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On the mass of the nucleons from "first principles"

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Summary
The mass of the nucleons is calculated from first principles by defining the quark as a non-gravitational particle, subject to Dirac's equation with non-canonical gamma matrices. Unlike the canonical electron-type, this Dirac particle has two real dipole moments, which allows a structural modelling of hadrons. It is shown how such modelling reveals striking correspondences and differences between dually related particles and properties like electrons and quarks, photons and gluons, pions and nucleons, spin and isospin and protons and neutrons. It is a stepping stone to the actual calculation, which competes in precision with lattice QCD. The appendix contains a Lorentz covariance proof of hadrons composed by such particular Dirac-type quarks.

Key words: lattice QCD; nucleon mass; quark confinement; gluon; Dirac's equation.

1. Introduction

One of the issues in particle physics is the challenge to calculate the masses of hadrons "from first principles" or "ab initio". It is commonly taken for granted that lattice QCD is the best way, if not the only one, to do so. The calculation of the proton mass is usually considered as a decisive proof of its capabilities. However, although the quark is considered as the ultimate building block of observable nuclear particles, lattice QCD is unable to use the attributes of quarks as the true reference for "ab initio", for the simple reason that a quark escapes from observability. For that reason, the real reference values for lattice QCD are the masses of the pion and the kaon. Although these mesons are not observable either, their masses can be measured as a result from their fermionic decay products. The lattice QCD quark model is subsequently used as a means to establish the relevant quark attributes for the calculation "from first principles". Several groups have claimed results from calculations on protons and other hadrons that are close to experimental results [1,2]. However, results from theory can still not compete in precision with results from experiments. Up to now, lattice QCD is unable to calculate the mass difference between a proton and a neutron. Moreover, the retrieved masses for the u and the d quark can only be established as an average over the two and lattice QCD calculations for the mass difference between a charged pion and a neutral pion are still missing. The lattice QCD calculations require highly-intensive computations. The results have to be believed on the reputation of the reporting scientists without any means for verification by the reader. It is my aim in this article to show that a more simple approach is capable to address these problems more adequately than lattice QCD can do. This will require, though, giving a motivation for choosing a different route. Similarly as in lattice QCD, the basic elements in this route are the quark and the gluon. And more particularly, their interrelationship, which will culminate into the statement that the quark-gluon relationship is the nuclear equivalent of the electromagnetic electron-photon relationship. This will require more precise definitions of the quark and the gluon than available in present theory. Before addressing the actual calculations, a review on the differences and correspondences between a quark and an electron will be instructive, as well as the differences and correspondences between a gluon and a photon (paragraphs 2 and 3). Next to those from spin-spin interactions (paragraph 5), their impact will be shown on other dualities, like pion and nucleon (paragraph 4), spin and isospin (paragraph 6), proton and neutron (paragraph 7). Prior to the discussion and conclusions (paragraph 8), the results of the mass calculations on nucleons are compared with those reported from lattice QCD.

2. The electron and the quark

Like all elementary fermions, electrons and quarks follow Fermi-Dirac statistics, obey the Pauli exclusion principle, have half integer spin and have distinct antiparticles. They can be modelled with the Dirac equation. The canonic formulation of Dirac's particle equation reads as [3,4],

$$(\mathrm{i}\hbar\gamma^{\mu}\partial_{\mu}\psi - \beta m_0 c\psi) = 0\,, \tag{1}$$

in which β is a 4 x 4 unity matrix and in which the 4 x 4 gamma matrices have different properties for electrons and quarks.

For electrons,

$$\gamma_{\mu}\gamma_{\nu} + \gamma_{\nu}\gamma_{\mu} = 0 \text{ if } \mu \neq \nu;\;\; \gamma_{\mu}\beta + \beta\gamma_{\mu} = 0;\; \gamma_0^2 = 1; \gamma_i^2 = -1;\; \beta^2 = 1\,. \tag{2}$$

while for quarks [5],

$$\gamma_{\mu}\gamma_{\nu} + \gamma_{\nu}\gamma_{\mu} = 0 \text{ if } \mu \neq \nu;\;\; \gamma_{\mu}\beta + \beta\gamma_{\mu} = 0;\; \gamma_0^2 = -1; \gamma_i^2 = -1;\; \beta^2 = -1\,. \tag{3}$$

In both cases Dirac's equation (1) is satisfied by the spinor,

$$\psi = u_{\mu} \exp\{\mathrm{i}(\mathbf{k}\cdot\mathbf{r} - \omega\tau)\};\;\; \mathbf{k} = \mathbf{p}/\hbar;\; \omega = W/\hbar\,, \tag{4}$$

in which $\mathbf{p}$ is the three-vector momentum and in which W is an energy relationship between the particle's rest mass and its motional energy. For electrons,

$$W^2 = (m_0 c^2)^2 + c^2|\mathbf{p}|^2\,, \tag{5}$$

while for quarks,

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$$W^2 = (m_0 c^2)^2 - c^2 |\mathbf{p}|^2 . \tag{6}$$

Note: It may seem that Dirac's equation (1) under the constraints (3) is not Lorentz covariant, because of the violation of the invariance of the space-time interval if W is equated with the Einsteinean energy E. This invariance is a basic theorem in Einstein's Relativity theory. Let us take into consideration, though, that this theorem applies to gravitational objects, in which energy is conceived as the sum of massive energy embodied in the rest mass and the kinetic energy of an object. Prior to Dirac's relativistic electron theory, the concept of negative energy has been considered as a violation of physical principles. After all, it was realized that the concept of negative energy did not violate the Einsteinean space-time invariance. Dirac himself proved the Lorentz covariance of his electron theory. But…if a particle with negative energy is physically viable, why would a particle that eats its kinetic energy from its rest mass not physically viable? It requires to identify such a particle as being different from a gravitational object and to redefine the space-time interval variance for such particles. As shown in the Appendix, such a non-gravitational object can be compliant with the Lorentz covariance.

