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Article

Space Theory

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Abstract: The author introduces a novel theoretical framework that suggests matter and energy are both converted from curved space. In Space Theory, gravitation is generated by the flow of space, instead of being transmitted by the graviton as in String Theory. This theory also suggests that Newton's gravitational constant, denoted as G, may not be truly constant but could vary over time. The equivalent equation of space is $S = Ec^2 = mc^4$, and the gravitational force formula is $S_{\mu\nu} = 4\pi Gm = (4/3)\pi((r+a)^3 - r^3)$. The Space Theory also predicts that the surface gravitational acceleration of the neutron star Crab Pulsar (PSR B0531+21) is approximately $8.21924883 \times 10^6\ m/s^2$.

Keywords: space; gravitation

Introduction

When a star dies, it undergoes a contraction and blows off its outer envelope, forming a planetary nebula. [3,4] If the star collapses to within its Schwarzschild radius, it forms a black hole. [5] Newton's law of universal gravitation [1] provides the equation for this force, which states that:

$$\mathbf{F}_1 = G\frac{m_1 m_3}{r^2}, \qquad \mathbf{F}_2 = G\frac{m_2 m_3}{r^2}.$$

Figure 1. A star forms a black hole.

Before the star collapses, the gravitational force between the planet and the star is represented by $\mathbf{F}_1$. After the collapse, the gravitational force between the planet and the black hole is represented by $\mathbf{F}_2$. In these equations, m_1 represents the mass of the star, m_2 represents the mass of the black hole, m_3 represents the mass of the planet, r represents the distance between their centers of mass, and G is the gravitational constant.

The gravitational force of a black hole is extremely strong and nothing, not even light, can escape it. [6] Therefore, $\mathbf{F}_2$ is greater than $\mathbf{F}_1$. As the star collapses into a black hole, it blows off its outer envelope and loses mass. [7] Assuming the star loses 0.2% of its mass during this process, the mass of the black hole can be represented as 99.8% of m_1. This can be rewritten as:

$$\mathbf{F}_1 < \mathbf{F}_2, \qquad G\frac{m_1 m_3}{r^2} < G\frac{0.998\, m_1 m_3}{r^2}.$$

When the common parameters are removed, the equation can be simplified to:

$$1 < 0.998.$$

How is it even possible that 1 is less than 0.998? As the mass decreases from m_1 to m_2 and the distance r between the objects remains unchanged, it suggests that the gravitational constant G has increased. In Einstein's theory of relativity [2], matter curves spacetime, and the Einstein field equations can be expressed in the following form:

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$$G_{\mu\nu} = \frac{8\pi G}{c^4} T_{\mu\nu}.$$

In this equation, $G_{\mu\nu}$ is the Einstein tensor and G is the gravitational constant. However, if the gravitational constant G is constantly changing, it raises the question of whether Einstein's theory is still accurate.

Model

What if the situation were reversed, and both matter and energy were converted from curved space?

Matter can release energy through annihilation, fission, and fusion [2], as matter and energy are different forms of the same thing. Therefore, matter curves spacetime because it is converted from curved space, and the flow of space released by matter creates gravitational force. The greater the amount of space released, the stronger the gravitational force generated. If one object releases much more space than another object, the flow of space will narrow the distance between the two objects. It is the space that moves, not the objects.

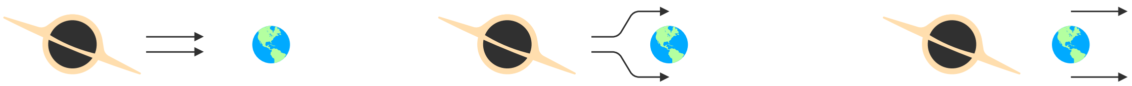

Figure 2. Gravitation.

When two galaxies are very far apart, the space they release accumulates in the middle, and this expands their distance. Therefore, the expansion of the universe and the phenomenon of redshift [8] are caused by the increase in space.

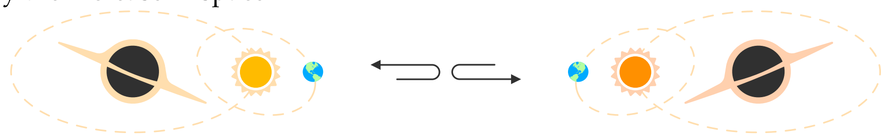

Figure 3. Expansion of the universe.

Thus, space curves in one dimension, converting matter and energy.

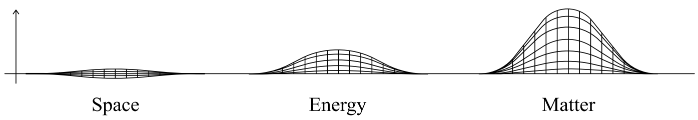

Figure 4. Curved space.

Since an electron is matter and there are only two directions in one dimension, matter curves in two different directions, creating two types of electric charges: positive and negative. Like charges repel each other because they occupy the same position in one dimension, while unlike charges attract each other because they occupy opposite positions in one dimension. Since they are all matter, they can only attract or repel each other, not annihilate.

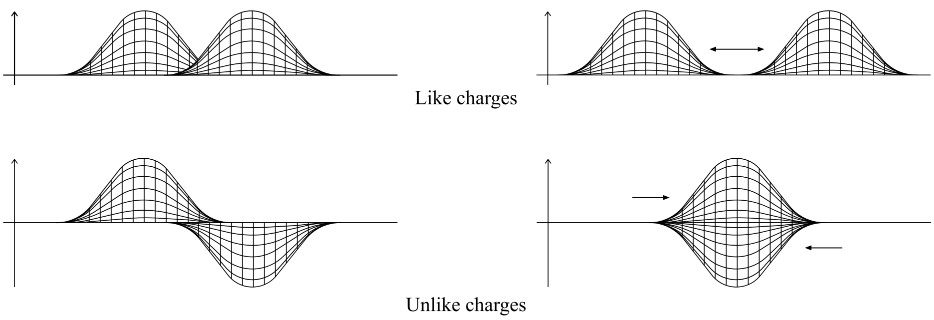

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Figure 5. Electric charges.

Quarks have three color charges [9]: red, green, and blue, which are converted by matter curving in three different dimensions. When a gluon is transferred between quarks, a color change occurs in both. For example, if a red quark emits a red–antigreen gluon, it becomes green; if a green quark absorbs a red–antigreen gluon, it becomes red. Quantum chromodynamics [10] proves that these three values of quark color can be converted into each other: they are different forms of the same thing.

