Article

The Effect of Negative Mass in Gravitating System

Edward Bormashenko

Ariel University, Ariel, 407000, Israel, Engineering Faculty, Department of Chemical Engineering, Biotechnology and Materials, edward@ariel.ac.il

Abstract: The effect of the negative effective mass emerging in the gravitating core-spring-shell system is considered. The effect appears when the entire system is exerted to the external harmonic force and the frequency of external force approaches to the critical frequency ω_0 from above. The critical frequency ω_0 depends on the density of the self-gravitating system only. The scaling law predicting the value of ω_0 for condensed phases is derived as: $\omega_0 = A^{\frac{1}{5}}\tilde{\omega}$, where A is the atomic weight and $\tilde{\omega}$ is the fundamental frequency. The generalization of the effect for the Coulomb-like forces is reported.

Keywords: core-spring-shell system; gravitating system; negative mass; Coulomb-like forces

1. Introduction

The broadly discussed effect of the "negative effective" mass emerges when the core-spring-shell system is substituted by the single effective mass m_{eff} [1]. Pseudo-negative mass introduced in Ref. 1 is quite different from the hypothetic "negative mass" suggested for the solutions of equations of general relativity [2-6] and enabling reasonable, however debatable, explanation for the "dark matter" problem [7]. The negative effective mass emerging in the core-spring-shell system, depicted in **Figure 1A** reflects the fact that the acceleration of the shell at certain conditions (to be discussed in detail below) is in the opposite direction to the applied force [1, 8-10]. Consequently, the chain of the core-spring-shell elements, depicted in **Figure 1A** demonstrates the paradoxical effective negative density [8, 11-14]. The effect of the negative effective density, in turn, opens the new pathways for the development of novel acoustic metamaterials [15]. Acoustic metamaterials are defined as engineered, artificial, periodic composites enabling altering mechanic properties of materials; thus, allowing properties that are not registered naturally [16]. In particular, metamaterials with controllable and negative bulk modulus were demonstrated, exploiting the effect of the negative mass [10, 11-13, 17]. A number of experimental exemplifications of the "negative mass effect" were already demonstrated [11-13, 18-19]. Use of the plasma oscillations of the free electron gas for the realization of the negative mass effect was suggested [20-21]. We demonstrate, how the gravitating system gives rise to the effect of the negative effective mass.

 doi:10.20944/preprints202204.0080.v1

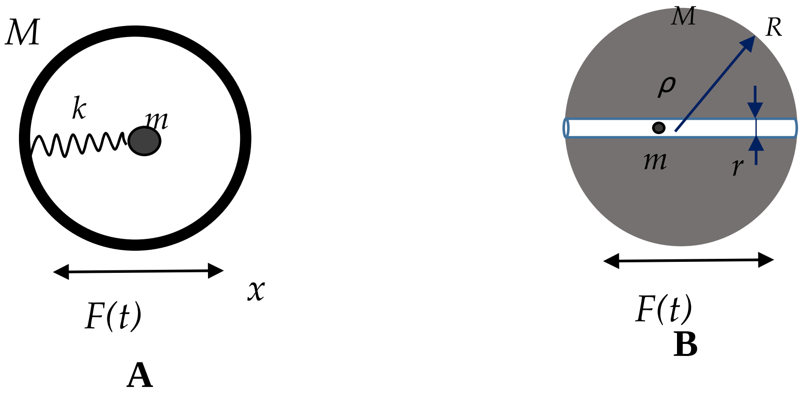

Figure 1. **A**. Mechanical scheme giving rise to the effect of the negative mass. The core-shell system, in which the core mass m is connected to the shell M *via* ideal Hookean spring k. **B**. Drilled gravitating ball R is depicted; density of the ball ρ is constant; the point mass m is located within the drilled channel r; $r \ll R$ is adopted. Both of systems are exerted to the harmonic external force $F(t) \sim e^{-j\omega t}$.

