The Complex Universe

Christopher A. Laforet

claforet@gmail.com

Windsor, ON, Canada

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In this paper, it is proposed that the correct metric for relativistic cosmology is one which has not only spatial curvature, but time curvature as well, and that it is the curvature of the time dimension that is the source of the accelerated expansion. It is argued that the FRW metric, whose time dimension is uncurved, is effectively a Newtonian approximation to the true cosmological metric and that the internal Schwarzschild metric is the true cosmological metric describing the 3D space of the Universe falling through the time dimension. The unknowns in the internal Schwarzschild metric are solved for using cosmological data and it is shown that the predictions it gives match observations without the need for a cosmological constant. The entire Schwarzschild metric in Kruskal-Sezekeres coordinates is examined and we see that it describes two CPT symmetric Universes moving in opposite directions in the time dimension. One Universe contains matter while the other contains antimatter. In section VIII, we discuss how the internal Schwarzschild metric can be understood as being made of imaginary time and space dimensions and these imaginary dimensions scale the real dimensions of the Universe. The Universe can be thought of as the imaginary counterpart of a galaxy with swapped space and time-like dimensions. The singularity is the point in time where the geodesics reverse their direction in time and begin to re-collapse toward each other. The matter and antimatter Universes annihilate with each other when they collide at the end of collapse, ultimately decaying into two new matter and antimatter Universes. The model also predicts that telescopes such as the JWST should find structures in the early Universe that are much older than expected or predicted by the current ΛCDM model.

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I. INTRODUCTION AND MOTIVATION

The current model of cosmology is based on the FRW metric, which, under the flat space assumption, is essentially the Minkowski metric in spherical coordinates whose space-like dimensions are scaled by a time-dependant scale factor. What is notable here is that for a Universe with a non-accelerating expansion, the FRW model makes the same predictions as a sperically symmetric cosmological model based on Newtonian gravity. But the expansion of the Universe is now known to be accelerating. To accommodate this acceleration, the cosmological constant is introduced into the field equations to effectively give empty space a pressure that creates an accelerated expansion. The problem with the cosmological constant is that it is just a measured number whose value is heretofore unpredictable via any currently existing theory, making the underlying cause of the accelerated expansion a mystery.

Another notable feature of the FRW metric is that it models the Universe as a continuous fluid and this fluid curves space via the scale factor, but leaves the time dimension uncurved. This is curious because we know that for a finite distribution of matter/energy, both space and time are curved, yet the FRW metric seems to suggest that the infinite matter and energy of the Universe has no effect on the curvature of the time dimension.

If we consider the gravitational field around a spherically symmetric mass, the external Schwarzschild metric provides the complete description of the gravitational field around this mass. The Newtonian potential is an approximation to the external Schwarzschild metric in the limit where objects are moving slowly and the curvature of space and time are small. This Newtonian approximation is found in General Relativity by making small perturbations to the Minkowski metric to get an approximated Schwarzschild solution.

It is proposed here that just as the Newtonian potential around a spherically symmetric body is an approximation to the external Schwarzschild metric, the FRW metric too is effectively a Newtonian approximation of cosmology, applicable over only very short durations over which the change in scale factor is negligible. As mentioned above, there is no reason to expect that the matter and energy of the Universe should leave cosmological time uncurved, and so if we expect that cosmological time should be curved by the Universe, then we must seek another metric that can account for that curvature. The cosmological constant in the Friedman equations effectively provides an additional constant amount of curvature that compensates for the lack of time curvature in the metric, but if the true cosmology has time curvature, then this constant would not adequately correct for the lack of curvature over the entire spacetime.

It will be argued in this paper that the metric properly describing both the space and time curvature of the Universe is the *internal* Schwarzschild metric. This metric is a spherically symmetric vacuum solution. But if we think of the external Schwarzschild solution, it also describes the worldlines of the particles on an infinitely thin shell collapsing toward a center in space. For the internal metric, we can imagine that the matter and energy in the Universe is isotropically distributed throughout infinite space (3D space in this case), but exists only at the present time (time is the radius in this case). As will be discussed, if we accept this assumption (motivation to support this assumption is discussed in later sections) and that the Universe is spherically symmetric, then according to Birkhoff's theorem, the internal metric is the *only* possible cosmological metric because the Schwarzschild solution is the only spherically symmetric vacuum solution in General Relativity.

In sections VIII and IX, the internal metric is interpreted as having complex space and time dimensions where the entire Universe behaves like a spherically symmetric distribution of matter exploding out of a horizon, reaching a maximum height, and then re-collapsing toward the horizon. The difference is that for the Universe, the expansion and collapse are happening through cosmological time rather than space. The Universe is the imaginary counterpart of a galaxy where the Big Bang horizon is the imaginary counterpart of the event horizon at the center of the galaxy. The imaginary radial dimension of the internal metric, which is timelike, effectively scales the space of the 3D Universe over time in the same way that the external metric would scale the space of a 2D shell of matter expanding or collapsing around its gravitational center in space.

We will begin the argument by examining the Schwarzschild metric in detail.

II.   THE SCHWARZSCHILD METRIC

The Schwarzschild metric is the simplest non-trivial solution to Einstein's field equations. It is a vacuum solution for the spacetime around a spherically-symmetric distribution of energy. The general form of the metric can be expressed as:

$$d\tau^2 = -\frac{u-r}{r}dt^2 + \frac{r}{u-r}dr^2 - r^2 d\Omega^2 \qquad (1)$$

Depending on the ratio $\frac{u}{r}$, we get three distinct descriptions of spacetime:

1. $u = 0$: This gives us the flat Minkowski metric of Special Relativity.

2. $\frac{u}{r} < 1$: This describes the metric for an eternally spherically-symmetric vacuum centered in space. This metric is also used to describe the vacuum outside a spherically symmetric object occupying a finite amount of space (like a star or planet).

3. $\frac{u}{r} \geq 1$: This describes the metric for a spherically symmetric vacuum centered on a point in time. Analogous to the second case, this metric should also describe a vacuum of time outside a spherically-symmetric object spanning infinite space. The center of the metric is everywhere in space, but at a single point in time (just like one could say that the vacuum described in the second case is centered at all times on a single point in space).

An important observation is that the internal metric describes a vacuum solution to the field equations. But the Universe is clearly filled with energy, so how can this solution apply? In order to satisfy the requirements of the metric, the Universe must be "a spherically-symmetric energy distribution occupying an infinite amount of space for a finite amount of time". For this metric to be a cosmological description, it must be that Universe only truly exists in the present and in a very real sense moves into the future. The surrounding vacuum is the future, and the Universe is freefalling through time toward the temporal center of the metric. The vacuum will be discussed further in section V.

Time being the radial dimension of the metric combined with the fact that the solution is a vacuum solution gives a mathematical justification for our intuitive notions of past, present, and future. The in-homogeneity along the radial direction gives us an arrow of time that distinguishes the 'past' and 'future' analogous to the way the external solution gives us an absolute distinction between 'up' and 'down'. And the vacuum as described above gives us a boundary between them, that boundary being the 'present' time, when the matter/energy of the Universe is actually positioned in the spacetime.

