Article

A Study of RANS Turbulence Models in Fully Turbulent Jets: A Perspective for CFD-DEM Simulations

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Abstract: This paper presents an analysis of linear viscous stress Favre-Averaged turbulence models for computational fluid dynamics (CFD) of fully turbulent round jets with a long straight tube geometry in the near field. Although similar work has been performed in the past with very relevant solutions, considerations were not given for the issues and limitations involved with coupling between an Eulerian and Lagrangian phase, such as in fully two-way coupled CFD-DEM. These issues include limitations on solution domain, mesh cell size, wall modelling, and momentum coupling between the two phases in relation to the particles size. Therefore, within these considerations, solutions are provided to the Navier-Stokes equations with various turbulence models using a three-dimensional wedge long straight tube geometry for fully developed turbulence flow. Simulations are performed with a Reynolds number of 15000 and 50000 using two different tube diameters. It is found that a modified $k - \varepsilon$ turbulence model achieved the most agreeable results for both the velocity and turbulent flow fields between these two flow regimes, while a modified $k - \omega$ SST/BSL also provided suitable results.

Keywords: Jet Flow; CFD; Turbulence; CFD-DEM

1. Introduction

RRound turbulent jets have a wide range of applications that include combustion, jet cutting, and surface coating and they have been studied extensively using experiments and numerically. The turbulent round jets can be categorized into three main groups: (i) particle-laden jets, (ii) impinging jets, and (iii) free round jets, each of which has distinct geometries.

A long straight tube nozzle is commonly found in particle-laden jets applications with the particle mixture injected into the inlet. The choice of the particle-laden flow geometry is driven by the need to achieve fully developed flow before the nozzle exit to help quantify spreading rates with variations of particle diameters and Stokes number with a higher accuracy. This is demonstrated in the papers given by [1–4] in which 93, 90, and 163 nozzle diameters are used for the lengths of the nozzles, and the flow fully develops by the nozzle exit. Similar long nozzles are also shown in many other studies including [5–9]. Using test cases that have fully developed flow provides consistency between the boundary conditions of the numerical simulations and input parameters of the experimental data.

The geometries for the application of impinging jets range from a smooth contraction to straight nozzles, and they may or may not have fully developed flow [10]. The free jets typically have contraction or sharp edged slots without fully developed flow [11].

There have been a plethora of numerical studies over the past years on jet turbulence modelling as shown in Table 1, but the geometries and operating parameters are not common across these studies. This difference is because the focus is on pure single-phase CFD simulations without the considerations and issues indicative of CFD-DEM coupling.

The most common way to model a jet in numerical simulations is using a fixed boundary condition taken from DNS or experimental data for pipe flow. This circumvents the use of a nozzle in the solution domain and wall functions or eliminates the requirement for resolving the boundary layer. Although this is the best solution for fundamental single-phase flow problems, determining an optimum nozzle shape for particle-laden jets using CFD-DEM simulations requires that the nozzle profile shape be included in the solution domain. This is due in part to the location and initial operating parameters for particle insertion. Particles could be inserted at the nozzle exit, but this would require experimental data for each change in nozzle shape, which would be unpractical for any type of industrial flow problem. Therefore, to tune these RANS turbulence models, a length of nozzle is included in the solution domain to ensure that the shear layer along the nozzle wall and the resulting boundary layer separation are being modelled correctly.

It is important to realize that with the current developments in computational resources, LES turbulence modelling is the ideal candidate for many different flow problems because of its ability to model anisotropic flow behaviors. However, for some applications, such as in fully coupled Eulerian-Lagrangian flow modelling, LES is impractical or impossible because of limitations related to mesh size for sufficient resolution of turbulent eddies and wall modelling. Therefore, for such applications, RANS turbulence models are a viable option. It should be noted that many studies have been performed using CFD-DEM and LES for jet flow, but a one-way coupled regime have been used. Although one-way coupled simulations may be adequate for dilute flows, particle-particle and particle-fluid interactions make a significant contribution to the physics of jets for dense flows [12] .

RANS turbulence models are derived purely from dimensional analysis and some intuition in theory. Coefficients are then used to provide accurate results to a variety of general flow regimes. It would be ideal if these coefficients were set from principle values, such as kinetic theory of gases, but instead, they are empirically set for a variety of general flow regimes. This has been proven to be accurate in many cases but has shown inaccuracy in capturing the spreading rates for round jets [13,14].

Researchers have implemented and tested many methods to circumvent this issue, such as adding vortex stretching, cross diffusion terms, and also modification of the empirical coefficients, and this has provided accurate results with a constant edged boundary condition. However, very little studies have been performed for long straight tube geometries that arise in fully coupled CFD-DEM simulations and optimization problems. Therefore, this present paper's contribution is modification and then testing of RANS turbulence models for the application of long straight tube jet flow that are required for CFD-DEM simulations to ensure that the results are agreeable to the experimental data.

2. Numerical Methodology

2.1. Background

In experiments, anisotropic turbulence behavior is observed in high-speed jet flow due to the strong boundary layer separation at the exit of the nozzle, which makes this type of flow difficult to model. RANS turbulence models studied in this paper use the very common linear viscous Boussinesq approximation that assumes that the momentum transfer caused by turbulence can be modelled by an eddy viscosity. In using this model, we assume that the eddy viscosity is independent of direction and, hence, the rate of turbulence generation is uniform in all directions. This is not to say there is no anisotropic

Table 1: Relevant RANS numerical studies on turbulent jets. Studies include fundamental work and modifications that were performed to help mitigate the problems associated with spreading rates

Study	Reynolds Number	Geometry	Nozzle Type	Turbulence Models	Key Features
Pope [13]	NA	Axi-Symmetric	Round jet*	Modified k-epsilon	Suggested changing $C_{\varepsilon1}$=1.6 and proposed vortex stretching term
Givi and Ramos [15]	373000	Axi-Symmetric	Round jet*	Modified standard $k-\varepsilon$ and $k-\omega$ turbulence models	Tested k-epsilon constants and $C_{\varepsilon1}$=1.52 with $C_{\varepsilon2}$=1.90 provided most agreeable spreading rates
Kim and Chung [16]	11600	Axi-Symmetric	Edge fixed value set from experiments	Standard k-epsilon, algebraic stress model	Highly agreement results for swirling jets
Morgans *et al.* [17]	50000	Axi-Symmetric	Fully developed straight tube round jet nozzle	Modified k-epsilon ($C_{\varepsilon1}$=1.6), $k-\omega$ Wilcox	Comparable results between $k-\omega$ Wilcox and modified k-epsilon
Quinn and Militzer [11]	208000	Axi-Symmetric	Sharp Edged and Smoothly Contoured	Modified k-epsilon ($C_{\varepsilon1}$=1.48)	Comparable results between experiments for velocity but not turbulence quantities.
Post *et al.* [18]	10000	Axi-Symmetric	Edge fixed value	Launder and Spalding, WF	Acheived results of lower entertainment from experiment
Georgiadis *et al.* [19]	610950	Axi-Symmetric	Convergent contraction	Many two-equation RANS turbulence models	Tam-Thies k-epsilon model demonstrated greatest agreement
Faghani *et al.* [20]	20800	Axi-Symmetric	Sharp edge , round, and contoured nozzles with edge fixed value set to experimental data nozzle exit conditions	Modified k-epsilon with $C_{\varepsilon1}$=1.5	Significant differences in velocity, turbulence intensity, and turbulence kinetic energy with a change in nozzle shape
Wang *et al.* [21]	20000	Impingement Three-Dimensional	Round jet with fully developed flow inlet	Many two-equation RANS turbulence models	$k-\omega$ SST demonstrated greatest agreement

turbulence when using these types of models, because the mean strain rate tensor of the velocity field in the Navier-Stokes equations is not necessarily isotropic. Suffice to say, there is an expected error in using RANS approaches because of anisotropy, but if we can capture an average quantification, then we can achieve results that are sufficient for understanding of the flow behavior while achieving adequate single-phase results before the addition of particles in unresolved fully coupled CFD-DEM simulations.

Turbulent round jets have been difficult to model numerically since the beginning of RANS turbulence modelling by producing spreading rates that are not agreeable. This is shown in many papers including Pope [13], Quinn and Militzer [11], Launder and Spalding [22], Morgans *et al.* [17], Givi and Ramos [15] and also stated in David C. Wilcox's famous book *Wilcox's Turbulence modelling for CFD* [14]. It is therefore very curious that some more recent studies in the past ten years obtain agreeable results with no modifications to the default Launder and Spalding [22] $k - \varepsilon$ turbulence model.