2. The effect of the negative effective mass in the gravitating system

Let us acquaint the contra-intuitive effect of the "negative mass". Consider the core-ideal-spring-shell units, shown in **Figure 1A**. The spring is supposed to be Hookean and massless. This system, as demonstrated in refs. 1, 8-9, gives rise to the "negative effective mass behavior". Assume that the mass of the shell is M, the mass of the core is m and the constant of the linear spring is k. When the core-shell system, depicted in **Figure 1A**, is subjected to the complex harmonic force $\hat{F} = \hat{F}_0\, e^{-j\omega t}$ the entire system may be substituted with a single mass m_{eff} defined according to Eq. 1, as suggested in refs 1, 8-9.

$$m_{eff} = M + \frac{m\omega_0^2}{\omega_0^2 - \omega^2} \quad (1)$$

where $\omega_0 = \sqrt{\frac{k}{m}}$. It is easily seen from Eq. 1 that, when the frequency ω approaches ω_0 from above the effective mass m_{eff} will be negative [11-16]. Eq. 1 arises from the Eq. 2, which enables to present the entire complex moment $\hat{P}$ of the core-spring-shell system, as follows (see Ref. 1):

$$\hat{P} = m_{eff}\hat{V}\ , \quad (2)$$

where $\hat{V}$ is the complex velocity of the shell M, and m_{eff} is given by Eq. 1. Huang *et al*. in ref. 8 noted that actually there is no, of course, the "negative mass", and the negativity of the effective mass is the result of the attempt to use a single mass to respresent a complex core-spring-shell system.

Consider now the drilled gravitating ball R, shown in **Figure 1B**, in which cylindrical channel r goes through the center of the ball and $r \ll R$ takes place. Assume that the density of the ball is constant $\rho =$

$const$, the mass of the ball is *M*. Small spherical mass *m* is placed in the channel, as shown in **Figure 1B**. We demonstrate that the aforemetioned system, depicted in **Figure 1B** is mechanically equivivalent to the core-spring-shell system shown in **Figure 1A**. Indeed, the gravitational force acting within the channel on the mass $\vec{F}_{gr}$ is given by Eq. 3:

$$\vec{F}_{gr}(r) = m\vec{g}(r), \tag{3}$$

where $\vec{g}$ is the gravitational field within the field. The Gauss's law for gravity is supplied by Eq. 4 (see ref. 22):

$$\iint_S \vec{g}\,(r) \cdot d\vec{S} = -4\pi GM, \tag{4}$$

where $G \cong 6.67 \times 10^{-11}\, m^3 s^{-2} kg^{-1}$ is the gravitational constant. Considering $r \ll R$ (keeping the spherical symmetry of the mass distrubution with the ball *R*); $M = \frac{4}{3}\pi\rho R^3$ and integration of Eq. 4 yields:

$$g(r) = -\frac{4}{3}\pi\rho G r \tag{5}$$

Thus, the gravitational force $\vec{F}_{gr}(r)$ acting on the point mass *m* is given by Eq. 6:

$$\vec{F}_{gr}(r) = -\frac{4}{3}\pi\rho G m r\hat{r} \ , \tag{6}$$

where $\hat{r}$ is the unit radius-vector. Eq. 6 represents the Hookean linear elastic force, $F_{el} = -kr$, where *k* is the stiffness coefficient, emerging immediately from Eq. 6:

$$k = \frac{4}{3}\pi\rho G m \tag{7}$$

Hence, the point mass *m* oscillates within the channel according to the law: $r \sim r_0 e^{-j\omega_0 t}$, where the "critical frequency" ω_0 is given by Eq. 8:

$$\omega_0 = \sqrt{\frac{4}{3}\pi\rho G} \tag{8}$$

Thus, the gravitating system, presented in **Figure 1B**, is mechanically equivalent to the core-spring-shell system, shown in **Figure 1A**, in which the spring stiffness *k* and the frequency of oscillations ω_0 are given by Eq. 7 and Eq. 8. It should be emphasized that the frequency of oscillations ω_0 is independent of the mass *m*, thus, illustrating the Einstein Equivalency Principle, postulating the equivalence of gravitational and inertial mass [23-24]. It is also noteworthy that the frequency of oscillations ω_0 is independent of the radius

Preprints (www.preprints.org) | NOT PEER-REVIEWED | Posted: 8 April 2022 doi:10.20944/preprints202204.0080.v1

of the ball, being dependent on its density ρ only. Thus, we conclude that the critical frequency ω_0 depends on the intensive parameters of the self-gravitating system only.