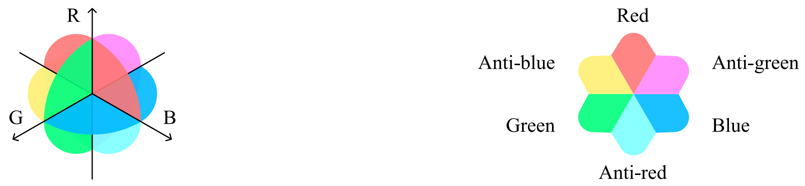

Figure 6. Quark color.

Since there are only three values of quark color, space has only three dimensions. Therefore, space curves in six different directions of three dimensions, creating six types of matter: ordinary matter, antimatter, two types of dark matter, and two types of anti-dark matter.

Matter and antimatter in opposite directions attract and annihilate each other, while matter and dark matter perpendicular to each other in another dimension have no electromagnetic force. Both matter, antimatter, dark matter, and anti-dark matter are affected by gravitation.

During the Big Bang, matter and antimatter moved in opposite directions, so antimatter ended up on the other side of the universe. Currently, there is only dark matter and anti-dark matter in the observable universe. Particle collision experiments [11] prove that ordinary matter can be converted into antimatter: they are different forms of the same thing.

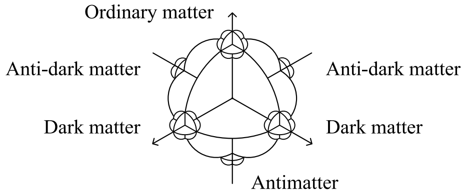

Figure 7. Six types of matter.

Matter is converted from curved space, and it also curves time.

In the double-slit experiment, an interference pattern emerges as the particles build up one by one. [12] This occurs because matter curves time, causing the particle from the past to interfere with the particle from the future.

In which-way experiment, if particle detectors are positioned at the slits, showing through which slit a photon goes, the interference pattern will disappear. [13] The mass of the observer is much greater than the particle, and when the observation occurs, the time of the observer engulfs the time of the particle, similar to how a black hole swallows a star, resulting in wave function collapse.

The Wheeler's delayed choice experiment demonstrates that extracting "which path" information after a particle passes through the slits can appear to retroactively alter its previous behavior at the slits. [14] Because matter curves time, the particle from the future can interfere with the particle from the past, meaning that the present behavior can have an impact on the past.

The quantum eraser experiment further shows that wave behavior can be restored by erasing or making permanently unavailable the "which path" information. [15] Since the time of the particle has no connection with the time of the observer, no wave function collapse occurs.

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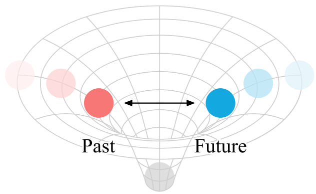

Figure 8. Matter curves time.

Every elementary particle is matter, and they are all converted from curved space, which gives them rest mass. However, the current commonly accepted physical theories imply or assume that the photon is strictly massless [16]. The Lorentz factor γ is defined as [17]:

$$\gamma = \frac{1}{\sqrt{1-(v/c)^2}}.$$

In Newton's law of universal gravitation [1], the gravitational acceleration is:

$$a = \frac{Gm_0/r^2}{\sqrt{1-(v/c)^2}}.$$

In Einstein's theory of relativity [2], the stress-energy tensor $T^{\alpha\beta}$ for a non-interacting particle with rest mass m_0 and trajectory $\mathbf{x}_p(t)$ is given by:

$$T^{\alpha\beta}(\mathbf{x},t) = \frac{m_0 v^\alpha(t) v^\beta(t)}{\sqrt{1-(v/c)^2}} \delta\left(\mathbf{x}-\mathbf{x}_p(t)\right).$$

If the photon has no rest mass, it would not be subject to gravitation and could escape from a black hole, contradicting observations. [6]

Of course, there is no doubt that the Standard Model is 100% accurate, so the blame is on the photon. The mistake, therefore, lies with the photon, which should behave precisely as predicted by scientists. The photon is both a wave and a particle, both matter and antimatter, and both has and does not have rest mass.

According to the standard model in particle physics [18], neutrino has zero rest mass and is a spin-half particle. Unfortunately, experimental observations by the Super-Kamiokande Observatory and the Sudbury Neutrino Observatories have shown that the neutrino actually has a non-zero rest mass, revealing the limitations of the Standard Model. [19,20,21]

Furthermore, as is widely known, dark matter possesses mass. Is this mass conferred by the Higgs field? If the answer is yes, does the Higgs boson composed of both matter, antimatter, and dark matter? If the answer is no, where does the mass of dark matter originate from?

Gravitation

Since there are only three values for the color charge of quarks, so the space has only three dimensions. Therefore, the space released by matter takes the form of a three-dimensional sphere. As this space flows outward, it forms a hollow sphere, and its volume can be written in the following form:

$$V_3 = \frac{4}{3}\pi r^3, \qquad S_{\mu\nu} = \frac{4}{3}\pi\left((r+\alpha)^3 - r^3\right).$$

where $S_{\mu\nu}$ is the space released by the matter, r is the radius of the sphere, and α is the degree of space curvature. The greater the degree of space curvature, the greater the gravitational acceleration of the object, which can be expressed as:

$$\alpha = at^2, \qquad t = 1, \qquad S_{\mu\nu} = \frac{4}{3}\pi\left((r+a)^3 - r^3\right).$$

When the outward flow of space occurs, its volume remains constant, allowing for the direct calculation of the gravitational acceleration a_2 at a different distance using the gravitational acceleration a_1 at a known distance. The equation can be rewritten to:

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$$S_{\mu\nu} = \frac{4}{3}\pi((r_1 + a_1)^3 - r_1{}^3) = \frac{4}{3}\pi((r_2 + a_2)^3 - r_2{}^3),$$

$$a_2 = \sqrt[3]{(r_1 + a_1)^3 - r_1{}^3 + r_2{}^3} - r_2.$$

The mass of the Earth is approximately 5.97237×10^{24} kg, and the average distance from its center to its surface is about $6.37123 \times 10^6\ m$. [22,23] According to Newton's law of universal gravitation, the value of the gravitational constant is approximately $6.67408 \times 10^{-11}\ m^3/\text{kg}/s^2$. [24] The gravitational acceleration a_1 at the Earth's surface can be calculated using this law and is given by:

$$a_1 = \frac{Gm}{r_1{}^2} \approx 9.81954911\ m/s^2.$$

If the distance up to $10^7\ m$, the gravitational acceleration a_2 is:

$$a_2 = \frac{Gm}{r_2{}^2} \approx 3.98600751\ m/s^2.$$

When the gravitational acceleration a_1 is equal to the Newtonian gravitational acceleration a_1 at r_1, the gravitational acceleration a_2 at r_2 is:

$$a_2 = \sqrt[3]{(r_1 + a_1)^3 - r_1{}^3 + r_2{}^3} - r_2 \approx 3.98601207\ m/s^2.$$

As you can see, the value of the gravitational acceleration a_2 calculated using the Space Theory is extremely close to the value calculated using Newton's law of universal gravitation. This confirms that the Space Theory can be used to accurately calculate the gravitational acceleration. However, when the distance is very large, it is advisable to use professional software to calculate the cube root, as calculators may return zero or negative values. The gravitational acceleration of the Earth at different distances in both models is shown in the following table:

Table 1. Gravitational acceleration of Earth.