The electron as well as the quark is a pointlike source of non-baryonic energy that erupts a scalar field Φ of non-baryonic energy to which an identical other electron or quark couples with a dimensionless coupling factor g. The fields are characterized by a Lagrangian density with the generic format

$$\mathrm{L} = -\frac{1}{2}\partial_\mu \Phi \partial^\mu \Phi + U(\Phi) + \rho\Phi , \tag{7}$$

in which $U(\Phi)$ is the potential energy of an energetic background field and in which $\rho\Phi$ is the source term. If the background energy and the source are known a spatial expression for Φ can be found as the solution of a field equation obtained from the Lagrangian density after application of the Euler-Lagange equation. If the source ρ is a scalar pointlike source in empty space, in which $U(\Phi)$ is zero, the result is the Coulomb field,

$$\Phi = \frac{\Phi_0}{r'} ; \quad r' = r\lambda , \tag{8}$$

If the source ρ is a scalar pointlike source in a background field that consists of polarizable dipoles, such that

$$U(\Phi) = U_{DB} = \lambda_{DB}^2 \frac{\Phi^2}{2} , \tag{9}$$

the field equation is inhomogeneous Helmholtz equation [6], which for a pointlike source ρ shows the solution

$$\Phi_{DB} = \Phi_0 \frac{\exp(-\lambda_{DB} r)}{\lambda_{DB} r} . \tag{10}$$

This is the shielded field from a charged particle in ionic plasma, known as the Debije effect [7].

Dirac's Hamiltonian analysis on his spinor equation has revealed that a pointlike source is more that a pointlike scalar. His analysis has shown that a pointlike particle is subject to elementary virtual motions that show up in an angular momentum ***ħ*** and in a vibration momentum ***ħ/c***. This means that the pointlike source is equipped with two dipoles. In the case of electrons, subject to gamma constraints (2), the former one shows up as the well known anomalous magnetic dipole moment $e\hbar/2m_0$, while the latter one remains hidden as the imaginary anomalous electric dipole moment $\mathrm{i}e\hbar/2m_0 c$. This is different for a quark. Unlike an electron, the quark, subject to the gamma constraints (3), shows two real dipole moments [8,9]. Hence, in qualitative terms, the potential field of a quark along the axis of the polarisable dipole, can be expressed as,

$$\Phi(\lambda x) = \Phi_0 \exp(-\lambda x)\{\frac{1}{(\lambda x)^2} - w\frac{1}{\lambda x}\} , \tag{11}$$

in which λ (with dimension m^{-1}) is a measure for the range of the nuclear potential, in which Φ_0 (in units of energy, i.e. joule) is a measure for the quark's "charge", and in which w is a dimensionless weigh factor that relates the strength of the monopole field to the dipole field. Because of the limited range of nuclear force there must be a Debije-type energetic background field, such that the field decays as $\exp(-\lambda x)/\lambda x$. As shown in [5], the similar result can be derived from the Lagrangian (7) in which the background field $U(\Phi)$ has the format,

$$U(\Phi) = -\frac{\mu_H^2}{2}\Phi^2 + \frac{\lambda_H^2}{4}\Phi^4 . \tag{12}$$

For positive values of λ_H^2 and μ_H^2, it is a broken field that is zero for

$$\Phi_0 = (\mu_H / \lambda_H)\sqrt{2} , \tag{13}$$

known as the vacuum expectation value. The field (12) is known as the Higgs field, which in the standard model has been adopted by axiom with a rather artificial interpretation [10]. Like discussed in this text it can be explained in terms of the quark's vibration momentum ***ħ/c*** in conjunction with a Debije-type background field.

3. The photon and the gluon

Similarly as the (fermionic) electron, the quark is a source of bosons. From an inspection of (11) it is obvious the quark field consists of a far field next to a near field. Whereas the far field is due to the quark's monopole properties, the near field is due to its dipole properties. The far field has the same properties as an electric charge in an energetic background field as described by (9). Applying the action principle on the Lagrangian (7) yields a Proca-type wave equation,

$$\frac{1}{c^2}\frac{\partial^2 r\Phi}{\partial t^2}-\frac{\partial^2}{\partial r^2}r\Phi+\lambda^2 r\Phi=\rho_H(r,t)\,, \tag{14}$$

in which $\rho_H(r,t)$ is a Dirac-type pointlike source that can be expressed as,

$$\rho_H(r,t)=4\pi r\frac{\Phi_0}{\lambda}\delta^3(r)H(t)\,, \tag{15}$$

in which $H(t)$ and $\delta(r)$, respectively, are Heaviside's step function and Dirac's delta function. Figure 1 shows the solution of this wave function in a graphical format. The upper part shows the field building up to the eventual steady state shape. The lower part shows the transient pulse. This transient pulse is the nuclear equivalent of a gamma photon. Unlike a gamma photon, this equivalent is subject to dispersion. The dispersion is due to the λ^2 term in the Proca wave equation (12). This term is a consequence of the energetic ambient field, known as the Higgs field. Whereas a photon can be seen as a component of a gamma photon, a gluon can be seen as a component of a "gamma gluon".

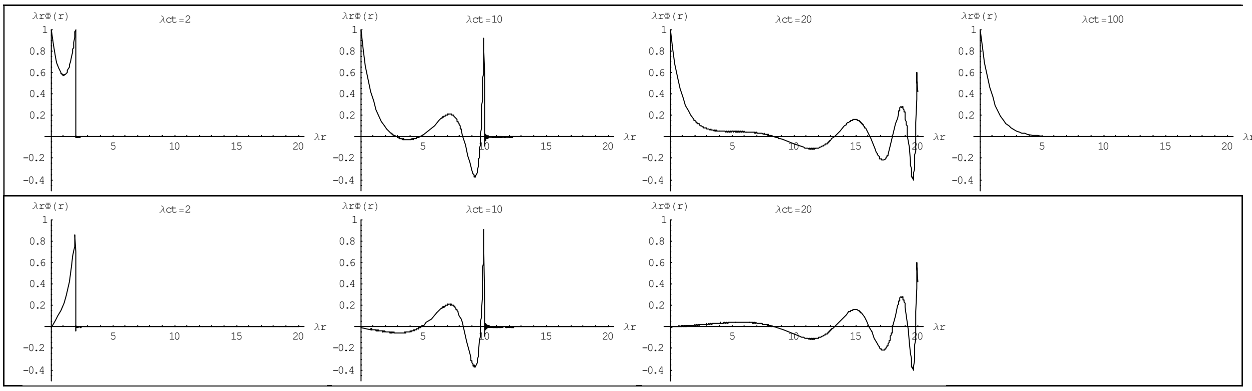

Fig.1. The building of the quark's potential far field as a result of a sudden energy eruption from its source. The field is the sum of the steady solution shown at the right and the transient pulse shown in the lower part of the figure. This pulse is a "gamma gluon". It propagates at light speed and it eventually disappears as a result of dispersion. If λ is zero, the transient is a never disappearing gamma photon and the stationary situation is shown by an unfinished rectangular shape of the upper most right graph. Note that the field is represented by $r\Phi(r)$.

