Article

About Adaptive Identification of Bouc-Wen Hysteresis

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Abstract: The adaptive identification method developed to evaluate the parameters of the Bouc-Wen hysteresis (BWH). The adaptive approach based on the use of adaptive observers. We synthesize adaptive identification algorithms using the second Lyapunov method. The boundedness of adaptive system processes shown in coordinate and parametric spaces. We prove exponential dissipativity of processes in an adaptive system. Estimating method proposed for signaling uncertainty in the system.

Keywords: hysteresis; Ben-Wien model; adaptive observer; exponential dissipativity; Lyapunov vector function; uncertainty

1. Introduction

Various models [1] apply to the description of the hysteresis. But the Bouc-Wen model (BWM) [2, 3] has the widest application. Many BWM modifications [4-18] proposed. Each model considered the features of the object. The BWM successful application depends on the identification of its parameters. Various algorithms use for BWM identification. An adaptive parameter identification method [14] was proposed for the BW hysteresis model. Identification of BW hysteresis parameters based on time data consider in [15]. The algorithm bases on the least squares method and the sensitivity analysis of the output.

In [15, 17], adaptive algorithms propose for the BWM parameters estimation with the data forgetting [8]. Paper [18] presents an adaptive on-line identification methodology with a variable trace method to adjust the adaptation gain matrix.

Most proposed approaches use the derivative measurement by the output of the system. This possibility does not always exist in practical applications. There are studies [19] when estimates of BWM parameters do not coincide with the results obtained for other input data. Explain it with the fact that the BWM should be stable and ensure the adequacy of a physical process [4].

The conditions to be satisfied by the Bouc-Wen model considered in [20]. The major difficulties of the BWM parameters estimation are (i) the ensuring model stability (ii) the input choice. The stability imposes restrictions on the ranges of changes in model parameters. The choice of parameters belonging to the stability domain does not always give an adequate BWM [20].

So, the set of algorithms and procedures proposed for the Bouc-Wen model parameters identification. The models reflect the features of the system under study. As a rule, the change area BWM parameters set a priori. This is also true for some system parameters. It is often assumed that all system derivatives are measured. This assumption is not always true, which makes the algorithms unrealizable. Most identification procedures are valid only in some area. Therefore, the design of identification algorithms is an urgent problem for BWH under uncertainty.

Below, the adaptive identification method based on adaptive observer application use for the problem solution of the model (3) stability. It is based on the approach proposed in [21, 22] and does not require measuring derivatives of the system output. We believe that only the input and output of the system measures. BMW modifications consider. They reduce the use of the model and remove the stability problem.

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2. Problem Statement

Consider the system S_{BW}

$$m\ddot{x}+c\dot{x}+F(x,z,t)=f(t), \tag{1}$$

$$F(x,z,t)=\alpha kx(t)+(1-\alpha)kdz(t), \tag{2}$$

$$\dot{z}=d^{-1}\left(a\dot{x}-\beta|\dot{x}||z|^{n}\,sign(z)-\gamma\dot{x}|z|^{n}\right), \tag{3}$$

$$y(t)=x(t), \tag{4}$$

where $m>0$ is mass, $c>0$ is damping, $F(x,z,t)$ is the recovering force, $d>0$, $n>0$, $k>0$, $\alpha\in(0,1)$, $f(t)$ is exciting force, a,β,γ are some numbers.

The set of the experimental data

$$\mathrm{I}_o=\{f(t),\, y(t),\, t\in J\}, \tag{5}$$

where $J\subset R$ is the given time interval. Denote the system parameters vector as $A=[m,c,a,k,\alpha,\beta,\gamma,n]^T$.

Problem: design the adaptive observer for vector A estimation to

$$\lim_{t\to\infty}|\hat{y}(t)-y(t)|\le\pi_y \tag{6}$$

where $\hat{y}\in R$ is the output of the adaptive observer, $\pi_y\ge0$.

3. System modifications S_{BW}

Various modifications of BWH have proposed (see, for example, [9, 11, 20]). They consider features and properties of the system. System (1) - (3) is the basis for modifications. The analysis shows that the last term in (3) guarantee "fine-tuning" the BW hysteresis in the saturation or switching areas. If this is not critical for the system, then by selecting parameters of the S1-system, this term in the equation (3) can compensate. In addition, some modifications simplify and increase the system (1) - (3) stability. The main purpose of making structural changes is to simplify the system and improve its properties. We propose the following modifications of the Beech-Vienna model (3) [22]

$$\mathcal{M}_{\rho\omega\mu\upsilon\beta n}:\dot{z}=-\rho z|\dot{x}|^{\omega}+\pi|\dot{x}|^{\mu}\,sign(\dot{x})-\beta|\dot{x}|^{\upsilon}|z|^{n}\,sign(z), \tag{7}$$

$$\mathcal{M}_{\mu\beta n}:\dot{z}=\pi|\dot{x}|^{\mu}\,sign(\dot{x})-\beta|\dot{x}||z|^{n}\,sign(z), \tag{8}$$

$$\mathcal{M}_{\mu\upsilon\beta n}:\dot{z}=\pi|\dot{x}|^{\mu}\,sign(\dot{x})-\beta|\dot{x}|^{\upsilon}|z|^{n}\,sign(z). \tag{9}$$

The linear component in $\mathcal{M}_{\rho\omega\mu\upsilon\beta n}$ increases the feasibility model and stability of the system. As the system is nonlinear, the function $|\dot{x}(t)|^{\omega}$ introduces to ensure the required hysteresis state. It guarantees a change z in the specified boundaries. Parameters $\rho>0,\omega\ge0$ are some numbers.

A comparison of the models (7) - (9) and (3) is shown in Fig. 1. The representation (Fig. 1) allows comparing model properties by generalized indicators in the "minimum-maximum" space. Notation in Fig. 1: z is model (3), z1 is model $\mathcal{M}_{\rho\omega\mu\upsilon\beta n}$, z2 is model $\mathcal{M}_{\mu\beta n}$

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, z3 is model $\mathcal{M}_{\mu\upsilon\beta n}$; ♦ is average value; — is median; ○ is the extreme value (end of the "saturation" region). Designate the model (3) as $\mathcal{M}$.

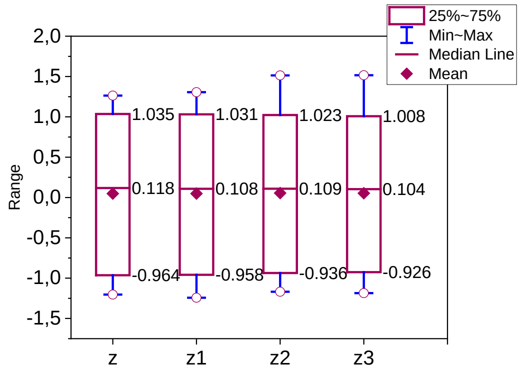

Figure 1. Comparison of hysteresis models (3), (42) - (44).

We have not tried to due reproduce BWH (3) using modifications (7) - (9). A detailed analysis of the models (7) - (9) parameters effect of on hysteresis given in [22].

4. About influence $f(t)$ on BWH parameters identifiability

The input choice is an important stage in the nonlinear systems identification. These issues are discussed in [22, 23]. The input $f(t)$ of the system must be constantly excited and have the property of S-synchronizability. These conditions are the basis for the structural identifiability of the system (1) – (3). They guarantee the system parameters evaluation using adaptive algorithms.

5. Design of adaptive observer

5.1. System S_{BW}

Let $d=1, a=1$. Substitute $F(x,z,t)$ in (1) and write it as

$$\left(s^2+\bar{a}_1 s+\bar{a}_2\right)x+\bar{a}_3 z=bf\ , \tag{10}$$

where

$$s=\frac{d}{dt},\ \bar{a}_1=\frac{c}{m},\ \bar{a}_2=\frac{\alpha k}{m},\ \bar{a}_3=\frac{(1-\alpha)k}{m},\ b=\frac{1}{m}.$$

Reduce (10) to an identification form on $\dot{x}$. Divide the left and right parts (10) into $s+\mu$, where $\mu>0$ does not coincide with roots of the polynomial $s^2+a_1 s+a_2$. Then (10)

$$\dot{x}=a_1 x+a_2 p_x+a_3 p_z+bp_f\ , \tag{11}$$

$$\begin{aligned}\dot{p}_x&=-\mu p_x+x,\\ \dot{p}_f&=-\mu p_f+f,\\ \dot{p}_z&=-\mu p_z+z,\end{aligned} \tag{12}$$

where

$$a_1=-\frac{c-\mu m}{m},\ a_2=-\frac{\alpha k-\mu(c-\mu m)}{m},\ a_3=-\frac{(1-\alpha)k}{m}.$$

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Equations (11), (12) contain only measurable variables except z. It complicates the identification of the system S_{BW} parameters.

