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Why do elementary particles have such strange mass ratios?

- The importance of quantum gravity at low energies -

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ABSTRACT

When gravity is quantum, the point structure of space-time should be replaced by a non-commutative coordinate geometry. This is true even for quantum gravity in the infra-red. Using the octonions as space-time coordinates, we construct a pre-spacetime, pre-quantum Lagrangian dynamics. We show that the symmetries of this non-commutative space unify the standard model of particle physics with $SU(2)_R$ chiral gravity. The algebra of the octonionic space yields spinor states which can be identified with three generations of quarks and leptons. The geometry of the space implies quantisation of electric charge, and leads to a theoretical derivation of the mysterious mass ratios of quarks and the charged leptons. Quantum gravity is quantisation not only of the gravitational field, but also of the point structure of space-time.

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I. WHEN IS QUANTUM GRAVITY NECESSARY?

Consider a massive object in a quantum superposition of it's two different classical position states A and B. The resulting gravitational field is also then in a superposition, of the field corresponding to position A and the field corresponding to position B. A clock kept at a field point C will not register a definite value of time, nor a measurement of the metric will yield a well-defined result [1]. Let us now imagine a thought experiment in which every object in today's universe is in a superposition of it's two different position states. The space-time metric will then undergo quantum fluctuations. Now, the Einstein hole argument shows that in order for space-time points to be operationally distinguishable, the manifold must be overlaid by a (classical) metric [2]. Therefore, in our thought experiment, the point structure of space-time is lost, even though the energy scales of interest are much smaller than Planck scale, and the gravitational fields are weak.

When we describe microscopic systems by the laws of quantum theory, we take it for granted that the universe is dominated by classical bodies, so that a background space-time can be achieved and is available for defining time evolution of quantum systems. However, if everything were to be quantised at once, in the sense of the afore-mentioned thought experiment, no classical time will be available, and yet we ought to be able to describe the dynamics. This is an example of quantum gravity in the infra-red: the action of the gravitational field is much larger than $\hbar$ (unlike for Planck scale quantum gravity), and yet the point structure of space-time is lost. The manifold has to be replaced by something non-classical: quantum gravity is quantisation not only of the gravitational field, but also of the point structure of space-time.

Since the energy scale is not a relevant criterion for deciding whether gravity is classical or quantum, we propose that a gravitational field is quantum in nature when one or more of the following three (energy independent) criteria are satisfied: (i) the time scales of interest are of the order of Planck time t_P; (ii) the length scales of interest are of the order of Planck length L_P; and (iii) every sub-system has an action of the order $\hbar$ (and is hence quantum and obeys quantum superposition). If (iii) holds but (i) and (ii) do not, we have quantum gravity in the infra-red. If (iii) holds along with (i) and (ii) then we have quantum gravity in the UV.

Put differently, there ought to exist a reformulation of quantum (field) theory, even at low

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energies, which does not depend on classical time. Such a reformulation is essential also for the standard model of particle physics. In fact we show that it helps us understand why the standard model has the symmetries it does, and why it's free parameters take the specific values they do, and also shows how to unify gravity with the other fundamental forces: electroweak and strong. We construct such a dynamics using Planck time t_P, Planck length L_P and Planck's constant $\hbar$ as the only three fundamental parameters in the theory. We note that in these units the low energy fine structure constant $\alpha_f = e^2/\hbar c \equiv e^2\ t_P/\hbar L_P \sim 1/137$ is order unity and hence quantum gravitational in origin (QG in IR). On the other hand, particles masses $m \sim \epsilon m_P \equiv \epsilon\hbar t_P/L_P^2$ are not, because $\epsilon \ll 1$. However, mass ratios (at low energies) can be, and in fact are, quantum gravitational in origin.