Distance of Earth	$\mathbf{0\ m}$	$\mathbf{1.15 \times 10^3\ m}$	$\mathbf{10^4\ m}$	$\mathbf{10^5\ m}$	$\mathbf{10^6\ m}$	$\mathbf{10^7\ m}$
Newton's law	∞	$3 \times 10^8\ m/s^2$	$3986007\ m/s^2$	$39860\ m/s^2$	$398.600\ m/s^2$	$3.986007\ m/s^2$
Space Theory	$106141\ m/s^2$	$104991\ m/s^2$	$96171\ m/s^2$	$29976\ m/s^2$	$398.442\ m/s^2$	$3.986012\ m/s^2$

The mass of the Moon is approximately 7.342×10^{22} kg, its mean radius is about $1.737 \times 10^6\ m$, and the time-averaged distance between the centers of the Earth and Moon is about $3.844 \times 10^8\ m$. [25,26,27] When considering different distances, the gravitational acceleration of the Moon in two different models is shown in the following table:

Table 2. Gravitational acceleration of Moon.

Distance of Moon	$\mathbf{0\ m}$	$\mathbf{1.27 \times 10^2\ m}$	$\mathbf{10^5\ m}$	$\mathbf{10^6\ m}$	$\mathbf{10^7\ m}$	$\mathbf{3.844 \times 10^8\ m}$
Newton's law	∞	$3 \times 10^8\ m/s^2$	$490.01\ m/s^2$	$4.900109\ m/s^2$	$0.04900\ m/s^2$	$0.00003316\ m/s^2$
Space Theory	$24496\ m/s^2$	$24369\ m/s^2$	$487.60\ m/s^2$	$4.899863\ m/s^2$	$0.04899\ m/s^2$	$0.00003316\ m/s^2$

The mass of the Sun is approximately 1.9885×10^{30} kg, with a mean radius of about $6.96342 \times 10^8\ m$, and the mean distance between the centers of the Earth and the Sun is about $1.496 \times 10^{11}\ m$. [28,29] The table below shows the gravitational acceleration of the Sun in two different models at varying distances:

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Table 3. Gravitational acceleration of Sun.

Distance of Sun	$0\ m$	$6.65 \times 10^5\ m$	$10^7\ m$	$10^9\ m$	$10^{10}\ m$	$1.496 \times 10^{11}\ m$
Newton's law	∞	$3 \times 10^8\ m/s^2$	$1327140\ m/s^2$	$132.71\ m/s^2$	$1.3271\ m/s^2$	$0.005929\ m/s^2$
Space Theory	$7356638\ m/s^2$	$6693449\ m/s^2$	$1181938\ m/s^2$	$132.71\ m/s^2$	$1.3271\ m/s^2$	$0.005929\ m/s^2$

When the distance is zero, Newton's gravitational acceleration becomes infinite, which is obviously incorrect. In contrast, the gravitation of the Space Theory is more accurate and does not require the use of Newton's constant of gravitation. The Schwarzschild radius [5] is a physical parameter that appears in the Schwarzschild solution to Einstein's field equations. It corresponds to the radius defining the event horizon of a black hole and can be expressed as:

$$r_s = \frac{2Gm}{c^2}.$$

The Schwarzschild radius of Earth is approximately $0.00887\ m$. However, when the distance is $1150\ m$, the Newton's gravitational acceleration exceeds approximately $3 \times 10^8\ m/s^2$, it is greater than the speed of light in vacuum, approximately $2.9979 \times 10^8\ m/s$, and this leads to the formation of a black hole, which is obviously wrong.

Expanding the formula, the gravitational acceleration of the Space Theory can be derived as:

$$S_{\mu\nu} = \frac{4}{3}\pi(3r^2a + 3ra^2 + a^3) = 4\pi r^2 a\left(1 + \frac{a}{r} + \frac{a^2}{3r^2}\right).$$

Then introduce a new variable β to represent $1 + a/r + a^2/(3r^2)$. The equation can be simplified to:

$$S_{\mu\nu} = 4\pi r^2 a\beta, \qquad \beta = 1 + \frac{a}{r} + \frac{a^2}{3r^2}, \qquad \lim_{r\to\infty} \beta = 1.$$

The expression implies that β can approach 1 as closely as desired by increasing the distance r to infinity. At the surface of the Earth, the ratio of the gravitational acceleration a to the distance r is approximately 0.00000153, which is negligible and can be omitted. The formula can be rewritten as:

$$S_{\mu\nu} = 4\pi r^2 a, \qquad \beta = 1.$$

Since the accelerations of the two formulas are equal at long distances, the formula can be simplified to:

$$a = \frac{S_{\mu\nu}}{4\pi r^2} = \frac{Gm}{r^2}.$$

After removing the same parameters, the gravitation of the Space Theory $S_{\mu\nu}$ thus takes the form:

$$S_{\mu\nu} = 4\pi Gm.$$

The result demonstrates that the space released by matter per kilogram is precisely equal to $4\pi G$, providing evidence that matter is converted from curved space, indicating that they are different forms of the same thing. In comparison to Newton's law of universal gravitation, the gravitation of the Space Theory has only one more variable, β, which is extremely close to 1 at long distances.

$$S_{\mu\nu} = 4\pi Gm = 4\pi r^2 a\beta, \qquad a = \frac{4\pi Gm}{4\pi r^2\beta} = \frac{Gm}{r^2}\frac{1}{\beta}.$$

The further the distance between the objects, the closer the values of the formulas.