Remark 1. Simplifications $d=1$ and $a=1$ do not affect the parameters (11) identification. Consideration d,a increases the number of estimated parameters.

Apply the model for parameters estimate of equation (11)

$$\dot{\hat{x}} = -k_x(\hat{x}-x) + \hat{a}_1 x + \hat{a}_2 p_x + \hat{a}_3 p_z + \hat{b} p_f \,, \tag{13}$$

where $k_x > 0$ is specified number, $\hat{a}_i(t)$, $i=1,2,3$ and $\hat{b}(t)$ are adjusted parameters.

Designate $e = \hat{x} - x$ and obtain the equation for the identification error from (11), (13)

$$\dot{e} = -k_x e + \Delta a_1 x + \Delta a_2 p_x + \Delta a_3 p_z + \Delta b p_f \,, \tag{14}$$

where $\Delta a_1 = \hat{a}_1(t) - a_1$, $\Delta a_2 = \hat{a}_2(t) - a_2$, $\Delta a_3 = \hat{a}_3(t) - a_3$, $\Delta b = \hat{b}(t) - b$.

The (14) is not solvable as the variable z is unknown in (12). Receive the current estimate for z. Consider the model

$$\dot{\hat{x}}_{\overline{z}} = -k_x(\hat{x}_{\overline{z}} - x) + \hat{a}_1 x + \hat{a}_2 p_x + \hat{b} p_f \,. \tag{15}$$

Determine the residual $\varepsilon_z = x - \hat{x}_{\overline{z}}$ and use it for the variable z estimation. Apply the model

$$\dot{\hat{z}} = -k_z\left(\hat{z} - \varepsilon_z\right) + \tilde{\dot{x}} - \hat{\beta}\left|\tilde{\dot{x}}\right|\left|\hat{z}\right|^n sign(\hat{z}) - \hat{\gamma}\tilde{\dot{x}}\left|\hat{z}\right|^n \,, \tag{16}$$

where $\tilde{\dot{x}} = \left(x(t+\tau) - x(t)\right)/\tau$; $k_z > 0$ is specified number; $\hat{\beta}$, $\hat{\gamma}$ are the hysteresis (3) parameters estimations; τ is the integration step.

Introduce the residual $\varepsilon = \hat{z} - \varepsilon_z$ and obtain the equation for ε

$$\dot{\varepsilon} = -k_z\varepsilon + \Delta\dot{x} + \Delta\beta\left|\tilde{\dot{x}}\right|\left|\hat{z}\right|^n sign\left(\hat{z}\right) + \beta\eta_\beta + \Delta\gamma\tilde{\dot{x}}\left|\hat{z}\right|^n + \gamma\eta_\gamma \,, \tag{17}$$

$$\eta_\beta = \left|\dot{x}\right|\left|z\right|^n sign\left(z\right) - \left|\tilde{\dot{x}}\right|\left|\hat{z}\right|^n sign\left(\hat{z}\right), \tag{18}$$

$$\eta_\gamma = \dot{x}\left|z\right|^n - \tilde{\dot{x}}\left|\hat{z}\right|^n \,, \tag{19}$$

where $\Delta\dot{x} = \tilde{\dot{x}} - \dot{x}$, $\Delta\beta = \beta - \hat{\beta}$, $\Delta\gamma = \gamma - \hat{\gamma}$.

Then the equation (13)

$$\dot{\hat{x}} = -k_x(\hat{x}-x) + \hat{a}_1 x + \hat{a}_2 p_x + \hat{a}_3 p_{\hat{z}} + \hat{b} p_f \,, \tag{20}$$

where

$$\dot{p}_{\hat{z}} = -\mu p_{\hat{z}} + \hat{z} \,. \tag{21}$$

Then (15)

$$\dot{e} = -k_x e + \Delta a_1 x + \Delta a_2 p_x + \Delta a_3 p_{\hat{z}} + \Delta b p_f \,. \tag{22}$$

Synthesize algorithms for tuning parameters of adaptive models. Consider the Lyapunov function (LF) $V_e(t) = 0{,}5e^2(t)$ and obtain for $\dot{V}_\varepsilon$

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$$\dot{V}_e = -k_x e^2 + e\left(\Delta a_1 x + \Delta a_2 p_x + \Delta a_3 p_{\hat{z}} + \Delta b p_f\right). \quad (23)$$

Obtain adaptive algorithms from the condition $\dot{V}_e \le 0$

$$\begin{aligned} \Delta\dot{a}_1 &= -\gamma_1 ex, \\ \Delta\dot{a}_2 &= -\gamma_2 e_x, \\ \Delta\dot{a}_3 &= -\gamma_3 ep_{\hat{z}}, \\ \Delta b &= -\gamma_b ep_f, \end{aligned} \quad (24)$$

where $\gamma_i > 0, i = 1,2,3$; $\gamma_b > 0$.

Synthesize algorithms for tuning model (16) parameters. Consider $V_\varepsilon(t) = 0,5\varepsilon^2(t)$. Then $\dot{V}_\varepsilon$

$$\dot{V}_\varepsilon = -k_z\varepsilon^2 + \varepsilon\left(\Delta\dot{x} + \Delta\beta|\tilde{x}||\hat{z}|^n sign(\hat{z}) + \beta\eta_\beta + \Delta\gamma\tilde{x}|\hat{z}|^n + \gamma\eta_\gamma\right). \quad (25)$$

We receive from (25)

$$\begin{aligned} \Delta\hat{\beta} &= -\chi_\beta\varepsilon|\tilde{x}||\hat{z}|^n sign(\hat{z}), \\ \Delta\dot{\gamma} &= -\chi_\gamma\varepsilon\tilde{x}|\hat{z}|^n, \end{aligned} \quad , \quad (26)$$

where $\chi_\beta > 0, \chi_\gamma > 0$ are parameters that ensure the algorithms convergence.

Several algorithms used to estimate the indicator n in (11). Their effectiveness depends on several factors. A simple algorithm has the form

$$\dot{\hat{n}} = \begin{cases} -\gamma_n\varepsilon\hat{\beta}|\hat{z}|^{\hat{n}-1}\hat{z}\tilde{x}, \text{если} \left|\dfrac{\varepsilon}{\varepsilon_z}\right| \in [\upsilon_0, \upsilon_1], \\ 0, \quad \text{если} \left|\dfrac{\varepsilon}{\varepsilon_z}\right| \notin [\upsilon_0, \upsilon_1], \end{cases} \quad (27)$$

where υ_0, υ_1 are set positive numbers, $\gamma_n > 0$.

So, equations (12), (17), (21), (22), (24), (26), (27) describe the adaptive identification system for the S_{BW} -system. Denote this system by AS_{BW}.