Observation has shown that the Universe is:

- Spherically Symmetric

- Homogeneous in space

- In-homogeneous across time

We will also make one further assumption in this paper:

- The Universe only ever occupies a single instant of Cosmic time and moves from one moment of cosmic time to the next where the time measured by observers between cosmic times depends on their respective motions.

Relativity of simultaneity does not prohibit the idea of the energy existing at a specific Cosmological time because of the nature of the metric. In Cosmology, we can determine absolute motion and absolute simultaneity because we have the Cosmic Microwave Background. For example, consider two events that are causally disconnected. If observers at each event see the CMB temperature to be uniform in all directions (the observers are co-moving), then if both observers measure the CMB to have the same temperature at both events, then we

know the events are absolutely simultaneous, even if a third observer in motion sees them as non-simultaneous. Any observer in motion through space, inertial or otherwise, will see a dipole on the CMB, and that dipole will provide all the info about the state of motion of the observer. Therefore, we can define past, present, future, and motion in an absolute sense. To put it another way, the fact that cosmological time is finite into both the past and future allows us to specify the distance of any event from either the beginning or end of time absolutely.

Let us call events the same distance away from us in time celestial spheres. We can classify these spheres into three types:

1. **Dynamic Spheres** – These are the spheres that galaxies reside on. Objects on these spheres maintain a constant coordinate distance from us and move forward in time. We are able to move toward or away from objects on these spheres by moving through space. If we fix our sights on a particular galaxy, the light we see from that galaxy is being emitted later in time as we ourselves move through time.

2. **Static Spheres** – These are spheres fixed in time. The Cosmic Microwave Background is the most obvious example of these spheres. Light from the CMB sphere is always emitted from the same cosmological time, but as we ourselves move through time, we see light from that time emitted from farther and farther away from us in space, giving the impression that the CMB sphere is growing. We cannot move toward or away from any objects on this sphere because it is frozen in time.

3. **The Dark Sphere** – The Dark Sphere is the Big Bang and lies beyond the CMB. It is in principle unobservable for two reasons. First, the CMB is opaque so that any light from the Big Bang cannot penetrate it. Second, even if the CMB was not blocking our view, any light from that sphere would be infinitely redshifted in the frame of all future observers since the scale factor on that sphere is zero.

These spheres are shown in terms of the internal Schwarzschild metric in Figure 1. Figure 1 shows the Schwarzschild coordinates of the internal metric plotted on the Kruskal-Szekeres coordinate plane [1]. In these coordinates, space is the 't' coordinate emanating from the center of the diagram (Big Bang) and time is the 'r' coordinate depicted as hyperbolas (time is flowing forward as r goes toward zero). The upper right quadrant of this diagram represents a single fixed direction ($\theta = const$, $\phi = const$). So each bold line representing a sphere would be a point on each sphere over time. Note that light on this diagram travels on 45-degree lines.
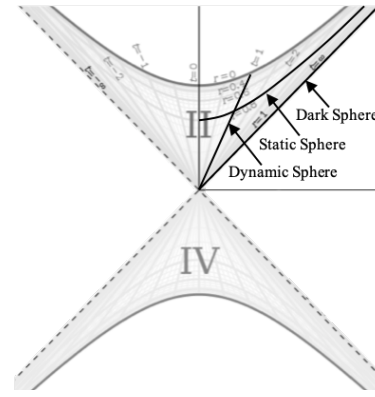

FIG. 1. Celestial Sphere Types on Kruskal-Szekeres Coordinate Chart

It is also notable from the metric that even though r is a timelike coordinate in this case, the $rd\Omega$ term is still spacelike, so objects on the celestial spheres at constant r are spacelike separated, which is what we expect.

III. THE SCALE FACTOR

Expressions for the proper time interval along lines of constant t and Ω and the proper distance interval along hyperbolas of constant r and Ω from Equation 1 are:

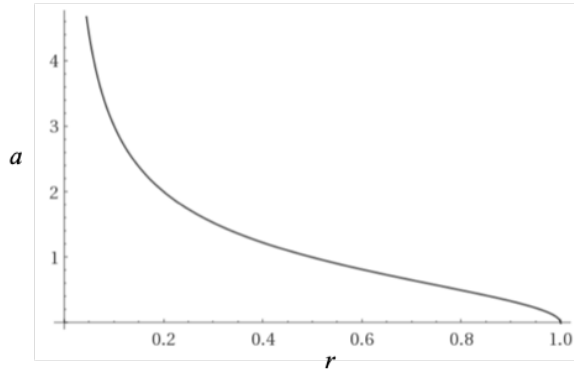$$\frac{ds}{dt} = \pm\sqrt{\frac{u-r}{r}} = \pm a \qquad (2)$$

$$\frac{d\tau}{dr} = \pm\sqrt{\frac{r}{u-r}} = \pm\frac{1}{a} \qquad (3)$$

And the coordinate speed of light is given by:

$$\left(\frac{dt}{dr}\right)_{light} = \pm\frac{r}{u-r} = \pm\frac{1}{a^2} \qquad (4)$$

Where a is the scale factor. First we should notice that none of the three equations depend on the t coordinate. This is good because the t coordinate marks the position of other galaxies relative to ours. Since all galaxies are freefalling in time inertially, the particular position of any one galaxy should not matter. The proper temporal velocity, proper distance, and coordinate speed of light only depend on the cosmological time r.

A plot of the scale factor vs. r (with $u = 1$) is given in Figure 2 below:

FIG. 2. Scale Factor vs. r for $u = 1$

IV. THE CO-MOVING OBSERVER

Let us take the center of our galaxy as the origin of an inertial reference frame. We can draw a line through the center of the reference frame that extends infinitely in both directions radially outward. This line will correspond to fixed angular coordinates (θ, ϕ). There are infinitely many such lines, but since we have an isotropic, spherically symmetric Universe, we only need to analyze this model along one of these lines, and the result will be the same for any line.

The radial distance in this frame is kind of a compound dimension. It is a distance in space as well as a distance in time. The farther away a galaxy is from us, the farther back in time the light we currently receive from it was emitted. Fortunately the $u/r \geq 1$ spacetime of the Schwarzschild solution plotted in Kruskal-Szekeres coordinates provides us with a method to understand this radial direction. Figure 1 showed the $u/r \geq 1$ solution on a Kruskal-Szekeres coordinate chart where, in this model, the hyperbolas of constant r represent spacelike slices of constant cosmological time and the rays of t represent spatial distances. We will focus on the upper half where the half represents an observer pointed in a particular direction and the positive t's represent the coordinate distance from the observer in that particular direction while the negative t's represent coordinate distance in the opposite direction.

