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Violation of the Relativistic Energy Conservation Law and Einstein's Principle of Relativity Caused by the Generation of Mechanical Transverse Waves in a Moving Medium

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Abstract

We study the effect of the generation of the mechanical transverse wave (MTW) travelling in the opposite direction (OD) to a moving medium (MM) on the relativistic energy conservation law (RECL). From the viewpoint of the relativity of simultaneity (RS), the time on the coordinate coinciding with the advance end of the wave (AEW) travelling toward the rear of the MM passes faster than that on the coordinate coinciding with the wave source (WS). Then the AEW in the MM travels forward compared to that in the rest frame of reference (RFR) which is stationary relative to the medium when the time on the coordinate coinciding with the WS is same for each inertial frame of reference (IFR). Hence, the coordinate interval (CI) between the AEW and WS in the MM is observed to be larger than that between them in the RFR. We show that this difference holds true for the CI of any portion having transverse velocities mutually converted by the Lorentz transformation (LT). This difference in the CI leads to that in the rest mass (RM). We demonstrate that the RM included in wave motion (WM) in the MM is larger than one included in WM in the RFR when comparing the portions having transverse velocities mutually converted by the LT. This relation holds true for all portions in WM. Therefore, the total coordinate interval of the portion (CIP) and total RM (TRM) included in WM in the MM (WMMM) are large compared to them included in WM in the RFR. Furthermore, we compare the relativistic kinetic energy (RKE) of the MTW travelling in the OD to the MM (ODMM) with that of the MTW propagating in the direction vertical to the moving direction of the medium. We prove that the CIP and RM included in the former MTW are larger than them included in the latter MTW when comparing each portion with the same transverse velocity (TV). Moreover, the total CIP and TRM included in the former MTW are also large compared to them included in the latter MTW. The reason for these is that the latter CIP and RM are equal to them in the RFR when comparing the portions having transverse velocities mutually converted by the LT. On the other hand, the energy supplied to generate each MTW is the same. From these, we demonstrate that the RKE of the MTW travelling in the ODMM can be larger than the total relativistic energy (TRE) of the MTW propagating in the direction vertical to the moving direction of the medium. Consequently, we propose a violation of the RECL and Einstein's principle of relativity (EPR) because the TRE is not necessarily conserved in the IFR in which the medium is moving.

Key words: relativistic energy, energy conservation law, Einstein's principle of relativity, rest mass in wave motion, relativity of simultaneity, time dilation, Lorentz contraction, mechanical transverse wave, special relativity

1. Introduction

It is well known as one of the basic laws of physics that the TRE is conserved in an isolated inertial frame of reference. According to EPR, furthermore, laws of physics take the same form in all frames of reference moving with constant velocity relative to one another. This is always true if transfer of energy is done at a point because an event at it occurs simultaneously in all frames of reference.

On the other hand, if transfer of energy is continuously done in a spatial spread, is the TRE conserved? Such example includes the generation of the MTW. A WS supplying the energy of the MTW is away from the places in the medium transferring its energy. Therefore, for the MM, we have to take the RS into consideration. As the result, the time supplying the energy is different from the times transferring it. What effect does this have on the RECL and EPR?

We could not find the previous works that examined the effect of the generation of the MTW on the RECL although a research for relativistic effects appearing at non-relativistic speeds considers the peculiarity of RE concomitant with generating the MTW in MM (MTWMM) [1]. Certainly, there are some works considering the energy conservation law in the special relativity (SR). They, however, focus only on microscopic phenomena with relativistic speeds such as retardation effect on energies of particles in electromagnetic field [2-4]. In this paper, we carry out a thought experiment concerning the generation of the MTWMM and thereby theoretically study whether the TRE is conserved.