4. The pion and the nucleon

Conceiving the pion as a structure in which a quark couples to the field of the antiquark by the generic quantum mechanical coupling factor g, the pion can be modeled as the one-body equivalent of a two-body oscillator, described by the equation for its wave function ψ. In its center of mass this wave equation is the non-relativistic approximation of the Dirac spinor equation (1). For its time-independent part, we may write [5],

$$-\frac{\hbar^2}{2m_m}\frac{\mathrm{d}^2\psi}{\mathrm{d}x^2}+\{U(d+x)+U(d-x)\}\psi = E\psi\,;\;\; U(x)=g\Phi(x)\,, \tag{16}$$

in which $\Phi(x)$ is the quark's scalar field as derived before and eventually expressed by (11), $2d$ the quark spacing, m_m the reduced mass that embodies the two massive contributions from the constituting quarks, $V(x)=U(d+x)+U(d-x)$ its potential energy, and E the generic energy constant, which is subject to quantization. It will be clear from (16) that the potential energy $V(x)$ can be expanded as,

$$V(x)=U(d+x)+U(d-x)=g\Phi_0(k_0+k_2\lambda^2x^2+....)\,, \tag{17}$$

in which k_0 and k_2 are dimensionless coefficients that depend on the spacing $2d$ between the quarks. To facilitate the analysis, (16) is normalized as,

$$-\alpha_0\frac{\mathrm{d}^2\psi}{\mathrm{d}x'^2}+V'(x')\psi = E'\psi\,, \tag{18}$$

in which $\alpha_0=\dfrac{\lambda^2\hbar^2}{2m_m g\Phi_0}$, $x'=x\lambda$, $d'=d\lambda$, $E'=\dfrac{E}{g\Phi_0}$, $U'(x')=\dfrac{U(\lambda x)}{g\Phi_0}$ and

$$V'(x')=U'(d'+x')+U'(d'-x')=k_0+k_2x'^2+.......$$

Invoking previous work [[11, eq. (24)] we get for α_0,

$$\alpha_0=\frac{k_0^2}{k_2}\,. \tag{19}$$

The two quarks in the meson settle in a state of minimum energy, at a spacing $2\lambda d=2d'_{\min}$, such that [11,12],

$$d'_{\min}=\lambda d=0.853;\; k_0=\text{-}1/2 \text{ and } k_2=2.36. \tag{20}$$

Figure 2 shows the structural pion configuration.

The archetype meson, the pion, is the two-quark oscillator in its ground state. The first excitation state transforms a pion into a kaon. The mass ratio between the two is the same as the mass ratio of the normalized energy constants $E' - k_0$. This is not trivial and it reflects the basic theorem of the theory. This theorem states that the energy wells of the two quarks are not massive. Instead, the mass attribute of two-quark junctions (mesons) and three-quark junctions (baryons) is made up by the vibration energy as expressed by the energy state of the quantum mechanical oscillator that they build. The distribution of this mass over constituent quarks is a consequence of this mechanism. Unfortunately, an analytical calculation of the $E' - k_0$ ratio of kaons over pions, is only possible for the quadratic approximation of the series expansion of the potential energy $V'(z')$. A more accurate calculation requires a numerical approach. A procedure to do so has been documented in [11, Appendix C]. It shows that some simple lines of code in Wolfram's *Mathematica* [13] may do the job. The numerically calculated ratio of the energy constants appears to be 3.57 instead of 3 as it would have been in the harmonic case. The result explains the excitation of the 137 MeV/c^2 pion mass to the 490 MeV/c^2 mass of the pseudoscalar kaon. This result gives a substantial support for the viability of the theory as developed in previous work.

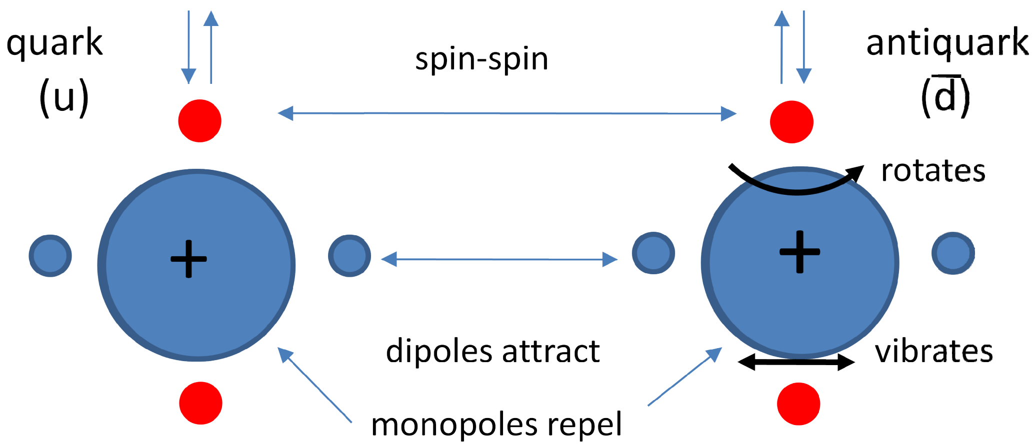

Fig. 2. A quark has two real dipole moments, hence two dipoles. One of these (horizontally visualized) is polarisable in a scalar potential field. The other one (vertically visualized) is not.. The polarity of the horizontal one is restrained by the bond: the horizontal dipoles are only oriented in the same direction: either inward to the centre or outward from the centre. The orientation of the dipole moments is unrelated from their isospin status.