RANS turbulence models have empirical coefficients that have been adjusted for generalized flow conditions. In an ideal world, these coefficients would be set from principle values, such as in kinetic theory of gases. Instead the developed RANS turbulence models are derived from dimensional analysis and some intuition on theory and, therefore, the coefficients are merely empirical providing agreeable results for a wide range of generalized flow conditions. Free shear flows, such as jet flows, are one of these generalized flow conditions.

In 1978, Pope suggested that changing the constant, $C_{\varepsilon 1} = 1.6$, in the $k - \varepsilon$ model will solve the issue with round/plane jets, but states that this goes against any kind of generality [13,17]. That being said, researchers have adopted the method of changing $C_{\varepsilon 1}$ and it is perfectly valid because, as stated before, these coefficients are merely empirical relations to fit to a generalized flow regime [14,15,17,20]. Instead, Pope proposed a modification to the $k - \varepsilon$ model by adding a vortex stretching variable to the destruction of turbulence term. The ε equation in the standard $k - \varepsilon$ model is modified by

$$C_{\varepsilon 2}\frac{\varepsilon^2}{k} \rightarrow \left(C_{\varepsilon 2} - C_{\varepsilon 3}\chi_p\right)\frac{\varepsilon^2}{k} \tag{1}$$

where, χ_p is the non-dimension measure of vortex stretching given by

$$\chi_p = \frac{\Omega_{ij}\Omega_{ik}S_{ki}}{(\varepsilon/k)^3} \tag{2}$$

and Ω_{ij} and S_{ij} are the mean-rotation and mean-strain rate tensors defined as

$$\Omega_{ij} = \frac{1}{2}\left(\frac{\partial u_i}{\partial x_j} - \frac{\partial u_j}{\partial x_i}\right) \tag{3}$$

$$S_{ij} = \frac{1}{2}\left(\frac{\partial u_i}{\partial x_j} + \frac{\partial u_j}{\partial x_i}\right) \tag{4}$$

In the OpenFOAM tensor library Eqs. 3 and 4 can be obtained by decomposing to the skew and symmetric parts of the gradient of velocity, U, respectively. That is: $\Omega_{ij} = skew(\nabla U)$ and $S_{ij} = symm(\nabla U)$. The theory is that the primary mechanism of turbulent energy dissipation is through stretching of the vortex rings in three-dimensions, but not in two-dimensions. Rubel [23] tested this model and determined that while this correction helps rectify the problems with round jet modelling, plane jet modelling was still experiencing problems. Because, exclusively, three-dimensional wedge simulations are considered in this work, the issues with plane jet modelling are ignored.

Contrasting from the $k - \varepsilon$ model, the Wilcox $k - \omega$ turbulence model produces spreading rates for the round nozzle that are comparable, but larger, than predicted spreading rates for the plane jets [24]. To solve this issue for the plane jets while conserving the correct predicted values for round jets, Wilcox realized that, when the constant β is reduced to 0.06 from 0.0708, the predicted spreading rates for both the round and plane jets were close to the experimental values. Rather then changing the value of the β constant, Wilcox proposed a more "general" solution that reformulates the $k - \omega$ model by

$$\beta = \beta_0 f_\beta \tag{5}$$

where

$$\beta_0 = 0.0708, \quad f_\beta = \frac{1 + 85\chi_\omega}{1 + 100\chi_\omega}, \quad \chi_\omega = \left| \frac{\Omega_{ij}\Omega_{jk}S_{ki}}{(\beta^*\omega)^3} \right| \tag{6}$$

Although this may solve the issue with the round/plane jet anomaly, the $k - \omega$ turbulence model is sensitive to freestream boundary conditions; this could present an issue in determining a solution that is invariant with different initial freestream conditions [24,25] and, to that affect, this turbulence model is not considered in this present work.

In this present work, we compare the velocity and kinetic energy profiles for various wall function RANS approaches to turbulence modelling. Two distinct test cases are considered with a Reynolds number of Re=15e3 for the test case given by Hardalupas *et al.* [1] and Re=50e3 for the test case given by Bogusławski and Popiel [26].

2.2. Numerical Model

The fluid flow in the present research is described by the Navier-Stokes equations given by

$$\frac{\partial u_j}{\partial x_j} = 0 \tag{7}$$

$$\frac{\partial u_i}{\partial t} + \frac{\partial}{\partial x_j}\left(u_j u_i\right) = -\frac{\partial}{\partial x_i}\left(\frac{p}{\rho_0}\right) + \frac{\partial}{\partial x_j}\left[\nu_m\left(\frac{\partial u_i}{\partial x_j} + \frac{\partial u_j}{\partial x_i}\right) - \frac{2}{3}\nu_m\frac{\partial u_k}{\partial x_k}\delta_{ij}\right] \tag{8}$$

where, u is the fluid velocity, p is the pressure, ρ_0 is the density of the fluid, ν_m is the viscosity of the fluid, and x is a unit in space. It should be noted that the assumption of an incompressible flow is made in this work since the considered velocities are far from the sonic velocity and, therefore, the following term becomes

$$-\frac{2}{3}\frac{\partial u_k}{\partial x_k}\delta_{ij} = 0 \tag{9}$$

but is still included in the formulation for solution boundedness.

Turbulent flows can be calculated for each point in the flow field by solving the Navier-Stokes equations, 7 and 8, directly; this approach is called direct numerical simulations (DNS). For this to be accurate, all the spatial and temporal turbulent scales need to be resolved, requiring an extremely fine mesh with a number of mesh points satisfying $N^3 \geq \text{Re}^{9/4}$, which is extremely computationally expensive. Less costly alternatives are to solve for the mean pressure and velocity fields using a Reynolds Averaged approach (RANS), or to solve for the large scales directly while modelling the subgrid scale turbulence by the use of a Large Eddy Simulation approach (LES).

Assuming that the velocity and pressure can be described by the mean and fluctuation components, $u = \bar{u} + u'$ and $p = \bar{p} + p'$, and with some lengthy reductions the Navier-Stokes equations become the RANS equations given by

$$\frac{\partial \bar{u}_i}{\partial t} + \frac{\partial}{\partial x_j}\left(\bar{u}_j \bar{u}_i\right) = -\frac{\partial}{\partial x_i}\left(\frac{\bar{p}}{\rho_0}\right) + \frac{\partial}{\partial x_j}\left[\nu_m\left(\frac{\partial \bar{u}_i}{\partial x_j} + \frac{\partial \bar{u}_j}{\partial x_i} - \frac{2}{3}\frac{\partial \bar{u}_k}{\partial x_k}\delta_{ij}\right) + \overline{u'_i u'_j}\right] \quad (10)$$

where, $\overline{u'_i u'_j}$ RANS Reynolds stress term. By examining the difference between the Navier-Stokes (Eqs. 7 & 8) and the RANS it can be realized that the difference between them is a Reynolds Stress term, and the use of mean pressure and mean velocities instead of instantaneous. This is an important realization during code development because we can merely specify an additional term when RANS or LES filtering is desired for turbulence modelling. It should be noted that many CFD codes use this method, including OpenFOAM. The Reynolds stress term, $\overline{u'_i u'_j}$, now needs to be modelled to close the RANS equations. We can calculate the Reynolds Stress term directly with the addition of six transport equations, but this approach is expensive. There are single transport equation models, such as the mixing-length and Sparlat-Allmaras, but their accuracy is lacking. The most common method, with a good balance in required computational resources and accuracy, is the use of an eddy viscosity turbulence model with two transport equations, such as the $k - \varepsilon$, $k - \omega$, and their variants.

