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A Critical Analysis of the Quantum Nonlocality Problem

Facultad de Ciencias Exactas y Naturales,Universidad Nacional de Asunción,Ruta Mcal. J. F. Estigarribia, Km 11 Campus de la UNA, San Lorenzo-Paraguay

Justo Pastor Lambare*

Abstract

The alleged nonlocal character of quantum mechanics is inextricably related to the formulation of the Bell theorem. However, as we shall see, that relation is commonly incorrectly assessed. The departure from the clear line of reasoning that John Bell tried to convey has led to a polarization of part of the scientific community into radical irreconcilable positions. We show how the correct appreciation of Bell's work calls for reinterpreting the usual significance given to the Bell theorem yielding a more rational perspective of the problem. Given the relevance of the Bell-type inequalities in quantum information technology and quantum foundations, further clarification of their relation to the nonlocality conundrum deserves due attention. The exposition is also of didactic value. It shows the problems arising from incorrect inferences and superfluous metaphysical ideas.

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*email: jupalam@gmail.com

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1 Introduction

John Bell is responsible for the two most relevant no-hidden-variables theorems in the foundations of quantum mechanics [1]. Those theorems give rise to contextuality and nonlocality, considered essential resources in quantum information [2–5]. However, nonlocality taken as quantum nonlocality remains a highly controversial subject [6–12].

Unfortunately, the arguments upholding the nonlocal character of quantum mechanics are often incorrectly posed, usually surrounded by an aura of unnecessary metaphysical conceptions. That situation promotes the extreme opposite position taken by localists. The current widespread perception regarding the locality problem of quantum mechanics reflects in localists expressions such as

> The terms "non-locality" or "quantum non-locality" are buzzwords in foundations of quantum mechanics and quantum information. Most of scientists treat these terms as a more handy expression equivalent to the clumsy "violation of Bell's inequalities". Unfortunately, some treat them seriously. [13]

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> For me the term nonlocality is so fraught with misinterpretation that I feel we'd all be better off if it were removed from discourse on quantum mechanics. [9]

On the other side, we find non-localists uttering assertions such as

> What Bell's theorem, together with the experimental results, proves to be impossible (subject to a few caveats we will attend to) is not determinism or hidden variables or realism but locality, in a perfectly clear sense. What Bell proved, and what theoretical physics has not yet properly absorbed, is that the physical world itself is non-local. [14]

We have a radical localist position asserting quantum mechanics is a local theory and that there exist no solid arguments to think otherwise [9, 13, 15]. On the other extreme, we find radical non-localists claiming the Bell theorem provides definitive proof of quantum nonlocality [10, 14, 16].

We shall see that a rational assessment of the arguments for either locality or nonlocality reveals that there is, so far, no definitive proof for sustaining any of them. Although we do not introduce new original ideas on the subject, we present a fresh perspective that does not seem to be widely appreciated.

For the sake of conciseness and given the subtlety and complexity sometimes involved, we do not discuss in detail every concept. Instead, we give references for the interested reader. We assume the reader is already familiar with at least one version of the Bell inequality. Probably the most popular version is the CHSH inequality [17]. References [18–22] discuss the CHSH inequality correct and incorrect derivations warning against the use of counterfactual reasoning. The derivation of the Bell inequality requires only elementary mathematics and is uncontroversial from the mathematical point of view. Only its interpretation and its relation to the nonlocal character of quantum mechanics is particularly problematic.

We begin by following the historical development of Bell's ideas because they clarify the nature of the existing problems. We shall argue that Bell's approach is still the correct one and that most of his non-localists followers got lost in the way.

As a disclaimer, we warn people fond of metaphysics that they shall find this piece particularly disappointing because we follow Bell's pragmatic and clear physical reasoning. We base our arguments only on predictions of actual experiments and the correct use of mathematical formalisms when necessary.

Since we are dealing with quantum mechanics interpretation, the existence of correct and coherent arguments upholding both contrary positions should not be so surprising.