5.2. System (1), (2) with hysteresis $\mathcal{M}_{\rho\omega\mu\upsilon\beta n}$, $\mathcal{M}_{\mu\beta n}$, $\mathcal{M}_{\mu\upsilon\beta n}$

1. Model $\mathcal{M}_{\rho\omega\mu\upsilon\beta n}$. Equations (16) - (19) have the form in this case

$$\dot{\hat{z}} = -k_z(\hat{z} - \varepsilon_z) - \hat{\rho}\hat{z}|\tilde{x}|^\omega + \hat{\pi}|\dot{x}|^\mu sign(\tilde{x}) - \beta|\tilde{x}|^\upsilon|\hat{z}|^n sign(\hat{z}), \quad (28)$$

$$\dot{\varepsilon} = -k_z\varepsilon - \Delta\rho\hat{z}|\tilde{x}|^\omega + \Delta\pi|\tilde{x}|^\mu sign(\tilde{x}) - \Delta\beta|\tilde{x}|^\upsilon|\hat{z}|^n sign(\hat{z}) + \rho\overline{\eta}_\rho + \pi\overline{\eta}_\pi + \beta\overline{\eta}_\beta, \quad (29)$$

$$\overline{\eta}_\rho = |\dot{x}|^\omega z - |\tilde{x}|^\omega\hat{z}, \quad (30)$$

$$\overline{\eta}_\pi = |\tilde{x}|^\mu sign(\tilde{x}) - |\dot{x}|^\mu sign(\dot{x}), \quad (31)$$

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$$\bar{\eta}_\beta = |\dot{x}|^\upsilon |z|^n \, sign(z) - |\tilde{x}|^\upsilon |\hat{z}|^n \, sign(\hat{z}). \tag{32}$$

Consider $\dot{V}_\varepsilon$

$$\dot{V}_\varepsilon = -k_z \varepsilon^2 + \varepsilon\left(-\Delta\rho \hat{z} |\tilde{x}|^\omega + \Delta\pi |\tilde{x}|^\mu \, sign(\tilde{x}) - \Delta\beta |\tilde{x}|^\upsilon |\hat{z}|^n \, sign(\hat{z}) + \rho\bar{\eta}_\rho + \pi\bar{\eta}_\pi + \beta\bar{\eta}_\beta\right) \tag{33}$$

and obtain algorithms

$$\begin{aligned} \Delta\dot{\beta} &= \chi_\beta \varepsilon |\tilde{x}||\hat{z}|^n \, sign(\hat{z}), \\ \Delta\dot{\pi} &= -\chi_\pi \varepsilon |\tilde{x}|^\mu \, sign(\tilde{x}), \\ \Delta\dot{\rho} &= -\chi_\rho \varepsilon \hat{z} |\tilde{x}|^\omega, \end{aligned} \tag{34}$$

where $\chi_\beta > 0, \chi_\rho > 0, \chi_\pi > 0$ are parameters guaranteed convergence of algorithms; $\Delta\rho = \hat{\rho}(t) - \rho$, $\Delta\pi = \hat{\pi}(t) - \pi$.

The structure of algorithms for estimating n, ω, μ coincides with (27).

2. Model $\mathcal{M}_{\mu\beta n}$. Equations (16) - (19) have the form

$$\dot{\hat{z}} = -k_z(\hat{z} - \varepsilon_z) + \hat{\pi}|\tilde{x}|^\mu \, sign(\tilde{x}) - \hat{\beta}|\tilde{x}||\hat{z}|^n \, sign(\hat{z}), \tag{16.2}$$

$$\dot{\varepsilon} = -k_z\varepsilon - \Delta\beta|\tilde{x}||\hat{z}|^n \, sign(\hat{z}) + \Delta\pi|\tilde{x}|^\mu \, sign(\tilde{x}) + \beta\tilde{\eta}_\beta + \pi\tilde{\eta}_\pi, \tag{17.2}$$

$$\tilde{\eta}_\beta = |\dot{x}||z|^n \, sign(z) - |\tilde{x}||\hat{z}|^n \, sign(\hat{z}), \tag{18.2}$$

$$\tilde{\eta}_\pi = |\dot{x}|^\mu \, sign(\dot{x}) - |\tilde{x}|^\mu \, sign(\tilde{x}). \tag{19.2}$$

3. Model $\mathcal{M}_{\mu\upsilon\beta n}$. Equations (16) - (19) have the form

$$\dot{\hat{z}} = -k_z(\hat{z} - \varepsilon_z) + \hat{\pi}|\tilde{x}|^\mu \, sign(\tilde{x}) - \hat{\beta}|\tilde{x}|^\upsilon |\hat{z}|^n \, sign(\hat{z}), \tag{16.3}$$

$$\dot{\varepsilon} = -k_z\varepsilon - \Delta\beta|\tilde{x}|^\upsilon |\hat{z}|^n \, sign(\hat{z}) + \Delta\pi|\tilde{x}|^\mu \, sign(\tilde{x}) + \beta\hat{\eta}_\beta + \pi\hat{\eta}_\pi, \tag{17.3}$$

$$\hat{\eta}_\beta = |\dot{x}|^\upsilon |z|^n \, sign(z) - |\tilde{x}|^\upsilon |\hat{z}|^n \, sign(\hat{z}), \tag{18.2}$$

$$\hat{\eta}_\pi = |\dot{x}|^\mu \, sign(\dot{x}) - |\tilde{x}|^\mu \, sign(\tilde{x}). \tag{19.3}$$

Algorithms (16.2) and (16.3) structurally coincide with (34).

6. Properties AS_{BW}

Evaluate properties of the AS_{BW}-system. Consider the subsystem AS_X described by equations (22), (24). Let $\Delta K(t) \overset{\Delta}{=} [\Delta a_1(t), \Delta a_2(t), \Delta a_3(t), \Delta b(t)]^T$,

$$V_K(t) \overset{\Delta}{=} 0,5\Delta K^T(t)\Gamma^{-1}\Delta K(t), \tag{35}$$

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$$V(t) = V_e(t) + V_K(t), \tag{36}$$

where $\Gamma = diag(\gamma_1, \gamma_2, \gamma_3, \gamma_b)$.

Assumption 1. The input of the system (1) – (3) is constantly excited and bounded, i.e. the condition

$$\mathcal{PE}_\eta^f : f^2(t) \geq \eta \tag{37}$$

is valid for $\exists \eta > 0$ and $\forall t \geq t_0$ on some interval $[0, T]$.

Theorem 1. *Let (i) functions $V_e(t) = 0.5e^2(t)$, $V_K(t)$ are positive definite and satisfy conditions $\inf_{|e|\to\infty} V_e(e) \to \infty$, $\inf_{\|\Delta K\|\to\infty} V_K(\Delta K) \to \infty$; (ii) assumption 1 for $f(t)$ satisfied. Then (a1) all trajectories of the system AS_X bounded, (a2) belong area*

$$\mathrm{G}_t = \{(e, \Delta K) : V(t) \leq V(t_0)\},$$

(a3) the estimation

$$\int_{t_0}^{t} 2k_x V_e(\tau) d\tau \leq V(t_0) - V(t) \tag{38}$$

is fair.

We give the proof of Theorem 1 in Appendix A.

Theorem 1 shows the restriction of adaptive system AS_X trajectories. Ensuring of asymptotic stability in the system demands to impose additional conditions. Consider these conditions. Let $P(t) \triangleq [x(t)\ p_x(t)\ p_{\dot{z}}(t) p_f(t)]^T$.

Definition 1. The vector P is constantly excited with a level ν or have property $\mathcal{PE}_\nu$ if

$$\mathcal{PE}_\nu : \ P(t)P^T(t) \geq \nu I_4 \tag{39}$$

fairly for $\exists \nu > 0$ and $\forall t \geq t_0$ on some interval $T > 0$, where $I_4 \in R^4$ is the unity matrix.

If the vector $P(t)$ has property $\mathcal{PE}_\nu$, then we will write $P(t) \in \mathcal{PE}_\nu$.

The system S_{BW} is stable, and the input $f(t)$ is restricted. Therefore, present the property $\mathcal{PE}_\nu$ for the matrix $B_P(t) = P(t)P^T(t)$ as

$$\mathcal{PE}_{\nu,\bar{\nu}} : \ \nu I_l \leq B_P(t) \leq \bar{\nu} I_l \quad \forall t \geq t_0, \tag{40}$$

where $\bar{\nu} > 0$ is some number.

Let the estimate for $V_K(t)$ be fair

$$0.5\ \beta_l^{-1}(\Gamma) \|\Delta K(t)\|^2 \leq V_K(t) \leq 0.5\ \beta_1^{-1}(\Gamma) \|\Delta K(t)\|^2, \tag{41}$$

where $\beta_1(\Gamma)$, $\beta_l(\Gamma)$ are minimal and maximum eigenvalues of the matrix Γ.

