda\to\infty}\frac{1}{a_\lambda\log\lambda}\Big\langle-\log(1-T^{r^\lambda})\,,M_1^{Y^\lambda}\otimes M_1^{Y^\lambda}\Big\rangle=0.$$

Using, Lemma 4.1 we have

$$\lim_{\lambda\to\infty}\frac{1}{\log\lambda}\Big\langle-\log\Big(T^\lambda/(1-T^\lambda)\Big)\,,M_2^{Y^\lambda}\Big\rangle=\Big\langle\mathbb{1}\,,t\mu\otimes\mathcal{K}\times\mu\otimes\mathcal{K}\Big\rangle$$

which concludes the proof of Theorem 2.2.

5. Proof of Theorem 2.3, Corollary 2.4, Corollary 2.5

For $\beta\in\mathcal{M}(\mathcal{W})$ we define the spectral potential of the marked SINR graph (Y^λ) conditional on the event $\{M_1^{Y^\lambda}=\beta\}$, $\rho_t(q,\beta)$ as

$$\rho_t(q,\beta)=\Big\langle-(1-e^q)\,,t\beta\otimes\beta\Big\rangle. \tag{5.1}$$

Note that remarkable properties of a spectral potential, see [1] or [9] holds for ρ_t.

For $\beta\in\mathcal{M}(\mathcal{W}\times\mathcal{W})$, we observe that $I_\beta(\phi)$ is the Kullback action of the marked SINR graph Y^λ.

Lemma 5.1. *The following hold for the Kullback action or divergence function $I_\beta(\phi)$:*

- $$I_\beta(\phi) = \sup_{g\in\mathcal{C}}\left\{\langle g,\, \phi\rangle - \phi_t(g,\beta)\right\}$$
- *The function $I_\beta(\phi)$ is convex and lower semi-continuous on the space $\mathcal{M}(\mathcal{W}\times\mathcal{W})$.*
- *For any real α, the set $\Big\{\phi\in\mathcal{M}(\mathcal{W}\times\mathcal{W}) : I_\beta(\phi)\le\alpha\Big\}$ is weakly compact.*

The proof of Lemma 5.1 is excluded from the article. Scholars of interest may infer to [10] for likewise proof for empirical measures of ‘ the supercritical marked SINR random network processes and/or the references therein for proof of the lemma for empirical measures on measurable spaces.

Note from Lemma 5.1 that, for any $\varepsilon>0$, there exists some function $q\in\mathcal{W}\times\mathcal{W}$ such that

$$I_\beta(\phi)-\tfrac{\varepsilon}{2} < \langle q\,,\,\beta\rangle - \phi_t(q,\phi).$$

We define the probability distribution of the powered R by P_β by

$$P_\beta(y) = \prod_{(u,v)\in E} e^{q(u,v)} \prod_{(u,v)\in\mathcal{E}} e^{g_\lambda(u,v)},$$

where

$$g_\lambda(u,v) = \frac{1}{a_\lambda}\log\Big[1-T^\lambda(u,v)+T^\lambda(u,v)e^{q(u,v)}\Big]$$

Then, clearly that

$$\begin{aligned}\frac{dP_\beta}{d\tilde{P}_\beta}(y) &= \prod_{(u,v)\in E} e^{-q(u,v)} \prod_{(u,v)\in\mathcal{E}} e^{-g_\lambda(u,v)a_\lambda}\\ &= e^{-\lambda^2 a_\lambda(\langle\frac{1}{2}q, M_2^{Y^\lambda}\rangle - \lambda^2 a_\lambda\langle\frac{1}{2}h_\lambda, M_1^{Y^\lambda}\otimes M_1^{Y^\lambda}\rangle) + \langle\frac{1}{2}g_\lambda, M_\Delta^\lambda\rangle}\end{aligned}$$

Now define the neighbourhood of ϕ, B_ϕ by

$$B_\phi := \Big\{\omega\in\mathcal{M}(\mathcal{W}\times\mathcal{W}) : \langle q,\omega\rangle - \rho_t(q,\beta) > \langle q,\phi\rangle - \rho_t(q,\phi) - \varepsilon/2\Big\}$$

Note that under the condition $M_2^{Y^\lambda}\in B_\phi$ we have

$$\frac{dP_\beta}{d\tilde{P}_\beta}(y) < e^{-\lambda^2 a_\lambda(\langle\frac{1}{2}g, L_2^\lambda\rangle - \lambda^2 a_\lambda\langle\frac{1}{2}h_\lambda, M_1^{Y^\lambda}\otimes M_1^{Y^\lambda}\rangle) + \langle\frac{1}{2}h_\lambda, M_\Delta^\lambda\rangle} < e^{-\lambda^2 a_\lambda I_{sc}(\nu) + \lambda^2 a_\lambda\varepsilon}$$

Thus, the study can deduce that

$$P_\beta\Big\{Y^\lambda\in\mathcal{G}\Big| M_2^{Y^\lambda}\in D_\phi\Big\} \le \int \mathbb{1}_{\{M_2^{Y^\lambda}\in D_\phi\}} d\tilde{P}_\beta(Y^\lambda) \le \int e^{-\lambda^2 a_\lambda I_{sc}(\beta)-\lambda\varepsilon} d\tilde{P}_\beta(Y^\lambda) \le e^{-\lambda^2 a_\lambda I_{sc}(\phi)-\lambda^2 a_\lambda\varepsilon}.$$

.