To achieve our goal, we build on Adler's pre-quantum theory, i.e. trace dynamics (TD) [3, 4]. Starting from classical Lagrangian dynamics, TD retains the classical space-time manifold, but all configuration variables and their canonical momenta are raised to the status of matrices (equivalently operators). This step is the same as in quantum theory; however the canonical Heisenberg commutation relations $[q, p] = i\hbar$ are not imposed. Instead, we have a matrix-valued Lagrangian dynamics, where the Lagrangian is the trace of a matrix polynomial made from matrix-valued configuration variables and their time derivatives (i.e. the velocities). A 'trace' derivative enables the derivation of Lagrange equations of motion, and a global unitary invariance of the trace Hamiltonian (this being an elementary consequence of invariance of the trace under cyclic permutations) implies the existence of the novel conserved Noether charge $\widetilde{C} \equiv \sum_i [q_i, p_i]$. The Hamiltonian of the theory is in general not self-adjoint, and dynamical evolution is not restricted to be unitary. Assuming this dynamics to hold on Planck time scale resolution, one asks what the averaged dynamics on lower energy scales will be, if one coarse-grains the dynamics on time scales much larger than Planck time. Using the techniques of statistical thermodynamics it is shown that if the anti-self-adjoint part of the Hamiltonian is negligible, the emergent dynamics is relativistic quantum (field) theory. The afore-mentioned Noether charge is equi-partitioned over all bosonic and fermionic degrees of freedom, and canonical commutation and anti-commutation relations emerge for the statistically averaged canonical variables, which obey the Heisenberg equations of motion. If the anti-self-adjoint part of the Hamiltonian becomes significant (this is enabled by large-scale quantum entanglement), spontaneous localisation results, leading to the quantum-to-classical transition and emergence of classical dynamics.

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II. REPLACING THE POINT STRUCTURE OF SPACETIME BY THE NON-COMMUTATIVE COORDINATE GEOMETRY OF THE OCTONIONS

Next, TD is generalised, so as to replace the 4D Minkowski space-time manifold by a higher dimensional non-commutative space-time, and incorporate matrix-valued pre-gravitation, thus taking TD to a pre-space-time, pre-quantum theory. Let us recall that in special relativity, given the four-vector $V^\mu = dt\,\hat{t} + dx\,\hat{x} + dy\,\hat{y} + dz\,\hat{z}$ connecting two neighbouring space-time points having a separation (dt, dx, dy, dz), one can define the line element $ds^2 = \eta_{\mu\nu}V^\mu V^\nu$ and the four-velocity dq^μ/ds of a particle having the configuration variable $q^\mu = (q^t, q^x, q^y, q^z)$. The action for the particle is $mc\int ds$ and the transition to curved space-time and general relativity is made by introducing the metric $g_{\mu\nu}$, i.e. $ds^2 = g_{\mu\nu}dx^\mu dx^\nu$, and writing down the action

$$S = \frac{c^4}{16\pi G}\int d^4x\,\sqrt{-g}\,R \;+\; \sum_i m_i c\int ds \;+\; S_{YM} \tag{1}$$

Here, S_{YM} stands for the action of Yang-Mills fields, and also includes their current sources.

We now generalise this action to construct a pre-spacetime, pre-quantum action principle [5] from which the sought for quantum theory without classical time emerges, and whose symmetries imply the standard model of particle physics and fix it's free parameters. The space-time coordinates (t, x, y, z) are replaced by a set $\{e_i,\; i = 0, 1, 2, ..., m-1\}$ of m non-commuting coordinates, to be specified shortly. The configuration variable q^μ for a particle is replaced by a matrix q_F whose entries are odd-grade Grassmann elements over the field of complex numbers (so as to represent fermions). q_F has m components q_F^i, one for each of the coordinates e_i, i.e. $q_F = (q_F^0\;e_0 + ...q_F^{(m-1)}\;e_{m-1})$. The point structure of space-time is lost; instead we have a non-commutative geometry, and the matrix-valued velocity is defined as $dq_F/d\tau \equiv \dot{q}_F$. Here, the newly introduced Connes time τ is a unique property of a non-commutative geometry; it is an absolute real-valued time parameter distinct from the non-commuting coordinates e_i, and is used to describe evolution [6].