The Boussinesq eddy viscosity assumption is now used to model the Reynolds stress tensor for both subgrid LES and RANS turbulence models. The Boussinesq eddy viscosity assumption for the Reynolds Stress with incompressibility is given by

$$\overline{u'_i u'_j} = \nu_t\left(\frac{\partial u_i}{\partial x_j} + \frac{\partial u_j}{\partial x_i} - \frac{2}{3}\frac{\partial u_k}{\partial x_k}\delta_{ij}\right) - \frac{2}{3}k\delta_{ij} \quad (11)$$

where the first term on the right hand side is the deviatoric part, and the second is the isotropic part. We then plug Eq. 11 into Eq. 10 to obtain the Navier-Stokes momentum equation in the OpenFOAM code.

$$\frac{\partial u_i}{\partial t} + \frac{\partial}{\partial x_j}\left(u_j u_i\right) = -\frac{\partial}{\partial x_i}\left(\frac{\tilde{p}}{\rho_0}\right) + \frac{\partial}{\partial x_j}\left[(\nu_m + \nu_t)\left(\frac{\partial u_i}{\partial x_j} + \frac{\partial u_j}{\partial x_i} - \frac{2}{3}\frac{\partial u_k}{\partial x_k}\delta_{ij}\right)\right] \quad (12)$$

$$\tilde{p} = \bar{p} + \frac{2}{3}k\delta_{ij} \quad (13)$$

where ν_t is the turbulent viscosity and $\tilde{p}$ is a modified mean pressure that has absorbed the isotropic part of the Reynolds Stress term.

It is now prudent to discuss the Navier-Stokes in relation to fully coupled unresolved CFD-DEM simulations to gain a better understanding on why a wall function approach with a coarser mesh is required. Coupling between the carrier phase and solid phase involves the use of volume fraction of solids with the Navier-Stokes equations given by

$$\nabla(\alpha_l \rho_l u_l) = 0 \quad (14)$$

$$\nabla \cdot (\rho_l \alpha_l \vec{u}_l \vec{u}_l) = -\alpha_l \nabla p + \nabla \cdot \left(\alpha_f \tau_l\right) - K_{sl}(\vec{u}_l - \vec{u}_s) + \alpha_{/}\rho_l \vec{g} + f \quad (15)$$

where α_l is the liquid phase volume fraction, $\vec{u}_l$ is the liquid fluid velocity, $\vec{u}_s$ is the mean solid particle velocity, τ_l is the liquid phase stress tensor, K_{sl} is the implicit momentum source term that is a volume average of all interacting forces acting on the solid phase due to the motion of the fluid in each cell, and f is any other explicit forces acting on the fluid.

As can be realized by the Eqs. 14 and 15, the momentum of the fluid is highly dependent on the volume fraction of solids, α_l, and hence, the calculation of this term is critical to obtain accurate physics. The most accurate solution to this would be to calculate the volume of the particle ratios directly in the carrier phase cells, but this is impractical because it would be extremely inefficient. Instead, a volume fraction model is used that allows efficient computation while providing reasonable results for accurate flow physics. To ensure that this model is correct, the particle size to cell size ratio must have minimal particle overlap [27,28]. It is, therefore, unfeasible to fully resolve the boundary layer and have an overall fine mesh in these types of simulations unless the desired particle size is extremely small

2.2.1. $k - \varepsilon$ Turbulence Models

The most common two-equation eddy viscosity turbulence models can be categorized into $k - \varepsilon$ and $k - \omega$ turbulence models with variants. Here, an equation for turbulent kinetic energy, k, and for a turbulence length scale, typically using ε or ω, are used. The $k - \varepsilon$ model is the more popular of the two, with the generalized formulation given by

$$\frac{\partial(k)}{\partial t} + \frac{\partial(u_j k)}{\partial x_j} = \tau_{ij}\frac{\partial u_j}{\partial x_j} - \varepsilon + \frac{\partial}{\partial x_j}\left[\left(\nu + \frac{\nu_t}{\sigma_k}\right)\frac{\partial k}{\partial x_j}\right] + L_k \tag{16}$$

$$\frac{\partial \varepsilon}{\partial t} + \frac{\partial(u_i \varepsilon)}{\partial x_j} = C_{\varepsilon 1} f_1 \frac{\varepsilon}{k}\tau_{ij}\frac{\partial u_i}{\partial x_j} - C_{\varepsilon 2} f_2 \frac{\varepsilon^2}{k} + \frac{\partial}{\partial x_j}\left[\left(\nu + \frac{\nu_t}{\sigma_\varepsilon}\right)\frac{\partial \varepsilon}{\partial x_j}\right] + L_\varepsilon \tag{17}$$

where eddy viscocity is given by $\nu_t = C_\mu f_\mu k^2/\varepsilon$ and the Boussinesq assumption is used to obtain

$$\tau_{ij} = \nu_t\left(2S_{ij} - \frac{2}{3}\frac{\partial u_k}{\partial x_k}\delta_{ij}\right) - \frac{2}{3}k\delta_{ij} \tag{18}$$

$$S_{ij} = \frac{1}{2}\left(\frac{\partial u_i}{\partial x_j} + \frac{\partial u_j}{\partial x_i}\right) = symm(\nabla U) \tag{19}$$

and k is the turbulent kinetic energy, ε is the turbulent dissipation rate, u is fluid velocity, f is a damping function, and the turbulence length scale is given by $l = C_\mu k^{3/2}/\varepsilon$. Table 2 shows the values or formulations of the generalized variables for the standard Launder and Spalding [22], variants, and modified $k - \varepsilon$ turbulence models.

For the Chien model we make an estimation for the non-dimensional wall distance, y^+. Wallin and Johansson [33] demonstrated that the non-dimensional wall distance as a function of kinetic energy is approximately equal to the non-dimensional wall distance as a function of fluid velocity for $y^+ \leq 100$; that is, $y^* \approx y^+$ for $y^+ \leq 100$. With this in mind, Wallin and Johansson [33] suggested using an alternative scaling of viscosity given by

$$y^* = C_{y1}\sqrt{\text{Re}_y} + C_{y2}\text{Re}_y^2$$
$$\text{Re}_y = \sqrt{k}y/\nu, \quad C_{y1} = 2.4, \quad C_{y2} = 0.003 \tag{20}$$

It should be noted that $f_\mu \approx 1$ at $y^+ = 300$. For the RNG turbulence model $\eta = Sk/\varepsilon$, $\eta_0 = 4.38$, $S = (2S_{ij}S_{ij})^{0.5}$.

For completeness, we will state the previously mentioned Pope correction to the standard Launder and Spalding [22] $k - \varepsilon$ turbulence model given by

$$C_{\varepsilon 2}\frac{\varepsilon^2}{k} \rightarrow (C_{\varepsilon 2} - C_{\varepsilon 3}\chi_p)\frac{\varepsilon^2}{k}, \quad \chi_p = \frac{\Omega_{ij}\Omega_{ik}S_{ki}}{(\varepsilon/k)^3} \tag{21}$$

Table 2: $k - \varepsilon$ turbulence models that are used in the present study. $\text{Re}_t = k^2/(\nu\varepsilon)$

Term	Launder and Spalding [22], mod1, mod2	Chien [29]	RNG Arbiter [30]	Launder and Sharma [31]	Thies and Tam [32]
$C_{\varepsilon 1}$	1.44, 1.50 ,1.52	1.35	1.42	1.44	1.40
$C_{\varepsilon 2}$	1.92,1.92,1.90	1.80	1.68	1.92	2.02
f_μ	1.0	$1 - e^{-0.0115 d^+}$	1.0	$\exp\dfrac{-3.4}{(1 + \text{Re}_t/50)^2}$	1.0
f_1	1.0	1.0	1.0	1.0	1.0
f_2	1.0	$1 - \dfrac{0.4}{1.8} e^{-\text{Re}_t/36}$	$C_{\varepsilon 2} + \dfrac{C_\mu \eta^3 (1 - \eta/\eta 0)}{1 + \beta\eta^3}$	$1 - 0.3 \exp\left[-\text{Re}_t^2\right]$	1.0
σ_k	1.0	1.0	0.7194	1.0	0.324
σ_ε	1.3	1.3	1.3	0.7194	0.377
L_k	0.0	$-2\dfrac{\nu k}{d^2}$	0.0	$2\nu\left(\dfrac{\partial\sqrt{k}}{\partial y}\right)^2$	0.0
L_ε	0.0	$-2\dfrac{\nu\varepsilon}{d^2} e^{-d^+/2}$	0.0	$2\nu\nu_t\left(\dfrac{\partial^2 u}{\partial y^2}\right)^2$	0.0