Apply (40), (41) and get estimations for $\dot{V}_e, \dot{V}_K$

$$\dot{V}_e \leq -k_x V_e + \frac{\bar{\nu}\beta_l(\Gamma)}{k_x} V_K, \tag{42}$$

$$\dot{V}_K \leq -\frac{3}{4} \vartheta \nu \beta_1(\Gamma) V_K + \frac{8}{3} \vartheta V_e, \tag{43}$$

where $\vartheta > 0$ is some number. We describe the method of obtaining estimates (42), (43) in [24].

Theorem 2. *Let conditions be satisfied (i) positive definite Lyapunov functions* $V_e(t) = 0.5e^2(t)$ *and* $V_K(t) = 0.5\Delta K^T(t)\Gamma^{-1}\Delta K(t)$ *allow the indefinitely small highest limit at* $|e(t)| \to 0$, $\|\Delta K(t)\| \to 0$; *(ii)* $P(t) \in \mathcal{PE}_{\nu,\bar{\nu}}$; *(iii) equality* $e\Delta K^T P = \vartheta\left(\Delta K^T B \Delta K + e^2\right)$ *is fair in the area* $O_v(O)$ *with* $0 < \vartheta$, *where* $O = \{0,\ 0^{3m}\} \subset R \times R^{3m} \times J_{0,\infty}$, O_v *is some neighborhood of the point* O; *(iv) the function* $V_K(t)$ *satisfies (41); (v)* $\dot{V}_\varepsilon, \dot{V}_K$ *satisfy the system of inequalities*

$$\begin{bmatrix} \dot{V}_e \\ \dot{V}_K \end{bmatrix} \le \underbrace{\begin{bmatrix} -k_x & \dfrac{\bar{\nu}\,\beta_l(\Gamma)}{k_x} \\ \dfrac{8}{3}\vartheta & -\dfrac{3\nu\vartheta\beta_1(\Gamma)}{4} \end{bmatrix}}_{A_V} \begin{bmatrix} V_e \\ V_K \end{bmatrix}; \tag{44}$$

(vi) the upper solution for $V_{e,K}(t) = [V_e(t)\ V_K(t)]^T$ *satisfies to the comparison equation* $\dot{S} = A_V S$

if

$$V_\rho(t) \le s_\rho(t) \quad \forall (t \ge t_0) \,\&\, \left(V_\rho(t_0) \le s_\rho(t_0)\right), \tag{45}$$

where $\rho = e, K$, $S = [s_e\ s_K]^T$, $A_V \in R^{2\times 2}$ *is M-matrix [25]. Then the system* AS_X *is exponentially stable with the estimation*

$$V_{e,K}(t) \le e^{A_V(t-t_0)} S(t_0), \quad V_{e,K} = [V_e\ V_K]^T, \tag{46}$$

if

$$k_x > 0, \quad k_x \ge \frac{4}{3}\sqrt{\frac{2\bar{\nu}\beta_l(\Gamma)}{\nu\beta_1(\Gamma)}}. \tag{47}$$

Theorem 2 shows if $P(t) \in \mathcal{PE}_{\nu,\bar{\nu}}$, then the adaptive system AS_X gives accurate estimates of system (11) parameters. The system parameters must satisfy condition (47). We suppose that the variable $p_{\hat{z}}$ bounded.

The boundedness of the variable $\hat{x}_{\bar{z}}$ follows from the system stability.

Consider subsystem AS_Z described by equations (17), (25) and (26). Introduce Lyapunov functions

$$V_{\varepsilon\beta\gamma}(t) = V_\varepsilon(t) + V_{\beta,\gamma}(t), \tag{48}$$

$$V_{\beta,\gamma}(t) = 0.5\chi_\beta^{-1}(\Delta\beta(t))^2 + 0.5\chi_\gamma^{-1}(\Delta\gamma(t))^2. \tag{49}$$

Theorem 3. *Let (1) functions* $V_\varepsilon(t) = 0.5\varepsilon^2(t)$, $V_{\beta,\gamma}(t)$ *(49) are positive definite and satisfy condition*

$$\inf_{|\varepsilon| \to \infty} V_\varepsilon(\varepsilon) \to \infty, \quad \inf_{\|[\Delta\beta, \Delta\gamma]\| \to \infty} V_{\beta,\gamma}(\Delta\beta, \Delta\gamma) \to \infty;$$

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(2) the function $V_{\varepsilon\beta\gamma}(t)$ *has the form (88); (3) the function*

$$\tilde{g}_1(t) = \sup_{\varepsilon\in\Omega} \frac{|\varepsilon|^{n+1}(t)}{V_\varepsilon(t,\varepsilon)}, \quad g_1 = \sup_{\varepsilon\in\Omega} \tilde{g}_1(t), \tag{50}$$

exists, where Ω *is the definition range of the subsystem* AS_Z*; (4)* $|\Delta\dot{x}| \le \delta_\Delta$, $\delta_\Delta \ge 0$, $|\dot{x}| \le \upsilon$, $\upsilon > 0$*; (5) the assumption 1 holds for the system (1)-(3).). Then (i) all trajectories of the system* AS_Z *bounded, (ii) trajectories belong in the area*

$$\mathrm{G}_\varepsilon = \left\{(\varepsilon,\Delta\beta,\Delta\gamma) : V_{\varepsilon\beta\gamma}(t) \le V_{\varepsilon\beta\gamma}(t_0)\right\},$$

(iii) the estimation

$$\int_{t_o}^{t} (k_z - \upsilon(\beta+\gamma)g_1) V_\varepsilon(\tau)d\tau + \frac{1}{2\left(k_z - \upsilon(\beta+\gamma)g_1\right)(t-t_0)}(\delta_\Delta)^2 \le V_{\varepsilon\beta\gamma}(t_0) - V_{\varepsilon\beta\gamma}(t), \tag{51}$$

is fair if

$$k_z > \upsilon(\beta+\gamma)g_1. \tag{52}$$

We give the proof of Theorem 1 in Appendix B.

So, the boundedness of trajectories in the adaptive system AS_{BW} proved. The trajectories limitation of the subsystem AS_Z is a more complex problem in the parametric and output spaces. The estimation (51) shows that the quality of AS_Z-system processes depends on the output derivative of the S_{BW}-system. The guarantee of the AS_Z-system stability is the fulfillment of the condition (52). This conclusion explains problems in implementing various procedures for BWM identifying. The following result gives more exact estimations for AS_Z-system.

Theorem 4. *Let (i) positive definite Lyapunov functions*

$$V_\varepsilon(t) = 0.5\varepsilon^2(t), \quad V_{\beta,\gamma}(t) = 0.5\chi_\beta^{-1}(\Delta\beta)^2 + 0.5\chi_\gamma^{-1}(\Delta\gamma)^2 \tag{53}$$

allow the indefinitely small highest limit at $|\varepsilon(t)| \to 0$, $\|[\Delta\beta(t),\Delta\gamma(t)]\| \to 0$*; (ii)* $P(t) \in \mathcal{PE}_{\nu,\bar{\nu}}$*; (iii)* $c_1 > 0, c_2 > 0$ *exist such that*

$$\begin{aligned} \varepsilon\Delta\gamma\tilde{\dot{x}}|\hat{z}|^n &= c_2\left[(\Delta\gamma)^2\left(\tilde{\dot{x}}|\hat{z}|^n\right)^2 + \varepsilon^2\right], \\ \varepsilon\Delta\beta\left|\tilde{\dot{x}}\right||\hat{z}|^n\, sign(\hat{z}) &= c_1\left[(\Delta\beta)^2\left(\left|\tilde{\dot{x}}\right||\hat{z}|^n\right)^2 + \varepsilon^2\right] \end{aligned} \tag{54}$$