Table I: meson excitation

Bottom level	E'_{bind} = - 1/2	mass ratio	mass in MeV/c^2
Ground state	$E'_0 - E'_{bind}$ = 0.84	1	137 (pion = 135-140)
First excitation	$E'_1 - E'_{bind}$ = 3.00	3.57	489 (kaon = 494-498)

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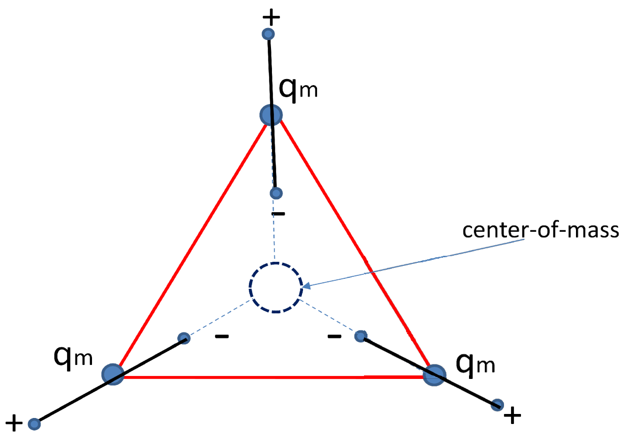
What about the nucleon? Figure 3 shows its basic configuration. It illustrates that the monopole fields of the quarks are balancing the fields of the polarisable dipole moments. Whereas a meson can be conceived as the one-body equivalent of a two-body harmonic oscillator, a baryon can be conceived as the one-body equivalent of a three-body harmonic oscillator. The one-body equivalent of the three-body quantum mechanical oscillator can be analyzed in terms of pseudo-spherical Smith Whitten coordinates [14]. The Smith-Whitten system of coordinates is six-dimensional. Next to a (hyper)radius ρ, the square of which is the sum of the squared spacings between the three bodies, there are five angles $\varphi, \vartheta, \alpha, \beta, \gamma$, in which φ and ϑ model the changes of shape of the triangular structure and in which α, β and γ are the Euler angles. The latter ones define the orientation of the body plane in 3D-space. The planar forces between three identical interacting bodies not only are the cause of dynamic deformations of the equilateral structure, but also are the cause of a Coriolis effect that result in vibra-rotations around the principal axes of inertia of the three-body structure [15].



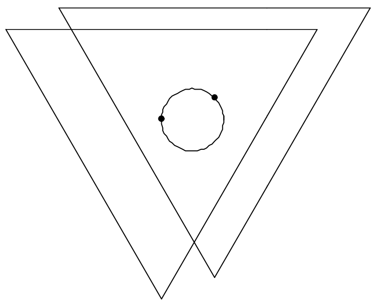


Fig.3: Left: the basic baryon structure as a harmonic oscillator. The polarisable dipole moments balance the fields of the monopoles. The vibra-rotations of the monopoles have an equivalent in the behavior of the center-of-mass. Right: illustration of the frame spin of the baryon.

The application of this approach for baryons has been documented by the author in [16], showing that the wave equation of the quasi-equilateral baryon structure can be formulated as

$$-\alpha_0 \{ \frac{d^2\psi}{d\rho'^2} + \frac{5}{\rho'}\frac{d\psi}{d\rho'} + \frac{R(m,v,k)}{\rho'^2}\psi \} + V'(\rho') = E'\psi ,$$

in which $\alpha_0 = \frac{\hbar^2\lambda^2}{6mg\Phi_0}$; $E' = \frac{E}{3g\Phi_0}$; $V' = \frac{V}{3g\Phi_0}$; $\rho' = \rho\lambda$, and

$$V(\rho') = 3g\Phi_0(k_0 + k_2\rho'^2 + ...) \qquad (21)$$

$$R(m,v,k) = 4m + |v-k|(4m + |v-k| + 4)$$

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This wave equation is the three-body equivalent of the pion's two-body wave equation shown in (46). In the ground state we have $m = 0$. Hence,

$$R = R(0, v, k) = l(l+4)\,;\; l = |v - k|\,. \tag{22}$$

The radial variable ρ is the already mentioned hyper radius. The potential field is just the threefold of the potential field in the wave equation of the pion. There are three quantum numbers involved. Two of those are left in the ground state, effectively bundled to a single one. The quantum number k allows a visual interpretation, while v is difficult to visualize. The impact of k is shown in the right hand part of figure 3. It illustrates the motion of the center of mass under influence of k. Note that this rotation is quite different from a rotation of the triangular frame around the center of mass. It is the center of mass itself that rotates, while the frame does not. Actually, the small motions of the individual quarks are responsible for this motion.

5. Spin-spin interactions

The simple anharmonic oscillator model described by (16-18) enables the mass spectrum calculation of the pseudoscalar mesons as excitations from the pion state. The excitation mechanism stops beyond the bottom quark due to the loss of binding energy. The mass spectrum calculation of the vector mesons requires the inclusion of the impact of the nuclear spin shown in the upper part of figure 2. A spin flip marks the difference between the pseudoscalar pion and the vector type sisters rho. The massive energy difference ΔE between the two types is a consequence of a spin-spin interaction process. It is of a similar nature as the analysis of the interaction process between the spin of electron and the spin of the proton nucleus in a hydrogen atom. Recognizing, though, that this is essentially a bosonic process, allows, in retrospect, a surprising simple approach. The step to be taken is conceiving the massive energy difference ΔE as a result of a bosonic interaction, mediated by Z bosons in virtual state. Because of the asymmetry in the spin-spin interaction ($-3\hbar^2/4$ and $+\hbar^2/4$), we have,

$$m'_\pi = 2m'_u - 3m''_Z \text{ and } m'_\rho = 2m'_u + m''_Z, \tag{23}$$

in which m'_π, m'_ρ, m'_u and m''_Z are the energies of, respectively, the rest masses of pion and the rho meson, the constituent massive energy of the u/d quark and the energy of the Z boson in virtual state in the rest frame of mesons. The statement that the energy of the rest mass of the pion is equal to the non-relativistic equivalent of the energy of the W boson enables to calculate the energy m''_Z of the Z boson in virtual state as,

$$m''_Z = m'_Z\left(\frac{m'_\pi}{m'_W}\right). \tag{24}$$

Under use of (23) and (24), the constituent rest mass energy m'_u of the u/d quark is calculated as

$$m'_u = \frac{1}{2} m'_\pi (1 + 3\frac{m'_Z}{m'_W}) . \qquad (25)$$

From (25) the energy of the constituent of the u/d quark is, under consideration of the energetic values of the weak interaction bosons m'_W = 80.4 GeV and m'_Z = 91.2 GeV , under adoption of the rest mass of the pion $m_\pi \approx$ 140 MeV/c^2, calculated as 308 MeV. Under use of this value, the energy of the rho meson is calculated from (61) and (62) as m'_ρ = 775 MeV. This is a perfect fit with experimental evidence!

