Preprints (www.preprints.org) | NOT PEER-REVIEWED | Posted: 12 October 2022 doi:10.20944/preprints202110.0082.v4

Information Theory of Gravity

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Abstract

A new model of gravity is presented here similar to the earlier work of Verlinde on Emergent Gravity using different thermodynamic assumptions. The theory does not use the main assumption of Verlinde on the nature of gravity as an entropic force using the First Law of Thermodynamics. Moreover, it does not use the Equipartition Theorem such that there is no need to define a thermal bath enclosed within a holographic screen. Instead of Equipartition Theorem, the theory uses $E = NE_p$, for the total energy of a massive object where E_p is the Planck Energy while N is the number of Planck Energy to represent the maximum possible density of information that can be extracted in any gravitating objects. The theory uses also the Holographic Principle as the basis for an information-theoretic approach to the nature of gravity. It is shown here that gravity emerges whenever there is an updating of the information within a given volume of space by the presence of matter.

1 Introduction

In Emergent Gravity(EG) theory, Verlinde conjectured in his first paper on EG [1] that ordinary surfaces are holographic screens that obey the First Law of Thermodynamics similar in Black Hole Physics [2]. He primarily used the equation, $F\Delta r = T\Delta S$, where Δr is the distance of the test particle from the holographic screen, T is the temperature in the screen and ΔS is the change in entropy S. He argued that as $\Delta r \to 0$, i.e., as the test particle touches the screen and increases the entropy, it induces gravitational force as a kind of entropic force. The test particle that enters a gravitational field is likened to a polymer molecule that enters a region where it is immersed in a thermal bath where under such conditions entropic force arises similar to what is observed at the molecular level. According to Verlinde, a test particle that enters a gravitational field is also undergoing similar conditions where the entropic force is the gravitational force. This analogy was heavily criticized and seems to be experimentally proven to be flawed [3, 4]. Wang et.al., [5], argued that horizons are indeed thermodynamic in nature but general ordinary surfaces that are considered in the Emergent Gravity program are not. In this paper, instead of using the First Law of Thermodynamics, results in the seminal work of Bekenstein and Hawking using the Second Law of Thermodynamics in Black Hole Physics will be used. One of which is the Bekenstein-Hawking area theorem, formulated in the early 1970s by Bekenstein who simply conjectured that the entropy S of a black hole is proportional to its area A, i.e., $S = \eta A$, where η is a constant of proportionality. Then Hawking was able to confirm such area-entropy relation using quantum field theory and determined the value of η. He came up with the equation $S = A/4l_p^2 = N_a/4$ where $l_p = \sqrt{\hbar G/c^3}$ is the Planck length, $N_a = A/A_p$ and A_p is the Planck area. This important result would later become the basis for what is now called as the Holographic Principle which states that the quantity N_a or the number of cells in the holographic screen is a measure of the density of information within the volume of space enclosed by the holographic screen. However, the question that Verlinde wanted to answer is how can this result be applicable to describe gravity in a non-black hole setting. The relation $A = N_a A_p$ seems to be the only central argument and the starting point in Verlinde's work. In this paper, we are also guided by the fact that the mass M of any gravitating object can be represented in units of Planck mass, $M_p = \sqrt{\hbar c/G}$, i.e., $M = NM_p$ and must also have a key role in describing gravity in a fundamental way. Hence, the main difference of our work with Verlinde's, is that the Energy Equipartition Principle, $E = \frac{1}{2}k_b NT$, will not be used here but to be replaced by the expression, $E = NE_p$, as a quantized representation of energy in terms of the Planck energy $E_p = M_p c^2$. We will also use here the relation of entropy to the mass given by the equation,

$$S = 4\pi M^2 \tag{1}$$

which is first derived by Hawking. This is a significant result in relation to how much information can be known when matter enters a black hole. In fact, it is well-established now that because of the no-hair theorem, all information inside a black hole can never be known except for its total mass, angular momentum, and its charge which are the only observable quantities to an outside observer. When a particle with mass m falls into a black hole, the entropy of the black hole increases where the increase of entropy is given by [6]:

$$\Delta S = 8\pi Mm + 4\pi m^2 \tag{2}$$

Lastly, the main objective of this paper is not to derive Newtonian Gravity as Verlinde had done in his original paper on EG, but to derive a new model of gravity similar to the approach of Modified Newtonian Dynamics (MOND) but with a physical motivation similar to Emergent Gravity.