2. A Thought Experiment

The consideration in this section is basically relied on Ref. [1]. Firstly, we presuppose two inertial frames of reference, S' and S, that move relative to each other. S', in which a box is stationary, moves at constant velocity V in the positive direction (PD) of the x-axis in S. In the box, there are two homogenous strings which are parallel to the x'-axis and at rest relative to S' in an initial state. One end of each string is fixed to the box and they are placed symmetrically with respect to the x'-axis. Each other end is on the opposite side of the box and they are also placed symmetrically with respect to the x' axis. They are not fixed to the box and hence we are able to move them freely along the y' axis vertical to the x'-axis. The density of mass of the uniform string is ρ'. Constant tension T' exists throughout the string although it is slightly extensible. The string is very long compared with the wavelength generated. We assume that there are no frictional forces as waves propagate along the strings. Here the effect of gravity is neglected.

Second, we simultaneously apply the same driving force to the non-fixed ends of the strings to move them along the y'-axis during a certain time t' respectively. Two MTWs with $1/2$-wavelength thereby are generated. These two waves are completely symmetrical to the x'-axis. The slope from the x'-axis of each string is very small. Two MTWs travel at constant velocity $-V'$ in the negative direction (ND) of the x'-axis, respectively. Then they are observed to be stationary in S. Hence, each AEW observed from S (AEWos) is stationary at the points generated from each WS. In contrast, two WSs move at constant V in the PD of the x-axis. After all, the wavelengths extend with the moving WSs, respectively.

Here, since the tension of each string along x'-axis is regarded as constant, the longitudinal forces acting on both sides of the box during generating the waves have the same magnitude but the direction of them are opposite. Further, the driving forces acting vertically to the x'-axis are also equal in magnitude and opposite in direction since two MTWs are generated symmetrically with respect to its axis. From these, the box remains equilibrium and therefore there is no change in energy of it.

Furthermore, we assume that, in the above box, there are two homogenous strings which are along the z'-axis vertical to the x'- and y'-axes and are stationary relative to S' in an initial state. One end of each string is fixed to the box and they are placed symmetrically with respect to the z'-axis. Each other end is in the opposite side of the box and they are also placed symmetrically with respect to the z'-axis. They are not fixed to the box and hence we are able to move them along the y'-axis vertical to the z'-axis. It is assumed that the properties and conditions of the strings are the same as those described above. We simultaneously apply the same driving force as above to the non-fixed ends of the strings to move them along the y'-axis during t' respectively. Two MTWs having $1/2$-wavelength thereby are generated respectively, and they travel in the PD of the z'-axis. These two waves are completely symmetrical to the z'-axis. Two MTWs travel at constant velocity $v_z{}'$ along the z'-axis, respectively. The velocity of each MTW observed from S is $(v_z{}'\sqrt{1-V^2/c^2})/(1+Vv_x{}'/c^2)$ according to the LT. It is observed to be $(v_z{}'\sqrt{1-V^2/c^2})$ in S since $v_x{}'=0$. For the same reason as above, the box remains equilibrium and hence there is no change in energy of it.

Here let $W_x{}'$ and $W_z{}'$ be each MTW travelling in the ND of the x'-axis and each MTW propagating in the PD of the z'-axis, respectively. Each total energy of $W_x{}'$ and $W_z{}'$ generated consists of the sum of kinetic and potential energy. As is well known, the kinetic energy (KE) of a minute portion of the string equals to $1/2\,\rho'dx'(\partial y'/\partial t')^2$ where $\rho'dx'$ is the mass of it and $\partial y'/\partial t'$ is the TV of it. On the other hand, the potential energy (PE) is equal to the work done by constant tension T' in extending the minute portion dx' to a new length $\mathrm{d}s'$ when the string is oscillating. Thus, it is written as $T'(ds'-dx')$. Moreover, it is known that each amount of KE and PE of the minute portion is the same in dynamical system in which linear restoring force acts. According to the energy conservation law, the total energy of the MTW generated is completely equal to that supplied from the WS, in other words, that lost in it. The energy of $W_x{}'$ equals to that of $W_z{}'$ since they are generated in the same way in the identical medium. Each energy supplied to generate $W_x{}'$ and $W_z{}'$ is also the same amount.