We must determine the paths of co-moving observers $(dt = d\Omega = 0)$ in the spacetime. For this we need the geodesic equations for the internal Schwarzschild metric [2] given in Equation 1. In these equations u represents a time constant (in Figure 1, the value of u is 1). The following equations are the geodesic equations for t and r $(0 \leq r \leq u)$ for $d\Omega = 0$:

$$\frac{d^2t}{d\tau^2} = \frac{u}{r(u-r)}\frac{dr}{d\tau}\frac{dt}{d\tau} = \frac{a^2+1}{a^2 r}\frac{dr}{d\tau}\frac{dt}{d\tau} \quad (5)$$

$$\frac{d^2r}{d\tau^2} = \frac{u}{2r^2} = \frac{a^2+1}{2r} \quad (6)$$

Looking at points $0 < r < u$, then by inspection of Equation 5 it is clear that an inertial observer at rest at t will remain at rest at t ($\frac{d^2t}{d\tau^2} = 0$ if $\frac{dt}{d\tau} = 0$). Also, we see that if an observer is moving inertially with some initial $\frac{dt}{d\tau}$, then if $\frac{dr}{d\tau} < 0$, the coordinate speed of the observer will be reduced over time (the coordinates are expanding beneath her) and if $\frac{dr}{d\tau} > 0$, the coordinate speed will be increased over time (the coordinates are collapsing beneath her).

V. THE VACUUM SOLUTION

As has been mentioned, in order for the Schwarzschild solution to represent the Cosmological spacetime, it is required that we do not exist in a block Universe, but rather one in which only matter and energy in the present truly exist. As mentioned in section II, relativity of simultaneity does not pose a problem here because absolute time and motion can be defined due to the fact that we can use the Cosmic Microwave Background mono-pole temperature to determine an exact time across the Universe. In other words, non-local clocks can be synchronized by all observers starting their clocks when the CMB reaches an agreed upon temperature. Likewise, if we are moving in any particular direction, the dipole temperatures of the CMB provides a way to determine absolute motion. We can think of each direction in the spherical spacetime as having a timelike and a spacelike character. When we move in a particular direction, we change our position in space, but our clocks are also dilated relative to co-moving observers. So the time dilation effectively means that the time an observer measures locally between increments of the CMB temperature is less than that of a co-moving observer, but all observers truly only exist at the same cosmological time.

The fact that there must be a vacuum in time follows from the symmetry of space and time in relativity. If a vacuum can exist in space, then there must also be a vacuum in time. In Figure 3 below, there are four points all representing the same observer (two in the external metric, two in the internal metric):
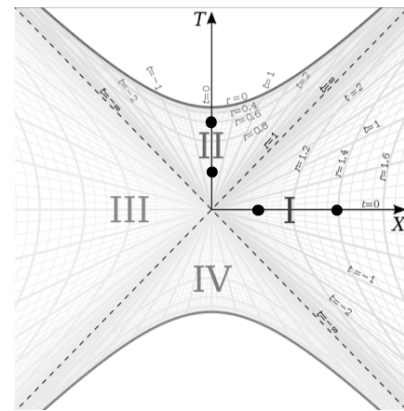

FIG. 3. Vacuum Coordinate Symmetry

Looking first at the external metric, an observer cannot exist at both of those points on the X axis. We know this because that would imply the observer is at both locations at the same time, which is not possible. And because of the symmetry of space and time in relativity, we can say the same is true for the internal metric where we have two points along the T axis. The argument here is that for the external case, we say that the observer can either be at one point or the other, but not both. In the internal case we simply enforce the same rule, that the observer can either be at one point or the other, but not both (the observer can move from one point to the other, but when it moves to the second point, it no longer exists at the first point). So in the external method, we're saying the observer cannot be at different locations at the same time, whereas for the internal metric, by symmetry, the observer cannot be at the same location at two different times. To put it generally, we say that an observer is at one place and time, *and then* at a different place and time where the "and then" is in reference to the observer's proper time.

This implies that we move through time just like we move through space. An observer does not exist at all parts of its worldline, only at the present. We know that everything in the Universe started together at the same point in time in the past and has been moving together through time since then. Different observers will disagree on how much time has elapsed due to the time dilation of the local gravitational fields and peculiar motion, but everything is falling together in the time dimension. As we will show, using the internal Schwarzschild solution as a model for Cosmology implies that while the universe is currently in the expansion phase, it will later recollapse over the same space and time coordinates. Thus, if this is the correct cosmological soultion, the matter and energy cannot exist at all times together because that would mean the Universe would collide with past events during the collapse.

Furthermore, if we accept that time is a vacuum and that the Universe is spherically symmetric, then the internal Schwarschild solution is **the only possible cosmological solution**. This is because Birckhoff's theorem states that the Schwarzschild metric is the only spherically symmetric vacuum solution in General Relativity and so that is the only spacetime a spherically symmetric Universe can fall in. It is notable, however, that the mathematics of General Relativity does not allow us to solve for a geometry where there is a vacuum in time, but not space. So what then is the meaning of all the non-vacuum solutions to GR? These must all be approximations that are only applicable for times over which the change in the curvature of the time dimension is negligible. These solutions do not account for the underlying cosmological changes to the spacetime over time. This includes the FRW metric, which, as has been discussed, cannot account for the accelerated expansion without the need for a mysterious 'Dark Energy' that is accounted for in the internal Schwarzschild metric by curvature of the time dimension.

In order to account for cosmology in non-vacuum metrics, one may need to reformulate the theory of General Relativity with complex numbers where the complex counterparts to the spacelike dimensions are timelike and vice versa. This will be elaborated on in section VIII.

We will discuss the worldlines in the Schwarzschild metric in section XI, but first we will compare cosmological data to the model and discuss the nature of the expansion.

VI. CALCULATION OF COSMOLOGICAL PARAMETERS

In order to compare this model to cosmological data, we must solve for u and find our current position in time (r_0) in the model. Reference [3] gives us transition redshift values ranging from $z_t = 0.337$ to $z_t = 0.89$, depending on the model used. We can use the expression for the scale factor in Equation 2 to get the expression for cosmological redshift from some emitter at r measured by an observer at r_0 [2]:

$$1 + z = \frac{a_0}{a} = \sqrt{\frac{r(u - r_0)}{r_0(u - r)}} \tag{7}$$

Furthermore, the decelration parameter is given by:

$$q = \frac{\ddot{a}a}{\dot{a}^2} = \frac{4r}{u} - 3 \tag{8}$$

By setting Equation 8 equal to zero, we find that the scale factor at the transition from decelerating to accelerating expansion a_t is:

$$a_t = \sqrt{\frac{4}{3} - 1} = \frac{1}{\sqrt{3}} \tag{9}$$

Using Equations 7, 9, and the transition redshift estimate, we can get an expression for the present scale factor:

$$a_0 = a_t(1 + z_t) = \frac{1 + z_t}{\sqrt{3}} \tag{10}$$

Next, we find expressions for u and our current radius r_0 by noting that the Universe has been found to be roughly 13.8 billion years old. Therefore, we can set $\alpha_{r_0} \equiv u - r_0 = 13.8$ and use Equations 2 and 10 to obtain the following for u and r_0:

$$r_0 = \frac{u - r_0}{a_0^2} = \frac{\alpha_{r_0}}{a_0^2} = \frac{3\alpha_{r_0}}{(1 + z_t)^2} \tag{11}$$

$$u = r_0 + \alpha_{r_0} = \alpha_{r_0}\left(\frac{3}{(1 + z_t)^2} + 1\right) \tag{12}$$