To introduce pre-gravitation into trace dynamics, we recall the spectral action principle of Chamseddine and Connes, which expresses the Einstein-Hilbert action in terms of the eigenvalues of the (regularised) Dirac operator D_B on a Riemannian manifold, in a truncated

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heat kernel expansion [7]

$$Tr\ [L_P^2\ D_B^2] \sim \int d^4x\sqrt{g}\ \frac{R}{L_P^2} + \mathcal{O}(L_P^0) \sim L_P^2 \sum_n \lambda_n^2 \tag{2}$$

Eigenvalues λ_n of the Dirac operator are dynamical variables of general relativity [8]. Every eigenvalue λ_n is raised to the status of a canonical matrix momentum: $\lambda_n \to p_{Bn} \propto q_{Bn}/d\tau \equiv D_B$, where the bosonic matrix q_B (with even grade Grassmann elements as entries) is the corresponding newly introduced configuration variable, and has m matrix components q_B^m over the non-commuting coordinates e_i. We hence have N copies of the Dirac operator (n runs from 1 to N, with $N \to \infty$). The matrix dynamics trace Lagrangian [space-time part] for the n-th degree of freedom is then $L_P^2\ Tr\ (dq_{Bn}/d\tau)^2$ and the total matrix dynamics action [space-time part] is $S \sim \sum_n \int d\tau\ L_P^2\ Tr\ (dq_{Bn}/d\tau)^2$. Yang-Mills fields are represented by the matrices q_{Bn}, pre-gravitation by the $\dot{q}_{Bn}$, fermionic degrees of freedom by fermionic matrices q_{Fn} and their 'velocities' $\dot{q}_{Fn}$. Each of the n degrees of freedom has an action of its own, which is given by [9]

$$\frac{S}{\hbar} = \frac{a_0}{2}\int \frac{d\tau}{\tau_{Pl}}\ Tr \left[\dot{q}_B^\dagger + i\frac{\alpha}{L}q_B^\dagger + a_0\beta_1\left(\dot{q}_F^\dagger + i\frac{\alpha}{L}q_F^\dagger\right)\right] \times \left[\dot{q}_B + i\frac{\alpha}{L}q_B + a_0\beta_2\left(\dot{q}_F + i\frac{\alpha}{L}q_F\right)\right] \tag{3}$$

where $a_0 \equiv L_P^2/L^2$. The total action of this generalised trace dynamics is the sum over n of N copies of the above action, one copy for each degree of freedom, and replaces (1) in the pre-theory. This total action defines the pre-spacetime, pre-quantum theory, with each degree of freedom [defined by the above action] being an 'atom' of space-time-matter [an STM atom]. L is a length parameter [scaled with respect to L_P; q_B and q_F have dimensions of length] characterising the STM atom, and α is the dimensionless Yang-Mills coupling constant. β_1 and β_2 are two unequal complex Grassmann numbers.

The further analysis of this pre-space-time, pre-quantum theory proceeds just as for the pre-quantum trace dynamics. Equations of motion are derived, and there is a conserved charge as before. Assuming that the theory holds at Planck time scales, the coarse-grained emergent low-energy approximation obeys quantum commutation rules and Heisenberg equations of motion, and is the sought for reformulation of quantum theory without classical time. This emergent theory is also the desired quantum theory of gravity in IR. If sufficiently many STM atoms get entangled, the anti-self-adjoint part of the Hamiltonian

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becomes significant, spontaneous localisation results, and the fermionic part of the entangled STM atoms is localised. There emerges a 4D classical space-time manifold (labelled by the positions of collapsed fermions), sourced by point masses and gauge fields, and whose geometry obeys the laws of general relativity given by (1). Those STM atoms which are not sufficiently entangled remain quantum; their dynamics is described by the low energy pre-theory itself, or approximately by quantum field theory on the 4D space-time background generated by the entangled, collapsed fermions [the macroscopic bodies of the universe].

Note that the non-commutative coordinate system $\{e_i \; ; \; i = 1, 2, ..., n\}$ is not impacted by the coarse-graining. The averaging takes place only over the time-scale τ and hence over energy; therefore the non-commuting coordinates e_i remain valid at low energies as well. What then, should we choose as our e_i, in place of the four real numbers (t, x, y, z) which label the 4D space-time manifold in classical physics? We take clue from the normed division algebras, i.e. number systems in which the four operations of addition, subtraction, multiplication and division can be defined. There are only four such number systems: real numbers $\mathbb{R}$, complex numbers $\mathbb{C}$, quaternions $\mathbb{H}$, and the octonions $\mathbb{O}$. A quaternion $H = (a_0 e_0 + a_1 \hat{i} + a_2 \hat{j} + a_3 \hat{k})$ is a generalisation of complex numbers, such that the a_i here are reals, $e_0^2 = 1$ and

$$\hat{i}^2 = \hat{j}^2 = \hat{k}^2 = -1 \; ; \quad \hat{i}\hat{j} = -\hat{j}\hat{i} = \hat{k}; \quad \hat{j}\hat{k} = -\hat{k}\hat{j} = \hat{i}; \quad \hat{k}\hat{i} = -\hat{i}\hat{k} = \hat{j} \tag{4}$$