Combining Eq. 1 and Eq. 8 yields for the effective mass of the gravitating system, shown in Figure 1B the following equation:

$$m_{eff} = M + \frac{\frac{4}{3}\pi\rho G m}{\frac{4}{3}\pi\rho G - \omega^2} \tag{9}$$

It will be instructive to calculate the explicit value of ω_0 arising from Eq. 8. Assuming $\rho \cong 1.0 \times 10^3 \frac{kg}{m^3}$ yields $\omega_0 \cong 5.0 \times 10^{-4} s^{-1}$. It is noteworthy, that the densities of condensed phases of different material are close, and they are confined within a relatively narrow range, as demonstrated in ref. 25. Victor Weisskopf in ref. 25 suggested the following expression for estimation of the density of the condensed phases:

$$\rho = \frac{A^{\frac{2}{5}} m_{pr}}{1.5^3 \frac{4\pi}{3} a_0^3}\ , \tag{10}$$

where A is the atomic or molecular weight, a_0 is the Bohr radius and m_{pr} is the mass of the proton. Substitution of Eq. 10 into Eq. 8 yields:

$$\omega_0 = A^{\frac{1}{5}} \sqrt{\frac{m_{pr} G}{(1.5 a_0)^3}} = A^{\frac{1}{5}} \widetilde{\omega}\ , \tag{11}$$

where $\widetilde{\omega} = \sqrt{\frac{m_{pr} G}{(1.5\, a_0)^3}} \cong 4.4 \times 10^{-4} s$ is the fundamental frequency, which is built of the fundamental physical constants only. It is useful to supply the following explicit expression for the fundamental frequency $\widetilde{\omega}$:

$$\widetilde{\omega} = \sqrt{\frac{m_{pr} m_e^3 c^3 \alpha^3 G}{(1.5\, \hbar)^3}}\ , \tag{12}$$

where m_e, c, α and $\hbar$ are the mass of electron, the speed of the light in vacuum, the fine structure constant and the reduced Planck constant correspondingly. Scaling Eq. 11 deserves the close inspection; indeed, it predicts a very weak dependence of the critical frequency ω_0 on the atomic weight, namely $\omega_0 \sim A^{\frac{1}{5}}$. In other words, the values of the critical frequency ω_0 are well-expected to be very close for very different condensed materials.

3. The effect of negative mass for homogeneous Coulomb-like spherically symmetrical systems

The aforementioned considerations are true for the similar systems governed by the Coulomb-like forces, which scale according to $F(r) \sim \frac{1}{r^2}$. Let us exemplify this fact with the electrically charged ball *R* drilled through it center, as shown in **Figure 2B**. The mass of the ball is *M*. Assume that the ball is homogeneously electrically charged with a volume density $\rho_{el} = const;\ [\rho_{el}] = \frac{C}{m^3}$. The point electrical charge *q* is placed within the cylindrical channel *r* as shown in **Figure 2B**; the mass of the charge carrier is *m*. Again, we adopt that the condition $r \ll R$ takes place, which keeps the spherical symmetry of the electrical charge distrubution within the ball. The electrical force $\vec{F}_{el}(r)$ acting on the point charge *q* is given by Eq. 13:

$$\vec{F}_{el}(r) = q\vec{E}(r), \quad (13)$$

where $\vec{E}(r)$ is an electrical field within the homogeneously charged ball *R*. The electrical field within the ball emerges from the Gauss theorem, and it is given by Eq. 14:

$$\vec{E}(r) = -\frac{\pi}{3\varepsilon_0}\rho_{el} r\hat{r}, \quad (14)$$

where $\varepsilon_0 = 8.854\frac{C}{Vm}$ is the vacuum permittivity. Thus, the Hookean linear electrical force acting on the charge is supplied by Eq. 15:

$$\vec{F}_{el}(r) = -\frac{\pi}{3\varepsilon_0} q\rho_{el} r\hat{r} \quad (15)$$

The stiffness coefficient *k* is now given by Eq. 16:

$$k = \frac{\pi}{3\varepsilon_0} q\rho_{el} \quad (16)$$

Thus, the frequency ω_0 is supplied by Eq. 17:

$$\omega_0 = \sqrt{\frac{\pi}{3\varepsilon_0}\frac{q\rho_{el}}{m}} \quad (17)$$

It is noteworthy, that the critical frequency ω_0 depends on the q/m ratio. The mechanical equivalence of the systems depicted in **Figures 2A** and **2B** is easily recognized, if the stiffness of the spring is given by Eq. 16. Now we exert the system shown in Figure 2B to the complex harmonic force $\hat{F} = \hat{F}_0\, e^{-j\omega t}$. The effective mass of the system is given by Eq. 18:

$$m_{eff} = M + \frac{\frac{\pi}{3\varepsilon_0} q\rho_{el}}{\frac{\pi}{3\varepsilon_0}\frac{q\rho_{el}}{m} - \omega^2} \quad (18)$$

And, again, the effective mass of the system becomes negative when the frequency of the external force ω approaches the critical frequency $\omega_0 = \sqrt{\frac{\pi}{3\varepsilon_0}\frac{q\rho_{el}}{m}}$ from above and in principle in the frictionless system it is unrestricted.

doi:10.20944/preprints202204.0080.v1

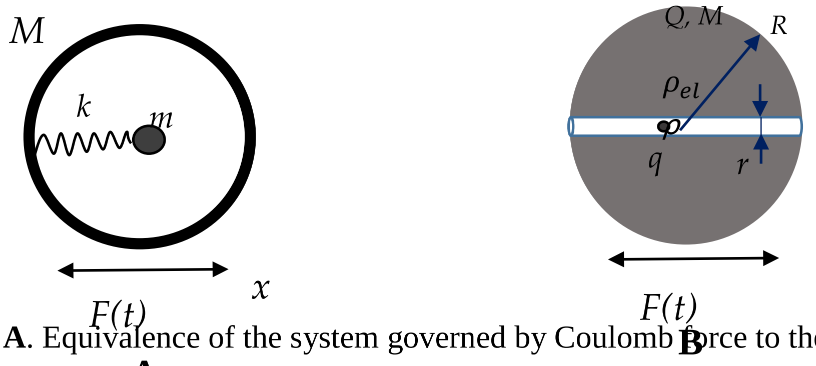

Figure 2. **A**. Equivalence of the system governed by Coulomb force to the core-spring-shell system is illustrated. The core-shell system, in which the core mass m connected to the shell M *via* ideal Hookean spring k is shown **B**. Drilled through it center electrically charged ball R is depicted; density of the electrical charge $\rho_{el} = const$; the point charge q is located within the drilled channel r; $r \ll R$ is assumed. Both of systems are exerted to the harmonic external force $F(t) \sim e^{-j\omega t}$.

4. Conclusions

The broadly discussed effect of the "negative effective mass" emerges in the core-spring-shell systems exerted to harmonic excitation [1, 8-10]. We addressed the effect of the negative effective mass emerging in the gravitating system, which is mechanically equivalent to the core-spring-shell system. The effect appears when the entire gravitating system is exerted to the external harmonic force and the frequency of external force approaches to the critical frequency ω_0 from above. The critical frequency $\omega_0 = \sqrt{\frac{4}{3}\pi\rho G}$ depends on the density of the self-gravitating system ρ only. The scaling law predicting the value of ω_0 for condensed phases is derived as: $\omega_0 = A^{\frac{1}{5}}\widetilde{\omega}$, where A is the atomic weight and $\widetilde{\omega} = \sqrt{\frac{m_{pr}G}{(1.5\, a_0)^3}}$ is the fundamental frequency, built of the fundamental physical constants. The values of the critical frequency ω_0 are close for different condensed materials. The generalization of the effect for the Coulomb-like forces is reported. In this case, the effective mass becomes negative, when ω approached to $\omega_0 = \sqrt{\frac{\pi}{3\varepsilon_0}\frac{q\rho_{el}}{m}}$ from above, and in principle, it is unrestricted.

References

[1] G. W .Milton, J. R. Willis, J. R. On modifications of Newton's second law and linear continuum elastodynamics. Proc. R. Soc. A 463 (2007) 855–880.
[2] H. Bondi, Negative Mass in General Relativity, Rev. Mod. Phys. 29 (1957) 423-429.