3. Two Physical Quantities Related to the RKE

Here let $W'_{x\,s}$ and $W'_{z\,s}$ be $W_x{}'$ and $W_z{}'$ observed from S, respectively. We analyze each RKE of $W'_{x\,s}$ and $W'_{z\,s}$. When analyzing the RKE, we need only to compare one $W'_{x\,s}$ to one $W'_{z\,s}$ since the RKE of each $W'_{x\,s}$ is the same, and that of each $W'_{z\,s}$ is the same. The physical quantities related to the RKE are the velocity and

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RM of each portion in WM (PWM). The consideration in this section is also based on Ref. [1].

3.1. Velocity

Here let v'_s be the velocity of a PWM measured from S. The corresponding velocity in S' is the TV v_y' since v_x' in the above MTW is zero. Further, the longitudinal velocity of the PWM measured from S is V since $v_x' = 0$.

The components of v'_s of a PWM in $W'_{x\,s}$ are $v_{x\,s}'$ and $v_y{}'_s$. By using the law of velocity addition in SR, they are written as

$$v_x{}'_s = \frac{V + v_x'}{1 + Vv_x'/c^2}, \qquad v_y{}'_s = \frac{v_y'\sqrt{1 - V^2/c^2}}{1 + Vv_x'/c^2}. \tag{1}$$

Since $v'_s = \sqrt{\left(v_x{}'_s{}^2 + v_y{}'_s{}^2\right)}$, replacing it by Eq. (1) gives the following equation

$$v'_s = \sqrt{\left(\frac{V + v_x'}{1 + Vv_x'/c^2}\right)^2 + \left(\frac{v_y'\sqrt{1 - V^2/c^2}}{1 + Vv_x'/c^2}\right)^2}$$

$$= \frac{\sqrt{V^2 + 2Vv_x' + v_x'^2 + v_y'^2(1 - V^2/c^2)}}{1 + Vv_x'/c^2}$$

$$= \frac{\sqrt{V^2 + 2Vv_x' + v_x'^2 + v_y'^2 - V^2 v_y'^2/c^2}}{1 + Vv_x'/c^2}. \tag{2}$$

Replacing v_x' in Eq. (2) by 0, we obtain

$$v'_s = \sqrt{V^2 + v_y'^2 - V^2 v_y'^2/c^2}. \tag{3}$$

The velocity components of each PWM in $W'_{z\,s}$ are also $v_x{}'_s$ and $v_y{}'_s$ because each PWM in it does not have $v'_{z\,s}$. As described above, the velocity of PWM related to each RKE of $W'_{x\,s}$ and $W'_{z\,s}$ is determined according to the LT. Then we need to consider whether the RM included in $W'_{x\,s}$ is equal to that included in $W'_{z\,s}$ for each portion with the same TV (STV) $v_y{}'_s$.

3.2. The difference in RM between W'_{xs} and W'_{zs}

Ref. [1] demonstrates a relativistic peculiarity with respect to the RM of a PWM (RMPWM). It regards the RMPWM as one corresponding to the CI of the wave.

At first, we compare the RMPWM in $W'_{x}{}_s$ with that in W_x'. Here we have to consider the RS. We assume that the synchronized clocks are placed along the x- and x'-axes respectively.

When observing $W'_{x}{}_s$, the clock on the x'-coordinate coinciding with the AEW at the rear of the MM always goes by fast compared with that on the x'-coordinate coinciding with the WS at the front of it. Then, if the CI between the AEW and WS in W_x' at a time t_1' is $|-x_1' - x_0'|$, then that between them in $W'_{x}{}_s$ at the same time becomes $|-x_2' - x_0'|$. The reason for this is that the AEWos has already passed through $-x_1'$ at t_1' and reaches $-x_2'$. The coordinate $-x_2'$ is located away from x_0' compared to $-x_1'$ and the time on it becomes t_2'. From these, the relation between the CI of PWM (CIPWM) in $W'_{x}{}_s$ and the CIPWM in W_x' is given by