Next we compute the CMB scale factor (a_{CMB}) and coordinate time (r_{CMB}) in this model where the redshift of the CMB (z_{CMB}) is currently measured to be 1100:

$$a_{CMB} = \frac{a_0}{1 + z_{CMB}} \qquad (13)$$

$$r_{CMB} = \frac{u}{1 + a_{CMB}^2} \qquad (14)$$

We can next derive the Hubble parameter equation using the scale factor. The Hubble parameter is given by (in units of $(Gy)^{-1}$):

$$H = \frac{\dot{a}}{a} = \frac{u}{2r(u - r)} \qquad (15)$$

Table I below gives the values of u, r_0, H_0, a_0, q_0, a_{CMB}, r_{CMB}, and q_{CMB} given the upper and lower bounds of z_t from [3] as well as the average of the upper and lower bound values and assuming $\alpha_{r_0} = 13.8$. All times are in Gy and H_0 is in $(km/s)/Mpc$.

z_t	α_{r_0}	u	r_0	H_0	a_0	q_0	a_{CMB}	r_{CMB}	q_{CMB}
0.337	13.8	37.0	23.2	56.6	0.77	-0.49	0.0007	36.95	0.99
0.614	13.8	29.7	15.9	66.2	0.93	-0.86	0.0008	29.65	0.99
0.89	13.8	25.4	11.6	77.6	1.09	-1.17	0.0010	25.35	0.99

TABLE I. Limiting Cosmological Parameter Values Based on z_t Measurement and a 13.8 Gy Age of the Universe

From the results in Table I, we see that the true transition redshift is likely between 0.614 and 0.89 given the fact that the current value of the Hubble constant is known to be in that range. Thus, more accurate measurements of the transition redshift are needed to increase the confidence of this model, though we do see that it is able to reproduce measured results.

Table II has the proper times from $r = u$ to the current time as well as the CMB for stationary, inertial observers ($dt = rd\Omega = 0$) by integrating Equation 1. The column τ_{tot} gives the time from $r = u$ to $r = 0$. The expression for τ_{tot} turns out to be quite simple:

$$\tau_{tot} = \frac{\pi}{2}u \qquad (16)$$

In Table II below, the column τ_{remain} gives the time between $r = r_0$ and $r = 0$.

z_t	α_{r_0}	τ_0	τ_{tot}	τ_{remain}	τ_{CMB}
0.337	13.8	42.2	58.1	15.9	8.6
0.614	13.8	37.1	46.7	9.6	2.4
0.89	13.8	33.7	39.9	6.2	2.3

TABLE II. Limiting Proper Times Based on z_t Measurements and an age of 13.8 Gy for the Universe (Time is in Gy)

Note that while the coordinate times for the current age of the Universe ($u - r_0$) are close to current estimates (for high z_t), the proper time τ_0 is actually much larger. And even though we are presently only about halfway through the "coordinate life" of the Universe (according to Table I), the amount of proper time remaining is actually much less than the amount of proper time that has already passed (according to Table II). This provides a measurable prediction from the model: as telescopes such as the JWST peer farther into the past with greater accuracy, we should expect to find stars, galaxies, and structures that are much older than expected because of the increased amount of proper time available for such things to form in the early Universe. Hints of this has already been found with the star HD 140283, whose age is estimated to be nearly the age of the Universe itself [4].

Next we would like to use the u and r_0 values found to create an envelope on a Hubble diagram to compare to measured supernova and quasar data. First we need to find r as a function of redshift. We can do this by solving for r in Equation 7:

$$r = \frac{u(1 + z)^2}{a_0^2 + (1 + z)^2} \qquad (17)$$

We can derive the expression for t vs. r along a null geodesic where the geodesic ends at the current time r_0 and $t = 0$ by setting $d\tau = rd\Omega = 0$ in Equation 1 and integrating:

$$t = \int_{r_0}^{r} \frac{r}{u - r} dr = u \ln\left(\frac{u - r_0}{u - r}\right) + r_0 - r \qquad (18)$$

Next we substitute Equation 17 into Equation 18 to get coordinate distance in terms of redshift:

$$t = r_0 + u\left[\ln\left(\frac{a_0^2 + (1 + z)^2}{1 + a_0^2}\right) - \frac{(1 + z)^2}{a_0^2 + (1 + z)^2}\right] \qquad (19)$$

We need to convert the distance from Equation 19 to the distance modulus, μ, which is defined as:

$$\mu = 5\log_{10}\left(\frac{D_L}{10}\right) \qquad (20)$$

Where D_L in Equation 20 is the luminosity distance. Luminosity distance is inversely proportional to brightness via the relationship:

$$B \propto \frac{1}{D_L^2} \qquad (21)$$

The brightness is affected by two things. First, the spatial expansion will effectively increase the distance between two objects at fixed co-moving distance from each other. This will reduce the brightness by a factor of $(1 + z)^2$ (because the distance in Equation 21 is squared). But there is also a brightening effect caused by the acceleration in the time dimension. We define $V \equiv \frac{d\tau}{dr} = \frac{1}{a}$ as the temporal velocity of the inertial observer at some

r and the speed of light at that r as $V_c \equiv \frac{dt}{dr} = \frac{1}{a^2}$. The ratio of these velocities gives us:

$$\frac{V_c}{V} = \frac{dt}{dr}\frac{dr}{d\tau} = \frac{dt}{d\tau} = \frac{a}{a^2} = \frac{1}{a} \tag{22}$$

Equation 22 tells us how far a photon travels over a given period of time measured by the inertial observer's clock. So we see that as light travels from the emitter to the receiver, this speed decreases. This decrease in the speed from emitter to receiver will result in an increased photon density at the receiver relative to the emitter, increasing the brightness. Therefore, this effect will increase the brightness by a factor of:

$$\frac{a_0}{a} = 1 + z \tag{23}$$

Taking these brightness effects into account, the total brightness will be reduced by an overall factor of $1 + z$ relative to the case of an emitter and receiver at rest relative to each other in flat spacetime. Equation 21 in terms of co-moving distance t and redshift z becomes:

$$B \propto \frac{1+z}{(t(1+z))^2} \rightarrow B \propto \frac{1}{t^2(1+z)} \tag{24}$$

Giving the luminosity distance as a function of co-moving distance t and redshift z:

$$D_L = t\sqrt{1+z} \tag{25}$$

Which gives us the final expression for the distance modulus as a function of co-moving distance and redshift:

$$\mu = 5\log_{10}\left(\frac{t\sqrt{1+z}}{10}\right) \tag{26}$$

A plot of distance modulus vs. redshift is shown in Figure 4 below plotted over data obtained from the Supernova Cosmology Project [5]. Curves calculated from all three values of z_t in Table I are plotted, giving an envelope for the model's prediction of the true Hubble diagram.
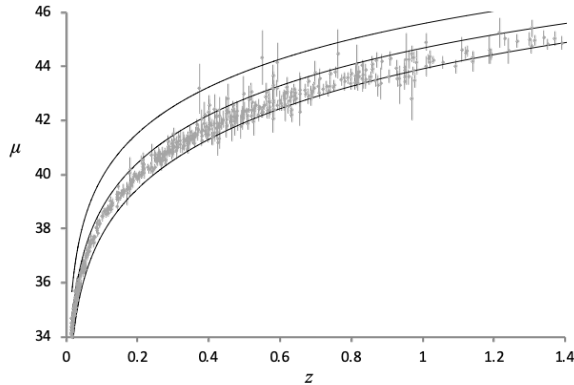

FIG. 4. Distance Modulus vs. Redshift Plotted with Supernova Measurements

Note that the middle curve corresponds to $z_t = 0.614$ and the lower curve corresponds to $z_t = 0.89$. The supernova data is better fit by a curve between these values. The curve halfway between (with $z_t = 0.75$) gives us $H_0 = 71.6$, $a_0 = 1.0$, $q_0 = -1.0$, $u = 27.3$, and $r_0 = 13.5$.