where the non-dimensional measure of vortex stretching term, χ_p, is computed by first performing the inner product of the two mean-rotation rate tensors and then by calculating the double inner product of the resultant and the mean-strain rate tensor. The coefficients for the Pope correction are equal to that of the standard $k - \varepsilon$ model with the additional constant $C_{\varepsilon 3} = 0.79$ providing the most agreeable results for the respective geometry and boundary condition tested in the original paper. The coefficients provided for these models are empirical relations that have been fit for generalized flow problems. Realizing this, along with the issues with the jet flow modelling, Tam and Ganesan [34] performed many jet flow experiments to determine a new set of coefficients that will allow accurate modelling of the jet flow for a variety of Reynolds numbers and geometries. This set of coefficients are given by $C_\mu = 0.0874$, $C_{\varepsilon 1} = 1.40$, $C_{\varepsilon 2} = 2.02$, $\sigma_k = 0.324$, $\sigma_\varepsilon = 0.377$, $C_{\varepsilon 3} = 0.822$. It is noted that the diffusion coefficients σ_k and σ_ε are significantly smaller than the standard coefficients and can be characterized by an increase in diffusion of k and ε because of their location in the denominators of Eq. 16 and 17. These coefficients will be used in the present work with the naming as Tam-Thies KE. Slight modifications to the empirical constants in the Launder and Spalding [22] are also used with "mod1" having the change of $C_{\varepsilon 1} = 1.50$ similarly performed by Quinn and Militzer [11] and Faghani *et al.* [20] and "mod2" having the change of $C_{\varepsilon 1} = 1.52$ and $C_{\varepsilon 2} = 1.90$ similarly performed by Givi and Ramos [15].

2.2.2. $k - \omega$ Turbulence Models

The standard $k - \omega$ model is given by a production term of turbulent kinetic energy, k, and frequency of dissipation, ω. Kolmogorov, in 1941, was the first to suggest the two-equation turbulence model for kinetic energy and dissipation per unit turbulence kinetic energy [35]. Unaware of Kolmgorgorov's work, Saffman [36] produced a $k - \omega$ model that had better agreement than Kolmgorgorov's, and has served as the basis of "modern" $k - \omega$ turbulence models. One of the most influential contributors to the $k - \omega$ turbulence model was the work performed by David C. Wilcox. A sequential timeline of Wilcox's work won't be stated, and since no modifications to this model have been performed in the present work, the reader is encouraged to review the paper given in Wilcox [24] for details on the model.

The goal of the $k - \omega$ SST and BSL turbulence models is to use the $k - \omega$ model due to
its robustness near the wall, and then transition to the $k - \varepsilon$ model for its effectiveness
in the far field. This is achieved using a blending function, F_1, that is a function of wall
distance with $k - \omega$ and "transformed" $k - \varepsilon$ models. When the value of F_1 is equal to
one, then the $k - \omega$ proposed by Wilcox [37] is used. When F_1 is equal to zero, then a
transformed version of the standard $k - \varepsilon$ model of Launder and Spalding [22] is used.
It should be noted here that the "transformed" $k - \varepsilon$ model is typically stated to be an
exact transformation of the $k - \varepsilon$ model, but this is erroneous to state because terms are
dropped in the derivation. This will be discussed further in this paper.

To better understand this model and determine its ability to adapt to turbulent jet flow
conditions, the $k - \omega$ BSL/SST turbulence models [38] with the updated modifications
[39] will first be stated, followed by an in depth study on the origins of the model to
determine if the coefficients are likely candidates for modification. The $k - \omega$ BSL model
is given by

$$\frac{Dk}{Dt} = P - \beta^* k\omega + \frac{\partial}{\partial x_j}\left[(\nu + \sigma_k \nu_t)\frac{\partial k}{\partial x_j}\right] \tag{22}$$

$$\frac{D\omega}{Dt} = \frac{\gamma}{\nu_t}P - \beta\omega^2 + + \frac{\partial}{\partial x_j}\left[(\nu + \sigma_\omega \nu_t)\frac{\partial \omega}{\partial x}\right] + 2(1 - F_1)\sigma_{\omega 2}\frac{1}{\omega}\frac{\partial k}{\partial x_j}\frac{\partial \omega}{\partial x_j} \tag{23}$$

$$F_1 = \tanh\left(\text{arg}_1^4\right)$$

$$\text{arg}_1 = \min\left[\max\left(\frac{\sqrt{k}}{0.09\omega y}, \frac{500\nu}{y^2\omega}\right), \frac{4\sigma_{\omega 2}k}{CD_{k\omega}y^2}\right] \tag{24}$$

$$CD_{k\omega} = \max\left(2\sigma_{\omega 2}\frac{1}{\omega}\frac{\partial k}{\partial x_j}\frac{\partial \omega}{\partial x_j}, 10^{-20}\right)$$

where, γ, β, σ_k, and σ_ω are constants, ϕ, that are calculated using the equation $\phi = F_1\phi_1 + (1 - F_1)\phi_2$. The $k - \omega$ model, ϕ_1, constants are taken directly from Wilcox [37] to
be $\sigma_{k1} = 0.5$, $\sigma_{\omega 1} = 0.5$, $\beta_1 = 0.0750$ and $\gamma_1 = \beta_1/\beta^* - \sigma_{\omega 1}\kappa/\sqrt{\beta^*} \cong 0.56$ by ensuring
that $\kappa = 0.41$. The constants for the "transformed" $k - \varepsilon$ model, ϕ_2, are given by $\sigma_{k2} = 1.0$,
$\sigma_{\omega 2} = 0.856$, $\beta_2 = 0.0828$, and $\gamma_2 = \beta_1/\beta^* - \sigma_{\omega 2}\kappa/\sqrt{\beta^*} \cong 0.44$ with $\nu_t = k/\omega$. These
are used to mimic the constants given by the standard $k - \varepsilon$ model of Launder and
Spalding [22].

As previously stated, the $k - \omega$ BSL/SST turbulence model uses the standard 1988 Wilcox
$k - \omega$ turbulence model in the near field and transitions to a "transformed" $k - \varepsilon$ in the
far field. The "transformed" $k - \varepsilon$ equation is derived by a change in dependent variables
from ε to ω using the relation defined by $\varepsilon = C_\phi k\omega$ with the standard $k - \varepsilon$ Launder and
Spalding [22] model. Specifically, for kinetic energy the model becomes

$$\frac{Dk}{Dt} = P - \varepsilon + \frac{\partial}{\partial x_j}\left[\left(\nu + \frac{\nu_t}{\sigma_k}\right)\frac{\partial k}{\partial x_j}\right]$$

$$\rightarrow \frac{Dk}{Dt} = P - C_\phi k\omega + \frac{\partial}{\partial x_j}\left[\left(\nu + \frac{\nu_t}{\sigma_k}\right)\frac{\partial k}{\partial x_j}\right] \tag{25}$$

and for rate of dissipation

$$\frac{D\varepsilon}{Dt} = C_{\varepsilon1}f_1\frac{\varepsilon}{k}P - C_{\varepsilon2}f_2\frac{\varepsilon^2}{k} + \frac{\partial}{\partial x_j}\left[\left(\nu + \frac{\nu_t}{\sigma_\varepsilon}\right)\frac{\partial\varepsilon}{\partial x_j}\right]$$

$$\rightarrow \frac{D\omega}{Dt} = (C_{\varepsilon1}f_1 - 1)\frac{\omega}{k}P - C_\phi(C_{\varepsilon2}f_2 - 1)\omega^2 + \frac{\partial}{\partial x_j}\left[\left(\nu + \frac{\nu_t}{\sigma_\varepsilon}\right)\frac{\partial\omega}{\partial x}\right] \qquad (26)$$

$$\frac{2}{k}\left(\nu + \frac{\nu_t}{\sigma_\varepsilon}\right)\frac{\partial k}{\partial x_j}\frac{\partial\omega}{\partial x_j} + \frac{\omega}{k}\frac{\partial}{\partial x_j}\left[\left(\frac{1}{\sigma_\varepsilon} - \frac{1}{\sigma_k}\right)\nu_t\frac{\partial k}{\partial x_j}\right]$$

If we then compare this fully derived transformed $k - \varepsilon$ formulation with the model given from Menter [25], it is observed that the equation for turbulent kinetic energy is equivalent, but that some terms are dropped in the rate of dissipation equation. Specifically, fluid viscosity in the cross diffusion term given by

$$\frac{2\nu}{k}\frac{\partial k}{\partial x_j}\frac{\partial\omega}{\partial x_j} \qquad (27)$$

and terms for turbulent diffusion given by

$$\frac{\omega}{k}\frac{\partial}{\partial x_j}\left[\left(\frac{1}{\sigma_\varepsilon} - \frac{1}{\sigma_k}\right)\nu_t\frac{\partial k}{\partial x_j}\right] \qquad (28)$$

235   In his book "Turbulence Modelling for CFD", Wilcox [14] states that *"focusing on free shear flow, we can ignore molecular viscosity"*. Perhaps the contribution of molecular viscosity has an insignificant contribution with the application to free sheer flows, but this does not explain why the model can be applied to applications other than free shear flows. Wilcox also states that if we assume $1/\sigma_\varepsilon = 1/\sigma_k$ for simplicity, then the terms for
240   turbulent diffusion given by Eq. 28 can be dropped. It can, therefore, be argued that this is not an "exact" transformation of the standard $k - \varepsilon$ model. That being said, this is not a problem, since the $k - \varepsilon$ is not anymore fundamental than the $k - \omega$ model; with the only difference in transformation method. Therefore, the modifications that have been performed for the standard $k - \varepsilon$ model in jet flow, both in this work and by other
245   researchers, do not necessarily carry over to the "transformed" $k - \varepsilon$ model.