$$|-x_2' - x_0'| > |-x_1' - x_0'|. \tag{4}$$

In addition, we can presuppose that the portion between the coordinates of $|-x_1' - x_0'|$ in S' has a TV v_y'. If we convert v_y' in S' into the corresponding TV $v_y{}'_s$ observed from S, from Eq. (1), $v_y{}'_s = v_y'\sqrt{1 - V^2/c^2}$ since v_x' is zero. Thus, the CIPWM having $v_y'\sqrt{1 - V^2/c^2}$ is $|-x_2' - x_0'|$ because the AEWos reaches $-x_2'$ at the moment that the portion between the coordinates of $|-x_1' - x_0'|$ in S' has v_y'. Consequently, the CIPWM with $v_y{}'_s$, $I_{v_y{}'_s W'_x{}_s}$, is larger than one having v_y' in S', $I_{v_y' W_x'}$, *i.e.*,

$$I_{v_y{}'_s W'_x{}_s} > I_{v_y' W_x'}. \tag{5}$$

Here we consider the RMPWM. The RMPWM is the quantity related to the CIPWM. Hence, the RMPWM having $v_y{}'_s$ in $W'_x{}_s$, $m_{v_y{}'_s W'_x{}_s}$, is defined as

$$m_{v_y{}'_s W'_x{}_s} = I_{v_y{}'_s W'_x{}_s}\rho', \tag{6}$$

where ρ' is RM per unit CI in S'. The RMPWM having v_y' in W_x', $m_{v_y' W_x'}$, is expressed as $m_{v_y'W_x'} = I_{v_y'W_x'}\rho'$. Since $I_{v_y'{}_sW_x'{}_s} > I_{v_y'W_x'}$ from inequality (5), we find that $m_{v_y'{}_sW_x'{}_s}$ is larger than $m_{v_y'W_x'}$ on condition that each PWM have the STV. The difference in the RM between $m_{v_y'{}_sW_x'{}_s}$ and $m_{v_y'W_x'}$ is due to $I_{v_y'{}_sW_x'{}_s}$:$I_{v_y'W_x'}$. Therefore, we obtain

$$m_{v_y'{}_sW_x'{}_s} = m_{v_y'W_x'}\frac{I_{v_y'{}_sW_x'{}_s}}{I_{v_y'W_x'}} > m_{v_y'W_x'}. \tag{7}$$

Inequality (5) is applied to any CIPWM with the STV. Hence, we get

$$TI_{W_x'{}_s} > TI_{W_x'} \tag{8}$$

where $TI_{W_x'{}_s}$ and $TI_{W_x'}$ are each total of the CIPWM (TCIPWM) in $W_x'{}_s$ and W_x'. Furthermore, since the inequality in expressions (7) is applied to any RMPWM with the STV, we obtain

$$TM_{W_x'{}_s} > TM_{W_x'}. \tag{9}$$

where $TM_{W_x'{}_s}$ and $TM_{W_x'}$ are each total of the RMPWM (TRMPWM) in $W_x'{}_s$ and W_x'.

Secondly, we compare the RMPWM in $W_z'{}_s$ with that in W_z'. We do not need to consider the RS because $W_z'{}_s$ is generated vertically to the travelling direction of S'. Thus, the coordinate on the z-axis coinciding with the AEW in $W_z'{}_s$ at an arbitrary time always equals to one on the z'-axis coinciding with the AEW in W_z'. Let $I_{v_y'{}_sW_z'{}_s}$, $m_{v_y'{}_sW_z'{}_s}$, $TI_{W_z'{}_s}$ and $TM_{W_z'{}_s}$ be the CIPWM having $v_y'{}_s$ in $W_z'{}_s$, the RMPWM with $v_y'{}_s$ in $W_z'{}_s$, the TCIPWM in $W_z'{}_s$ and the TRMPWM in $W_z'{}_s$, respectively. Then we have $I_{v_y'{}_sW_z'{}_s} = I_{v_y'W_z'}$, $m_{v_y'{}_sW_z'{}_s} = m_{v_y'W_z'}$, $TI_{W_z'{}_s} = TI_{W_z'}$ and $TM_{W_z'{}_s} = TM_{W_z'}$ where $I_{v_y'W_z'}$, $m_{v_y'W_z'}$, $TI_{W_z'}$ and $TM_{W_z'}$ denote the CIPWM in W_z' having v_y' corresponding to $v_y'{}_s$, the RMPWM with its v_y' in W_z', the TCIPWM in W_z' and the TRMPWM in W_z', respectively. In addition, $TI_{W_z'} = TI_{W_x'}$ and $TM_{W_z'} = TM_{W_x'}$ since W_z' and W_x' are generated in the same way in the identical medium.