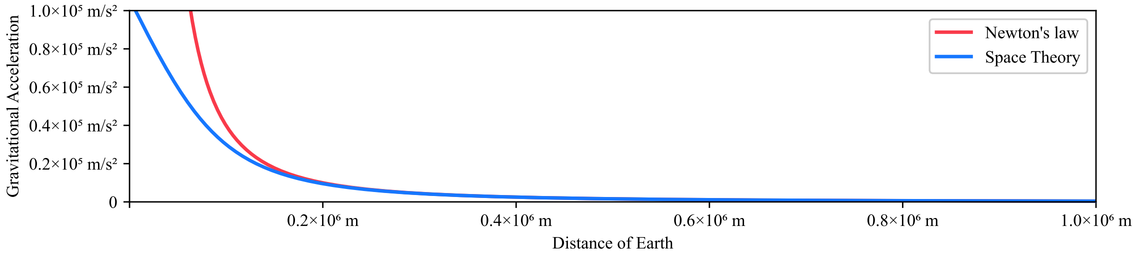

Figure 9. Gravitational acceleration of Earth.

In Einstein's theory of relativity [2], matter curves spacetime, and the Einstein field equations can be expressed in the following form:

$$G_{\mu\nu} = \frac{8\pi G}{c^4} T_{\mu\nu}, \qquad \kappa = \frac{8\pi G}{c^4}.$$

where $G_{\mu\nu}$ is the Einstein tensor, $T_{\mu\nu}$ is the stress-energy tensor, c is the speed of light in vacuum, and κ is the Einstein constant of gravitation. In the geometrized unit system, the value of $4\pi G$ is set equal to unity. Therefore, it is possible to remove the Newton's constant of gravitation from the equations to avoid mistakes, especially since the Newton's law of universal gravitation does not apply to black holes. The equation can be rewritten as:

$$4\pi G = 1, \qquad \kappa = \frac{2}{c^4}.$$

Since the theory is that matter and energy are both converted from curved space, the form should also be reversed. As space S is proportional to mass m and the fourth power of the speed of light in vacuum c^4, the equivalent equation for space S can be expressed as:

$$S = Ec^2 = mc^4, \qquad S_{\mu\nu} = \frac{4\pi G}{c^4} S.$$

Of course, under ordinary circumstances, if the Newton's "constant" of gravitation doesn't change too much, it can still be used to calculate the gravitational acceleration, the form can be rewritten to:

$$S_{\mu\nu} = 4\pi Gm = \frac{4}{3}\pi((r+a)^3 - r^3), \qquad a = \sqrt[3]{3Gm + r^3} - r.$$

On the surface of Earth, the gravitational acceleration is:

$$a_1 = \frac{Gm}{r^2} \approx 9.81954911\ m/s^2,$$

$$a_2 = \sqrt[3]{3Gm + r^3} - r \approx 9.81953396\ m/s^2.$$

And the relative error is:

$$\eta = \left|1 - \frac{a_2}{a_1}\right| \approx 0.00000154284.$$

As you can see, this value is still extremely close to the original. What's more, this formula does not introduce any new variables. Although this formula solves the problem at short distances, when the distance becomes too large, the software calculator may return zero or a negative value, so the author has made improvements to the original equation.

In 1665, Newton extended the binomial theorem to include real exponents, expanding the finite sum into an infinite series. [1] To achieve this, he needed to give binomial coefficients a definition with an arbitrary upper index, which could not be accomplished through the traditional factorial formula. Nonetheless, for any given number n, it is possible to define the coefficients as:

$$\binom{n}{k} = \frac{n(n-1)(n-2)\cdots(n-k+1)}{k!} = \frac{(n)_k}{k!}.$$

The Pochhammer symbol $(\cdot)_k$ is used to represent a falling factorial, which is defined as a polynomial:

$$(n)_k = n^{\underline{k}} = n(n-1)(n-2)\cdots(n-k+1) = \prod_{k=1}^{n}(n-k+1).$$

This formula holds true for the usual definitions when n is a nonnegative integer. For any complex number n and real numbers x and y with $|x| > |y|$, the following equation holds:

$$(x+y)^n = \sum_{k=0}^{\infty}\binom{n}{k}x^{n-k}y^k = x^n + nx^{n-1}y + \frac{n(n-1)}{2!}x^{n-2}y^2 + \frac{n(n-1)(n-2)}{3!}x^{n-3}y^3 + \cdots.$$

The series for the cube root can be obtained by setting $n = 1/3$, which gives:

$$(x+y)^{\frac{1}{3}} = x^{\frac{1}{3}} + \frac{1}{3}x^{-\frac{2}{3}}y - \frac{1}{9}x^{-\frac{5}{3}}y^2 + \frac{5}{81}x^{-\frac{8}{3}}y^3 + \cdots.$$

At long distances where $|r| > \left|\sqrt[3]{3Gm}\right|$, the equation becomes:

$$a = \sqrt[3]{r^3 + 3Gm} - r = r + \frac{1}{3}r^{-2}3Gm - \frac{1}{9}r^{-5}(3Gm)^2 + \frac{5}{81}r^{-8}(3Gm)^3 + \cdots - r,$$

$$a = \frac{Gm}{r^2} - \frac{1}{r}\left(\frac{Gm}{r^2}\right)^2 + \frac{5}{3r^2}\left(\frac{Gm}{r^2}\right)^3 + \cdots.$$

Then introduce a new variable β, the equation can be simplified to:

$$a = \frac{Gm}{r^2} + \beta, \qquad \beta = -\frac{1}{r}\left(\frac{Gm}{r^2}\right)^2 + \frac{5}{3r^2}\left(\frac{Gm}{r^2}\right)^3 + \cdots, \qquad \lim_{r\to\infty}\beta = 0.$$

At the surface of the Earth, the variable β is approximately -0.00001513417, which is negligible and can be omitted. This is the reason why it is extremely close to Newton's value at long distances.

At short distances where $\left|\sqrt[3]{3Gm}\right| > |r|$, the form of the equation is different:

$$a = \sqrt[3]{3Gm + r^3} - r = (3Gm)^{\frac{1}{3}} + \frac{1}{3}(3Gm)^{-\frac{2}{3}}r^3 - \frac{1}{9}(3Gm)^{-\frac{5}{3}}r^6 + \frac{5}{81}(3Gm)^{-\frac{8}{3}}r^9 + \cdots - r,$$

$$a = \sqrt[3]{3Gm} - r\left(1 - \frac{1}{3}\left(\frac{r}{\sqrt[3]{3Gm}}\right)^2 + \frac{1}{9}\left(\frac{r}{\sqrt[3]{3Gm}}\right)^5 - \frac{5}{81}\left(\frac{r}{\sqrt[3]{3Gm}}\right)^8 + \cdots\right).$$

In summary, formulas inversely proportional to the distance r raised to the power of n can all be replaced with the hollow sphere model:

$$\lim_{r\to\infty}\frac{x}{r^n} = \lim_{r\to\infty}\sqrt[n+1]{(n+1)x + r^{n+1}} - r.$$

In Chapter 7 of the book "Why String Theory?", it is mentioned that there is no direct experimental evidence for the String Theory. [30] To avoid encountering the same issue of unverifiability as String Theory, the author makes a prediction about the surface gravity of neutron star.