In [6], the authors analyze a large sample of quasar data to obtain distance moduli at higher redshifts than is possible with supernova data. Figure 5 shows the same predicted envelope from Figure 4 for the Hubble diagram plotted out to higher redshifts with the quasar data from [6] also shown with error bars. The black diamonds in the figure are the 18 high-luminosity XMM-Newton quasar points described in [6].
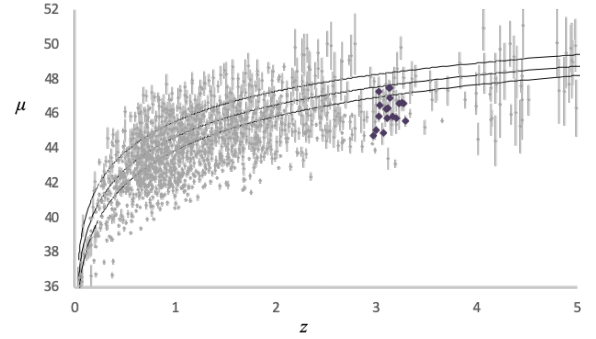

FIG. 5. Distance Modulus vs. Redshift Plotted with Quasar Measurements

Finally, by subtracting r_0 from Equation 17 we can calculate the lookback time for a given redshift. Figure 6 shows the lookback time vs. redshift for the three transition redshifts.
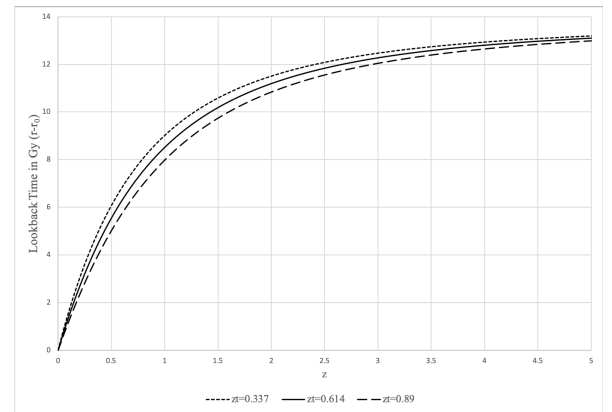

FIG. 6. Lookback Time vs. Redshift

VII.   THE ANTIMATTER UNIVERSE

Figure 7 shows the full Schwarzschild metric in Kruskal-Sezekeres coordinates. The diagram can be split in two along the diagonal where in the top right half, forward time points up while in the bottom right half,

forward in time points down. Left and right are also swapped when looking at the upper and lower halves.

We can therefore conjecture that the diagram is describing both a matter Universe expanding up from the center and an antimatter Universe expanding down from the center, each one moving toward a singularity. The reason we expect an antimatter Universe is because the directions of both time and space are reversed relative to each other and therefore, we expect the particles of the second Universe to have opposite charges relative to the first. Thus, the pair of Universes (or 'Duoverse') satisfies CPT symmetry and the Kruskal coordinates T and X represent cardinal directions of space and time.
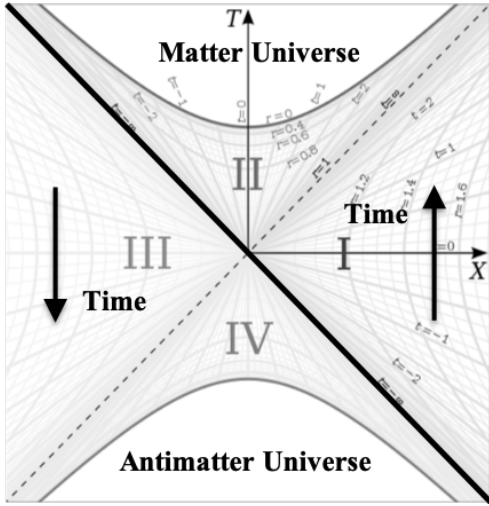

FIG. 7. Matter and Antimatter Universes

VIII. COMPLEX SPACETIME

Notice that the dr and $rd\Omega$ terms in Equation 1 have opposite signs. As is the case in the external Schwarzschild and FRW metrics, we would expect the angular and pure radius terms to have the same sign. We can remedy this by changing Equation 1 to:

$$d\tau^2 = \frac{u-r}{r}d\left(it\right)^2 - \frac{r}{u-r}d\left(ir\right)^2 - r^2 d\Omega^2 \qquad (27)$$

Equation 27 implies that the imaginary counterpart of the time coordinate is spacelike and the imaginary counterpart of the spatial (radius) coordinate is timelike. We can even see how the timelike r coordinate and spacelike t coordinate retain some of the time and spacelike characteristics of their real counterparts.

To see how the imaginary t coordinate, which is spacelike in the internal metric retains some timelike properties, we need to observe a surface of constant r with changing t. The Cosmic Microwave Background is such a surface. If one were to observe a patch of the CMB over billions of years, one would see it change over time with a given patch in the sky having slight changes in temperature distribution over time. But the surface is not actually changing over time because it is by definition a surface at a fixed time. The changes we would observe would come from the fact that we are seeing the CMB in the direction we are observing it at greater distances from us over time, as if the surface we see is moving through the Universe itself at that fixed time. So even though the CMB would appear to change over time, it isn't actually changing with time, we are only seeing it at greater coordinate distances as we move through time. There is even an 'arrow of time' characteristic to the t coordinate when observing the CMB. If we look at the CMB in one direction, we see it change from time A to time B (our time A and time B, the CMB time does not change) because we're seeing it at a greater distance from us at time B than we did at time A. We might initially think that if we move in the opposite direction, we should eventually be able to observe the CMB the way it was at our time A since the differences we see in the CMB at times A and B are not changes in the CMB over time, but rather we are observing different locations of the CMB at the same time. But in order to see the CMB at the location we saw it at time A, we would need to move faster than the speed of light in the opposite direction. Whether we remain co-moving or move with arbitrary peculiar velocity, we will always see the CMB change in the same way. The only difference our peculiar motion will make is how quickly we see it change. This can be seen by looking at the 'Static sphere' hyperbola in Figure 1 and imagining an observer moving along $t = 0$. If you draw a 45 degree line from the hyperbola to $t = 0$ (the hyperbola will represent the CMB), you can see that once the observer at $t = 0$ has moved forward in time (up the diagram), that 45 degree line is outside the future light cone of the observer for the rest of the expansion, no matter which direction the observer moves in. Thus, our observations of the CMB over time are not reversible during expansion. We cannot move in a way to go back and see previously observed states of the CMB even though the different states are due to changes in space, not time. This is analagous to the arrow of time, but in a spacelike dimension.