2 Modified Newtonian Gravity

As mentioned in the previous section, in Verlinde's theory, he primarily used N_a to represent the number of bits that can occupy within the holographic screen. However, it only gives the number of fundamental units of space that the energy associated with gravity can occupy. One must also consider the amount

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of information that resides within all matters in any gravitating system. For example, in a 2-body system, such quantity can be represented by the quantity $N = \frac{M}{M_p} + \frac{m}{M_p} = N_1 + N_2$, where $N_1 = M/M_p$, and $N_2 = m/M_p$ represent the maximum possible density of information that can be stored within each gravitating matter. It is define by the mass of each gravitating object, M and m and by the Planck mass $M_p = \sqrt{\hbar c/G}$. From this physical definition of the quantity N, we aim to achieve here a purely information-theoretic approach to gravity where its magnitude will not be dependent on the amount of heat or curvature of spacetime within the gravitational field but solely on the amount of information that resides in space and matter within any gravitational system. Hence, the magnitude of gravity F should only be dependent on the value of N and N_a. The former represents the amount of information that resides in a gravitating matter and the latter, by Holographic Principle, represents the amount of information within a given volume of space that can be occupied by any amount of energy within the gravitational system. Gravity therefore would only be proportional to the information density. To quantify this idea, we consider the dimensionless form of Newton's Law of Gravity in terms of Planck scale units, i.e.,

$$N_F = \frac{F}{F_p} = \frac{\left(\frac{M_1}{M_p}\right)\left(\frac{M_2}{M_p}\right)}{r^2/l_p^2} = \frac{N_1 N_2}{N_a} \tag{3}$$

where $F_p = c^4/G$ is the Planck force. This expression, however, must be modified in cases when the magnitude of gravity is very large, either because the gravity is generated by a black hole or by a large number of gravitating bodies that collectively generate a significant gravitational effect like in the case of a galaxy. For a black hole with mass M, its initial entropy S, changes upon the introduction of a test particle with mass m. The change in entropy can be expressed by the transformation below using Eq.(2),

$$S \to S + \Delta S = 4\pi(M^2 + 2Mm + m^2) = 4\pi M_p^2 \left(\frac{M}{M_p} + \frac{m}{M_p}\right)^2 = 4\pi M_p^2 N^2 \tag{4}$$

On a galactic scale, the magnitude of gravity is also large since it involves a large number of matter within the galaxy. Hence, a similar transformation above must also be done and this can be expressed by adding an additional term in Eq.(3), i.e.,

$$N_F \to \tilde{N}_F = \frac{F_m}{F_p} = \frac{N_1 N_2}{N_a} + \frac{N_1^2 + N_2^2}{2N_a} = \frac{(N_1 + N_2)^2}{2N_a} \tag{5}$$

In terms of mass, we can rewrite Eqn.(5) to derive the modified magnitude F_m of gravity as follows,

$$F_m = G\frac{Mm}{r^2} + \frac{G}{2}\left(\frac{M^2 + m^2}{r^2}\right) \tag{6}$$

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where M represents the mass of the entire galaxy and m is the mass of the object acted upon by the gravity of the galaxy. The equation above can be simplified further as follows,

$$F_m = G\frac{Mm}{r^2}f \tag{7}$$

where $f = f\left(\frac{M}{m}\right) = 1 + \alpha$ and $\alpha = \frac{1}{2}\left(\frac{M}{m} + \frac{m}{M}\right)$. Another way to derive this result is by considering that the individual entropy of all gravitating bodies need not to be multiplied but to be added up where the sum is proportional to the square of the total mass $\mu = M + m$, which is consistent with Eq.(1) for black holes. It can be conjectured therefore that the entropy associated with matter is not an exponential entropy that can be multiplied but an entropy of information-bearing states that adds up. Such kind of entropy obeys an extended form of Landauer's Principle as suggested in the recent work of Vopson [7] which shows that information has corresponding mass or energy.

The modification of Newton's law of gravity given by Eqn.(7) is not something new. Similar models also exist that try to modify Newton's Law of Gravity by adding additional terms. However, most of the modifications were done arbitrarily with the aim of fitting the model to the observed data and reconciling it with the dark matter hypothesis. In [8], different types of such models were discussed. This non-Newtonian law of gravity approaches, according to [8], "Although... an old idea that could appear rudimentary...and it is mostly abandoned in modern literature, we think that a reconsideration of this approach could motivate further research in the area of modified gravity theories." On the other hand, the most commonly used approach in introducing a new theory of gravity nowadays is to generalize the Einstein-Hilbert action, $S = \int \sqrt{-g}Rd^4x$, by imposing additional parameters into the action, such as scalar, vector, tensor and spinor fields for the purpose of making the action conformally invariant and to produce field equations that might explain the dark energy and dark matter problems. One of the well-known examples of this, is the Tensor–Vector–Scalar (TeVeS) gravity theory by Bekenstein [9] as a relativistic generalization of MOND paradigm of Milgrom[10]. Although energy and information are related as suggested by Landauer's Principle, the Lagrangian method of modifying our conventional description of gravity will not be used here since the model presented here will focus more on the relation of gravity with information density rather than on the energy density of a gravitational system.

