Class 1 heating cycles: A new class of thermodynamic cycles

Hong-Rui Li[1, *] and Hua-Yu Li[2, *]

[1]Key Laboratory of Efficient Utilization of Low and Medium Grade Energy (Ministry of Education), Tianjin University, Tianjin 300350, China.

[2]Department of Energy and Power engineering, China University of Petroleum (East China), Qingdao, Shandong 266580, China.

[*]Corresponding authors: lhr@tju.edu.cn (H.-R.L.); lihy1963@upc.edu.cn (H.-Y.L.)

ORCID: 0000-0003-0249-7260 (H.-R.L.); 0000-0002-8229-019X (H.-Y.L.)

Supplementary Information

Figure S1 shows three typical examples of class 1 heating cycles (HC-1s), which have been presented in the main text. Without loss of generality, they are all in basic form A (HC-1A). Here we derive their coefficients of performance (COPs, the ratio of the cycle's output to input) for medium-temperature heating or low-temperature cooling when the cycle's net work output $W_{\mathrm{net}} = 0$. The former two cycles are regarded as internally reversible [1].

1. The HC-1A with isothermal heat transfer processes (Figure S1a)

According to the law of conservation of energy, we have

$$W_{\mathrm{net}} = Q_{\mathrm{in,\,H}} + Q_{\mathrm{in,\,L}} - Q_{\mathrm{out}} = 0 \tag{S1}$$

where $Q_{\mathrm{in,\,H}}$ is the amount of heat absorbed from the high-temperature heat source by the cycle, $Q_{\mathrm{in,\,L}}$ is the amount of heat absorbed from the low-temperature heat source by the cycle, and Q_{out} is the amount of heat rejected to the medium-temperature heat sink by the cycle.

The cycle's net entropy change should be zero. Thus,

$$\oint dS = \frac{Q_{in,H}}{T_H} + \frac{Q_{in,L}}{T_L} - \frac{Q_{out}}{T_M} = 0 \tag{S2}$$

Combining Eq. (S1) and Eq. (S2), we can express the COPs as

$$COP_H = \frac{Q_{out}}{Q_{in,H}} = \frac{T_M(T_H - T_L)}{T_H(T_M - T_L)} \tag{S3}$$

$$COP_C = \frac{Q_{in,L}}{Q_{in,H}} = \frac{T_L(T_H - T_M)}{T_H(T_M - T_L)} \tag{S4}$$

where COP_H and COP_C are the cycle's COPs for medium-temperature heating and low-temperature cooling respectively.

2. The HC-1A with isobaric heat transfer processes, employing an ideal gas as its working medium (Figure S1b)

Notice that state point 7 follows the rules of both heat rejection and pressurization. Dividing the medium-temperature heat Q_{out} into two parts, and regarding the ideal gas's isobaric specific heat c_p as constant [1], we obtain

$$W_{net} = Q_{in,H} + Q_{in,L} - Q_{out} = Q_{in,H} + Q_{in,L} - \left(Q_{out}^{(1)} + Q_{out}^{(2)}\right) = 0 \tag{S5}$$

$$\begin{aligned} Q_{in,H} &= H_3 - H_2 = mc_p(T_3 - T_2) \\ Q_{in,L} &= H_1 - H_6 = mc_p(T_1 - T_6) \\ Q_{out}^{(1)} &= H_4 - H_7 = mc_p(T_4 - T_7) \\ Q_{out}^{(2)} &= H_7 - H_5 = mc_p(T_7 - T_5) \end{aligned} \tag{S6}$$

where H is the working medium's enthalpy at each state point, m is the working medium's mass, and T is the working medium's thermodynamic temperature at each state point.