Even more obvious is how the imaginary r coordinate, which is timelike, still retains spacelike characteristics. When we look out at the Universe, it almost seems like we are looking at galaxies surrounding us in space in the present time. But it is more useful to measure those distances in time because the Universe is homogenous in space at a given time, so specifying the distance in spatial coordinates is less useful because the differences we see in the structure of the Universe at different distances are due to their separation from us in time, not space. Therefore, it feels like the farther away we look in space the more different the Universe looks, it is not the distance in space that is responsible for the differences but rather the distance in time.

It is notable that the $d\Omega$ term is unchanged in the internal metric relative to the external metric where surfaces of constant r are spacelike in both cases. This makes

sense because surfaces of constant r in spherically symmetric Universes are at a constant distance in both space and time in all cases and so the angular dimension of the metric has no imaginary counterpart.

Finally, even the proper time and space quantities $d\tau$, ds have dual characteristics in the internal metric. In section IX we will discuss how the proper time of the worldline in the internal metric behaves more like a height in the internal metric as opposed to a time. We can also say that the proper distance ds has a timelike quality in the internal metric in that the change in proper distance between distant regions of the Universe over time gives us the scale factor, which is the main indicator what time in the Universe we are observing.

So if we put the observer at the center of a spherically symmetric space, we can say that every direction has both real and imaginary space and time dimensions associated with it. If we again consider the CMB in a particular direction, no matter how long we travel in that direction, we will not reach the CMB because it is separated from us in the imaginary spatial dimension, which is related to the timelike radius of the internal metric. The imaginary cosmological metric, whose main attribute is the scale factor of the Universe, determines the scaling of the real metric of the Universe (the FRW metric) throughout cosmological time.

Looking at Figure 7, let us imagine a complex plane perpendicular to the page whose real axis is coincident with the T axis of Figure 7. Setting $u = 1$, in Kruskal coordinates the relationship between T and r along $t = 0$ is:

$$T = \pm\sqrt{(1-r)e^r} \tag{28}$$

$$r = 1 + W_0\left(-\frac{T^2}{e}\right) \tag{29}$$

Where W_0 is the Lambert W function. Therefore, we can plot the relationship between T and ir on the aforementioned complex plane in Figure 10 for both the matter and antimatter Universes:
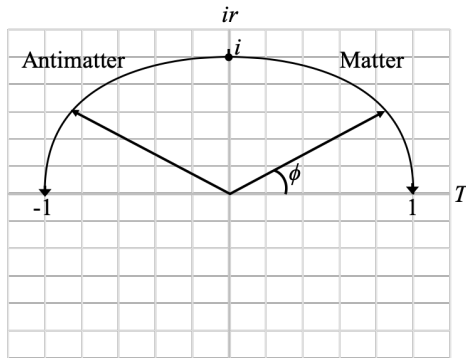

FIG. 8. Imaginary Radius to Real Radius for the Matter (Right) and Antimatter (Left) Universes

In Figure 10, we see two oblong curves, the right one for the matter Universe and the left one for the antimatter Universe with a vector whose projections give the magnitudes of the real and imaginary radii at a given time. The two Universes are coincident at i, representing the event horizon/Big Bang era (in the rest of this paper, the Big Bang will be referred to as Annihilation). Here, we can say the matter and antimatter Universes have annihilated with each other and new pairs of matter and antimatter are created from the annihilation, creating the two Universes travelling in opposite directions of time. Over time, the imaginary radii of the Universes decrease while the real radii increase up to the singularity, where the imaginary radii are zero and the real radii are 1.

The antimatter Universe moves in the opposite direction of time relative to the matter Universe, and so we expect their vectors on this plane to rotate in opposite directions as shown.

But looking at Figure 10, one can't help but be tempted to complete the curves by mirroring them in the real axis. Doing so would indicate that right as the Universes reach maximum expansion, the geodesics reverse in time and the Universes begin to re-collapse toward each other. This creates a discontinuity in the geodesics, resulting in the singular nature of $r = 0$, which we will dissect further in the next section.

We can think of the Universe as the imaginary counterpart of a galaxy. Let's imagine starting form the center of a galaxy and moving out through space at the center. At the center of the galaxy, we have a spherically symmetric event horizon. The 'Dark Sphere' of Figure 1 is the imaginary counterpart of the galaxy's event horizon. As we move out radially, we first have a dense accretion disk around the horizon. The CMB is the imaginary counterpart of this disk. As we move out farther and farther radially, the galaxy becomes less and less dense. The scale factor of the Universe is the imaginary counterpart of this reduction in density at greater radii of the galaxy. Eventually we get to a maximum radius of the galaxy with minimum density. The singularity is the imaginary counterpart of the galaxy's edge. It has been observed that the size of a galaxy's black hole is related to the overall size of the galaxy itself. This is analogous to the fact the the radius of the internal solution determines the timescale of the expansion phase of the Universe and the maximum size of the observable Universe. Finally, the many galaxies in our Universe are analogous to the repeated expansion and collapse cycles of the Universe where each of the galaxies is an independent cycle of the Universe. We will discuss observable Universes and the repeating cycles in later sections.

IX.   NEWTONIAN ANALOG

This entire system is the temporal equivalent of two masses initially moving apart from one another until they reach a maximum separation distance u. At that point

they will start falling toward each other again due to mutual gravitational attraction. When they meet at their common center, they annihilate, creating new pairs of matter/antimatter particles and begin moving away from each other again, as if they've bounced off each other. It is equivalent to the exchange of potential and kinetic Energy, but in the time dimension. Looking at the equation $E^2 = m^2 + p^2$, we can say that this process conserves E by converting p into m during expansion (cosmological redshift is a consequence of the loss of momentum) and vice versa during the collapse. This will be further explored in section XI.

Now consider the Newtonian example of a ball in a gravitational field rising to a maximum height h and then falling back to the ground. $\frac{dh}{dt}$ will be positive on the way up, negative on the way down and zero at max height. But this also means that $\frac{dt}{dh}$ will be infinite at the maximum height because $dh = 0$ there. We might think that when comparing this to the present case, $t \to \tau$ and $h \to r$, but this is incorrect. We know that r is our time coordinate and τ is the distance along the geodesic, so $h \to \tau$ and $t \to r$. So from Equation 3, we see that, just like in the Newtonian example, $\frac{d\tau}{dr} = 0$ and $\frac{dr}{d\tau} = \infty$ at the singularity because in this case $d\tau = 0$ at the turnaround.

When the Newtonian ball falls back to the ground, if the ball and ground were perfectly rigid and the collision perfectly elastic, there would be an infinite impulse during the collision where the ball would shatter and the fragments would once again start rising up into the air. This is analogous to the matter and antimatter Universes annihilating after the collapse and then re-expanding.