We may go a step further by conceiving the spin-spin interaction energy as an add-on to the monopole interaction energy between the quarks. Doing so as described in Griffith's textbook, but now supported by theory rather than by empirics, we may compose a mass table for the light sector as shown in Table I and illustrated by figure 4. The constituent mass value for the s quark can now be calculated from the excitation result for the kaon as shown in Table I and the spin-spin interaction result as shown in Table II as,

$$m'_\kappa = m'_u + m'_S - 3\frac{m'^2_u}{m'_u m'_s} m''_Z = 3.57 m'_\pi \rightarrow m'_s = 489 \text{ MeV}. \qquad (26)$$

Table II: Mass formulae for mesons in the light sector

excit.		ms	pseudoscalar	mass	ms	vector	mass
ground state	$u\bar{u}$	π	$2m'_u - 3m''_Z$	140 (138)	ρ	$2m'_u + m''_Z$	780 (775)
		η	??	(549)	ω	$2m'_u + m''_Z$	780 (783)
first level (strange)	$u\bar{s}$	K	$m'_u + m'_s - 3\frac{m'^2_u}{m'_u m'_s} m''_Z$	484 (496)	K^*	$m'_u + m'_s + \frac{m'^2_u}{m'_u m'_s} m''_Z$	896 (892)
	$s\bar{s}$	η'	??	(958)	φ	$2m'_s + \frac{m'^2_u}{m'^2_s} m''_Z$	1032 (1020

Calculated values in MeV/c^2, actual values between brackets.

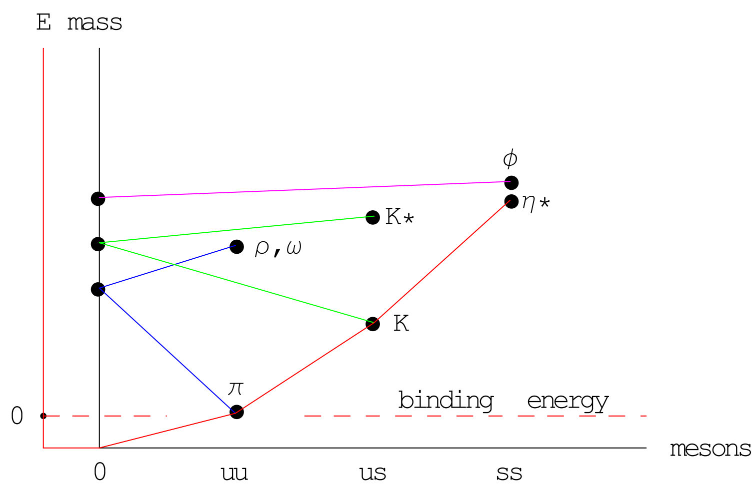

Figure 4. Illustration of the excitation mechanism of mesons and the scaling mechanism of quarks in the light sector.

Figure 3 is a graphical representation of Table II. It has to be emphasized that the constituent masses of the quarks in the mesons don't represent the (bare) gravitational masses. The constituent masses represent the massive equivalent of the lab frame value of the meson energy distributed over the quarks. Doing the same for baryons, the constituent masses of the quarks in the baryon may show different values. Similarly as in the case of mesons the baryon mass is the result of the monopole interaction energies between the constituent quarks corrected by spin-spin interaction energy carried by the Z boson in virtual state. This allows to compose the mass table for baryons in the light sector, like shown in Table III, as motivated in [16]. The application of the one-body anharmonic oscillator model for the three-quark structure as shown in (21) enables the assessment of quantitative values for the baryonic constituent masses by theory instead of doing so by an empirical numerical fit on measured mass values. To do so, the energy ratio between the ground state value and the first (orbital) excitation ($(m=0; l=0)$ is calculated from (21) by the numerical model documented in [15]. This ratio happens to be

$$\frac{(E'-k_0)|_{l=1}}{(E'-k_0)|_{l=0}} = \frac{10.697}{7.254} = 1.475 \tag{27}$$

This ratio happens to be quite accurately the mass ratio of the nucleon in ground state (938.3 and 939.6 MeV/c^2) over the Σ^* baryon in excited orbital state (1882.7, 1383.7 and 1387.2 MeV/c^2). It shows a similar accuracy as the mass ratio calculation from the pion state into kaon state. (Why the nucleon excites to Σ^* instead of Σ is not obvious, though). It gives an anchor point for the mass calculations shown in table III. The major one, however, is the calculation of the nucleon mass. Let us proceed by considering that in absence of spin-spin interaction, the excited three m'_u baryon would be equivalent with the ground state of a three m'_s baryon. Hence,

$$\frac{m'_s}{m'_u} = 1.475. \tag{28}$$

In the actual situation of spin-spin interaction, however, the baryon ground state, excites under the same ratio into the Σ^* state. Hence,

$$\frac{2m'_u + m'_s + (1 + \frac{2m'_u}{m'_s})\frac{m''_Z}{3}}{3m'_u - m''_Z} = 1.475. \tag{29}$$

From (28) and (29),

$$m'_u = \frac{(3a + 2/a + 1)}{2(a-1)}\frac{m''_Z}{3}; \quad a = 1.475. \tag{30}$$