Consequently, we find the following relational expressions for $W_x'{}_s$ and $W_z'{}_s$:

$$I_{v_y'{}_sW_x'{}_s} > I_{v_y'{}_sW_z'{}_s} = I_{v_y'W_z'} = I_{v_y'W_x'}, \tag{10}$$

$$m_{v_y'{}_sW_x'{}_s} > m_{v_y'{}_sW_z'{}_s} = m_{v_y'W_z'} = m_{v_y'W_x'}, \tag{11}$$

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$$TI_{W_x'{}_s} > TI_{W_z'{}_s} = TI_{W_z'} = TI_{W_x'} \tag{12}$$

$$TM_{W_x'{}_s} > TM_{W_z'{}_s} = TM_{W_z'} = TM_{W_x'}. \tag{13}$$

4. The Comparison of RKE between $W_x'{}_s$ and $W_z'{}_s$

We compare the RKE of $W_x'{}_s$ to that of $W_z'{}_s$. Let $m_1W_x'{}_s$ and $m_1W_z'{}_s$ be the RM of a PWM in $W_x'{}_s$ and that of a PWM in $W_z'{}_s$, respectively. Each portion has the STV $v_y'{}_s$. From Eq. (3), v'_s of each PWM observed from S equals to $\sqrt{V^2 + v_y'^2 - V^2 v_y'^2/c^2}$. Hence, the RKE of $m_1W_x'{}_s$, $RKE_{m_1W_x'{}_s}$, and that of $m_1W_z'{}_s$, $RKE_{m_1W_z'{}_s}$, are given by

$$RKE_{m_1W_x'{}_s} = m_1W_x'{}_s c^2 \left\{ \frac{1}{\sqrt{1 - \left(\sqrt{V^2 + v_y'^2 - V^2 v_y'^2/c^2}\right)^2 / c^2}} - 1 \right\} \tag{14}$$

and

$$RKE_{m_1W_z'{}_s} = m_1W_z'{}_s c^2 \left\{ \frac{1}{\sqrt{1 - \left(\sqrt{V^2 + v_y'^2 - V^2 v_y'^2/c^2}\right)^2 / c^2}} - 1 \right\}. \tag{15}$$

From the inequality in expressions (11), $m_1W_x'{}_s > m_1W_z'{}_s$. By contrast, the velocity related to each RKE is identical. Therefore, we find

$$RKE_{m_1W_x'{}_s} > RKE_{m_1W_z'{}_s}. \tag{16}$$

Let $TRKE_{W_x'{}_s}$ and $TRKE_{W_z'{}_s}$ be the total RKE of $W_x'{}_s$ and that of $W_z'{}_s$, respectively. We get the following equation for $TRKE_{W_x'{}_s}$

$$TRKE_{W_x's} = RKE_{m_1W_x's} + RKE_{m_2W_x's} + RKE_{m_3W_x's} + \cdots = \sum_{i=1}^{n} RKE_{m_iW_x's}. \quad (17)$$

Likewise, $TRKE_{W_z's}$ is expressed as

$$TRKE_{W_z's} = RKE_{m_1W_z's} + RKE_{m_2W_z's} + RKE_{m_3W_z's} + \cdots = \sum_{i=1}^{n} RKE_{m_iW_z's}. \quad (18)$$

The RKE of each term in Eq. (17) is large compared to that of the corresponding term in Eq. (18) because inequality (16) is applied to any PWM with the identical velocity. Consequently, we find the following inequality

$$TRKE_{W_x's} > TRKE_{W_z's}. \quad (19)$$

This is due to the difference in each RMPWM concomitant with the difference in each CIPWM having the STV.