3 Mass Correction and Tully-Fisher Relation

The new model of gravity presented here would lead to a small correction to the mass of a test object with a motion that is under the influence of gravity. The derivation of the correction term will be used later to derive the Tully-fisher Relation. For simplicity, we consider a test object with mass m, that has a circular orbit around the center of the source of gravity with mass M.

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We note of the fact that any object that is under the influence of gravity will react by accelerating and experiencing a force that is a fictitious one, known as the centrifugal force. It has a magnitude that is equal to the magnitude of the centripetal force F_c which is usually associated with the source of gravity. The magnitude is given by $F_c = mv^2/r$, where r is the distance from the source of gravity and v is the rotational velocity of the test object. Equating this to Eqn.(7) we yield, $M \approx \frac{rv^2}{G}\gamma$, where $\gamma = f^{-1}$. For the case of a binary system where $M \approx m$, $\gamma \approx \frac{1}{2}$ and $M \approx \frac{1}{2}\frac{rv^2}{G}$. For $M >> m$, $\gamma \approx (1 + \frac{M}{2m})^{-1} = (\frac{2m+M}{2m})^{-1} \approx (\frac{1}{2}\frac{M}{m})^{-1}$ which gives us $M \approx \frac{rv^2}{G}\frac{2m}{M}$. For both cases, the correction term is too small that the model approximates the results of known classic theories of gravity. The correction term γ can be computed by getting the mass ratio without necessarily measuring the individual masses and dividing it. There are already known methods that can be used to measure the mass ratio in a two-body system. For non-luminous objects in a 2-body system, the distances R_1 and R_2 from the barycenter can be used. Since each force felt by both bodies acts only along the line joining the centers of the masses and both bodies must complete one orbit in the same period, the centripetal forces can be equated using Newton's 3rd law, such that we can have the relation $\frac{M}{m} = \frac{R_1}{R_2}$. On the other hand, to get the mass ratio of distant luminous objects in a two-body system like in a binary star, one can use an approximation via the mass-luminosity relationship, $\frac{M}{m} \approx \left(\frac{L_M}{L_m}\right)^{\frac{1}{3.9}}$, which applies for main sequence stars. For galaxy with mass M and a star that revolves around it with mass $m_\odot$ equal to one solar mass, we can use the work of Vale et.al.[11] that relates the luminosity and mass of the galaxy via a double power law equation. It will give us the relation $\frac{M}{m_\odot} \approx (\frac{L}{L_\odot})^{\frac{1}{b}}$, where L is the observed luminosity of the galaxy and the range $0.28 \leq b \leq 4$, is for galaxies with galactic halo mass that ranges form high-mass end to low-mass end. If one is to get the square of the rotational velocity v of the revolving star, the mass-luminosity relation will yield us, $v^2 = \frac{GM}{2r}\frac{M}{m_\odot} \approx \frac{GM}{2r}\left(\frac{L}{L_\odot}\right)^{\frac{1}{b}}$, since $f \approx \frac{1}{2}\frac{M}{m}$, for $M >> m$. This implies that $L \sim v^{2b}$ which for the average, $b \approx 2$, the equation give us the Tully-Fisher relation $L \sim v^4$.

4 On External Field Effect and Dark Matter

It should be realized that any local measurement of the magnitude of gravity is not absolute. It will always depend on the external gravity of other masses. This is known in MOND theories as the External Field Effect (EFE)[12]. Although MOND theories were the first to suggest the existence of EFE, as far as we know, it was never translated into concrete mathematical terms. In this section, we wanted to express EFE, mathematically, based on the results from the previous section. For large-scale gravity which involves a larger group of gravitational sources we now have, $N = N_1 + N_2 + \ldots + N_k$, corresponding to the total mass , $\mu = M_1 + M_2 + \ldots + M_k$, for k number of gravitating objects. Squaring N, we have, $N^2 = (N_1)N_1 + (N_1+N_2)N_2 + \ldots (N_1+N_2+\ldots+N_k)N_k$. Distributing and rearranging terms, we can have a more compact expression using the summation symbol, i.e., $N^2 = \sum_{i<j}^{k} N_i N_j + \sum_i^k N_i^2$, which can be expanded as follows:

$$N^2 = \left(\sum_{i<j}^{k-1} N_i N_j + N_k \sum_{i}^{k-1} N_i\right) + \left(\sum_{i}^{k-1} N_i^2 + N_k^2\right) \tag{8}$$