Moreover, from (24),

$$m''_Z = (\frac{m'_Z}{m'_W})m'_\pi. \tag{31}$$

Table III: Mass formulae for baryons in the light sector

sym	Spin 1/2 (octet)	m	sym	Spin 3/2 (decuplet)	m
N (2)	$3m'_u - m''_Z$	939 (939)	Δ (4)	$3m'_u + m''_Z$	1246 (1232)
Λ^0 (1)	$2m'_u + m'_s - m''_Z$	1113 (1116)			
Σ (3)	$2m'_u + m'_s + (1 - 4\frac{m'_u}{m'_s})\frac{m''_Z}{3}$	1179 (1190)	Σ^* (3)	$2m'_u + m'_s + (1 + 2\frac{m'_u}{m'_s})\frac{m''_Z}{3}$	1386 (1385)
Ξ (2)	$m'_u + 2m'_s + \frac{m'^2_u}{m'^2_s}(1 - 4\frac{m'_s}{m'_u})\frac{m''_Z}{3}$	1324 (1320)	Ξ^* (2)	$m'_u + 2m'_s + \frac{m'^2_u}{m'^2_s}(1 + 2\frac{m'_s}{m'_u})\frac{m''_Z}{3}$	1532 (1530)
			Ω^- (1)	$3m'_s + \frac{m'^2_u}{m'^2_s}m''_Z$	1683 (1672)

Calculated values in MeV/c^2, actual values between brackets.

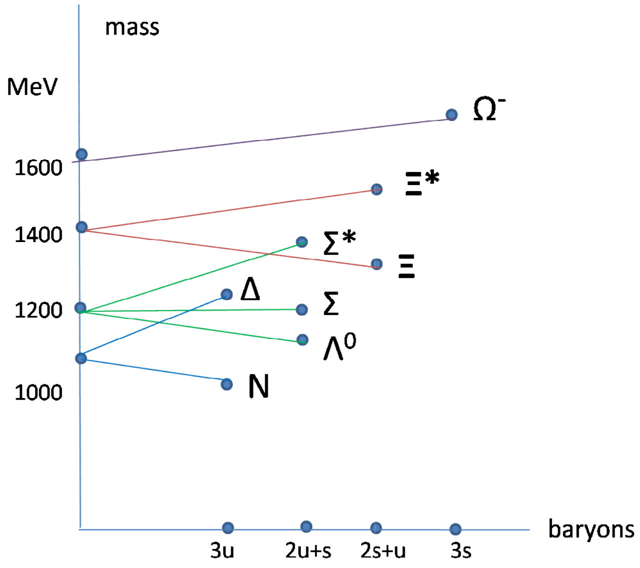

Figure 5. Illustration of the excitation mechanism of baryons in the light sector. The dots on the vertical axis represent the baryonic constituent masses values 3u, 2u +s, 2s + u, 3s.

From (30) and (31) the nucleon mass as shown in Table II can be related with the mass of the pion m'_π as,

$$m'_N = \frac{a+2/a+3}{2(a-1)}\left(\frac{m'_Z}{m'_W}\right)m'_\pi; \quad a = 1.475. \tag{32}$$

This expression relates the nucleon mass with the mass of the pion. It is not clear whether the neutral pion mass should be taken as reference or the mass of the charged pion. With the neutral pion mass (m'_π = 135 MeV) and the weak interaction bosons (m'_W = 80.5 GeV) and (m'_Z = 91.2 GeV), the nucleon mass is calculated from (32) as 939 MeV/c^2. It means that the mass value of the nucleon can be derived from the pion's rest mass as reference.

6. Spin and isospin

While the nuclear spin-spin interaction has a significant impact on the mass attribute of mesons and baryons, it has no impact on the charge attribute. Charge is determined by the quark's isospin. In the standard model of particle physics, this isospin is an axiomatic property of the quark without a known physical interpretation. Because it has similar properties as nuclear spin and because nuclear spin is related with the quark's angular dipole moment **ħ**, one may expect that isospin is related with the quark's linear dipole moment **ħ/c**. As discussed in paragraph 2, this property has remained unknown up to its description in 2021 [7,8]. However, whereas the nuclear spin state allows a direct interpretation as the spatial orientation of the angular dipole moment, the spatial orientation of the linear dipole moment is structurally bound. This seems to prevent the statistical freedom of isospin in the case of a one-to-one relationship between linear dipole moment and isospin. The relationship is however not as simple as that. Because of another issue in this context, namely the not yet considered phenomenon that the nuclear spin-spin interaction as discussed in the previous paragraph is a Chinese copy of the Maxwellian spin-spin interaction between the spins of the electron and the proton in atomic Hydrogen. It is therefore a logic step to adhere Maxwellian properties to a quark. A straightforward way to do so is to conceive the quark as a Maxwellian magnetic monopole with a real electric dipole moment and a real magnetic dipole moment. This real magnetic dipole moment reveals the presence of an elementary pointlike amount of electric charge associated with the magnetic quark monopole. If so, the spin of this electric kernel may either belong to negative or to a positive electric kernel. This ambiguity is equivalent to assign the isospin status to the linear dipole moment.

It mIght be seen as a revival of Schwingers's suggestion from 1969 that a quark is a *dyon*, i.e., both an electric monopole and a magnetic monopole [17]. However, whereas Schwinger proposed his dyon to explain the (quasi) stable hadron structures from a balance between an attracting magnetic force and a repelling electric force (or vice versa), the balance in the structure shown in figure 2 is obtained by the balance between the monopole field and the field evoked by the polarizable dipole moment under a scalar potential. Electric charge pops up as a minor side effect, hence not as a gluing force. This marks a fundamental difference between Schwinger's dyon (abandoned and replaced by QCD) and the structure shown in figure 2.