We can dive into this analysis further by relating the empirical coefficients between the standard and "transformed" $k - \varepsilon$ turbulence models. It is realized that the coefficients chosen for the "transformed" $k - \varepsilon$ model directly mimic that of the standard $k - \varepsilon$ model for λ_2 and β_2, where $\sigma_{\omega2} = 0.856$ for the "transformed" and $\sigma_{\omega2derived} = 1/\sigma_\varepsilon = 0.769$
250   for the derived version. It is still unclear whether this adjustment makes a significant contribution to a solution, but it should be noted nevertheless.

All this being said, if the standard $k - \varepsilon$ model produces spreading rates that are not agreeable, then it makes sense that the $k - \omega$ BSL model will produce spreading rates similar to the standard $k - \varepsilon$ and, therefore, some modification of the empirical coef-
255   ficients need to be performed to provide "more" agreeable results. To directly mimic the modification of the the $k - \varepsilon$ model, we set $C_{\varepsilon1} = 1.5$ and relate the constants from the $k - \varepsilon$ to the transformed $k - \varepsilon$ model by $C_{\varepsilon1} - 1 = 0.5 = \gamma_2$. This will be aptly named *mod*1 for the $k - \omega$ SST/BSL models in the solution results. It should be noted that although other researchers have modified the empirical constants for the $k - \varepsilon$ in
260   the past [11,15,17,20], to the present authors' knowledge there has been no one that has modified the constants in $k - \omega$ SST/BSL models other than the developer of the turbulence model themselves. Therefore results for the modified $k - \omega$ SST/BSL models should not be indicative as a general solution for a variety of jet flows unless further testing is performed.

The $k - \omega$ SST model is the most commonly used model proposed by Menter [38] that is differentiated from the BSL model through a change in coefficients, turbulence viscosity, and blending functions. The coefficients for the $k - \omega$ SST turbulence model are given by

$$\sigma_{k1} = 0.85, \quad \sigma_{\omega1} = 0.5, \quad \beta_1 = 0.0750, \quad a_1 = 0.31$$

$$\beta^* = 0.09, \quad \kappa = 0.41, \quad \gamma_1 = \beta_1/\beta^* - \sigma_{\omega1}\kappa \big/ \sqrt{\beta^*} \cong 0.55 \tag{29}$$

and turbulent viscosity is given by

$$\nu_t = \frac{a_1 k}{\max(a_1 \omega, \Omega F_2)}$$

$$F_2 = \tanh\left(\text{arg}_2^2\right) \tag{30}$$

$$\text{arg}_2 = \max\left(2\frac{\sqrt{k}}{0.09\omega y}, \frac{500\nu}{y^2\omega}\right)$$

where, Ω is the vorticity magnitude calculated as $\Omega = \sqrt{2}\sqrt{\Omega_{ij} : \Omega_{ij}}$, and y is the distance from the center of the cell to the wall. It should be noted that because of the change in eddy viscosity relative to the BSL model ($\nu_t = k/\omega$), the production term in the dissipation equation for $k - \omega$ SST model is equal to $(\gamma\omega/k)P$ instead of $(\gamma/\nu_t)P$.

There have since been updates to the original [38] model given in Menter *et al.* [39] that provide more agreeable solutions in industrial flow applications and is considered the "standard" model, so we will name this model SST std. to avoid confusion. The most significant change is that of eddy viscosity, from the use of the magnitude of vorticity to the magnitude of the strain invariant.

$$\nu_t = \frac{a_1 k}{\max(a_1 \omega, S F_2)} \tag{31}$$

where, $S = \sqrt{2S_{ij}S_{ij}} = \sqrt{2}\sqrt{S_{ij} : S_{ij}}$. A turbulent production limiter is also used to avoid turbulence production in the stagnation regions for both k and ω, as given by

$$P = \min(P, 10\beta^*\rho\omega k) \tag{32}$$

$CD_{k\omega}$ is changed by the use of 10^{-10} instead of 10^{-20},

$$CD_{k\omega} = \max\left(2\sigma_{\omega2}\frac{1}{\omega}\frac{\partial k}{\partial x_j}\frac{\partial \omega}{\partial x_j}, 10^{-10}\right) \tag{33}$$

and some constants are changed, specifically, $\gamma_1 = 5/9$ and $\gamma_2 = 0.44$. The default $k - \omega$ SST that is implemented in OpenFOAM is a modification of the original $k - \omega$ SST model. The origins of these modifications are unclear because of lack of a reference, but it is very similar to the original SST model and will be labeled SST def. in all the results.

3. Simulation Setup

Two distinct cases are considered that are separated by geometry and flow conditions. Namely, cases that follow the work performed by Hardalupas *et al.* [1] and Bogusławski and Popiel [26]. The operating parameters chosen for the simulations are taken directly from the inputs in the respective papers and if an assumption was needed then an explanation and justification will be shown.

3.1. Hardalupas et al. [1] Test Case

Hardalupas *et al.* [1] use a phase-Doppler anemometer to measure the velocity and flux distributions in both clean jet (with seeding powders) and particle-laden high-speed air

jets. The flow is fully developed in a $15\,mm$ diameter nozzle of 93 nozzle diameters in length. An average velocity of $13\,m/s$ is used that correlates to a Reynolds number of $1.3e4$.

3.2. Bogusławski and Popiel [26] Test Case

The experiments performed in the study of Bogusławski and Popiel [26] use an air jet issued from a long straight tube of $38.5\,mm$ in diameter and $50D$ in length. The velocity and turbulence characteristics are measured using a constant temperature thermo-anemometer. This paper defines $u_{\text{inf}} = 20m/s$ that corresponds to a Reynolds number, $\text{Re}_{dh} = 5.10e4$, but then uses this same u_{inf} as the maximum center-line velocity at the nozzle exit. This is somewhat of an ambiguity because the mean freestream velocity in a nozzle won't be the maximum centerline velocity and therefore to avoid confusion the freestream velocity in this case will be considered the maximum centerline velocity, $u_{\text{inf}} = u_{0m} = 20m/s$. The fixed value inlet boundary condition for the numerical simulation similarly follow the Hardalupas *et al.* [1] test case in that the velocity profile varies radially to the power law for turbulent jet flow with an exponent, $n = 6.6$. The boundary conditions were set to that of Table 3, where $I = 0.16\text{Re}_{dh}^{-1/8} \approx 0.04$ with $\text{Re}_{dh} = 5.10e4$.

3.3. Model Setup

Two domains are created by taking a 5 degree wedge slice of a circular 3D domain. Figure 1 shows the geometry for both the Hardalupas *et al.* [1] and Bogusławski and Popiel [26] test cases. A nozzle length of 100D is used with a freestream domain half width of 4D. A nozzle wall thickness is include in the domain with thickness D/6 and extended out towards the nozzle inlet by D/2. This is to potentially model any affects caused by boundary layer attachment at the nozzle wall. Discretization is done using hexahedral cells generated in Ansys® ICEM CFD. Uniformly space cells of size 0.00075, 0.0006, and 0.0005 are used with a single cell being used for the width in the wedge direction.