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in area $O_v(O)$, *where* $O=\{0,\ 0^2\}\subset R\times R^2\times J_{0,\infty}$, O_v *is some neighborhood of a point* O; *(iv) the inequality* $(\varepsilon-\varepsilon_z)^{2n}\geq c_z$ *holds for almost all* t, *where* $c_z\geq 0$; *(v)* $\pi_x\geq 0$ *u* $\omega>0$ *exist such that* $(\tilde{x})^2\geq\pi_x$ *and* $|\varepsilon-\varepsilon_z|\leq\omega|\varepsilon|$; *(vi) the function*

$$g_2(t)=\sup_{\varepsilon\in\Omega}\frac{|\varepsilon|^{2(n+1)}(t)}{V_\varepsilon(t,\varepsilon)},\ \ g_2=\sup_{\varepsilon\in\Omega}\tilde{g}_2(t), \tag{55}$$

exists, where Ω *the definition range of the subsystem; (vii)* $\dot{V}_\varepsilon$, $\dot{V}_{\beta,\gamma}$ *satisfy the system of inequalities*

$$\begin{bmatrix}\dot{V}_\varepsilon\\ \dot{V}_{\beta,\gamma}\end{bmatrix}\leq\underbrace{\begin{bmatrix}-(k_z-2\tilde{\upsilon}g_1-\omega\upsilon g_2) & \lambda\chi\omega\upsilon\\ c & -\dfrac{d_s}{2}\end{bmatrix}}_{A_\varepsilon}\begin{bmatrix}V_\varepsilon\\ V_{\beta,\gamma}\end{bmatrix}+\underbrace{\begin{bmatrix}\dfrac{1}{2k_z}\\ 0\end{bmatrix}}_{B_\varepsilon}(\delta_\Delta)^2. \tag{56}$$

(viii) the upper solution for $V_{\varepsilon,\beta,\gamma}=\left[V_\varepsilon(t)\,V_{\beta,\gamma}(t)\right]^T$ *satisfies to the equation*

$$\dot{\tilde{S}}=A_\varepsilon\tilde{S}+B_\varepsilon(\delta_\Delta)^2, \tag{57}$$

if

$$V_{\tilde{\rho}}(t)\leq\tilde{s}_{\tilde{\rho}}(t)\quad\forall(t\geq t_0)\,\&\,\left(V_{\tilde{\rho}}(t_0)\leq\tilde{s}_{\tilde{\rho}}(t_0)\right), \tag{58}$$

where $\tilde{S}=\left[\tilde{s}_\varepsilon\ \tilde{s}_{\beta,\gamma}\right]^T$, $\tilde{\rho}=\varepsilon,(\beta,\gamma)$, $A_\varepsilon\in R^{2\times 2}$ *is M-matrix. Then the system* AS_Z *is exponentially dissipative with the estimate*

$$V_{\varepsilon,\beta,\gamma}(t)\leq e^{A_\varepsilon(t-t_0)}\tilde{S}(t_0)+(\delta_\Delta)^2\int_{t_0}^{T}e^{A_\varepsilon(t-\tau)}B_\varepsilon d\tau, \tag{59}$$

if

$$k_z>2\tilde{\upsilon}g_1-\omega\upsilon g_2,\ \ (k_z-2\tilde{\upsilon}g_1-\omega\upsilon g_2)d_s>2c\lambda\chi\omega\upsilon,\ \ d_s>0,$$

where

$$\overline{\chi}=\min(\chi_\beta,\chi_\gamma),\ \ \overline{c}=\min(c_1,c_2),\ \ \chi=\max\left(\chi_\beta,\chi_\gamma\right),\ \ d_s=\chi\pi_x\overline{c}c_z.$$

So, the system AS_Z is exponentially dissipative. The dissipativity area depends on the informational set I_o of the S_{BW}-system.

Get results show the possibility of using adaptive observers to parameters identification of the S_{BW} -system.

7. Simulation results

Consider the engine control system (1) - (3) with parameters: $n=1.5$, $c=2$, $m=1$, $\beta=0.5$, $\gamma=0.2$, $\alpha=0.7$, $k=0.6$. Let $d=a=1$. Exciting force $f(t)=2-2\sin(0.15\pi t)$. The system is modeled with initial conditions $x(0)=1$, $\dot{x}(0)=0$, $z(0)=1$. Form the set I_o. The system phase portrait and output of the hysteresis shown in Fig. 2.

Estimate the structural identifiability of the system (1) - (3). Construct the structure $S_{\tilde{e}y}$ (Fig. 3) using the method [26]. A variable $\tilde{e}\in R$ is $\tilde{e}=\dot{x}-\hat{\dot{x}}_h$. $\hat{\dot{x}}_h$ is an estimation of the steady state (process) in the S_{BW} -system for $\forall t\geq 9.85$ s, and $\tilde{e}$ is the nonlinearity estimation in the corresponding space.

As follows from Fig. 2, 3, definition areas z and $\tilde{e}$ coincide. Analysis $S_{\tilde{e}y}$ shows that the system S_{BW} is structurally identifiable, and input $f(t)$ is S-stabilizing.

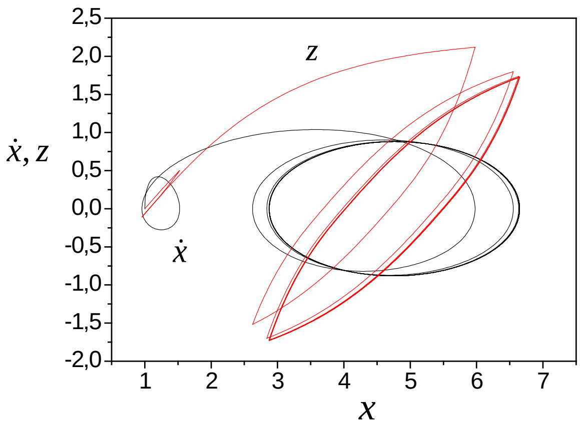

Figure 2. System phase portrait and hysteresis change.

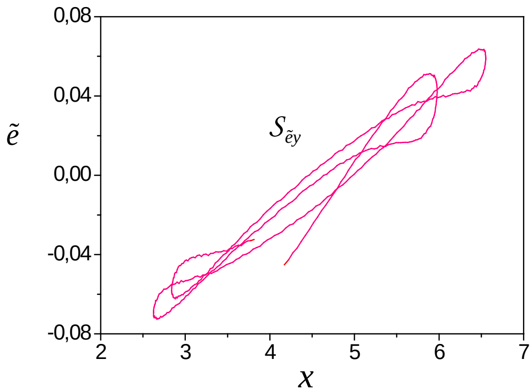

Figure 3. Structure $S_{\tilde{e}y}$ for assessing possibility of solving identification problem.

Consider the system parameters identification. Determine the parameter μ of the system (13) using the transient process analysis for $\tilde{e}$ and $t<9.85$ s. Calculate Lyapunov exponents (LE) [27]. The estimation for the maximum LE is –0.9. Therefore, we set $\mu=0.8$. Initial conditions in (12) are equal to zero.

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Adaptive system work results presented in Figures 4 – 6. Parameters k_x, k_z equal to 2.5 and 0.75. The tuning process of $\mathcal{AS}_X$ -systems parameters (the model (12)) shown in Fig. 4. Fig. 5 showed the model (16) parameters tuning.

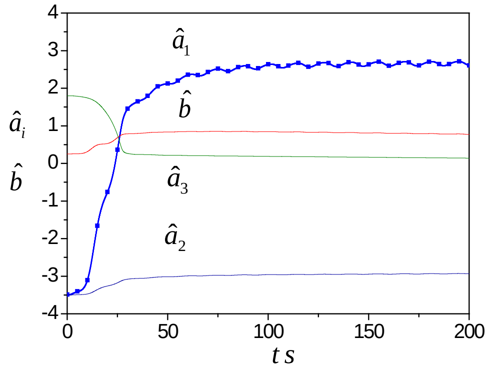

Figure 4. Tuning of model (13) parameters.

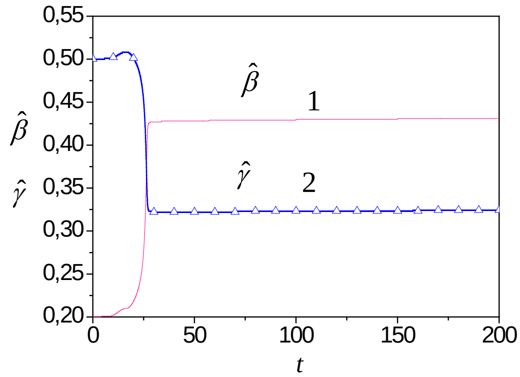

Figure 5. Tuning of model (16) parameters: 1 is tuning $\hat{\beta}$, 2 is tuning $\hat{\gamma}$.