By using Eqn. (3) and the convention $G = \hbar = c = 1$, the magnitude of the gravity of a galaxy, F_G, acting on kth star, would be

$$F_G = \left(\frac{N_k \sum_{i}^{k-1} N_i}{r_{cg}^2}\right) + \left(\frac{\sum_{i<j}^{k-1} N_i N_j}{r_{cg}^2}\right) + \frac{1}{2}\left(\frac{\sum_{i}^{k-1} N_i^2}{N_l^2}\right) + \frac{1}{2}\left(\frac{N_k^2}{N_l^2}\right) = F_{NG} + F_{HG} \tag{9}$$

where $N_l = r_{cg}/L_p$ and r_{cg} is the distance of separation of the kth object from the center of gravity of all other stars within tha galaxy. The number of stars with gravity acting on the kth star is given by $k - 1$. The "stars" mentioned here include black holes which usually have a greater contribution to the overall gravity of the galaxy than the usual stars. Contributions of planets, asteroids, and other matter in the overall gravity of a galaxy would probably be just equal to the gravity of one or two black holes. The first term in Eqn. (9) can be associated with Newtonian Gravity (F_{NG}) while the last three terms are with the "Hidden Gravity" (F_{HG}). It should be noticed that F_{HG} is exceedingly larger than F_{NG} which means that the influence of the gravity of the galaxy extends not just on stars at the edge of the visible galaxy but beyond it, even up to the edge of the galactic halo that surrounds the visible part of the galaxy. If one is to understand this result from the perspective of Newtonian or Einsteinian theory of gravity, such an amount of gravity would be associated with a mass greater than the visible mass within the galactic halo. Such is the very reason why the so-called Dark Matter was hypothesized by thinking within such a paradigm. The observed flat rotation curve of galaxies is therefore explained here not by an unobservable additional matter within the galaxy halo but by the excess gravity that was not accounted for when one is using the classical theory of gravity of Newton or Einstein.

5 On Mach's Principle

Another advantage of Eqn.(9) is that, it can also be used at the cosmological scale not just at the galactic scale. Instead of considering all the stars within a galaxy, we can consider all the galaxies within the Universe. The simplicity of the mathematical formalism of the new theory presented here should not be considered as its weakness but in fact its advantage. It removes any mathematical problem like the existence of singularities that may result to various interpretations. Furthermore, although it is not as sophisticated as the mathematical formalism of Einstein's theory, the most important aspect of the mathematical formalism of the theory is it naturally includes Mach's Principle which is not apparent in the mathematics of General Relativity (GR). Historically, according to Pais [13], Einstein strongly believed that to have a "satisfactory theory of gravity", Mach's Principle must be included along with the Principle of General Covariance and Principle of Equivalence. In GR, the latter two were incorporated mathematically but not Mach's Principle. In our theory, when $\sum N_i = 0$, i.e., the total density of all possible information that can be contained within all matter in the Universe is zero, gravity is totally non-existent. This must also be true for inertia if one is to uphold the Principle of Equivalence. In comparison with GR, its field equations allow for matter-free solutions which seems to suggest that it is incompatible with Mach's Principle. The theory presented here incorporates Mach's Principle where the interpretation of Mach's Principle that we used here is aligned with the interpretation of de Sitter [14]. Although de Sitter's interpretation is just one out of many possible interpretations of Mach's Principle according to Bondi [15], the fact remains that the theory of gravity presented here incorporates Mach's Principle and must be considered as its advantage over other theories of gravity. Lastly, the Principle of General Covariance will not be violated by the theory since one can use the Planck scale quantities in the formulation of physical laws where the measurements of those quantities by observers in different frames of reference can be correlated without ambiguity. The quantities like the Planck length and Planck mass can be considered as invariant quantities which are even more fundamental than other known constants in Nature, like the speed of light, since their values will not be varying at the Planck scale.

6 Conclusions and Recommendations

We presented an emergent theory of gravity that incorporates Mach's Principle and lead to results similar to a modified Newtonian Gravity. Gravity is not described by the amount of curvature of spacetime (*à la* Einstein) or as a force that emerges in a thermal bath (*à la* Verlinde), but by the density of information that can be contained within a gravitational system. Also, the theory neither introduced a new baryonic particle as suggested by the Dark Matter hypothesis nor introduced a new field as suggested by MOND theories. It modifies Newtonian gravity by using the fundamental role of information in the description of gravity.

7 Acknowledgment

I would like to thank the Board of Trustees and Administration of New Era University and my wife for all the support during this time of the pandemic.

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