The implicit second order backward scheme is used for temporal discritization. OpenFOAM second order linearUpwind scheme is used for all velocity and turbulent fields. This scheme uses upwind interpolation weights depending on the gradient of the fluid velocity [40]. The widely used pressure-based PISO (pressure-implicit split-operator) algorithm proposed by Issa [41] is used for the fluid solver. The OpenFOAM PCG solver with a GAMG pre-conditioner is used for pressure and PBiCG with DILU pre-condioner used for all other fields [40]. Final solver tolerance is set to $1e-8$ for all scalar and vector fields.

Table 3 shows the boundary conditions for the Hardalupas *et al.* [1] test case. The eddy viscosity near the wall is calculated from a velocity based continuous wall function that follows the Spalding's equation [42]. The inlet velocity is set to the power law for fully developed turbulent pipe flow with the exponent, n, set equal to 6.6 and a maximum velocity of $u_{\text{max}} = 14.7$. A "calculated" entry is used to signify that the turbulent viscosity is calculated from k, ϵ, and ω in the turbulence model. In Table 3, the turbulence intensity is equal to $I = 0.16\text{Re}_{dh}^{-1/8} \approx 0.05$ with $\text{Re}_{dh} = 1.3e4$. Entrainment boundary conditions are used for pressure and velocity at the domain outlet. Pressure is set to zero for outflow and set to $-0.5|U^2|$ for inflow. Velocity is set to zero gradient at all times except the tangential component is set to zero. This is used to ensure that the inflow velocity can "find" the normal component, facilitating a proper entrainment boundary.

There are many ways to calculate an initial ω such as $\omega = C_\mu^{-1/4}\sqrt{k}\big/l \cong 17.9$, but the most certain way is to a run a $k - \varepsilon$ simulation and use the freestream results to calculate the initial freestream ω for the $k - \omega$ with $\omega = \varepsilon/(kC_\mu)$. It was determined that an average value of 550 will suffice for the initial freestream values. It should be noted
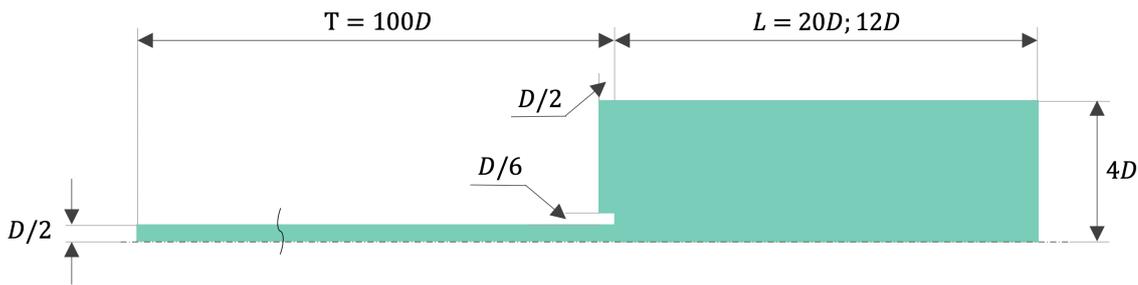
Figure 1. Slice of the 5 degree wedge simulation domain. D is the diameter of the nozzle, T is the length of the nozzle, and L is equal to 20D for Hardalupas *et al.* [1] test case and 12D for Bogusławski and Popiel [26] test case.

Table 3: Boundary conditions for Hardalupas *et al.* [1]

Location	Velocity	Pressure	Eddy Viscosity	Kinetic Energy	Epsilon	Omega
Wall	noSlip	Zero Gradient	Spalding Wall Function	Zero Gradient	Epsilon Wall Function	Omega Wall Function
Inlet	$u_{\max}\left(1 - \frac{r}{R}\right)^{1/n}$	Zero Gradient	Calculated	$1.5(I\lvert u\rvert)^2$	$C_\omega^{3/4}\left(k^{3/2}\big/l\right)$	$k^{0.5}\big/\left(C_\mu^{0.25}l\right)$
Outlet	Entrainment Velocity	Total Pressure	Calculated	Zero gradient	Zero gradient	Zero gradient
Initial freestream	Uniform 0.0	Uniform 0.0	0	$k = 1.5(I\lvert u\rvert)^2$	$C_\omega^{3/4}\left(k^{3/2}\big/l\right)$	550

that this value is difficult to discern because of the boundary layer separation at the exit of the nozzle that may cause the $k - \omega$ Wilcox turbulence model to be inaccurate from sensitivity to freestream input conditions [25]. The initial freestream values were set to 0.1 for kinetic energy, k, that corresponds to a turbulent intensity of 0.05

3.4. Wall Modelling Concerns and Mesh Resolution

It is the present authors' position that this work will serve as a basic for future work in multiphase particle-laden flows. Therefore, the meshing thought process utilized in this research has the consideration of limitations in mesh size relative to a particle diameter and the primary focus of this paper is the use of wall function approaches. In eddy viscosity RANS turbulence models, it is required to either fully integrate to the wall, or use wall function to estimate the near wall properties where viscous affects cannot be ignored. Fully integrating to the wall requires the first cell height to produce a $yplus$ of about 1 or less, and the first 8-10 cells located below a $yplus$ of about 11.5. This can cause issues with the addition of particles because of the momentum coupling between the two phases. Therefore, a good option is to use a wall function approach that will allow a larger cell near the wall with effective momentum transfer between the two phases.

The first wall functions were constructed using the Launder and Spalding methods in which it is required that the first cell height produce a yplus in the logarithmic layer ($30 < yplus < 300$) [22]. Newer adaptive wall functions have since been proposed, which allow a wide range of yplus values that include the more illusive buffer region. OpenFOAM uses adapative wall function approaches proposed by Kalitzin *et al.* [43]. These wall functions were tested in the same work and it was realized that solutions were agreeable through a spread of $yplus$ values from $11 - 111$, with the highest error found at $5 < yplus < 11$. This is the wall function used in the present work.

A zero gradient boundary condition is used for kinetic energy at the wall, with wall functions for both ε and ω. In OpenFOAM, the epsilon wall function provides a constraint at the wall for both turbulent kinetic energy, k, and dissipation, ϵ [44]. The Omega wall function uses a blending of the viscous and logarithmic layer with a blending function

360 and, theoretically, can be accurate in the buffer region. That being said, much is still unknown about the buffer region that is between the viscous and log-layer and, therefore, it is the best practice to stay out of this region as much as possible. A continuous wall function for turbulent viscosity is used, which is modelled from Spalding's equation that fits a curve of the relationship between $u+$ and $y+$ [42] for the full range of the boundary
365 layer. It should be noted that this is a function that is based on velocity or shear stress at the wall and not kinetic energy.

4. Results and discussion

4.1. Grid independence

A grid sensitivity analysis is performed using the Hardalupas *et al.* [1] test case to deter-
370 mine if there are errors associated with the discretization of the fluid domain into a mesh and that the solution is indpendent of the mesh cell size. Three systematically refined uniformly sized meshes of 52, 85, and 100 thousand hexahedral cells are generated in ICEM$^{©}$ CFD meshing software with a resulting yplus based on velocity of 18.5 15, and 13 respectively.

375 The default $k - \varepsilon$ turbulence model available in OpenFOAM are used for the grid independence with the simulation setup as described in Section 3 of this paper. It should be noted that the default $k - \varepsilon$ turbulence model is the model proposed by Launder and Spalding [22]. Richardson extrapolation was going to be used, as recommended by the Journal of Fluids Engineering described in Celik *et al.* [45], but the solutions for both
380 fluid velocity and kinetic energy showed very little difference and therefore Richardson extrapolation was of little quantitative value. The mesh of $100,000$ was chosen for all proceeding simulations because it was the finest mesh available that allows a reasonable yplus value for accurate wall function modelling.