According to the behavior of the ideal gas [1] and Eq. (S6), we have

$$\left(\frac{p_{\mathrm{H}}}{p_{\mathrm{M}}}\right)^{\frac{k-1}{k}}=\frac{T_3}{T_4}=\frac{T_2}{T_7}=\frac{T_3-T_2}{T_4-T_7}=\frac{Q_{\mathrm{in,\,H}}}{Q_{\mathrm{out}}^{(1)}} \tag{S7}$$

$$\left(\frac{p_{\mathrm{M}}}{p_{\mathrm{L}}}\right)^{\frac{k-1}{k}}=\frac{T_7}{T_1}=\frac{T_5}{T_6}=\frac{T_7-T_5}{T_1-T_6}=\frac{Q_{\mathrm{out}}^{(2)}}{Q_{\mathrm{in,\,L}}} \tag{S8}$$

where k is the ideal gas's specific heat ratio.

Combining Eq. (S5), Eq. (S7) and Eq. (S8), we can express the COPs as

$$\mathrm{COP_H}=\frac{Q_{\mathrm{out}}}{Q_{\mathrm{in,\,H}}}=\frac{Q_{\mathrm{out}}^{(1)}+Q_{\mathrm{out}}^{(2)}}{Q_{\mathrm{in,\,H}}}=\frac{p_{\mathrm{M}}^{\frac{k-1}{k}}\left(p_{\mathrm{H}}^{\frac{k-1}{k}}-p_{\mathrm{L}}^{\frac{k-1}{k}}\right)}{p_{\mathrm{H}}^{\frac{k-1}{k}}\left(p_{\mathrm{M}}^{\frac{k-1}{k}}-p_{\mathrm{L}}^{\frac{k-1}{k}}\right)} \tag{S9}$$

$$\mathrm{COP_C}=\frac{Q_{\mathrm{in,\,L}}}{Q_{\mathrm{in,\,H}}}=\frac{p_{\mathrm{L}}^{\frac{k-1}{k}}\left(p_{\mathrm{H}}^{\frac{k-1}{k}}-p_{\mathrm{M}}^{\frac{k-1}{k}}\right)}{p_{\mathrm{H}}^{\frac{k-1}{k}}\left(p_{\mathrm{M}}^{\frac{k-1}{k}}-p_{\mathrm{L}}^{\frac{k-1}{k}}\right)} \tag{S10}$$

or

$$\mathrm{COP_H}=\frac{Q_{\mathrm{out}}}{Q_{\mathrm{in,\,H}}}=\frac{T_7\left(T_2-T_1\right)}{T_2\left(T_7-T_1\right)}=\frac{T_3T_5-T_4T_6}{T_3\left(T_5-T_6\right)} \tag{S11}$$

$$\mathrm{COP_C}=\frac{Q_{\mathrm{in,\,L}}}{Q_{\mathrm{in,\,H}}}=\frac{T_1\left(T_2-T_7\right)}{T_2\left(T_7-T_1\right)}=\frac{T_6\left(T_3-T_4\right)}{T_3\left(T_5-T_6\right)} \tag{S12}$$

3. The HC-1A with isobaric heat transfer processes, employing a phase-change working medium (Figure S1c)

$$\mathrm{COP_H} = \frac{Q_{\mathrm{out}}}{Q_{\mathrm{in,\,H}}} = \frac{H_4 - H_5}{H_3 - H_2} = \frac{h_4 - h_5}{h_3 - h_2} \tag{S13}$$

$$\mathrm{COP_C} = \frac{Q_{\mathrm{in,\,L}}}{Q_{\mathrm{in,\,H}}} = \frac{H_1 - H_6}{H_3 - H_2} = \frac{h_1 - h_6}{h_3 - h_2} \tag{S14}$$

where h is the working medium's enthalpy per unit of mass at each state point. These two formulae cannot be further simplified because the behavior of the phase-change working medium is much more complex than that of the ideal gas.

When $W_{\mathrm{net}} \neq 0$, we can also obtain the cycles' COPs in a similar way. However, since heat and power differ in grade, the meanings of such formulae are not clear.