5. The Numerical Analysis of each RE of $W_x's$ and $W_z's$

Here we perform the numerical analysis of each RKE of $W_x's$ and $W_z's$ on the basis of the effect of time dilation and Lorentz contraction in SR and furthermore compare $TRKE_{W_x's}$ to the TRE of $W_z's$, $TRE_{W_z's}$. We assume that the travelling velocity in W_x' generated during a certain time t' is $-v_x' = -V'$ and its wavelength, $1/2\,\lambda_x'$, equals to $|-V' \times t'|$. When measuring this from S, the AEW in W_x' is at rest and the WS in it moves at V in the PD of the x-axis in S.

Firstly, we need to take time dilation in S' observed from S into account. Let t be the passage of time, observed from S, corresponding to t'. For the relation between t and t', we have

$$t = \frac{t'}{\sqrt{1 - V^2/c^2}} > t'. \quad (20)$$

Hence, we find that the clocks in S go fast by a factor $(1 - V^2/c^2)^{1/2}$ compared to them in S' moving at V. In other words, the time of the generation of $W_x{'}_s$ is longer than that generating $W_x{'}$ in S'. If we focus only on time dilation, since $t > t'$, we get the following relational expressions between $1/2\,\lambda_x{'}_s$ observed from S and $1/2\,\lambda_x{'}$

$$1/2\,\lambda_x{'}_s = Vt > 1/2\,\lambda_x{'} = |-V't'| \tag{21}$$

where $|V| = |-V'|$.

Secondly, we consider the effect of Lorentz contraction. The length of a body in the direction of its motion with uniform velocity V is reduced by a factor $\sqrt{1-V^2/c^2}$. Thus, the length of $1/2\,\lambda_x{'}_s$ is reduced by a factor $\sqrt{1-V^2/c^2}$ relative to the corresponding length of the medium in S'. In other words, the length of $1/2\,\lambda_x{'}_s$ corresponds to $1/2\,\lambda_x{'}_s/\sqrt{1-V^2/c^2}$ in S' if we convert it to the length of the medium in S'. Let $1/2\,\lambda_x{'}_{ss'}$ be the length of medium in S' corresponding to $1/2\,\lambda_x{'}_s$. Then the relation between $1/2\,\lambda_x{'}_{ss'}$ and $1/2\,\lambda_x{'}$ becomes as follows

$$1/2\,\lambda_x{'}_{ss'} = \frac{Vt}{\sqrt{1-V^2/c^2}} > 1/2\,\lambda_x{'}. \tag{22}$$

This also means that the TCIPWM included in $1/2\,\lambda_x{'}_{ss'}$ is larger than that included in $1/2\,\lambda_x{'}$.

Here we substitute a value, $V = 0.9$, into expressions (20). Then we assume that $t' = 1.0$ and $c = 1$, we have

$$t = \frac{t'}{\sqrt{1-V^2/c^2}} = \frac{1.0}{\sqrt{1-0.9^2/1^2}} \fallingdotseq 2.5 > t' = 1.0. \tag{23}$$

Substituting the values mentioned above into expressions (22), we obtain

$$1/2\,\lambda_x{'}_{ss'} = \frac{0.9 \times 2.5}{\sqrt{1-0.9^2/1^2}} = 5.6 > 1/2\,\lambda_x{'} = |-0.9 \times 1.0| = 0.9. \tag{24}$$

The ratio of $1/2\,\lambda_x{'}_{ss'}$ to $1/2\,\lambda_x{'}$, in other words, the ratio of the TCIPWM of the former to that of the latter is as follows

$$\frac{1/2\,\lambda_x{'}_{ss'}}{1/2\,\lambda_x{'}} = \frac{5.6}{0.9} \fallingdotseq 6.2. \tag{25}$$

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Consequently, we find that ${W_x}'_s$ with $1/2\,{\lambda_x}'_{ss'}$ has about 6.2 times more the TRM than ${W_x}'$ with $1/2\,\lambda_x'$, *i.e.*,

$$TM_{{W_x}'_s} \fallingdotseq 6.2TM_{W_x'} \tag{26}$$

because the difference in the TCIPWM corresponds to that in the TRM. Also, from expressions (11), each RMPWM in ${W_x}'_s$ having ${v_y}'_s$ is large compared to that in ${W_x}'$ having ${v_y}'$ corresponding to ${v_y}'_s$. The former has about 6.2 times more the RM than the later.