The Crab Pulsar (PSR B0531+21) has a mass of 1.4 solar masses (M☉) and a radius of approximately $10500\ m$. [31] The solar mass (M☉) is a standard unit of mass in astronomy, equal to approximately 1.98847×10^{30} kg. [28] On the surface of Crab Pulsar, the gravitational acceleration is:

$$a_1 = \frac{Gm}{r^2} \approx 1.68523274 \times 10^{12}\ m/s^2,$$

$$a_2 = \sqrt[3]{3Gm + r^3} - r \approx 8.21924883 \times 10^6\ m/s^2.$$

Newton's prediction of $1.68523274 \times 10^{12}\ m/s^2$, which is significantly greater than the speed of light in vacuum, $2.9979 \times 10^8\ m/s$, and this leads to the formation of a black hole, clearly deviates from what is expected. In contrast, the predicted value from the Space Theory, $8.21924883 \times 10^6\ m/s^2$, aligns more closely with reality.

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In addition, the distance can also be determined by the gravitational acceleration, it can be rewritten as:

$$r_1 = \sqrt{\frac{Gm}{a}},$$

$$r_2 = \sqrt{\frac{Gm}{a} - \frac{1}{12}a^2} - \frac{1}{2}a.$$

And the product of variables, G and m, becomes:

$$Gm_1 = ar^2,$$

$$Gm_2 = ar^2 + a^2r + \frac{1}{3}a^3.$$

As you can see, when the acceleration is very small, the results between the two formulas will be very close. Due to the modification of the Newton's law, the orbital velocity also requires a corresponding adjustment, and it can be expressed in the following form:

$$\lim_{r\to\infty}\frac{x}{r} = \lim_{r\to\infty}\sqrt{2x + r^2} - r, \qquad v = \sqrt{\sqrt{2Gm + r^2} - r}.$$

The mass of the Earth is approximately 5.97237×10^{24} kg, and the mass of the Moon is approximately 7.342×10^{22} kg. [23,25] The time-averaged distance between the centers of the Earth and Moon is about $3.844 \times 10^8\ m$. [27] The orbital velocity is:

$$v_1 = \sqrt{\frac{G(m_1 + m_2)}{r}} \approx 1024.54383\ m/s,$$

$$v_2 = \sqrt{\sqrt{2G(m_1 + m_2) + r^2} - r} \approx 1023.84606\ m/s.$$

And the relative error is:

$$\eta \approx 0.0006815.$$

The modified formula resolves the issue of approaching infinity as the distance nears zero. Similarly, the distance of the orbit can also be determined by velocity, it can be rewritten as:

$$(v^2 + r)^2 = 2Gm + r^2, \qquad r = \frac{Gm}{v^2} - \frac{1}{2}v^2.$$

When the orbital velocity is $1024\ m/s$, the distance is:

$$r_1 = \frac{G(m_1 + m_2)}{v^2} \approx 3.84808 \times 10^8\ m,$$

$$r_2 = \frac{G(m_1 + m_2)}{v^2} - \frac{1}{2}v^2 \approx 3.84284 \times 10^8\ m.$$

And the relative error is:

$$\eta \approx 0.0013643.$$

Electromagnetism

The same formula can also be used to calculate the electrostatic force. Interpret the formula as describing the propagation of photons in three-dimensional space, extending as a hollow sphere, with the number of particles remaining constant, and the density decreasing with distance, causing changes in the electric field. Therefore, the electric field can also be represented using the formula:

$$\lim_{r\to\infty}\frac{x}{r^2} = \lim_{r\to\infty}\sqrt[3]{3x + r^3} - r, \qquad \mathbf{E} = \sqrt[3]{3k_e|q| + r^3} - r.$$

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Where $\mathbf{E}$ is the electric field of photons, k_e is the Coulomb constant, and q is the quantity of each charge. The Coulomb constant is about $8.98755179 \times 10^9\ Nm^2/C^2$, and the electric charge of a single electron is approximately $1.602176634 \times 10^{-19}\ C$ [32]. If the distance of an electron is $1\ m$, the electric field is:

$$\mathbf{E}_1 = k_e \frac{|q|}{r^2} \approx 1.43996426069 \times 10^{-9}\ N/C,$$

$$\mathbf{E}_2 = \sqrt[3]{3k_e|q| + r^3} - r \approx 1.43996425861 \times 10^{-9}\ N/C.$$

The further the distance between the objects, the closer the values of the formulas.

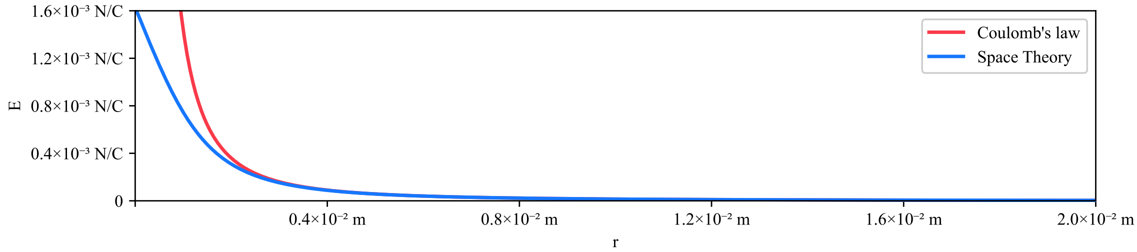

Figure 10. The electric field of a proton.