Referring back to section VIII, we can also think of the collapse phase of the Universe as being analogous to a spherically symmetric shell of matter starting from rest (at the singularity) and collapsing through space toward an event horizon, but in this case, it is the 3D spherically symmetric Universe falling through time toward the horizon where it annihilates with the antimatter Universe. The expansion phase is simply the time-reversed version of this.

X.   THE NATURE OF EXPANSION

From Equation 4 we can calculate the angle of the cosmological light cone as $\theta = \arctan \frac{1}{a^2}$. At the beginning, when $\theta = \frac{\pi}{2}$, the speed of light is infinite which means all fractions of the speed of light are infinite, and that manifests itself as space having zero size. As time progresses, the light cone closes. The closing of the light cone manifests as an expansion of space since this means the cosmological speed of light is getting smaller, so all fractions of it also get proportionally smaller. The cosmological redshift and dimming come from the fact that the present Universe is accelerating away from past events through time. So if you set off to another galaxy at some time with a constant velocity, over time that velocity will

effectively slow as the light cone closes even though no forces have acted on the observer. Since the observer does not feel any change in their velocity, they will describe this as an expansion of space since it will take them longer to reach their destination. As θ goes to zero, the light cone closes completely meaning nothing can move in space, manifesting itself as an infinite scale factor. The Universe has lost all momentum and the momentum has been converted into inertia and this increase in inertia manifests as spatial expansion.

XI.   SCHWARZSCHILD WORLDLINES

We have so far focused on co-moving worldlines when analyzing the internal metric. Let us now consider what the worldlines of general inertial particles might look like on the spacetime diagram.

Let's first consider an inertial observer moving with some non zero $\frac{dt}{d\tau}$ as it approaches, and then leaves, the singularity. From Equation 5, we see that as $r \to 0$, $\frac{d^2t}{d\tau^2}$ goes to infinity. This means that $\frac{dt}{d\tau}$ will be forced to zero as the observer approaches the singularity since, during expansion, the acceleration is opposite in direction to the velocity. This means that the worldline will be parallel to whichever t coordinate the observer is at as they reach the singularity. This also means that the worldline will be parallel to the t coordinate as it leaves the singularity at the beginning of collapse.

But during the.collapse phase, $\frac{d^2t}{d\tau^2}$ and $\frac{dt}{d\tau}$ are in the same direction, so even though the worldline begins parallel to the t coordinate at the beginning of collapse, this position is unstable because if the particle is perturbed at all, giving it some non-zero velocity, Equation 5 shows that the acceleration, and therefore the velocity, will be increased over time by the collapse of the t coordinates. This acceleration also goes to infinity as the worldline approaches $r = u$, meaning the worldline becomes a null geodesic perpendicular to the horizon at the annihilation event. By symmetry, we can likewise say that the worldline emerging from the annihilation event at the beginning of expansion will also be null and perpendicular to the horizon.

The slopes of the worldlines at the annihilation event and singularity support the idea that the energy of the Universe is pure momentum at the annihilation event and pure inertia at the singularity. An example worldline on the spacetime diagram is given in Figure 9:
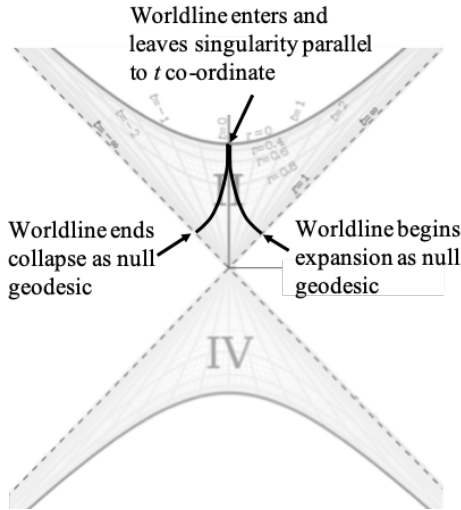
FIG. 9. Example Internal Worldline

Next, we consider the external solution under the same conditions. In this case, the $t < 0$ coordinates of the external solution cover the expansion phase, while the $t > 0$ coordinates are for the collapsing phase:
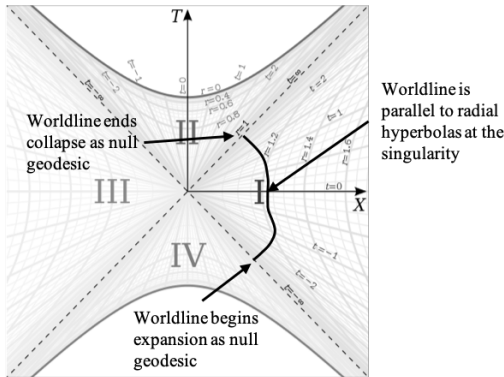

FIG. 10. Example External Worldline

For this, we will assume the Universe has a quasi black hole that the observer falls towards. Just as with the internal solution, the worldline will start out null as it emerges from annihilation. During expansion, it will gravitate toward the black hole, but because of the cosmological effects, the line will curve to be parallel to the tangent of the closest r hyperbola at $t = 0$. This corresponds to the singularity when the Universe begins collapse. After the singularity, it will continue to fall until it reaches the horizon at the next annihilation event.

It is also notable that at according to the external metric, all worldlines that intersect the horizon will do so simultaneously. This is because all worldline that intersect $r = 1$ are at the same location with proper distance/time between them equal to zero (because the line $r = 1$ is a null geodesic). So no matter when different observers start falling toward the horizon, even if they start at the same location at different times, they will all reach the horizon simultaneously. This is consistent with the collapsing Universe where the horizon represents the end of collapse where the Universe becomes infinitely dense (the same is true at the beginning of expansion).

And for an observer at constant r, remaining at the constant r will become more and more difficult as the Universe collapses because of equation 5. Once stable orbits will become less and less stable as the Universe collapses and at some point during the collapse, it will become practically impossible to maintain a stable orbit and they too will fall to the horizon. Therefore, we conclude that the black hole will never form because the matter will not reach the event horizon radius until the entire Universe has re-collapsed to the Annihilation event, at which point all matter in both Universes will meet at the event horizon and annihilate. So in a sense, the whole Universe will 'fall into a black hole', where once the matter in it reaches the Annihilation event after the Universe collapses (which corresponds to the event horizon), it re-emerges into a new expanding spacetime that is the next cycle of the expanding Universe. All gravitational event horizons are surfaces of future time that all matter will fall to at the end of re-collapse as it is destroyed and remade in an effectively new, expanding Universe.

The expansion and contraction of the spatial coordinates also demonstrates a counter-intuitive fact. During expansion, gravitational systems become increasingly stable because as the singularity is approached, it becomes increasingly difficult to change coordinate positions in space until at the instant of the singularity, all spatial positions become fixed. Orbits will also become more and more circular as the singularity is approached since objects in circular orbits remain at fixed spatial coordinates. Then during collapse, as the spatial coordinates contract, orbits become less stable over time due to equation 5 leading to a more chaotic Universe toward the end of the collapse phase. This makes sense however if we consider that the collapse is the time reversed expansion. We know the early Universe was chaotic and disorganized and as the expansion progressed, stable stars, galaxies, and gravitating structures in general formed. So the collapse will essentially undo the order gained from the expansion, returning the Universe to the same kind of state it was in at the beginning of expansion when it reaches the end of the collapse.