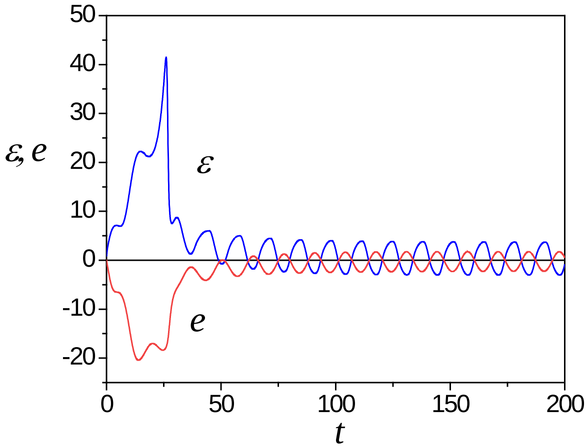

Figure 6. Outputs modification of systems AS_X, AS_Z.

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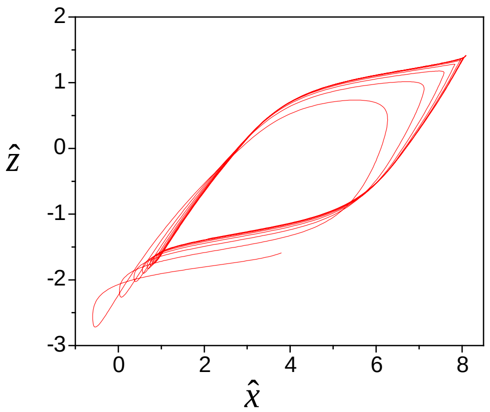

Figure 7. Hysteresis estimation at adaptation of AS_{BW} -system.

Show the modification of identification errors e, ε in Fig. 6. We see that the accuracy of obtained estimations depends on the numbers of tuned parameters, and the level $\dot{x}$ and properties $f(t)$. Obtained results confirm statements of theorems 3 and 4. The AS_Z -system work results influence the tuning processes in the AS_X -system. Gain coefficients in (25), (26) and (27) are $\chi_\beta = 0.0000002$, $\chi_\gamma = 0.0000002$, $\gamma_4 = 0.00005$ $\gamma_1 = 0.0002$, $\gamma_2 = 0.00001$, $\gamma_3 = 0.00002$. The parameter n in (16) is 1.5.

Remark 2. Modeling results of the system AS_{BW} with the algorithm (27) showed that the algorithm is sensitive to various perturbations, increases the adaptation time and requires further study.

The hysteresis output estimation shown in Fig. 7. Comparison of determination coefficients $r_{xz} = 0.864$ for the reference BWH in (Fig. 2) and the resulting BWH (Fig. 7) $r_{\hat{x}\hat{z}} = 0.764$ confirms the effectiveness of the proposed approach.

Fig. 8 presents comparing results estimates $\hat{z}$ and ε_z, obtaining in subsystems AS_X and AS_Z on the interval [25; 70]s. We analyze the dependence $\varepsilon_z(\hat{z})$ and show the approach effectiveness as the coefficient of determination is $r_{\hat{z},\varepsilon_z} = 0.91$. In Fig. 8, we represent the secant $\hat{\varepsilon}_z(\hat{z})$. Results confirm the adequacy of the obtained estimate $\hat{z}$.

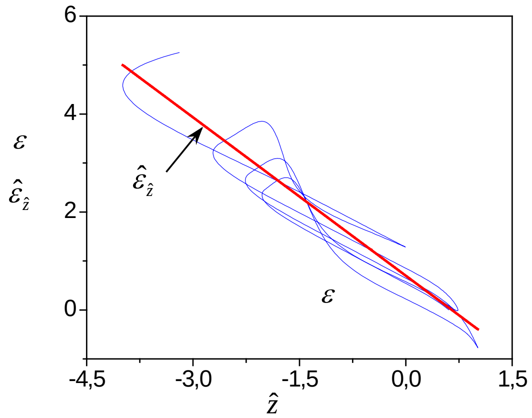

Figure 8. Comparison of estimates $\hat{z}$ and uncertainty ε_z.

So, simulation results confirm the exponential dissipativity of the designed system.

8. Conclusion

We propose the adaptive identification method of system parameters with the Bouc-Wen hysteresis. We relate the fundamental problem of the BWH identification to ensuring the stability of the assessment system. The proposed identification method is based on the use of adaptive observers. Algorithms for the adaptive observer designed and the trajectories limitation in the adaptive system shown. An approach proposed to estimate the uncertainty about the hysteresis state. This estimation uses to adjust the parameters of the hysteresis model. We consider BWH modifications and propose adaptive algorithms for estimating their parameters. The Lyapunov vector function method use to evaluate the identification system quality in coordinate and parametric spaces. We prove exponential dissipativity of processes in an adaptive system. We show that the exponential dissipation domain of the system determines by the level of derivative output.

Appendices A. Proof of Theorem 1

Consider the Lyapunov function $V(t)$ (36). Then $\dot{V}(t)$

$$\dot{V} = -k_x e^2 + \dot{V}_K - \dot{V}_K \leq -2k_x V_e . \tag{A.1}$$

Apply the condition (i) theorem 1. As $\dot{V}(t) < 0$, the AS_X -system is stable. Integrate $\dot{V}(t)$ on the time and obtain

$$V(t_0) - 2k_x \int_{t_0}^{t} V_e(\tau) d\tau \geq V(t) . \tag{A.2}$$

Get from (A.2) to all trajectories of the system AS_X belong to the area $\mathrm{G}_t = \left\{ (e, \Delta K) : V(t) \leq V(t_0) \right\}$. We get an estimate for the AS_X -system

$$\int_{t_0}^{t} 2k_x V_e(\tau) d\tau \leq V(t_0) - V(t) . \blacksquare \tag{A.3}$$

Appendices B. Proof of Theorem 3

Determine $\dot{V}_{\varepsilon,\beta,\gamma}$

$$\begin{aligned} \dot{V}_{\varepsilon\beta\gamma} &= -k_z \varepsilon^2 + \varepsilon\left(\beta\eta_\beta + \gamma\eta_\gamma + \Delta\dot{x}\right) + \dot{V}_{\beta,\gamma} - \dot{V}_{\beta,\gamma} = \\ & -k_z \varepsilon^2 + \varepsilon\left(\beta\eta_\beta + \gamma\eta_\gamma + \Delta\dot{x}\right). \end{aligned} \tag{B.1}$$

Since, $\tilde{\dot{x}}$ is function x, $\tilde{\dot{x}} = \sigma \dot{x}$, where $\sigma \approx 1$. We have showed that ε_z is the estimation z. Therefore, present η_β as

$$\eta_\beta = |\dot{x}||z|^n \, sign(z) - |\tilde{\dot{x}}||z|^n \, sign(z) \cong |\dot{x}||\varepsilon|^n , \tag{B.2}$$

for $\forall t > t_\varepsilon$. Similarly

$$\eta_\beta = |\dot{x}||z|^n \, sign(z) - |\tilde{\dot{x}}||z|^n \, sign(z) \cong |\dot{x}||\varepsilon|^n , \tag{B.2}$$

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Considering the assumption 1 and the boundedness of trajectories AS_X -system, we obtain $|\dot{x}| \le \upsilon$ for $\forall t > t_0$, where $\upsilon > 0$. Then

$$\begin{aligned} \dot{V}_{\varepsilon\beta\gamma} &\le -k_z\varepsilon^2 + \beta|\varepsilon||\eta_\beta| + \gamma|\varepsilon||\eta_\gamma| + |\varepsilon||\Delta\dot{x}| \\ &\le -k_z\varepsilon^2 + \beta\upsilon|\varepsilon|^{n+1} + \gamma\upsilon|\varepsilon|^{n+1} + |\varepsilon||\Delta\dot{x}|, \\ &\le -k_z\varepsilon^2 + \upsilon(\beta+\gamma)|\varepsilon|^{n+1} + \delta_\Delta|\varepsilon| \end{aligned} \tag{B.4}$$

where $|\Delta\dot{x}| \le \delta_\Delta$, $\delta_\Delta \ge 0$.