In the geometrized unit system, where $\hbar = e = G = k_e = 1$ and $\varepsilon_0 = 1/(4\pi)$. For the convenience of calculation, here Z represents the atomic number of the atom, and the quantity of charge is set to 1. The Bohr radius, symbolized a_0, is approximately $52.9\ pm$. When the distance is $100\ pm$, the electric field of the hydrogen atom $Z = 1$ is:

$$\mathbf{E}_1 = \frac{Z}{(r/a_0)^2} \approx 0.27984099,$$

$$\mathbf{E}_2 = \sqrt[3]{3Z + (r/a_0)^3} - r/a_0 \approx 0.24634396.$$

When the distance is equal to the Bohr radius, the electric field is:

$$\mathbf{E}_1 = \frac{Z}{(r/a_0)^2} = 1,$$

$$\mathbf{E}_2 = \sqrt[3]{3Z + (r/a_0)^3} - r/a_0 \approx 0.58740105.$$

The variation is illustrated in the diagram:

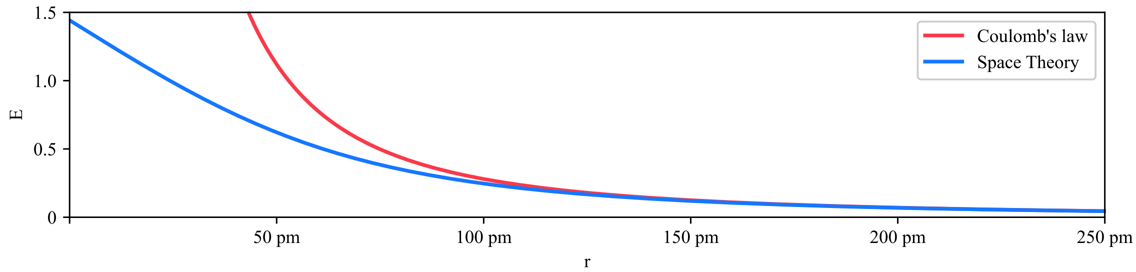

Figure 11. The electric field of the Hydrogen atom.

Carbon, hydrogen, nitrogen, oxygen, phosphorus, and sulfur are the six most important chemical elements whose covalent combinations make up most biological molecules on Earth.[33] The following table shows atomic radius, Van der Waals radius and covalent bond from theoretical models. [34,35,36,37,38]

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Table 4. Atomic radius from theoretical models.

Number	Symbol	Name	Radius			Bond		
			Calculated	Empirically	Van der Waals	Single	Double	Triple
1	H	Hydrogen	53 pm	25 pm	120 pm	32 pm		
2	He	Helium	31 pm	120 pm	140 pm			
6	C	Carbon	67 pm	70 pm	170 pm	75 pm	67 pm	60 pm
7	N	Nitrogen	56 pm	65 pm	155 pm	71 pm	60 pm	54 pm
8	O	Oxygen	48 pm	60 pm	152 pm	63 pm	57 pm	53 pm
15	P	Phosphorus	98 pm	100 pm	180 pm	111 pm	102 pm	94 pm
16	S	Sulfur	88 pm	100 pm	180 pm	103 pm	94 pm	95 pm

And the typical bond lengths and bond energies are: [39]

Table 5. Typical bond lengths and bond energies.

Bond	Length	Energy	Bond	Length	Energy	Bond	Length	Energy
C−O	143 pm	358 kJ/mo	N−C	147 pm	305 kJ/mo	N−O	144 pm	201 kJ/mo
C=O	123 pm	745 kJ/mo	N=C	127 pm	615 kJ/mo	N=O	120 pm	607 kJ/mo
C≡O	113 pm	1072 kJ/mo	N≡C	115 pm	891 kJ/mo	O−O	148 pm	146 kJ/mo
C−C	154 pm	347 kJ/mo	N−N	147 pm	160 kJ/mo	O=O	121 pm	495 kJ/mo
C=C	133 pm	614 kJ/mo	N=N	124 pm	418 kJ/mo	O−H	96 pm	467 kJ/mo
C≡C	120 pm	839 kJ/mo	N≡N	110 pm	941 kJ/mo	H−H	74 pm	436 kJ/mo
C−H	109 pm	413 kJ/mo	N−H	101 pm	391 kJ/mo			

The Lennard-Jones potential characterizes intermolecular interactions within pairs. This potential model accounts for both soft repulsive and attractive (van der Waals) forces and is applicable to electronically neutral atoms or molecules. The commonly employed expression for the Lennard-Jones potential is: [40,41,42]

$$V(r) = 4\varepsilon\left[\left(\frac{\sigma}{r}\right)^{12} - \left(\frac{\sigma}{r}\right)^{6}\right].$$

where r represents the distance between two interacting particles, ε represents the depth of the potential well, commonly known as the 'dispersion energy', and σ represents the distance at which the particle-particle potential energy $V(r)$ equals zero, often termed the 'size of the particle'. To simplify the formula the potential energy term can be rewritten as:

$$x^2 - x = \left(x - \frac{1}{2}\right)^2 - \frac{1}{4}, \qquad x = \left(\frac{\sigma}{r}\right)^6, \qquad V(r) = 4\varepsilon\left[\left(\left(\frac{\sigma}{r}\right)^6 - \frac{1}{2}\right)^2 - \frac{1}{4}\right].$$

However, as the distance approaches zero, the result tends toward infinity. Therefore, the formula is rewritten using the hollow sphere model:

$$\lim_{r\to\infty}\frac{x}{r^6}=\lim_{r\to\infty}\sqrt[7]{7x+r^7}-r, \qquad V(r)=4\varepsilon\left[\left(\sqrt[7]{7\sigma^6+r^7}-r-\frac{1}{2}\right)^2-\frac{1}{4}\right].$$

The bond length of the O-H molecule is $96\,pm$, so the σ is:

$$\sigma=r\sqrt[6]{\frac{1}{2}}\approx 85.52627\,pm,$$

$$\sigma=\sqrt[6]{\frac{1}{7}\left(\left(\frac{1}{2}+r\right)^7-r^7\right)}\approx 85.74948\,pm.$$

When r is $96\,pm$ and σ is $85.52\,pm$, the result is:

$$\frac{V(r)}{\varepsilon}=4\left[\left(\left(\frac{\sigma}{r}\right)^6-\frac{1}{2}\right)^2-\frac{1}{4}\right]\approx -0.99999,$$

$$\frac{V(r)}{\varepsilon}=4\left[\left(\sqrt[7]{7\sigma^6+r^7}-r-\frac{1}{2}\right)^2-\frac{1}{4}\right]\approx -0.99975.$$

When r is $25\,pm$, the result is:

$$\frac{V(r)}{\varepsilon}=4\left[\left(\left(\frac{\sigma}{r}\right)^6-\frac{1}{2}\right)^2-\frac{1}{4}\right]\approx 10264104.8201,$$

$$\frac{V(r)}{\varepsilon}=4\left[\left(\sqrt[7]{7\sigma^6+r^7}-r-\frac{1}{2}\right)^2-\frac{1}{4}\right]\approx 4713.4013.$$