We will consider the above consequence for $1/2\,{\lambda_x}'_{ss'}$ from another perspective. When observing from S, $1/2\,\lambda_x'$ is reduced due to Lorentz contraction and becomes $1/2\,\lambda_x'\sqrt{1-V^2/c^2}$. This $1/2\,\lambda_x'\sqrt{1-V^2/c^2}$ is the length from the AEW in ${W_x}'$ to the WS. The position of the WS at the beginning of the generation of ${W_x}'$ equals to the stationary AEWos. Then the WS and the AEW located $1/2\,\lambda_x'\sqrt{1-V^2/c^2}$ away from it travel at V in the PD of the x-axis. Thus, the time until the AEW in ${W_x}'$ advances to the stationary AEWos and coincides with it becomes as follows

$$\frac{1/2\,\lambda_x'\sqrt{1-V^2/c^2}}{V}. \tag{27}$$

The WM in S' is occurring in the medium with the length corresponding to $1/2\,\lambda_x'$ during the above time. Nevertheless, the supply of energy to the medium from the WS continues. The time that the generation of ${W_x}'$, observed from S, finishes is t. From these, we obtain

$$\frac{1/2\,\lambda_x'\sqrt{1-V^2/c^2}}{V} : 1/2\,\lambda_x' = t : 1/2\,{\lambda_x}'_{ss'}. \tag{28}$$

Solving for $1/2\,{\lambda_x}'_{ss'}$ gives

$$1/2\,{\lambda_x}'_{ss'} = \frac{1/2\,\lambda_x'\,t}{(1/2\,\lambda_x'\sqrt{1-V^2/c^2})/V} = \frac{Vt}{\sqrt{1-V^2/c^2}}. \tag{29}$$

We get the same conclusion as Eq. (22) again.

On the other hand, we assume that the propagating velocity and wavelength of ${W_z}'$

generated during the same time t' are $v_z{}'$ and $1/2\,\lambda_z{}' = v_z{}'t'$, respectively. When observing this from S, $t = t'/\sqrt{1-V^2/c^2} \fallingdotseq 2.5t'$, since the LT for time is same as above. For the velocity of $W_z{}'_s$, $v_z{}'_s$, since $v_x{}' = 0$, we have

$$v_z{}'_s = v_z{}'\sqrt{1-V^2/c^2}. \tag{30}$$

Then $1/2\,\lambda_z{}'$ observed from S, $1/2\,\lambda_z{}'_s$, is given by

$$1/2\,\lambda_z{}'_s = \left(v_z{}'\sqrt{1-V^2/c^2}\right)\frac{t'}{\sqrt{1-V^2/c^2}} = v_z{}'t' = 1/2\,\lambda_z{}'. \tag{31}$$

After all, $1/2\,\lambda_z{}'_s$ is equal to $1/2\,\lambda_z{}'$. This is natural because the distance vertical to the travelling direction is invariant with respect to the LT. Also, from expressions (12), $TI_{W_z{}'_s} = TI_{W_z{}'}$ and $TI_{W_z{}'} = TI_{W_x{}'}$. Moreover, from expressions (13), $TM_{W_z{}'_s} = TM_{W_z{}'}$ and $TM_{W_z{}'} = TM_{W_x{}'}$.