[3] W. B. Bonnor, Negative mass in general relativity. Gen. Relat. Gravit. 21 (1989) 1143–1157.
[4] G. Manfredi, J.-L. Rouet, B. Miller, G. Chardin, Cosmological structure formation with negative mass, Phys. Rev. D 98 (2018) 023514.
[5] S. Nájera, A. Gamboa, A. Aguilar-Nieto, C. Escamilla-Rivera, On negative mass cosmology in General Relativity, Astronomy & Astrophysics, 651(2021) L13.
[6] V. Veselý, M. Žofka. Pinball spring model, Phys. Rev. D 104 (2021) 104033.
[7] H. Socas-Navarro, Can a negative-mass cosmology explain dark matter and dark energy, Astronomy & Astrophysics 626 (2019) A5.
[8] H. H. Huang, C. N. Sun, G. L Huang, G. L. On the negative effective mass density in acoustic metamaterials. Int. J. Eng. Sci. 47 (2009) 610-617.
[9] C. T. Chan, J. Li, K. H. Fung, On extending the concept of double negativity to acoustic waves. JZUS A 7 (2006) 24-28.
[10] P. Sarathi, N. K. Pathak, Application of exact solution of complex Morse potential to investigate physical systems with complex and negative masses. J. Phys. Commun. 5 (2021) 065006.
[11] X. N. Liu, G. K. Hu, G. L. Huang, C. T. Sun, C. T. An elastic metamaterial with simultaneously negative mass density and bulk modulus. Appl. Phys. Lett. 98 (2011) 251907.
[12] S. Sang, A. Mhannawee, Z. Wang, A design of active elastic metamaterials with negative mass density and tunable bulk modulus. Acta Mech. 230 (2019) 1003–1008.
[13] X.N. Liu, G. K. Hu, G.L Huang, C. T. Sun, An elastic metamaterial with simultaneously negative mass density and bulk modulus. Appl. Phys. Lett. 98 (2011) 251907.
[14] M. Jaberzadeh, B. Li, K. T. Tan, Wave propagation in an elastic metamaterial with anisotropic effective mass density, Wave Motion, 89 (2019) 131-141.
[15] Z.-H. He, Y.-Z. Wang, Y.-S. Wang, Active feedback control of effective mass density and sound transmission on elastic wave metamaterials, Int. J. Mech. Sci. 195 (2021) 106221.
[16] C. Ding, Y. Dong, Y. Wang, J. Shi, S. Zhai, X. Zhao, Acoustic metamaterials and metasurfaces composed of meta-atoms and meta-molecules, J. Phys. D: Appl. Phys. 55 (2022) 253002.
[17] F. Liu, Z. Wang, M. Ke, Z. Liu, Metafluids beyond the Bulk Modulus, Phys. Rev. Lett. 125 (2020 185502.
[18] J. Lončar, B. Igrec, D. Babić, Negative-Inertia Converters: Devices Manifesting Negative Mass and Negative Moment of Inertia, Symmetry, 14(3) (2022) 529.
[19] G. Hong, J. Li, J. Dong, Y. Ning, J. Pan, Frequency-dependent characteristics of grain-beam system: Negative mass and jump behavior, Int. J. Mech. Sci. 209 (2021) 106706.
[20] Ed. Bormashenko, I. Legchenkova, I. Negative effective mass in plasmonic systems. Materials 13 (8) (2020) 1890.
[21] Ed. Bormashenko, I. Legchenkova, M. Frenkel, Negative Effective Mass in Plasmonic Systems II: Elucidating the Optical and Acoustical Branches of Vibrations and the Possibility of Anti-Resonance Propagation. Materials 13(16) (2020) 3512.
[22] G. B. Arfken, H. J. Weber, Mathematical Methods for Physicists, 5th ed, Academic Press, San Diego, USA, 2001.
[23] P. G. Roll, R. Krotkov, R. H. Dicke, R.H. The equivalence of inertial and passive gravitational mass. Ann. Phys. 26 (1964) 442–517.
[24] Ohanian, H. What is the principle of equivalence? Am. J. Phys. 45 (1977) 903.

doi:10.20944/preprints202204.0080.v1

[25] V. F. Weisskopf, Of Atoms, Mountains, and Stars: A Study in Qualitative Physics, Am. J. Phys. 187 (4177) (1975) 605-612