2 The 1964 Bell Theorem

We propose a reading of Bell's 1964 theorem [23] different from the traditional non-localists' one. We claim our proposal is more according to what Bell intended in 1964 and clarifies the origin of the conflicts between localists and non-localists.

The traditional non-localists' reading asserts that Bell formulated his inequality to prove that quantum mechanics is not a local theory, presenting the Bell theorem as a quantum nonlocality theorem. On the other hand, we assert that an objective reading of Bell's 1964 reveals that he introduced his inequality only after taking for granted that orthodox quantum mechanics is nonlocal.

That means the conclusion of Bell's theorem is not, as usually reported, that quantum mechanics is not local but that we cannot complete it with hidden parameters to turn it into a local theory. As we shall see, Bell made that point even more evident in his later formulations.

Bell conceived his 1964 theorem inspired by Einstein-Podolski-Rosen's (EPR) [24] previous argument. According to EPR, if quantum mechanics is local, it must be incomplete

$$quantun\, locality \implies incompleteness \tag{1}$$

Bell reproduced an EPR-like argument to derive the necessity of a deterministic local theory that completes quantum mechanics. Then proved that a local completion is untenable under very reasonable physical assumptions.

Where does quantum nonlocality fare in that picture? Well, since orthodox quantum mechanics is already complete, according to (1), it must be nonlocal

$$\neg\, incompleteness \implies \neg\,(quantun\, locality) \tag{2}$$

That is also Bell's 1964 reasoning, so his inequality does not enter the argument for nonlocality. Bell's ground-breaking 1964 contribution is not quantum nonlocality but the impossibility of deterministic local hidden variables. That is why physicists convinced of the completeness and weirdness of quantum mechanics, like Richard Feynman, were not impressed by Bell's result

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> It is not an important theorem. It is simply a statement of something we know is true – a mathematical proof of it. [25]

The controversy around the classical or non-classical nature of the Bell inequality and the esoteric tied-up issue of realism is a red herring that only diverts from the actual nonlocality argument [6, 7, 9, 11–13, 26–28], namely the EPR reasoning.

Bohr rejected the EPR reasoning, and orthodox quantum physicists agree with Bohr. Conflating the EPR nonlocality argument with the Bell inequality without clearly separating their roles in the reasoning is particularly convenient for an easy dismissal of quantum mechanics' nonlocal character. Indeed, given that the Bell inequality contains parameters foreign to quantum mechanics, it is easier to dismiss it as classical, which would purportedly save quantum locality. On the other hand, rejecting the EPR reasoning is less evident. Thence Bohr's famous obscure and difficult response [29].

We shall see that Bell's arguments evolved over the years. But for some reason, most commentators ignore them, insisting on getting stuck with an incorrect reading of Bell's 1964 formulation and advertising the Bell theorem as a quantum nonlocality theorem.

3 Bell's Proof of Quantum Nonlocality

In 1975 Bell [1] wrote an improved version of his seminal 1964 paper. The new version has four outstanding characteristics that were missing in 1964:

- A rigorous definition of locality he called *local causality* (LC).
- A proof that quantum mechanics violates LC, therefore, is nonlocal.
- A physical justification for assuming what later became known as the *statistical independence* (SI) hypothesis. In 1964, SI was an *ad hoc* implicit assumption.
- An absence of any reference to the EPR paper.

Next, we briefly address each of these characteristics to see that his new formulation is quite different from the 1964 one. Now the argument for quantum nonlocality and the formulation of his inequality are more clearly separated.

[1] Bell's work is reproduced in [30].

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So their mixing resulting in obscure and poorly supported arguments can not be attributed to John Bell.