Given that $I^2(\phi)=0$ means Theorem 2.2 (ii), hence it is enough us to obtain that the result is true for a probability distribution of the form $\phi=e^q\beta\otimes\beta$ and for $I^2(\phi)=\frac{1}{2}\mathcal{H}(\phi\|t\beta\otimes\beta)$, where $\beta=\mu\otimes\mathcal{K}$. Fix any number $\varepsilon>0$ and any neigbourhood $B_\phi\subset\mathcal{M}(\mathcal{W}\times\mathcal{W})$. Now define the sequence of sets

$$\mathcal{G}^\lambda = \Big\{y^\lambda\in\mathcal{G} : M_2^{y^\lambda}\in B_\phi\Big|\langle q, M_2^{y^\lambda}\rangle - \rho_t(q,\beta)\Big| \le \tfrac{\varepsilon}{2}\Big\}.$$

Note that for all $q\in\mathcal{G}^\lambda$ we have

$$\frac{dP_\beta}{d\tilde{P}_\beta}(y) > e^{-\lambda^2 a_\lambda \langle \frac{1}{2} q, \phi\rangle + \lambda^2 a_\lambda \phi_t(q,\beta) + \lambda^2 a_\lambda \frac{\varepsilon}{2}}.$$

This yields

$$P_\pi(\mathcal{G}^\lambda) = \int_{\mathcal{G}^\lambda} dP_\beta(y) \geq \int e^{-\lambda^2 a_\lambda \langle \frac{1}{2} g, \nu\rangle + \lambda^2 a_\lambda \rho_t(g,\beta) + \lambda^2 a_\lambda \frac{\varepsilon}{2}} d\tilde{P}_\beta(y) \geq e^{-\lambda^2 a_\lambda \frac{1}{2}\mathcal{H}(\nu \| t\beta\otimes\beta) + \lambda^2 a_\lambda \varepsilon} \tilde{P}_\beta(\mathcal{G}^\lambda).$$

Applying the law of large numbers, we have that $\lim_{\lambda\to\infty} \tilde{P}_\beta(\mathcal{G}^\lambda) = 1$. This completes of the Theorem.

Proof of Corollary 2.4

The proof of Corollary 2.4 follows from the definition of the Kullback action and Theorem 2.3 if we set $\beta = \mu \otimes \mathcal{K}$ and $\lambda\beta\otimes\beta(a,b) = \|\lambda\beta\otimes\beta\|$, for all $(a,b) \in \mathcal{Y}\times\mathcal{Y}$.

Proof of Corollary 2.5

In this scenario, the result was obtained by Lemma 3.3 the law of empirical link measure is exponentially tight. Moreover, without loss of generality, we can assume that the set U in Corollary 2.5(ii) above is relatively compact. If the study chooses any $\varepsilon > 0$; then for each functional $\phi \in U$ the researchers can find a weak neighborhood such that the estimate of Theorem 2.3(u) above holds. From all these neighborhood, the study select a finite cover of $\mathcal{G}$ and sums up over the value in Corollary 2.5(u) above to obtain

$$\limsup_{\lambda\to\infty} \frac{1}{\lambda} \log \mathbb{P}_\beta\Big\{ Y^\lambda \in \mathcal{G} \,\Big|\, M_2^\lambda \in U \Big\} \leq -\inf_{\phi\in U} I_\beta(\phi) + \varepsilon, \quad \text{where } \beta = \mu\otimes\mathcal{K}.$$

As ε was arbitrarily chosen and the lower bound in Theorem 2.1(ii) means in the lower bound in Theorem 2.5 holds, the study obtains the desired results which completes the proof.

6. Conclusion

The study provided a joint large deviation principle for the empirical power measure and the empirical connectivity measure of telecommunication networks in the $\tau-$ topology. Adopting the concept of the large deviations,we have proved Shannon-McMillian Breiman Theorem for the telecommunication network modelled as the sub-critical SINR network model. In addition, we have proved a local large deviation principle for the empirical connectivity measure given the empirical power measure and from this result;we have obtained the classical McMillian theorem and for a given PPP. Finally, we have obtained an asymptotic bound on the set of all possible sub-critical SINR network processes . Conclusively, we have presented large deviation principles for the sub-critical SINR networks. Note, that our results may form the bases for designing an anomaly inference algorithms for subcritical wireless telecommunication network models.

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