The TRE of $W_z{}'_s$, $TRE_{W_z{}'_s}$, consists of the total RKE of it, $TRKE_{W_z{}'_s}$, and the total PE of it, $TPE_{W_z{}'_s}$. Also, the $TRE_{W_z{}'_s}$ is equal to the value obtained by converting the total energy of $W_z{}'$, $TE_{W_z{}'}$, according to the LT. Let $TKE_{W_z{}'}$ and $TPE_{W_z{}'}$ be the total KE and PE of $W_z{}'$, respectively. Then $TE_{W_z{}'} = TKE_{W_z{}'} + TPE_{W_z{}'}$ and moreover $TE_{W_z{}'} = 2TKE_{W_z{}'}$ since we presuppose that $TKE_{W_z{}'} = TPE_{W_z{}'}$. Similarly, $TRE_{W_z{}'_s} = 2TRKE_{W_z{}'_s}$ because we can assume that $TRKE_{W_z{}'_s} = TPE_{W_z{}'_s}$.

Each energy supplied to generate $W_x{}'$ and $W_z{}'$ in S' is the same. The RE supplied to generate $W_x{}'_s$, $RESG_{W_x{}'_s}$, also equals to one supplied to generate $W_z{}'_s$, $RESG_{W_z{}'_s}$, because the RE supplied to generate each MTW comes from the same driving force vertical to the moving direction of the medium. On the other hand, from expressions (13), $TM_{W_x{}'_s} > TM_{W_z{}'_s} = TM_{W_z{}'} = TM_{W_x{}'}$. Furthermore, since $TM_{W_x{}'_s} \fallingdotseq 6.2TM_{W_x{}'}$, we obtain

$$TM_{W_x{}'_s} \fallingdotseq 6.2TM_{W_z{}'_s}. \tag{32}$$

Also, from expressions (11), each RMPWM in $W_x{}'_s$ with $v_y{}'_s$ is large compared to that in $W_z{}'_s$ with the STV. The former has about 6.2 times more the RM than the latter.

From these, we get the same conclusion, *i.e.*, $TRKE_{W_x{}'_s} > TRKE_{W_z{}'_s}$ as inequality (19). Moreover, the relation between the former and the latter in this case is as follows

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$$TRKE_{W_x{'}_s} \fallingdotseq 6.2TRKE_{W_z{'}_s} \tag{33}$$

although the energy supplied to generate each MTW is the same. Therefore, we obtain

$$TRKE_{W_x{'}_s} > 2TRKE_{W_z{'}_s} = TRE_{W_z{'}_s}. \tag{34}$$

Consequently, we find the following expressions:

$$TRKE_{W_x{'}_s} > RESG_{W_x{'}_s} = RESG_{W_z{'}_s} = TRE_{W_z{'}_s}. \tag{35}$$

We can conclude that, for $W_x{'}_s$, the generated RKE is larger than the RE supplied to generate it. This conclusion shows a violation of the RECL and EPR resulting from the generation of the MTWMM.

6. Conclusions

We studied the effect of the generation of MTW, $W_x{'}_s$, travelling in the ODMM on the RECL. The time on the x'-coordinate coinciding with the AEW travelling toward the rear of the MM passes faster than one on the x'-coordinate corresponding to the WS. Therefore, the AEW is positioned away from the WS compared to that in the RFR S' when the time on the WS is same for S and S'. As the result, the CI between the AEW and WS in the MM is observed to be larger than that between them in S'. This means that the TRMPWM observed from S is larger than one in S'.

In addition, we compared $TRKE_{W_x{'}_s}$ with $TRKE_{W_z{'}_s}$ propagating in the direction of the z' axis vertical to the x'- and y'-axes. We showed that $TM_{W_x{'}_s}$ is large compared to $TM_{W_z{'}_s}$ and also, for each portion with the STV, the RM in $W_x{'}_s$ is larger than that in $W_z{'}_s$. Therefore, we found that $TRKE_{W_x{'}_s}$ is larger than $TRKE_{W_z{'}_s}$ although the energy supplied to generate each MTW is the same. Furthermore, on the basis of the numerical analysis of each RE of $W_x{'}_s$ and $W_z{'}_s$, we demonstrated that $W_x{'}_s$ has more the RE than one supplied to generate it, in other words, the TRE is not necessarily conserved in S. Consequently, we proposed a violation of the RECL and EPR resulting from the generation of the MTWMM.

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