3.1 Local Causality

Bell's definition of LC is a formalization of the idea that, according to relativity theory, interactions can happen only at a finite speed. It means that causes cannot have an immediate effect on distant events. *He formulated LC so that it can be applied to not deterministic theories like quantum mechanics. It is a locality argument that avoids a purportedly classical EPR-like reasoning. One directly applicable to orthodox quantum mechanics without distoring its nature.*

For the particular case that concerns us, i.e., the singlet state correlations in a Bell-type experiment, LC takes the following form. Let $P(A, B \mid a, b)$ be the probability of a joint measurement giving the results $A, B \in \{-1, +1\}$ conditional on the respective measurements directions a, b. The laws of probabilities require

$$P(A, B \mid a, b) = P(A \mid B, a, b)P(B \mid a, b) \tag{3}$$

So far, it is just about probabilities. Let us now add some physics and assume that both observers, Alice and Bob, choose their measurement directions at the last moment so that both measurements are spacelike separated events. Then LC requires that neither the results A, B nor the measurement settings a, b made on one side can affect the state of affairs on the other side. However, we cannot exclude the existence of correlations. In the r.h.s of (3), we can have that

$$P(A \mid B, a, b) \neq P(A \mid a) \tag{4}$$
$$P(B \mid a, b) \neq P(B \mid b) \tag{5}$$

notwithstanding that events A and a are spacelike separated from B and b. However, relativistic causality requires the correlations implied by (4) and (5) to be explained by local common causes λ. They are local because they are supposed to lie at the intersection of the backward light cones of the measurement events. Once the common causes λ are specified, the inclusion of spacelike separated parameters in the l.h.s of (4) and (5) become redundant

$$P(A \mid B, a, b, \lambda) = P(A \mid a, \lambda) \tag{6}$$
$$P(B \mid a, b, \lambda) = P(B \mid b, \lambda) \tag{7}$$

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Including λ in (3)

$$P(A, B \mid a, b, \lambda) = P(A \mid B, a, b, \lambda)P(B \mid a, b, \lambda) \tag{8}$$

Replacing (6) and (7) in (8)

$$P(A, B \mid a, b, \lambda) = P(A \mid a, \lambda)P(B \mid b, \lambda) \tag{9}$$

The last equation is also known as the screening-off condition.[2] It is the formal expression of the intuitive idea behind relativistic locality and is Bell's definition of LC for the case at hand.

The common cause λ is usually called "hidden variables"; however, it is somewhat misleading to believe the λ variables are necessarily unknown parameters. The only condition they need to comply with is lying at the intersection of the backward light cones of the measuring events to constitute a local explanation of the correlations. It is also utterly misleading to think they are EPR elements of physical reality; on the contrary, their role is to eliminate any EPR-like argument. Furthermore, local causality is independent of the stochastic properties of the common causes.[3]

Although Bell did not mention Reichenbach, his λ variables are according to Reichenbach's common cause principle [31]. The last point is relevant because, as we shall see later, one possibility to block the argument in favor of quantum nonlocality is to reject Reichenbach's principle of common causes [32].

3.2 Quantum Nonlocality

After defining local causality, Bell gave an argument explaining why, when considered complete, quantum mechanics violates it. Bell's argument is similar to the one given by Einstein in 1927.[4]

We can recast Bell's and Einstein's arguments in more formal terms through the mathematical formulation of local causality. The crucial point is that (9) avoids the polemic around an EPR-like classical argument. If quantum mechanics is complete and local, the locally causal explanation of its correlations must lie within the quantum state. In our case

$$\mid \psi\rangle = \frac{1}{\sqrt{2}}(\mid +\rangle \otimes \mid -\rangle - \mid -\rangle \otimes \mid +\rangle) \tag{10}$$

[2]For a more precise definition of the terms in the r.h.s of (9), see Appendix 8.

[3]More concretely, they are independent of the statistical independence hypothesis.

[4]Einstein's argument is reproduced by Laudisa in [33]

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Thus, if locally causal, ordinary quantum mechanics must satisfy (9) when

$$\lambda = \mid \psi\rangle \tag{11}$$

However, choosing $a = b$, $A = 1$, and $B = -1$, an elementary quantum mechanical calculation gives

$$\begin{array}{rcl} P(1, -1 \mid a, a, \mid \psi\rangle) & = & P(1 \mid a, \mid \psi\rangle) P(-1 \mid a, \mid \psi\rangle) \\ \frac{1}{2} & \neq & \frac{1}{2}\frac{1}{2} = \frac{1}{4} \end{array} \tag{12}$$

Given that (12) does not seem to be widely known as a non-classical argument for quantum nonlocality, Appendix 7 explains it in detail.