Let

$$\tilde{g}_1(t) = \sup_{\varepsilon\in\Omega} \frac{|\varepsilon|^{n+1}(t)}{V_\varepsilon(t,\varepsilon)}, \quad g_1 = \max_t \tilde{g}_1(t). \tag{B.5}$$

Then $|\varepsilon|^{n+1}(t) \le g_1\varepsilon^2(t)$ and transform (B.4) to the form

$$\begin{aligned} \dot{V}_{\varepsilon\beta\gamma} &\le -k_z\varepsilon^2 + \upsilon(\beta+\gamma)g_1\varepsilon^2 + \delta_\Delta|\varepsilon| = \\ &-(k_z - \upsilon(\beta+\gamma)g_1)\varepsilon^2 + \delta_\Delta|\varepsilon|, \end{aligned} \tag{B.6}$$

where $k_z - \upsilon(\beta+\gamma)g_1 > 0$.

Apply the inequality

$$-aq^2 + bq \le -\frac{a}{2}q^2 + \frac{b^2}{2a}. \tag{B.7}$$

Then (B.6)

$$\begin{aligned} \dot{V}_{\varepsilon\beta\gamma} &\le -(k_z - \upsilon(\beta+\gamma)g_1)\varepsilon^2 + \delta_\Delta|\varepsilon| \\ &\le -\frac{k_z - \upsilon(\beta+\gamma)g_1}{2}\varepsilon^2 + \frac{1}{2(k_z - \upsilon(\beta+\gamma)g_1)}(\delta_\Delta)^2 \\ &\le -(k_z - \upsilon(\beta+\gamma)g_1)V_\varepsilon + \frac{1}{2(k_z - \upsilon(\beta+\gamma)g_1)}(\delta_\Delta)^2. \end{aligned} \tag{B.8}$$

Integrate (B.8) and obtain the estimation

$$\int_{t_o}^{t}(k_z - \upsilon(\beta+\gamma)g_1)V_\varepsilon(\tau) + \frac{1}{2(k_z - \upsilon(\beta+\gamma)g_1)(t-t_0)}(\delta_\Delta)^2 \le V_{\varepsilon\beta\gamma}(t_0) - V_{\varepsilon\beta\gamma}(t). \tag{B.9}$$

The left part (B.9) is nonnegative and $V_\varepsilon(t)$ satisfies conditions of theorem 3. Therefore, all trajectories AS_Z -the system is limited. ■

Appendices B. Proof of Theorem 4

Consider $\dot{V}_\varepsilon$

$$\dot{V}_\varepsilon = -k_z\varepsilon^2 + \varepsilon(\beta\eta_\beta + \gamma\eta_\gamma + \Delta\dot{x}) + \varepsilon\left(\Delta\beta|\tilde{\dot{x}}||\hat{z}|^n sign(\hat{z}) + \Delta\gamma\tilde{\dot{x}}|\hat{z}|^n\right). \tag{C.1}$$

Evaluate the second and third summands on the right side (C.1).

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Lemma C1. *The estimation*

$$\left|\eta_{\beta}\right| \le \upsilon \left|\varepsilon\right|^{n}$$

is fair for $\eta_{\beta} = \left|\dot{x}\right|\left|z\right|^{n} sign(z) - \left|\tilde{\dot{x}}\right|\left|\hat{z}\right|^{n} sign(\hat{z})$.

Lemma C1 proof.

As $\left|\dot{x}\right| \le \upsilon$, then

$$\begin{gathered}\left|\eta_{\beta}\right| \cong \upsilon\left|\left|z\right|^{n} sign(z) - \sigma\left|\hat{z}\right|^{n} sign(\hat{z})\right| \le \\ \upsilon\left(\left|z-\hat{z}\right|^{n} + \left|\hat{z}\right|^{n} - \left|\hat{z}\right|^{n}\right) \le \upsilon\left|z-\hat{z}\right|^{n} = \upsilon\left|\varepsilon\right|^{n}.\end{gathered} \blacksquare$$

Similarly, the estimation has the form $\left|\eta_{\gamma}\right| \le \upsilon\left|\varepsilon\right|^{n}$ for $\eta_{\gamma} = \dot{x}\left|z\right|^{n} - \tilde{\dot{x}}\left|\hat{z}\right|^{n}$. It is based on the proof of the lemma C1. Then

$$\left|\varepsilon\left(\beta\eta_{\beta} + \gamma\eta_{\gamma} + \Delta\dot{x}\right)\right| \le \tilde{\upsilon}\left(\beta+\gamma\right)\left|\varepsilon\right|^{n+1} + \delta_{\Delta}\left|\varepsilon\right|, \tag{C.2}$$

where $\tilde{\upsilon} = \upsilon\left(\beta+\gamma\right)$.

Consider the last item in the right member (C.1).

Lemma C2. *The estimation*

$$\left|\varepsilon\left(\Delta\beta\left|\tilde{\dot{x}}\right|\left|\hat{z}\right|^{n} sign(\hat{z}) + \Delta\gamma\tilde{\dot{x}}\left|\hat{z}\right|^{n}\right)\right| \le \omega\upsilon\left|\varepsilon\right|^{n+1}\left\{\left|\Delta\beta\right| + \left|\Delta\gamma\right|\right\}, \tag{C.3}$$

is fair for $\varepsilon\left(\Delta\beta\left|\tilde{\dot{x}}\right|\left|\hat{z}\right|^{n} sign(\hat{z}) + \Delta\gamma\tilde{\dot{x}}\left|\hat{z}\right|^{n}\right)$, *where* $\omega > 0$ *is such that* $\left|\varepsilon - \varepsilon_{z}\right| \le \omega\left|\varepsilon\right|$.

Lemma C2 proof. Transform (C.3) to the form

$$\left(\varepsilon\Delta\beta\left|\tilde{\dot{x}}\right|\left|\hat{z}\right|^{n} sign(\hat{z}) + \varepsilon\Delta\gamma\tilde{\dot{x}}\left|\hat{z}\right|^{n}\right)\Big|_{\hat{z}=\varepsilon-\varepsilon_{z}} = \varepsilon\left|\tilde{\dot{x}}\right|\left|\varepsilon-\varepsilon_{z}\right|^{n}\left(\Delta\beta sign\left(\varepsilon-\varepsilon_{z}\right) + \Delta\gamma sign\left(\tilde{\dot{x}}\right)\right). \tag{C.4}$$

Let $\omega > 0$ exist such that $\left|\varepsilon - \varepsilon_{z}\right| \le \omega\left|\varepsilon\right|$. Then (C.4)

$$\begin{aligned}&\left|\varepsilon\right|\left|\tilde{\dot{x}}\right|\left|\varepsilon-\varepsilon_{z}\right|^{n}\left|\Delta\beta sign\left(\varepsilon-\varepsilon_{z}\right) + \Delta\gamma sign\left(\tilde{\dot{x}}\right)\right| \\ &\le \omega\upsilon\left|\varepsilon\right|^{n+1}\left|\Delta\beta sign\left(\varepsilon-\varepsilon_{z}\right) + \Delta\gamma sign\left(\tilde{\dot{x}}\right)\right| \\ &\le \omega\upsilon\left|\varepsilon\right|^{n+1}\left\{\left|\Delta\beta\right| + \left|\Delta\gamma\right|\right\}.\end{aligned} \tag{C.5}$$

(C.3) is result (C.5). ■

Consider (C.5) and transform $\omega\upsilon\left|\varepsilon\right|^{n+1}\left\{\left|\Delta\beta\right| + \left|\Delta\gamma\right|\right\}$ into the form

$$\omega\upsilon\left|\varepsilon\right|^{n+1}\left\{\left|\Delta\beta\right| + \left|\Delta\gamma\right|\right\} \le 0.5\varepsilon^{2(n+1)} + 0.5\lambda\left(\Delta\beta^{2} + \Delta\gamma^{2}\right), \tag{C.6}$$

where $\lambda > 1$.