Figures

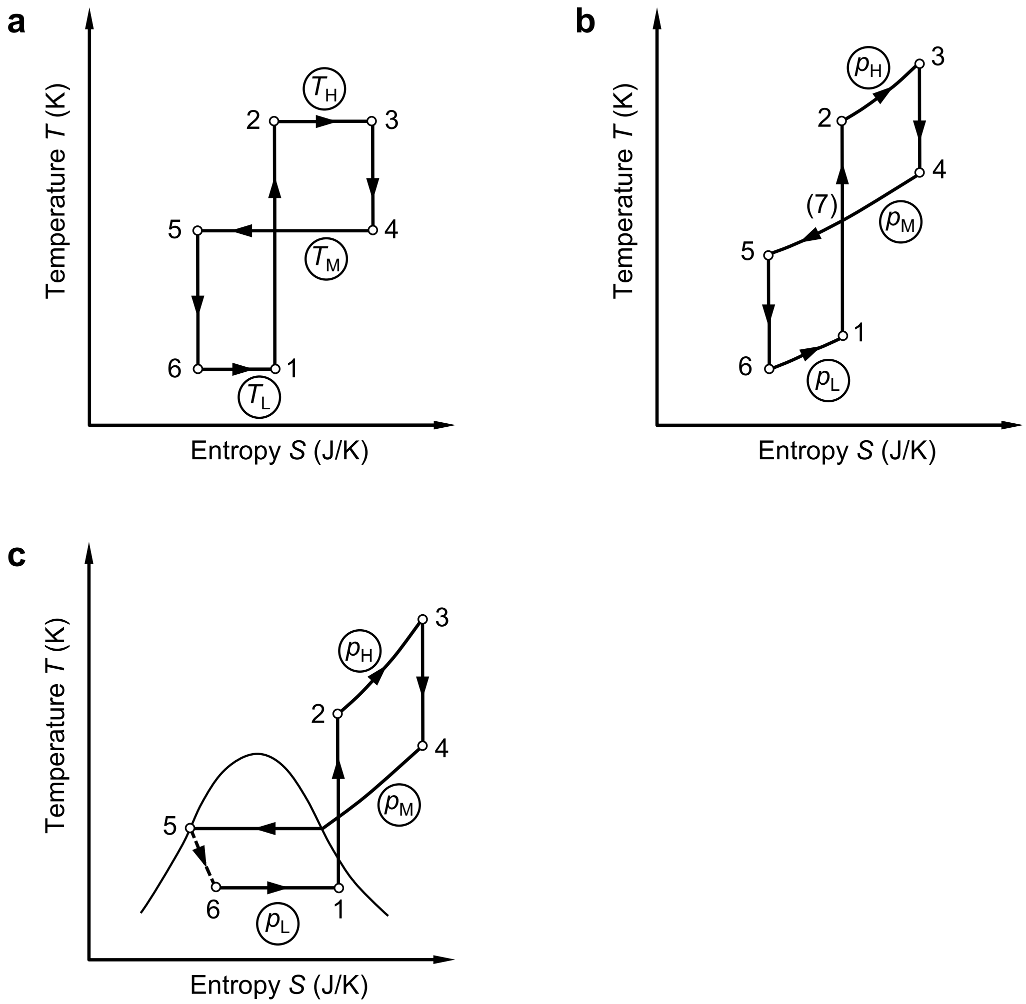

Figure S1. Three typical examples of HC-1s. (a) An HC-1A with isothermal heat transfer processes. T_H, T_M and T_L are the working medium's thermodynamic temperatures during high-temperature heat absorption, medium-temperature heat rejection, and low-temperature heat absorption, respectively. **(b)** An HC-1A with isobaric heat transfer processes, employing an ideal gas as its working medium. State point 7 is the state passed through by both process 1-2 and process 4-5. **(c)** An HC-1A with isobaric heat transfer processes, employing a phase-change working medium. The depressurization (throttling) process 5-6 is internally irreversible and thus expressed as a dotted line. In (b) and (c), p_H, p_M and p_L are the working medium's pressures during high-temperature heat absorption, medium-temperature heat rejection, and low-temperature heat absorption, respectively.

References

[1] Çengel YA, Boles MA, Kanoğlu M. Thermodynamics: An engineering approach. 9 ed. New York: McGraw-Hill Education, 2019.