Quaternions are used to describe rotations in three dimensions, and the automorphism group formed by the three imaginary directions is $SO(3)$. Complex quaternions $\mathbb{C} \times \mathbb{H}$ generate the Lorentz algebra $SL(2, \mathbb{C}) \sim SO(1, 3)$ and the Clifford algebra $Cl(2)$, if one of the three quaternionic imaginary directions is kept fixed. If no direction is kept fixed, they generate the Lorentz algebra in 6D : $SL(2, \mathbb{H}) \sim SO(1, 5)$ and the Clifford algebra $Cl(3)$. However, they are not a big enough number system for unifying all the standard model symmetries with the Lorentz symmetry. Whereas, the octonions seem to be just right for that purpose!

An octonion is defined as $O = a_0 e_0 + a_1 e_1 + a_2 e_2 + a_3 e_3 + a_4 e_4 + a_5 e_5 + a_6 e_6 + a_7 e_7$ such that the a_i are reals, $e_0^2 = 1$, each of the seven imaginary directions $(e_1, e_2, ..., e_7)$ squares to -1, these directions anti-commute with each other, and their multiplication rule is given by the so-called Fano plane. Octonionic multiplication is non-associative. The imaginary directions form the automorphism group G_2, which is the smallest of the five exceptional

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Lie groups G_2, F_4, E_6, E_7, E_8 all of which have to do with the symmetries of the octonion algebra. F_4 is the automorphism group of the exceptional Jordan algebra: the algebra of 3×3 Hermitean matrices with octonionic entries, and E_6 is the automorphism group of the complexified exceptional Jordan algebra [10]. The octonions are our sought for non-commuting coordinates e_i on which the action principle (3) is constructed. They generate 10D space-time : $SL(2, \mathbb{O}) \sim SO(1,9)$. The coordinate geometry of the octonions dictates the allowed symmetry groups, and definition and properties of fermions such as quantisation of electric charge [11], value of the low energy fine structure constant [12], and mass-ratios [13]. The parameter L and the coupling constant α in (3) are determined by the algebra of the octonions, not by the dynamics of q_F and q_B. This way, not only does the geometry tell matter how to move, it also tells matter what to be. The dynamical variables (q_B, q_F) curve the flat geometry $\{e_i\}$; however even before the dynamics is switched on, the low-energy standard model of particle physics is fixed by the e_i, unlike when space-time is $\mathbb{R}^4$. The transition $e_i \rightarrow q_F^i e_i + q_B^i e_i$ is akin to the transition $\eta_{\mu\nu} x^\mu x^\nu \rightarrow g_{\mu\nu} x^\mu x^\nu$, with the important difference that the former transition takes place at the 'square-root of metric' level, as if for tetrads, and the matrices q_B and q_F incorporate standard model forces besides gravity, and also fermionic matter. In fact, with the redefinition $\dot{\tilde{Q}}_B \equiv (i\alpha q_B + L\dot{q}_B)/L$; $\dot{\tilde{Q}}_F \equiv (i\alpha q_F + L\dot{q}_F)/L$ the Lagrangian in (3) can be brought to the elegant and revealing form, as if describing a two-dimensional (because $\beta_1 \neq \beta_2$) free particle:

$$\frac{S}{\hbar} = \frac{a_0}{2} \int \frac{d\tau}{\tau_{Pl}}\, Tr \bigg(\dot{\tilde{Q}}_B^\dagger + \frac{L_p^2}{L^2} \beta_1 \dot{\tilde{Q}}_F^\dagger \bigg) \bigg(\dot{\tilde{Q}}_B + \frac{L_p^2}{L^2} \beta_2 \dot{\tilde{Q}}_F \bigg) \tag{5}$$

In this fundamental form of the action, the coupling constant α is not present. In fact α, along with mass ratios, emerges only after (left-right) symmetry breaking segregates the unifying dynamical variable $\dot{\tilde{Q}}_B$ into its gravitational part $\dot{q}_B$ and Yang-Mills part q_B.