When the distance is zero, the result is:

$$\frac{V(r)}{\varepsilon}=4\left[\left(\left(\frac{\sigma}{r}\right)^6-\frac{1}{2}\right)^2-\frac{1}{4}\right]\approx \infty,$$

$$\frac{V(r)}{\varepsilon}=4\left[\left(\sqrt[7]{7\sigma^6+r^7}-r-\frac{1}{2}\right)^2-\frac{1}{4}\right]\approx 14070.5219.$$

The variation is illustrated in the diagram:

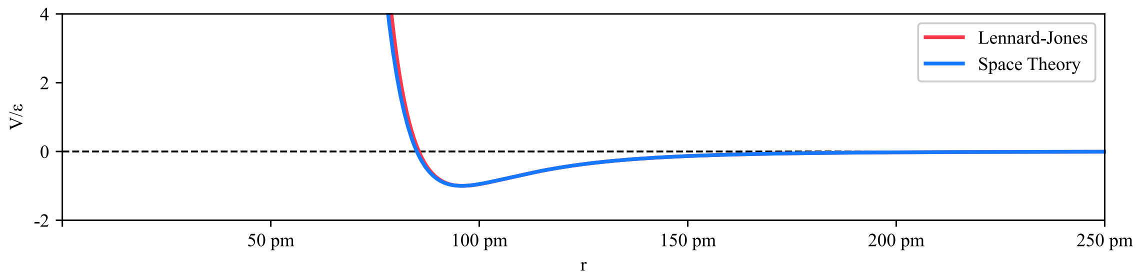

Figure 12. Lennard-Jones potential of a hydrogen bond.

The formula can also be further modified:

$$\lim_{r\to\infty}\frac{x}{r^{12}}=\lim_{r\to\infty}\sqrt[13]{13x+r^{13}}-r, \qquad V(r)=4\varepsilon\left[\sqrt[13]{13\sigma^{12}+r^{13}}-\sqrt[7]{7\sigma^6+r^7}\right].$$

When r is $96\,pm$ and σ is $85.52\,pm$, the result is:

$$\frac{V(r)}{\varepsilon}=4\left[\left(\frac{\sigma}{r}\right)^{12}-\left(\frac{\sigma}{r}\right)^6\right]\approx -0.99999,$$

$$\frac{V(r)}{\varepsilon}=4\left[\sqrt[13]{13\sigma^{12}+r^{13}}-\sqrt[7]{7\sigma^6+r^7}\right]\approx -0.98473.$$

When r is $25\,pm$, the result is:

$$\frac{V(r)}{\varepsilon} = 4\left[\left(\frac{\sigma}{r}\right)^{12} - \left(\frac{\sigma}{r}\right)^{6}\right] \approx 10264104.8201,$$

$$\frac{V(r)}{\varepsilon} = 4\left[\sqrt[13]{13\sigma^{12} + r^{13}} - \sqrt[7]{7\sigma^{6} + r^{7}}\right] \approx 56.6111.$$

When the distance is zero, the result is:

$$\frac{V(r)}{\varepsilon} = 4\left[\left(\frac{\sigma}{r}\right)^{12} - \left(\frac{\sigma}{r}\right)^{6}\right] \approx \infty,$$

$$\frac{V(r)}{\varepsilon} = 4\left[\sqrt[13]{13\sigma^{12} + r^{13}} - \sqrt[7]{7\sigma^{6} + r^{7}}\right] \approx 56.6872.$$

Through correction, the Lennard-Jones potential at short distances has been limited.

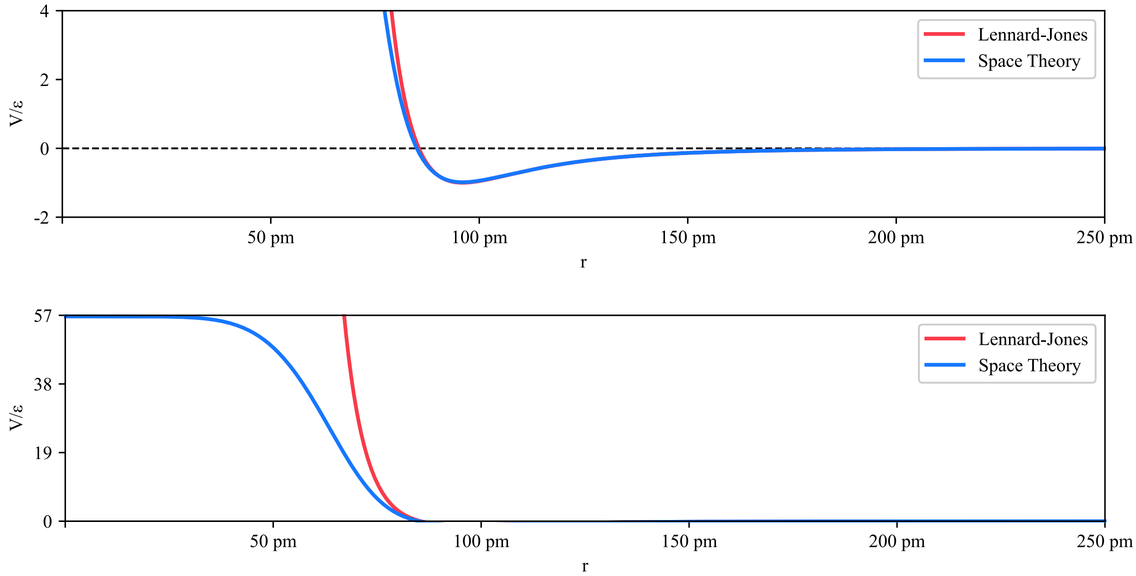

Figure 13. Lennard-Jones potential of a hydrogen bond.

The Schrödinger equation for a hydrogen atom can be solved by separation of variables. [43] In this case, spherical polar coordinates are the most convenient.

$$\Psi(r,\theta,\varphi) = R(r)Y_l^m(\theta,\varphi) = R(r)\Theta(\theta)\Phi(\varphi).$$

The Spherical Harmonic functions indicate the electron's position around the proton, and the radial function shows the electron's distance from the proton. The solutions to the radial differential equation are products of the associated Laguerre polynomials, an exponential factor, and a normalization factor.

$$R(r) = N_{n,l}\left(\frac{r}{a_0}\right)^{l} \times L_{n,l}(r) \times e^{-\frac{r}{na_0}}.$$

The first six radial functions are provided in Table below.