7. The proton and the neutron

An intriguing problem in particle physics theory is the one how to explain the mass difference between a charged pion and a neutral pion on the one hand and the mass difference between the charged nucleon (proton) and the neutral nucleon on the other hand. It is too simple pointing to a difference in bare mass between an u quark and a d quark, if such a difference would exist. Lattice QCD is unable to prove a mass difference between an u quark and a d quark, because it is only the average mass over the two that can be retrieved. Therefore, the differentiation between the mass values is hypothesized on the basis of their (supposed) charge difference. Accepting the difference, would make the neutron (with two d and one u) heavier than the proton (with two u and one d). This, however, cannot explain why a charged pion (with two u and one d) is heavier than a neutral pion (with two d and one u). Before discussing this difference, let us first discuss the mass difference between the proton and the neutron. Under the acceptance of the existence of a second dipole moment next to the angular related one, the structural representation of baryons is shown by figure 3. As discussed in the previous paragraph, the quarks in this picture have isospin statistics that are independent from the structural orientation their dipoles. Associating electric charge with isospin and considering the interaction between the electric charges along the peripheral axes, a net amount of electric charge will cause a tiny increase of the baryon's (hyper)radius ρ. As a consequence, the energetic state of the baryon is reduced. It makes a charged baryon lighter than a neutral baryon. Although this phenomenon introduces some asymmetry in the symmetrical structure shown in figure 3, the Smith Whitten model allows to model asymmetrical effects as an equivalent symmetrical one. This consideration allows to distribute the charge of the proton uniformly along the axes of the structure, like shown in the left-hand part of figure 6.

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The pion case is illustrated in the right-hand part. Here, the difference between the charged pion and the neutral pion is modeled by a distribution (+e/2, +e/2) for a charged pion. Unlike as in the representation for baryons, the picture shows a spin arrow associated with the charge. This is to symbolize the spin-spin interaction between isospins. Electric interaction at extreme short spacing is more than Coulombian. Electric charge kernels evoked by the quark's second dipoles have their own (magnetic) spin, which can be seen as the manifestation of isospin. These spins interact. At extreme short spacing the interaction energy involved is larger than the Coulombian interaction energy. In charged pions, the isospins are parallel. This explains why charged pions are in a state of energy higher than neutral pions in spite of the opposite effect due to Coulombian interaction. A calculation of its quantitative effect can be found in [5]. It is beyond the scope of this article.

Although this isopin interaction effect is present in the baryon as well, the net effect of it is the same for protons and neutrons (their isospin sum is the same). Being interested in the mass difference between protons and neutrons only, there is no need for further elaboration on the isospin interaction. Let us model the impact of the electrical interaction instead. Concluding that the electromagnetic interaction increases the interaction strength $F_F(r)$ of the far field of a quark, we may write for the latter,

$$F_F(r) = g\frac{\Phi_0}{\lambda r}\exp(-\lambda r) + p\frac{e^2}{4\pi\varepsilon_0 r^2}, \tag{33}$$

in which p is a measure for the effectiveness of the electric interaction force along the axes of the baryon. Under adoption of the symmetrical charge distribution as shown in figure 6, we have $p =$ 1/36. Let us rewrite (33) in terms of the electromagnetic fine structure relationship,

$$e^2 = 4\pi\varepsilon_0\hbar c g_e^2, \tag{34}$$

in which g_e^2 is the well-known fine structure constant $g_e^2 = \alpha_{em} \approx 1/137$. Hence, from (34) and (33), the the far field potential $\Phi_F(r)$ can now be written after including the influence of the electric interaction as,

$$\Phi_F(r) = \frac{\Phi_0}{\lambda r}\exp(-\lambda r) + \frac{p}{4}\frac{g_e^2\lambda\hbar c}{\lambda r}. \tag{35}$$

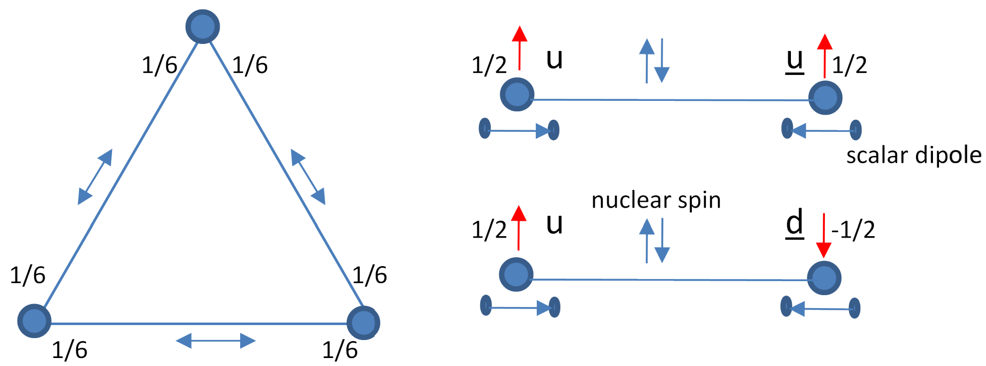

Fig.5. The left-hand part shows the proton's structural model with equal charge distribution along the axes. The upper part at the right shows the model for the charged pion, the lower part shows the model for the neutral pion. The nuclear dipoles are shown as up/down arrows in the middle, the second dipoles are shown as horizontal arrows. These latter dipoles evoke kernels of electric charge with their own (iso)spin. In parallel condition ($u\overline{u}$) their spin-spin interaction increases the energetic state of the pion by an amount larger than the increase of electric interaction as present in the neutral pion.

Now, the potential $\Phi(x')$ of the field built up by the quarks with inclusion of an electrical potential $\Phi_{em}(x')$, can be written as,

$$\Phi(x') = \Phi_N(x') - \Phi_F(x') - \Phi_{em}(x') \text{ with } x' = \lambda x,$$

$$\Phi_N(x') - \Phi_F(x') = \Phi_0(k_0 + k_2 x'^2 + ...),$$

$$\Phi_{em}(x') = pw\frac{g_e^2}{4g}\frac{(\hbar c)\lambda}{d'}(2 + \frac{2}{d'^2}x'^2 + ...);\ d' = d\lambda . \tag{36}$$

It is not difficult to modify the numerical procedure of the anharmonic oscillator accordingly, Doing so, we find

$$\frac{(E' - k_0)|_{neutron}}{(E' - k_0)|_{proton}} = \frac{7.2541}{7.2418} = 1.00169 \tag{37}$$

This corresponds with a mass difference of (1.00169 – 1) 936 = 1.58 MeV/c^2. It is slightly above the known experimentally established value of 1.29 MeV. It is close to the 1.51 MeV/c^2 value obtained by lattice QCD [2], which is built up as a QED correction of about 1 MeV/c^2 on the retrieved masses of the u quark (2.3 MeV/c^2) and the d quark (4.8 MeV/c^2) [1,2]. Curiously, whereas in present canonical theory the u/d masses are no longer be expressed in terms of constituent masses, it is still the case for the other quark flavors [1]. Within the view on quarks as presented in this article a retrieval of quark masses from the reference rest masses of the pion and the kaon is not required. Instead, it is based upon the theoretical view exposed in previous work [5], in which the charge asymmetry between the u quark and the d quark is challenged.