III. SPINOR STATES FOR QUARKS AND LEPTONS, FROM THE ALGEBRA OF THE COMPLEX OCTONIONS

The automorphism group G_2 of the octonions has two maximal sub-groups $U(3) \sim SU(3) \times U(1)$ and $SO(4) \sim SU(2) \times SU(2)$, the first of which is the element preserver group of the octonions, and the second is the stabiliser group of the quaternions inside the octonions

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[14]. The two groups have a $U(2) \sim SU(2) \times U(1)$ intersection. Keeping one of the seven imaginary directions, say e_7, fixed, the remaining six directions can be used to form an MTIS (maximally totally isotropic subspace) and the following generators (along with their adjoints) for the Clifford algebra $Cl(6)$:

$$\alpha_1 = \frac{-e_5 + ie_4}{2}, \quad \alpha_2 = \frac{-e_3 + ie_1}{2}, \quad \alpha_3 = \frac{-e_6 + ie_2}{2} \tag{6}$$

(This is a covariant choice as all the imaginary directions are equivalent and interchanging any of them does not change the analysis or results). From here, one can construct spinors as minimum left ideals of the algebra, by first constructing the idempotent $\Omega\Omega^\dagger$ where $\Omega = \alpha_1\alpha_2\alpha_3$. The eight resulting spinors are

$$\begin{aligned} &\mathcal{V} = \Omega\Omega^\dagger; \quad V_{ad1} = \alpha_1^\dagger \mathcal{V}; \quad V_{ad2} = \alpha_2^\dagger \mathcal{V}; \quad V_{ad3} = \alpha_3^\dagger \mathcal{V}; \\ &V_{u1} = \alpha_3^\dagger\alpha_2^\dagger \mathcal{V}; \quad V_{u2} = \alpha_1^\dagger\alpha_3^\dagger \mathcal{V}; \quad V_{u3} = \alpha_2^\dagger\alpha_1^\dagger \mathcal{V}; \quad V_{e+} = \alpha_3^\dagger\alpha_2^\dagger\alpha_1^\dagger \mathcal{V} \end{aligned} \tag{7}$$

After defining the operator $Q = (\alpha_1^\dagger\alpha_1 + \alpha_2^\dagger\alpha_2 + \alpha_3^\dagger\alpha_3)/3$ as one-third of the $U(1)$ number operator we find that the states $\mathcal{V}$ and V_{e+} are singlets under $SU(3)$ and respectively have the eigenvalues $Q = 0$ and $Q = 1$. The states $V_{ad1}, V_{ad2}, V_{ad3}$ are anti-triplets under $SU(3)$ and have $Q = 1/3$ each, whereas the states V_{u1}, V_{u2}, V_{u3} are triplets under $SU(3)$ and each have $Q = 2/3$. These results allow Q to be interpreted as electric charge, and the eight states represent a neutrino, three anti-down quarks, three up quarks and the positron having the standard model symmetries $SU(3)_{color} \times U(1)_{em}$. Anti-particle states are obtained by complex conjugation. The eight $SU(3)$ generators can also be expressed in terms of the octonions and represent the eight gluons, whereas the $U(1)$ generator is for the photon. We hence see the standard model of particle physics emerging from the symmetries of the physical octonionic space, and the quantisation of electric charge is a consequence of the coordinate geometry of the octonions [11].

To see how the weak force (and electroweak) and chiral gravity emerge from the other maximal sub-group $SO(4) \sim SU(2) \times SU(2)$ we must consider three fermion generations and the larger exceptional Lie group E_6 because these symmetries are shared pair-wise across fermion generations, as shown in Fig. 1. Furthermore, the neutrino will be assumed to be Majorana, because only then the correct values of mass ratios are obtained [13]. Also, notably, E_6 is the only one of the exceptional groups which has complex representations.