Table 6. Radial wave functions for wavefunctions of the first three shells.

n	l		$\Phi(\varphi)$	$\Theta(\theta)$	$R(r)$
1	0	1s	$\frac{1}{\sqrt{2\pi}}$	$\frac{1}{\sqrt{2}}$	$2\left(\frac{Z}{a_0}\right)^{\frac{3}{2}} e^{-\frac{Zr}{a_0}}$
2	0	2s	$\frac{1}{\sqrt{2\pi}}$	$\frac{1}{\sqrt{2}}$	$\frac{1}{2\sqrt{2}}\left(\frac{Z}{a_0}\right)^{\frac{3}{2}}\left[2 - \frac{Zr}{a_0}\right] e^{-\frac{Zr}{2a_0}}$

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2	1	2p	$\frac{1}{\sqrt{2\pi}}$	$\frac{\sqrt{6}}{2}\cos\theta$	$\frac{1}{2\sqrt{6}}\left(\frac{Z}{a_0}\right)^{\frac{3}{2}}\left[\frac{Zr}{a_0}\right]e^{-\frac{Zr}{2a_0}}$
3	0	3s	$\frac{1}{\sqrt{2\pi}}$	$\frac{1}{\sqrt{2}}$	$\frac{2}{81\sqrt{3}}\left(\frac{Z}{a_0}\right)^{\frac{3}{2}}\left[27-18\frac{Zr}{a_0}+2\left(\frac{Zr}{a_0}\right)^2\right]e^{-\frac{Zr}{3a_0}}$
3	1	3p	$\frac{1}{\sqrt{2\pi}}$	$\frac{\sqrt{6}}{2}\cos\theta$	$\frac{4}{81\sqrt{6}}\left(\frac{Z}{a_0}\right)^{\frac{3}{2}}\left[6\frac{Zr}{a_0}-\left(\frac{Zr}{a_0}\right)^2\right]e^{-\frac{Zr}{3a_0}}$
3	2	3d	$\frac{1}{\sqrt{2\pi}}$	$\frac{\sqrt{10}}{4}(3\cos^2\theta-1)$	$\frac{4}{81\sqrt{30}}\left(\frac{Z}{a_0}\right)^{\frac{3}{2}}\left(\frac{Zr}{a_0}\right)^2 e^{-\frac{Zr}{3a_0}}$

The radial probability density function is:

$$P(r) = |\Psi|^2 4\pi r^2.$$

For the $1s$ orbital, the radial wave function is given as:

$$P(r) = \left[\frac{1}{\sqrt{\pi}}\left(\frac{Z}{a_0}\right)^{\frac{3}{2}} e^{-\frac{Zr}{a_0}}\right]^2 4\pi r^2 = 4\left(\frac{Z}{a_0}\right)^3\left(re^{-r\frac{Z}{a_0}}\right)^2.$$

Remove the constant terms, take the derivative with respect to r, and set it to zero to obtain the radius of maximum probability.

$$(uv)' = u'v + uv' = 0, \qquad (r)' = 1, \qquad \left(e^{-r\frac{Z}{a_0}}\right)' = -\frac{Z}{a_0}e^{-r\frac{Z}{a_0}}.$$

Which gives:

$$\left[1 - r\frac{Z}{a_0}\right]e^{-r\frac{Z}{a_0}} = 0, \qquad r = \frac{a_0}{Z}.$$

Slater's rules [44] provide a value for the screening constant, denoted by s, which relates the effective and actual nuclear charges as:

$$Z_{\text{eff}} = Z - s.$$

Stars obtained their energy from nuclear fusion of hydrogen to form helium. [45] In this case, the radius for maximum probability will change from $52.9\,pm$ to $31.1\,pm$. [34] This means that as the distance between the two atoms changes, the radius for maximum probability will also change. Set the distance between the two atoms as x, the formula can be modified to:

$$\alpha = \sqrt[3]{1 + (x/a_0)^3} - x/a_0, \qquad 0 \le \alpha \le 1, \qquad Z = 1 + (1 - s)\alpha.$$

The shielding constant s of a $1s$ electron is 0.3 [44], and the bond length in a H_2 molecule is $74\,pm$ [39], so the radius for maximum probability of a H_2 molecule is:

$$Z \approx 1.1071, \qquad r = \frac{a_0}{Z} \approx 47.7825.$$

The variation is illustrated in the diagram:

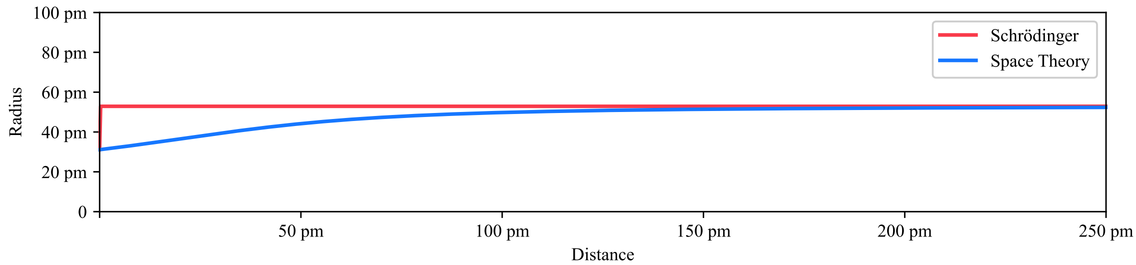

Figure 14. The radius for maximum probability (nuclear fusion).

And the radial probability density function $P(r)$ is:

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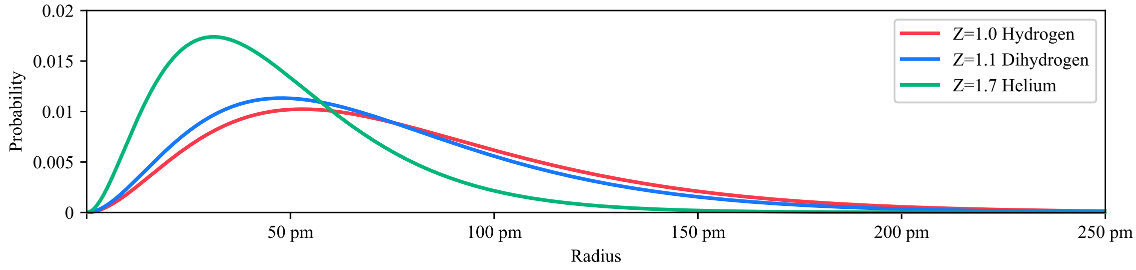

Figure 15. The radial probability density (1s).

Discussion

This paper is not yet complete. To ensure the accuracy of calculations, the author recommends using the Decimal Module in Python, and suggests setting precision to 100 digits. As of this writing, graviton has not been found yet.

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