4.2. Fluid Velocity

385 The potential core of the jet is the region in which there is a small difference in the velocity of the bulk and jet regions. The core is characterized by a conical shape narrowing from the nozzle exit to the end of the developing region. After the potential core region, the flow develops into what is known as the fully established flow field [46]. Figure 2a illustrates the velocity as a function of axial non-dimensional distance from the nozzle
390 exit for the Hardalupas *et al.* [1] test case. Although it is difficult to discern exactly where the potential core ends for the experimental data because of the limited number of sampling points in the axial direction, if an extrapolation line is fitted to follow the available points, we can comment that the potential core has a length that is the most similar to the standard $k - \varepsilon$, Pope Corr. $k - \varepsilon$, and $k - \omega$ BSL turbulence model. The
395 $k - \varepsilon$ mod1 model with $C_{\varepsilon 1} = 1.52$ and $C_{\varepsilon 1} = 1.90$ is the most similar throughout the entire range of velocities, but has a slightly longer potential core as shown by Givi and Ramos [15]. The conclusion, therefore, is that if merely the modelling of the potential core is desired and not the fully established flow field, then a variety of turbulence models can be used with sufficient accuracy. Nonetheless, if the full range of the near
400 field, from the potential core through to the fully established flow ($0 \geq x/D \geq 20$), is modelled, then the $k - \varepsilon$ mod1 turbulence model is the most desired with the $k - \omega$ SST mod1 and $k - \omega$ BSL mod1 coming in a close second. This is shown for the axial and radial velocity profiles by observing Figure 2.

The $1/7^{th}$ Power law has been shown to be an accurate and simple model for profiles
405 of fully developed turbulence pipe flow for a variety of Reynolds numbers. It was observed that the flow profiles for the Hardalupas *et al.* [1] test cases follow the $1/7^{th}$ Power law well for the inner region of the flow at the nozzle exit. However, the profile has a slight "U" shape, which is characteristic of a more laminar flow without a boundary layer. By observing Figure 2b it is realized that the numerical results agree more with the
410 Hardalupas *et al.* [1] test case then the Power law. The Bogusławski and Popiel [26] test
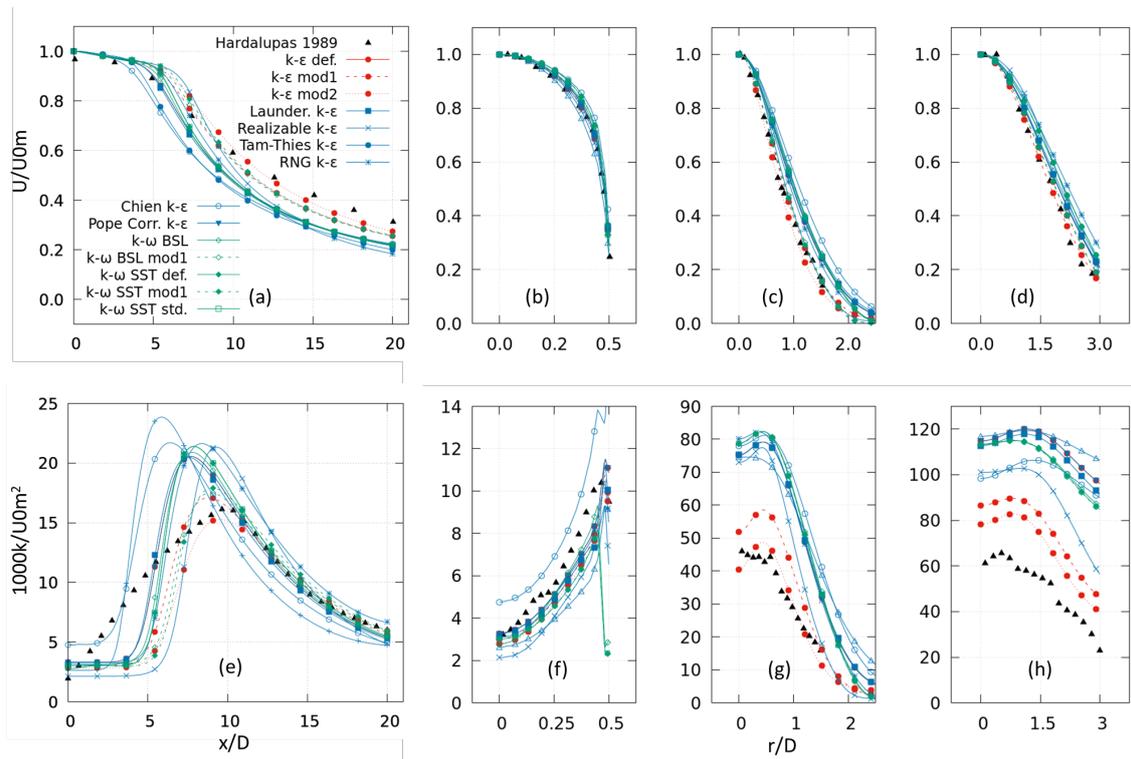
Figure 2. Test case Hardalupas *et al.* [1] normalized velocity profiles at (a) axial and (b) 0.1D, (c) 10D, and (d) 20D normal planes, and normalized turbulent kinetic energies at (e) axial, and (f) 0.1D, (g) 10D, and (h) 20D normal planes.
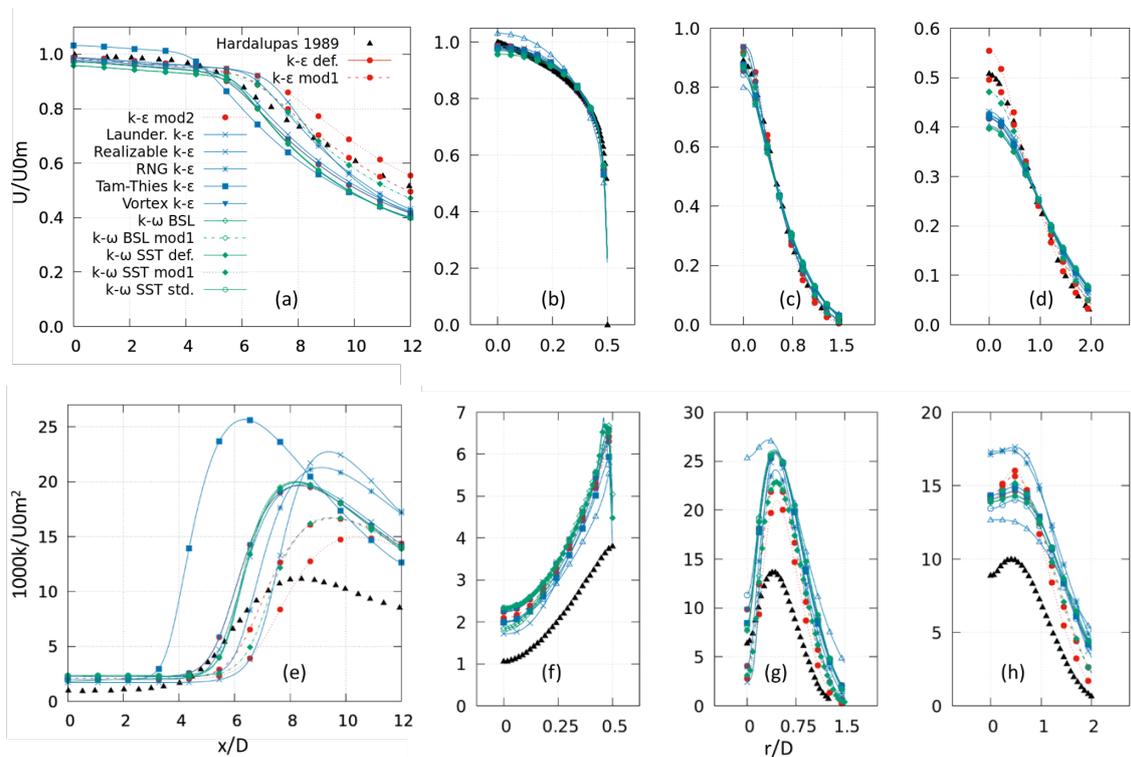

Figure 3. Test case Bogusławski and Popiel [26] normalized velocity profiles at (a) axial and (b) 0D, (c) 6D, and (d) 12D normal planes, and normalized turbulent kinetic energies at (e) axial, and (f) 0D, (g) 6D, and (h) 12D normal planes.

case exhibits different behavior in that the velocity at the nozzle exit follows the $1/7^{th}$ Power law for both the experimental and numerical results.

It was originally thought that $k - \omega$ SST turbulence model, with its variants, would be the best option for these types of flow because of its ability to model the near field effectively using $k - \omega$ and the outer field using $k - \varepsilon$. This was found to be untrue because of the issues with empirical coefficients associated with these models. In the derivation of the $k - \omega$ SST/BSL turbulence models some terms are dropped and seem to be absorbed into the cross diffusion empirical coefficient of the transformed $k - \varepsilon$ given by $\sigma_{\omega 2} = 0.856$ that are provided in the $k - \omega$ BSL/SST turbulence models.