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8. Discussion and conclusion

It has already been concluded at the end of paragraphs 5 and 7 that the capabilities to calculate the masses of the nucleons and the mass difference between a proton and a neutron on the basis of a rather simple structural model have a similar accuracy as those claimed from lattice QCD. This article is a summary, application and extension of previous work [5,8,9,11,12,16]. It is based upon a somewhat different view on quarks as compared to canonical theory. In this view the quark is a non-gravitational particle without baryonic mass that sources a nuclear field capable to bind other quarks in mesons and baryons, which behave as quantum mechanical oscillators in a state of baryonic energy. The quark is a Dirac particle, but in an unrecognized mode. This mode allows to interpret isospin as the physical manifestation of a real second dipole moment. This second dipole moment (which has an imaginary value in electron-type Dirac particles) enables to give hadrons a clear structural interpretation as a substitute for the axiomatically conceived electroweak GWS model (Glashow, Weinberg, Salam). The article is a rebuttal to opponents who have put the Lorentz covariance of the novel Dirac particle mode into doubt and who doubted about the capability to calculate the masses of nucleons from the same reference values as lattice QCD may do.

Appendix: Lorentz covariance of a non-gravitational object

Whereas a gravitational object is subject to the Einsteinean energy expression,

$$W_G^2 = (m_0 c^2)^2 + c^2 |\mathbf{p}|^2 , \qquad \text{(A-1)}$$

the quark, as a non-gravitational object, is subject to

$$W_Q^2 = (m_0 c^2)^2 - c^2 |\mathbf{p}|^2 . \qquad \text{(A-2)}$$

Defining $W_G^2 = (m_0 \frac{\mathrm{d}ct}{\mathrm{d}\tau})^2$, (A-3)

the Einsteinean energy expression (A-1) is conveniently expressed as,

$$(m_0 \frac{\mathrm{d}ct}{\mathrm{d}\tau})^2 = (m_0 c^2)^2 + (m_0 \frac{\mathrm{dx}}{\mathrm{d}\tau})^2 + (m_0 \frac{\mathrm{dy}}{\mathrm{d}\tau})^2 + (m_0 \frac{\mathrm{dz}}{\mathrm{d}\tau})^2 . \qquad \text{(A-4)}$$

which is equivalent with,

$$(\mathrm{d}ct)^2 - \{(\mathrm{dx})^2 + (\mathrm{dy})^2 + (\mathrm{dz})^2\} = (c\mathrm{d}\tau)^2 . \qquad \text{(A-5)}$$

Note: τ is proper time.

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Transforming this property to a different space-time frame (ξ,η,ζ,τ'), related by the Lorentz transform

$$x=\frac{1}{w}(\xi+v\tau')\,;\;\; t=\frac{1}{w}(\tau'+\frac{v\xi}{c^2})\,;\;\; w^2=1-\frac{v^2}{c^2}\,;\;\; y=\eta\,;\;\; z=\zeta\,, \qquad \text{(A-6)},$$

we get, after substitution of (A-6) into (A-5),

$$-\frac{1}{w^2}(1-\frac{v^2}{c^2})(\mathrm{d}\xi^2-c^2\mathrm{d}\tau'^2)-\mathrm{d}\eta^2-\mathrm{d}\zeta^2=(c\mathrm{d}\tau)^2\,. \qquad \text{(A-7)}$$

Note that τ' is different from proper time τ.

This result (A-7) has the same format as (A-4). It proves the Lorentz covariance of the Einsteinean energy expression.

In a similar view on the non-gravitational object (A-2),

$$(m_0\frac{\mathrm{d}ct}{\mathrm{d}\tau})^2=(m_0c^2)^2-\{(m_0\frac{\mathrm{dx}}{\mathrm{d}\tau})^2+(m_0\frac{\mathrm{dy}}{\mathrm{d}\tau})^2+(m_0\frac{\mathrm{dz}}{\mathrm{d}\tau})^2\}\,. \qquad \text{(A-8)}$$

which is equivalent with,

$$(\mathrm{d}ct)^2+\{(\mathrm{d}x)^2+(\mathrm{dy})^2+(\mathrm{dz})^2\}=(c\mathrm{d}\tau)^2\,. \qquad \text{(A-9)}$$

Transforming under the Lorentz transform gives

$$\begin{aligned}&\frac{1}{w^2}\mathrm{d}(c\tau+\frac{v\xi}{c})^2+\frac{1}{w^2}\mathrm{d}(\xi+v\tau')^2+\mathrm{d}\eta^2+\mathrm{d}\zeta^2=\\&\frac{1}{w^2}(1+\frac{v^2}{c^2})(\mathrm{d}\xi^2+c^2\mathrm{d}\tau'^2)+\mathrm{d}\eta^2+\mathrm{d}\zeta^2+\frac{4}{w^2}v\mathrm{d}\xi\mathrm{d}\tau'=(c\mathrm{d}\tau)^2\end{aligned} \qquad \text{(A-10)}$$

It is clear that the last term of the left hand part of (A-10) violates the Lorentz covariance. This seems being a show stopper for the existence of a non gravitational particle with the property shown in (A-2), as required for the existence of quarks with two real dipole moments. It is known however that quarks in isolation don't exist, such as formalized in the confinement axiom. Taking this into consideration, it might be that the Lorentz violating term in (A-10) disappears for two and three quarks in conjunction. The structural meson (an)harmonic oscillator model, shown in figure 2, demonstrates that the two quarks vibrate in opposite direction under a stationary position of their center-of-mass. It is therefore quite probable that the Lorentz violation of one of the quarks is cancelled by the opposite Lorentz violation of the other quark. This would make mesons composed by quarks with two real dipole moments Lorentz covariant in spite of the Lorentz covariance violation of quarks in isolation.

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