Supporting Information

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Methods S1: **Revisiting Gersani *et al.* (2001)'s model**

Gersani *et al.* (2001) analyzed their game root tragedy of the commons (RToC) model in a particular scenario in which both the total number of plants ($n = 10$) and the total rooting volume available to these plants were fixed. They considered three particular cases: ($N = 1$) the soil is partitioned in 10 equal compartments, with each plant having access to one of them (equivalent to their control treatment); ($N = 2$) the soil is partitioned in five equal compartments shared by pairs of plants (equivalent to their interaction treatment); and ($N = 10$) the soil is not partitioned, with all plants having access to all rooting volume. The value of N indicates the number of plants sharing each soil partition.

A formal definition to the $H(x)$ and $C(u)$ functions from **Eq. 1** should first be established to be consistent with the equations that Gersani *et al*. (2001) may have used, with their graphical results as a reference. Let's assume that the nutrient uptake function $H(x)$ is a saturating function of the form:

$$H(x) = \frac{\varphi - \varphi e^{-\theta x}}{\theta} \qquad [3]$$

and the cost function is a quadratic equation of the form

$$C(u) = \alpha u^2 + \beta u \qquad [4]$$

As the ESS is the solution that, if adopted by all coexisting plants—$u^* = u_i$ for any i—maximizes the resource net gain with respect to u_i, and must satisfy $\partial G / \partial u_i = 0$, we can write

$$\frac{N-1}{N} \frac{(\varphi - \varphi e^{-\theta n u^*})}{\theta n u^*} + \frac{1}{N} \varphi e^{-\theta n u^*} = 2\alpha u^* + \beta \qquad [5]$$

Using these equations, we can accurately reproduce all results from Gersani et al. (2001). Concretely, **Eq. 3** reproduces the root nutrient uptake as shown in **Fig 4a**, and the left and right sides of **Eq. 5** reproduce the nutrient uptake and cost respectively per unit root increase, as shown in **Fig 4b**. Unfortunately, there is no analytical solution that can be derived for u^* in **Eq. 5**. But we can approach the solutions numerically for parameter values of *φ=0.3*, *θ=0.25*, *α=0.025*, and *β=0.025*, obtaining for the control treatment $u^*_{N=1} \approx 2.6181\ Units\ of\ Root\ (UR)$, the interaction treatment $u^*_{N=2} \approx 3.0174\ UR$, and the ten plants sharing a soil volume $u^*_{N=10} \approx 3.3959\ UR$. These values represent the total amount of roots one plant produces in the total soil volume.

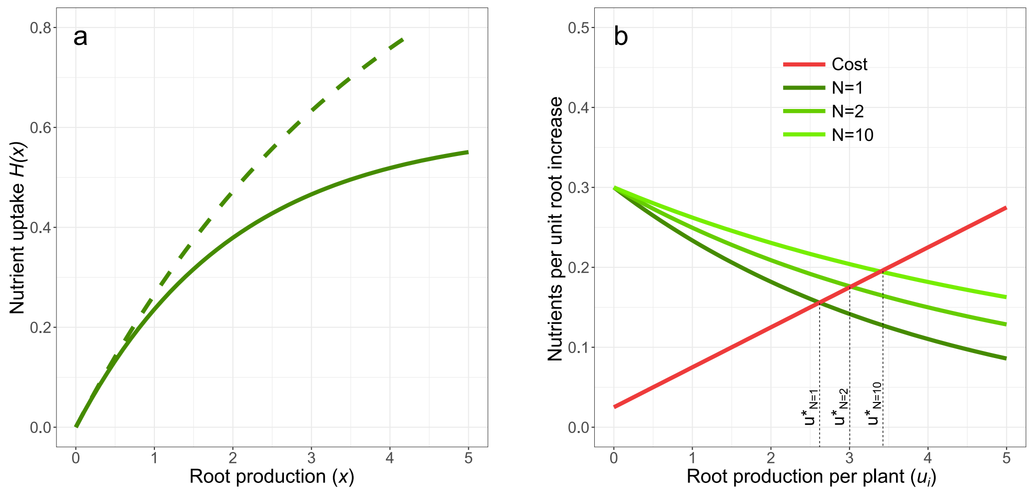

Figure S1: Reproduction of Gersani et al.'s figure 1 for **a-** nutrient uptake function assuming $\varphi = 0.3$ and $\theta = 0.5$ (green solid line) or $\theta = 0.25$ (green dashed line). **b-** Nutrient uptake (green lines) and costs per unit root increase (red line) for $\theta = 0.25$, $\alpha = 0.025$, and $\beta = 0.025$.

Model results do not control for rooting volume in the manner assumed by researchers (i.e., accounting for the rooting volume actually available for each plant in each "pot"). However, this can be calculated by defining a "pot" or unit of rooting volume (v) as a tenth of the total soil volume. Thereafter, m can be defined as the number of rooting volume units per compartment (note that $m = N$), while plant root density d can be defined as the units of root for each plant in each soil compartment (UR/v). We can calculate each plant's root density in equilibrium using

$$d^* = \frac{u^*}{m} \qquad [6]$$

Although root production per plant in the total soil volume increases with competition intensity (N), the root density per plant actually decreases; for the control treatment, root density per plant is $d_{N=1}^{*} \approx 2.618\, UR/v$, and for the interaction treatment, this value is $d_{N=2}^{*} \approx 1.509\ UR/v$. Nonetheless, such result does not mean that plants are not engaging in an RToC. The optimal collective rooting strategy x^{*} satisfying $dG_T/dx = 0$ (the optimal root proliferation for any amount of plants sharing a unit of soil volume), where

$$G_T(\mathrm{x}) = H(x) - C(x) \qquad [7]$$

is equivalent to the optimal root production of a plant owning a unit of soil volume when plants follow the strategy u^{*} satisfying $dG_i/du_{i\ (N=1)} = 0$. This equality indicates that the maximum collective gain is reached when the root production per plant is equivalent to the root production of plants in individual soil compartments. More generally, this equality indicates that, given the choice of parameters, a total root density of $2.6181\ UR/v$ is optimal, regardless of the number of plants growing roots. Collective gain is optimized at this density, as confirmed by plugging the values in the resource net gain equation: The collective net gains are $G_{N=1}$=0.3396, $G_{N=2}$=0.2333, and $G_{N=10}$=0.2032 resources per plant. In the N=2 and N=10 scenarios, plants are engaging in an RToC by overproliferating their roots with respect to the collective optimal, and are thus inefficiently overexploiting the common resource.

References

Gersani, M., Brown, J.S., O’Brien, E.E., Maina, G.M. & Abramsky, Z. (2001). Tragedy of the Commons as a Result of Root Competition. *J. Ecol.*, 89, 660–669.