It was found that the default $k - \omega$ SST model compared very well to the standard $k - \varepsilon$ model in the far field. It is stated by Menter [25] that the blending function goes to zero at the log region. We observed that the yplus value for the mesh and Reynolds numbers was about 13 for the Hardalupas *et al.* [1] test case and 30 for the Bogusławski and Popiel [26] test case. With this yplus, the wall functions are fully modelling the flow field up to, or very close to, the log region for the Bogusławski and Popiel [26] test case and into the buffer region for the Hardalupas *et al.* [1]. Hence for the Hardalupas *et al.* [1] the flow field is calculated slightly with the $k - \omega$ turbulence model and the rest with the "transformed" $k - \varepsilon$ while the flow field is calculated almost exclusively from the "transformed" $k - \varepsilon$ for the Bogusławski and Popiel [26] test case.

Hence, the flow field is calculated from the transformed $k - \varepsilon$ turbulence model for the $k - \omega$ SST turbulence model. The derivation of the $k - \omega$ SST model shows that two terms are dropped, and this does affect the results slightly with the difference between the modified and $k - \varepsilon$ equation. It is thought that the constants can be further modified to account for this slight change. No previous research has investigated and reported if modifying the constants of the $k - \omega$ SST model would gain any benefit in the predicted flow field, and an extensive study would be required to fully understand the behavior with this type of modification and this particular aspect was not researched in this study. That being said, sufficiently accurate results using the modified $k - \varepsilon$ turbulence model were achieved, which can be used for future work in round turbulence dense CFD-DEM turbulence jet flows.

It should be noted that the stated inlet velocity for the experiments performed by Bogusławski and Popiel [26] is $U_\infty = 20 m/s$ with the profile at the nozzle exit following the $1/7^{th}$ Power law given by

$$U/U_\infty = (1 - 2r/D)^{\frac{1}{7}} \tag{34}$$

Typically the $1/7^{th}$ Power law velocity is normalized with the maximum centerline velocity at the nozzle exit, U_{0m}, and not the infinite, or mean, freestream velocity, U_∞, and, therefore, the maximum velocity is slightly greater in magnitude than the normalized velocity. The data given from the experiments demonstrate that the freestream velocity is, actually, the maximum centerline velocity at the nozzle exit, that is $U_\infty = U_{0m}$, because the plots never go above unity. Therefore, going forward, we will normalize the numerical data with the maximum centerline velocity, U_{0m}. This data treatment allows consistent normalization across each of the turbulence models because the maximum centerline velocity changes slightly between turbulence models with a constant freestream velocity value at the inlet.

4.3. Turbulence properties

As outlined before, when Favre averaging the Navier-Stokes equations, an additional Reynolds stress term is added that accounts for the fluctuations in the flow field. The Reynolds Stress is given as a tensor by

$$\overline{u'_i u'_j} = \begin{bmatrix} \overline{u'_x u'_x} & \overline{u'_x u'_y} & \overline{u'_x u'_z} \\ \overline{u'_y u'_x} & \overline{u'_y u'_y} & \overline{u'_y u'_z} \\ \overline{u'_z u'_x} & \overline{u'_z u'_y} & \overline{u'_z u'_z} \end{bmatrix} \tag{35}$$

This is a symmetric tensor and so we have gained six additional scalar transport equations in addition to the Navier-Stokes. Instead of modelling each of these six transport equations, we choose a more heuristic approach by modelling the Reynolds stress directly with the Boussinesq approximation with a linear function of the mean-strain-rate tensor using an eddy viscosity that was previously stated by Eq. 11 but will also be stated here for completeness

$$\overline{u'_i u'_j} = \nu_t \left(\frac{\partial u_i}{\partial x_j} + \frac{\partial u_j}{\partial x_i} \right) - \frac{2}{3} k \delta_{ij} \tag{36}$$

where k is the turbulent kinetic energy defined by the one-half of trace of the Reynolds Stress tensor

$$k = \frac{1}{2} trace \left(\overline{u'_i u'_j} \right) \tag{37}$$

Therefore, the kinetic energy can be expressed by one-half of the sum of the variances, or one-half the mean square magnitude of the velocity fluctuations given as

$$k = \frac{1}{2} \overline{u'_i u'_i} = \frac{1}{2} \left(\overline{(u'_x)^2} + \overline{(u'_y)^2} + \overline{(u'_z)^2} \right) \tag{38}$$

It should be noted that the eddy viscosity is isotropic but not necessarily homogeneous and, therefore, the turbulence itself is not necessarily isotropic because the mean-strain-rate tensor of the Navier-Stokes equations is not isotropic. This is an important distinction because eddy viscosity is merely independent of direction.

To calculate turbulence kinetic energy for the experiments we use the provided root-mean-square (RMS) components of velocity given in the Hardalupas *et al.* [1] paper's notation as $u' = RMS(u) = \sqrt{u^2}$, which is the square root of the variance equivalent to the present author's paper notation as $\sqrt{(u'_{axial})^2}$ and $\sqrt{(u'_{radial})^2}$. Only the radial and axial velocity components are given and, therefore, kinetic energy is in "cylindrical" coordinates and transformation can be challenging. One estimation is the so-called two-dimension approximation that is proposed by Rodriguez *et al.* [47]

$$k = \frac{3}{4} \left(\left(u'_{axial} \right)^2 + \left(u'_{radial} \right)^2 \right) \tag{39}$$

where, to obtain Eq. 39, we have a third component of turbulent kinetic energy distribution given by $(u'_\theta)^2 = \left((u'_{axial})^2 + (u'_{radial})^2 \right)^{0.5}$. This is assuming that the turbulent kinetic energy values are the same magnitude in the θ direction, which is not necessarily the case. It is observed by Fig. 2(e-h) that this approximation provides results that are agreeable for the $k - \varepsilon$ mod1 turbulence model but it would be unwise to make a definite conclusion with the two-dimensional assumption and, therefore, no definite conclusion for kinetic energy can be realized purely from this test case.

The turbulent kinetic energy for both cases drastically changes from a "U" shape to a "W" shape between -0.1D to 0.1D as the flow leaves the nozzle exit. It is interesting to note that a "W" shape is achieved at the nozzle exit within the range of a single nozzle

diameter, 1D, in the numerical results. However, the experimental results demonstrate a "W" shape, but outside the range of 1D. This is a very important observation because it appears that the location of the hump in the W shape is not significantly different between the numerical and experimental results further in the flow field.

470   From Figures 3(e-h) it is observed that the turbulent kinetic energy in the experimental results of Bogusławski and Popiel [26] follows the overall trend well but the magnitude is significantly lower then the numerical results for all of the turbulence models tested. This was also observed in the work performed in Faghani *et al.* [20] and Quinn and Militzer [11]. Contrasting to this, the use of the two-dimensional approximation causes

475   a higher magnitude in results for the Hardalupas *et al.* [1] test case. It is thought that this approximation is more for swirling flows causing more turbulent kinetic energy generation in the θ direction. The turbulence intensity in the axial direction lined up well with the experimental data, but the intensity in the other directions was about half of those achieved with the numerical results. This can be expected because of the isotropic

480   assumption of the linear viscous RANS turbulence models ensures that the turbulence modulation is equal in all directions. Therefore, it is up to the researcher to determine if the benefits of using this type of model outweigh the potential errors. This notion is true for the use of any linear viscous RANS turbulence model because our goal is to capture the average rather then the instantaneous flow behavior.

485   ## 5. Conclusion

It was realized that a slight modification of the empirical constant $C_{\varepsilon 1}$=1.5 in the standard Launder and Spalding [22] $k - \varepsilon$ model provided the most agreeable results for both test cases for fluid velocity. A change in coefficients of the $k - \omega$ SST v1/BSL turbulence models that mimic the $C_{\varepsilon 1}$=1.5 turbulence model by setting γ=0.5 provided suitable

490   results as well. It is unclear whether RANS models in general will accurately model the turbulent kinetic energy in all directions for jet flow but, as a general conclusion, the kinetic energy from the numerical results will be higher in magnitude then the experiments because this is observed for the Bogusławski and Popiel [26] test case and the work performed previously by Faghani *et al.* [20] and Quinn and Militzer [11]. It is

495   therefore up to the researcher to weigh the costs and benefits of using these models and if merely with considerations to help quantify these errors along with considerations on if merely a velocity field is desired.

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