XII. THE MANY WORLDS

To this point we have described the spacetime dynamically, but there is still an open issue regarding the angle in the internal Schwarzschild metric at which a given event takes place. As we will see, the answer to this question is that it depends on from which location it is being measured from.

Let's consider the Universe at $r = 0$, the singularity. Imagine a 3D flat space where every point in this space

is an observer in the Universe at $r = 0$. If we pick out one such observer, when they look out at the Universe (we will ignore the redshift for this argument and assume the entire past light cone for the observer is visible), this observer will see the Universe much like we see it today: a dense plasma at the farthest distance followed by stars and galaxies with decreasing densities as the radius gets smaller. A 2D representation of this is shown in Figure 11 below where the observer is at the center of the circle.
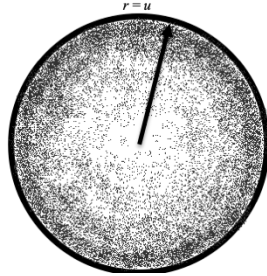

FIG. 11. Observable Universe at $r = 0$

So each observer in the 3D flat space has a sphere like this mapped to it. We will refer to these spheres as observable Universes. But the radius of the sphere is not in the 3D space but is instead the 4th dimension. There is also an antimatter sphere at each point that intersects the matter radius at the $r = 0$ points and extend into the negative direction of this 4th dimension. This is a static picture, but dynamically, we can imagine the spheres growing out from the $r = 0$ points in the 3D space as time progresses. Thus, the r in Figure 11 is the real radius from Figure 10 which grows as the imaginary radius becomes real. So this model can be said to have 3 flat dimensions of space and 3 spherical dimensions of time (though the 3 dimensions of space and two angular dimensions of time are dependant, so this can still be reduced to a 4D spacetime). Furthermore, all light beams in a given observable Universe converge at the center of the time sphere, meaning that every point in the 3D space has null geodesics converging to them from all directions as the geodesics approach $r = 0$, which satisfies the singularity theorem. We would normally imagine light converging to the center of a volume, but that is not the case here. In this scenario, every single point in the volume has its own set of geodesics converging to it from all directions.

Let us consider our current position in the Universe where we sit at some $r = r_0$. Imagine we send out light from our current location in all directions. Assuming none of the light is absorbed in transit, the light will reach a spherical surface around us in the 3D space as the light beams reach $r = 0$. Therefore, the angle at which we reside in the internal Schwarzschild metric depends on which observable Universe we are measuring our position from because we will be visible to all the observable Universes that lie on that aforementioned 2D shell in the 3D space. Each of those observable Universes see us from a

different direction, and the direction from which a given observable Universe sees us determines our angle in the internal Schwarzschild metric. Another way to put it is that each of the infinite observable Universes at $r = 0$ corresponds to a unique infinite set of null geodesics (one geodesic for each direction) that converge at a given observable Universe's t at $r = 0$.

These quasi 3+3 dimensional matter and antimatter Universes contain all the events for a single expansion from beginning to end (these dimensions are smooth and continuous). However, the matter and antimatter Universes then re-collapse and eventually result in new expansions. Therefore, we can think of each successive expansion and contraction of the Universes as happening along another dimension which is discrete. This dimension essentially labels the different countably infinite random Universes.

Since each Duoverse begins with annihilation, this means each Duoverse begins with a random configuration after annihilation. Therefore, there is no cause and effect relationship between Duoverses from cycle to cycle. This means the cycles cannot be ordered sequentially because there is no way to know which cycle preceded or will follow the current cycle. If we cannot order the cycles in a sequence, then we can think of them all as being parallel to each other. While events within a cycle can have cause and effect relationships (i.e. the events 'happen' at given times), the various cycles themselves do not 'happen', they just exist along side all other cycles. Thus we can think of the annihilation events as being *a single* event from which infinite Duoverses emerge and to which they return. This implies that finding ourselves in a particular Duoverse is completely probabilistic where the probability that we find ourselves in a Duoverse with a particular configuration depends on how likely that configuration is across all possible configurations (where many configurations are similar enough to be effectively indistinguishable from each other). This gives us the many worlds that have been invoked to explain quantum probability in the Everett many worlds interpretation of QM.

XIII.   THE CHARGE AND SPIN HYPOTHESIS

Given that the matter and antimatter Universes are moving in opposite directions in time, we can hypothesize that the electric charge of a particle is related to the orientation of the particle's velocity vector in time. The sign of the charges of matter particles would indicate that the temporal velocities of these particles are oriented parallel to the time radius of the matter Universe. The antimatter particles have opposite sign and so their vectors are oriented anti-parallel to the time radius of the matter Universe (or parallel to the time radius of the antimatter Universe). This could be perhaps understood as differences in the directions of group and phase velocities of the wave function in time:

1. *Matter particles in matter Universe*: Group and

phase velocities pointed in the same direction toward positive time.

2. *Antiparticles in matter Universe*: Group velocity pointed in positive time direction, phase velocity pointed in negative time direction.

3. *Antimatter particles in antimatter Universe*: Group and phase velocities pointed in the same direction toward negative time.

4. *Matter particles in antimatter Universe*: Group velocity pointed in negative time direction, phase velocity pointed in positive time direction.

Consider the turnaround point at the singularity as the Universe transitions from expansion to collapse. On the way into the singularity, the phase and group velocity vectors of matter particles are pointing toward the singularity. At the singularity, the velocity vectors disappear because of the turnaround and all matter becomes instantaneously chargeless. Photons also converge at every point in space at the singularity as discussed in the previous section. Once the collapse starts, the photons re-emerge from every point in space and the matter group and phase velocity vectors are pointed away from the singularity, flipping the charges of all charged particles. Therefore, relative to the expanding Universe, the collapsing Universe is an antimatter Universe moving backwards in time (and this is mirrored in the other antimatter Universe).

We can extend this hypothesis further by considering the spin of Fermions. Fermions can be measured to be spin up or spin down. We could interpret the spin to be a physical spin about the time radius with, for instance, spin up indicating the spin vector is parallel to the time radius of the matter Universe, and spin down indicating the spin vector is anti-parallel to the time radius of the matter Universe. Treating Quantum spin as a rotation about the time axis could be seen as a necessary consequence of relativity: if space and time are equivalent, then the possibility of rotations about an axis in space implies that it is also possible to rotate about an axis of time.

More generally, we can posit that the imaginary parts of the quantum wave functions are vibrations of the wave function along the radial time dimension.

[1] Figures 1, 7, 9, and **??** are modifications of: 'Kruskal diagram of Schwarzschild chart' by Dr Greg. Licensed under CC BY-SA 3.0 via Wikimedia Commons - http://commons.wikimedia.org/wiki/File: Kruskal_diagram_of_Schwarzschild_chart.svg#/media /File:Kruskal_diagram_of_Schwarzschild_chart.svg.

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