IV. PRE-GRAVITATION AND MASS RATIOS FROM A LEFT-RIGHT SYMMETRIC EXTENSION OF THE STANDARD MODEL OF PARTICLE PHYSICS

The 78 dimensional exceptional Lie group E_6 is the automorphism group of the complexified Jordan algebra, and admits the sub-group structure shown in Fig. 1, as motivated by the discussion in [15]. E_6 contains three intersecting copies of $Spin(9,1) \sim SL(2,\mathbb{O})$ which have an $SO(8)$ intersection, and the triality property of $SO(8)$ motivates that there are exactly three fermion generations. In order to account for the symmetries of E_6 and to obtain chiral fermions, we now work with split bioctonions (instead of octonions, which are used in (5) i.e. before symmetry breaking) [16]. Embedded in the three $Spin(9,1)$ are

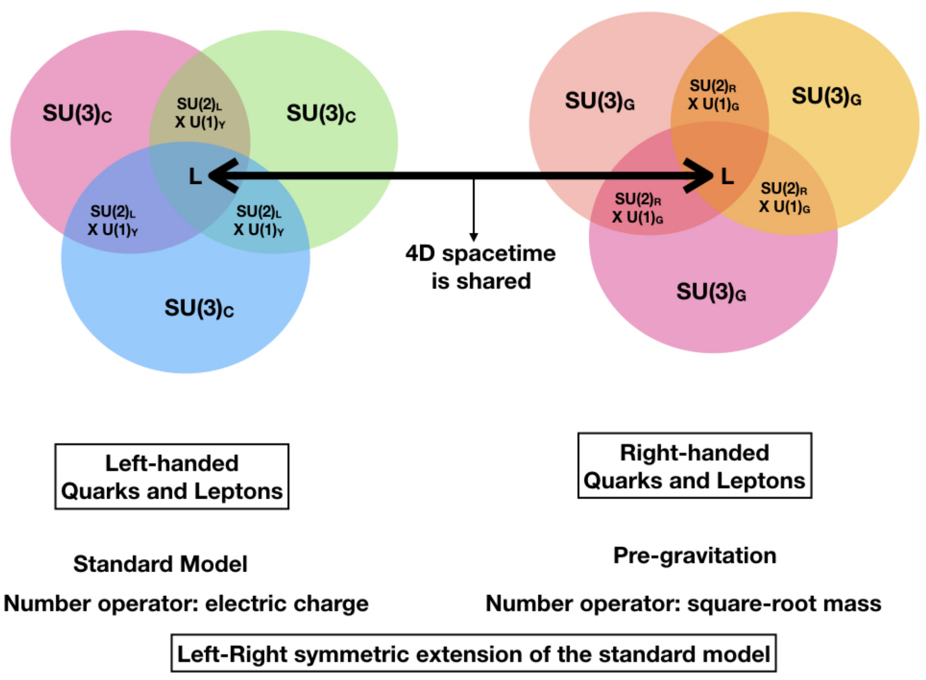

FIG. 1. Unification from symmetries of E_6

three copies of $SU(3)$, one of which is $SU(3)_{color}$ and one is for generational symmetry - this is shown in the left part of Fig. 1. There is a pairwise intersection amongst a pair of generations, which is the electroweak group $SU(2)_L \times U(1)_Y$ from which the weak interaction and electromagnetism can be obtained. There is a three way intersection marked L which is

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the 4D Lorentz group $SO(3,1)$. The $Cl(4)$ generators of $SU(2)_L$ are made from the $Cl(6)$ of $SU(3)_{color}$ and $Cl(2)$ of the Lorentz algebra, and it can be shown that $SU(2)_L$ acts only on left-handed fermions. The spinor states for the LH quarks and leptons of one generation are constructed analogously to those in (7), by using the left-handed active Majorana neutrino as the idempotent, with complex conjugation giving the corresponding antiparticles. The spinor states for the second and third generation are respectively obtained by applying two successive $2\pi/3$ rotations on the eight states of the first generation while staying in the plane defined by the form $(e_i + ie_j)$ of a given first generation particle [SO(8) symmetry implies eight independent great circles on an 8-sphere, one for each of the eight particles, and three particles of three generations]. There are a total of 24 LH fermions and their 24 anti-particles, and 12 gauge bosons. The unified symmetry group of Lagrangian (5) is E_6.