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As

$$(\Delta\beta)^2+(\Delta\gamma)^2=2\cdot 0.5\left[\chi_\beta^{-1}\chi_\beta(\Delta\beta)^2+\chi_\gamma^{-1}\chi_\gamma(\Delta\gamma)^2\right]\le 2\chi V_{\beta,\gamma},$$

where $\chi=\max\left(\chi_\beta,\chi_\gamma\right)$, then (C.6) receives the form

$$\omega\upsilon|\varepsilon|^{n+1}\{|\Delta\beta|+|\Delta\gamma|\}\le 0.5\omega\upsilon\varepsilon^{2(n+1)}+\lambda\chi\omega\upsilon V_{\beta,\gamma}. \quad (C.7)$$

As

$$-k_z\varepsilon^2+\delta_\Delta|\varepsilon|\le-\frac{1}{2}k_z\varepsilon^2+\frac{1}{2k_z}(\delta_\Delta)^2, \quad (C.8)$$

then (C.1), considering lemmas C1 and C2, and inequalities (C.7) and (C.8), we present as

$$\begin{aligned}\dot V_\varepsilon&\le-k_z\varepsilon^2+\tilde\upsilon|\varepsilon|^{n+1}+\delta_\Delta|\varepsilon|+0.5\omega\upsilon\varepsilon^{2(n+1)}+\lambda\chi\omega\upsilon V_{\beta,\gamma}\\&\le-\frac{1}{2}k_z\varepsilon^2+\tilde\upsilon|\varepsilon|^{n+1}+0.5\omega\upsilon\varepsilon^{2(n+1)}+\lambda\chi\omega\upsilon V_{\beta,\gamma}+\frac{1}{2k_z}(\delta_\Delta)^2.\end{aligned} \quad (C.9)$$

Consider functions $\tilde g_1(t)$ (B.5) and

$$g_2(t)=\sup_{\varepsilon\in\Omega}\frac{|\varepsilon|^{2(n+1)}(t)}{V_\varepsilon(t,\varepsilon)},\quad g_2=\max_t\tilde g_2(t).$$

Then (C.9)

$$\dot V_\varepsilon\le-\frac{1}{2}(k_z-2\tilde\upsilon g_1-\omega\upsilon g_2)V_\varepsilon+\lambda\chi\omega\upsilon V_{\beta,\gamma}+\frac{1}{2k_z}(\delta_\Delta)^2. \quad (C.10)$$

Obtain the estimation for the derivative $V_{\beta,\gamma}(t)=0.5\chi_\beta^{-1}(\Delta\beta)^2+0.5\chi_\gamma^{-1}(\Delta\gamma)^2$:

$$\dot V_{\beta,\gamma}=-\varepsilon\Delta\beta|\tilde x||\hat z|^n\,sign(\hat z)-\varepsilon\Delta\gamma\tilde x|\hat z|^n \quad (C.11)$$

Let $c_1>0, c_2>0$ exist such that

$$\varepsilon\Delta\gamma\tilde x|\hat z|^n=c_2\left[(\Delta\gamma)^2\left(\tilde x|\hat z|^n\right)^2+\varepsilon^2\right], \quad (C.12)$$

$$\varepsilon\Delta\beta|\tilde x||\hat z|^n\,sign(\hat z)=c_1\left[(\Delta\beta)^2\left(|\tilde x||\hat z|^n\right)^2+\varepsilon^2\right]. \quad (C.13)$$

Then (C.11)

$$\dot V_{\beta,\gamma}=-c\varepsilon^2-\left(\tilde x|\hat z|^n\right)^2\left(c_1(\Delta\beta)^2+c_2(\Delta\gamma)^2\right), \quad (C.14)$$

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where $c = c_1 + c_2$.

Let $\overline{c} = \min(c_1, c_2)$. Then

$$c_1(\Delta\beta)^2 + c_2(\Delta\gamma)^2 \geq \overline{c}\left((\Delta\beta)^2 + (\Delta\gamma)^2\right) \geq 2\overline{c}\,\overline{\chi} V_{\beta,\gamma}. \tag{C.15}$$

where $\overline{\chi} = \min(\chi_\beta, \chi_\gamma)$.

Apply (C.14) and (C.15) write as

$$\dot{V}_{\beta,\gamma} \leq -c\varepsilon^2 - 2\overline{c}\,\overline{\chi}\left(\tilde{x}|\hat{z}|^n\right)^2 V_{\beta,\gamma}. \tag{C.16}$$

$\tilde{x}$ bounded $(\tilde{x})^2 \geq \pi_x \geq 0$, therefore,

$$\begin{gathered}\dot{V}_{\beta,\gamma} \leq -c\varepsilon^2 - 2\overline{c}\chi\left(\tilde{x}|\hat{z}|^n\right)^2 V_{\beta,\gamma} \leq -c\varepsilon^2 - 2\overline{c}\chi\pi_x|\hat{z}|^{2n} V_{\beta,\gamma}, \\ \dot{V}_{\beta,\gamma} \leq -c\varepsilon^2 - 2\chi\pi_x\overline{c}(\varepsilon - \varepsilon_z)^{2n} V_{\beta,\gamma}.\end{gathered} \tag{C.17}$$

The variable ε boundedness follows from theorem 3, and the boundedness ε_z is the boundedness result ε. Therefore, $(\varepsilon - \varepsilon_z)^{2n} \geq c_z$, where $c_z \geq 0$. So

$$\dot{V}_{\beta,\gamma} \leq -c\varepsilon^2 - 2\chi\pi_x\overline{c}c_z V_{\beta,\gamma}. \tag{C.18}$$

Let $d_s \triangleq \chi\pi_x\overline{c}c_z$. Then (C.14) write as

$$\dot{V}_{\beta,\gamma} \leq -d_s V_{\beta,\gamma} + 2\sqrt{cd_s}\,\varepsilon\sqrt{V_{\beta,\gamma}}. \tag{C.19}$$

Use the inequality (B.7) and obtain the estimation for $\dot{V}_{\beta,\gamma}$

$$\dot{V}_{\beta,\gamma} \leq -\frac{d_s}{2} V_{\beta,\gamma} + cV_\varepsilon. \tag{C.20}$$

So, the following system of inequalities is fair for the AS_Z-system

$$\begin{bmatrix}\dot{V}_\varepsilon \\ \dot{V}_{\beta,\gamma}\end{bmatrix} \leq \underbrace{\begin{bmatrix}-(k_z - 2\tilde{\upsilon}g_1 - \omega\upsilon g_2) & \lambda\chi\omega\upsilon \\ c & -\dfrac{d_s}{2}\end{bmatrix}}_{A_\varepsilon} \begin{bmatrix}V_\varepsilon \\ V_{\beta,\gamma}\end{bmatrix} + \underbrace{\begin{bmatrix}\dfrac{1}{2k_z} \\ 0\end{bmatrix}}_{B_\varepsilon} (\delta_\Delta)^2. \tag{C.21}$$

Let $V_{\tilde{\rho}}(t) \leq \tilde{s}_{\tilde{\rho}}(t) \quad \forall(t \geq t_0) \,\&\, \left(V_{\tilde{\rho}}(t_0) \leq \tilde{s}_{\tilde{\rho}}(t_0)\right)$, $\tilde{\rho} = \varepsilon, (\beta, \gamma)$, and $\tilde{S} = \left[\tilde{s}_\varepsilon \; \tilde{s}_{\beta,\gamma}\right]^T$. The comparison system for (C.17) has the form

$$\dot{\tilde{S}} = A_\varepsilon \tilde{S} + B_\varepsilon (\delta_\Delta)^2. \tag{C.22}$$

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We have the estimation for the system AS_Z from (C.22)

$$V_{\varepsilon,\beta,\gamma}(t) \le e^{A_\varepsilon(t-t_0)}\tilde{S}(t_0) + (\delta_\Delta)^2 \int_{t_0}^{T} e^{A_\varepsilon(t-\tau)} B_\varepsilon d\tau , \quad (C.23)$$

where $V_{\varepsilon,\beta,\gamma} = \left[V_\varepsilon \; V_{\beta,\gamma}\right]^T$ if

$$k_z > 2\tilde{\upsilon} g_1 - \omega\upsilon g_2 , \; (k_z - 2\tilde{\upsilon} g_1 - \omega\upsilon g_2) d_s > 2c\lambda\chi\omega\upsilon , \; d_s > 0 . \blacksquare$$

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