Similarly, three generations of RH fermions are obtained by using split octonions and the three RH sterile Majorana neutrinos as the idempotent. We identify the associated $SU(3)$ with $SU(3)_{grav}$ - a newly introduced gravitational sector; and identify the $U(1)$ number operator with [± square-root] of the mass of a quark / lepton (in Planck mass units), and the eight respective spinor states of one generation are: the sterile neutrino, three positrons of three different gravi-colors, three RH up quarks of three colors same as $SU(3)_{color}$, and one down quark which is a singlet under $SU(3)_{grav}$, along with the singlet sterile neutrino. These obtain the respective square-root mass number $(0, 1/3, 2/3, 1)$ explaining why the down quark is nine times heavier than the electron. The $SU(2)_R$ is RH chiral gravity (LQG?) [17] which reduces, in the classical limit induced by spontaneous localisation, to general relativity. The Lorentz group (whose Casimir invariant is the introduced mass number) is common with the LH particles, and its 6 dimensions, together with the 24 RH fermions and 12 new gauge bosons, when added to the LH sector, give the correct count of 78 for E_6. The split complex number gives a scalar field which acts as the Higgs mediating between the LH charge eigenstates, and RH mass eigenstates. Is $U(1)$ gravity the dark energy?

The group E_6 is also the symmetry group for the Dirac equation in 10D [15] for three fermion generations (either LH or RH). The eigenvalue and eigenmatrix problem for the Dirac equation is in fact the same as $J_3(8)X = \lambda X$ where $J_3(8)$ is the exceptional Jordan algebra with symmetry group F_4. Substituting the above-mentioned spinor states of LH fermions (these being eigenstates of electric charge) and solving this eigenvalue problem expresses the LH charge eigenstates as superpositions of the RH mass eigenstates (thus fixing

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α and L in (3)), and the ratios of the eigenvalues yield mass ratios of charged fermions as shown in Fig. 2; these exhibit very good agreement with the mysterious mass ratios [13].

E_6 has three copies of 10D spacetime. We never compactify the extra six complex dimensions - these represent the standard model internal forces which determine the geometry of these extra dimensions. Quantum systems do not live in 4D spacetime. They live in E_6 and their true dynamics is the generalised trace dynamics, with evolution given by Connes time. Only classical systems live in 4D spacetime, where they descend as a result of spontaneous localisation of highly entangled fermions (compactification without compactification). This overcomes the troublesome non-unique compactification problem of string theory.

Mass ratios: Square root of the mass of a charged fermion with respect to the down quark

Down quark	Strange quark	Bottom quark
1	$\frac{1+\sqrt{3/8}}{1-\sqrt{3/8}}$	$\frac{1+\sqrt{3/8}}{1-\sqrt{3/8}} \times \frac{1+\sqrt{3/8}}{1-\sqrt{3/8}} \times \frac{1+\sqrt{3/8}}{1}$
Up quark	**Charm quark**	**Top quark**
$2/3$	$\frac{2}{3} \times \frac{\frac{2}{3}+\sqrt{3/8}}{\frac{2}{3}-\sqrt{3/8}}$	$\frac{2}{3} \times \frac{\frac{2}{3}+\sqrt{3/8}}{\frac{2}{3}-\sqrt{3/8}} \times \frac{\frac{2}{3}}{\frac{2}{3}-\sqrt{3/8}}$
Electron	**Muon**	**Tau lepton**
$1/3$	$\frac{1}{3} \times \frac{1+\sqrt{3/8}}{1-\sqrt{3/8}} \times \frac{1/3+\sqrt{3/8}}{\lvert 1/3-\sqrt{3/8}\rvert}$	$\frac{1}{3} \times \frac{1+\sqrt{3/8}}{1-\sqrt{3/8}} \times \frac{1+\sqrt{3/8}}{1-\sqrt{3/8}} \times \frac{1/3+\sqrt{3/8}}{\lvert 1/3-\sqrt{3/8}\rvert}$

FIG. 2. Square-root mass ratios of charged elementary fermions [13]

In quantum theory, even at low energies, assuming a point structure for spacetime is an approximation; it is because of this approximation that the standard model of particle physics has so many unexplained free parameters. When we replace this approximate description by a non-commutative spacetime, we find evidence that these parameter values get fixed. In particular, we derive the low energy fine structure constant [12] and mass ratios of charged fermions [13] from first principles. We do not need experiments at ever higher energies to understand the low energy standard model. Instead, we need a better understanding of the quantum nature of spacetime at low energies, such that the quantum spacetime is consistent with the principle of quantum linear superposition.

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