Since in (12) $1/2 \neq 1/4$, ordinary quantum mechanics lacks a locally causal explanation of its correlations, i.e., *the quantum state alone cannot screen-off events on one side from spacelike separated events on the other far away side. Hence, it conspicuously fails the LC locality criterion.*

Note that (12) is not an EPR-like argument. It relies exclusively on quantum mechanical objective predictions. It is independent of the wave function interpretation and wave function collapse. In particular, it is independent of the ontic or epistemic nature of the quantum state, depending only on the quantum formalism irrespective of any interpretation. It is an argument in line with the Copenhagen approach, an operational definition that does not rely on metaphysical assumptions.

Formally, that is the counterargument against claims asserting the singlet correlations find a local common cause explanation in their preparation with the same generating event [15]. There is no doubt they find an explanation in their preparation. Unfortunately, that explanation is not locally causal because all we know from its preparation is its quantum state, and as (12) proves, it does not contain a common cause explanation. Nor does the magic of superposition justify those correlations, at least in a locally causal way [9].

Note that Bell never invoked his inequality for proving quantum nonlocality. Either in 1964, or 1975, Bell introduced his inequality only after taking for granted that quantum mechanics is not a local theory. The logically dubious approach of using a “classical inequality” to derive properties of quantum theory is not ascribable to John Bell. Unfortunately, often the same does not apply either to his followers or detractors [6, 9, 10, 12–14, 16], sometimes leading to heated debates [6–8].

Although the inequality does not make sense unless we assume “additional” hidden variables and is not a valid (or compelling) argument for

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quantum nonlocality, it is worth noting its 1975 derivation relies on only two hypotheses, namely, local causality and statistical independence. It is a stochastic derivation avoiding any polemics regarding determinism

$$LC \wedge SI \rightarrow Bell \quad inequality \tag{13}$$

We shall refer to this version of the Bell inequality as the stochastic Bell inequality. Although the inequality per se does not prove quantum nonlocality, it proves we cannot cure the nonlocality disease, not even by jettison determinism. Later, in 1982, A. Fine [34] proved that a stochastic hidden variable model exists if, and only if, a deterministic model does, so determinism was not, after all, a limiting constraint.

3.3 Statistical Independence

As we mentioned above, in Bell's 1964 paper, he implicitly assumed the hidden variables distribution function[5] $P(\lambda)$ was not conditional on the experimental settings a and b.

$$P(\lambda \mid a, b) = P(\lambda) \tag{14}$$

We can justify (14) by requiring the experimental settings to be independent of the same common factors λ affecting the results

$$P(a, b \mid \lambda) = P(a, b) \tag{15}$$

According to Bayes theorem we have

$$P(a, b \mid \lambda)P(\lambda) = P(\lambda \mid a, b)P(a, b) \tag{16}$$

Then from (15) and (16) we get (14). The ansatz (15) seems to be a reasonable assumption justifying (14).

Thus, (15) and (14) are equivalent and are known as *statistical independence*, *measurement independence*, *freedom*, or *no-conspiracy*. We shall come back to SI in sect. 4.3.

[5]Note that the distribution function of the λ common causes is irrelevant for the definition of local causality. $P(\lambda)$ is necessary only to derive the Bell inequality.

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3.4 The EPR Paper

Although Bell conceived his 1964 paper as a continuation of the EPR argument, one of the virtues of his 1975 formulation is not referencing the EPR paper. Besides presumably being a classical-like argument, the EPR reasoning contains an unnecessary metaphysical construction that has been the source of much confusion, namely, the *elements of physical reality*.

The reality criterion has a highly metaphysical burden because it assumes the existence of physical magnitudes from the mere possibility of predicting their values, notwithstanding that we do not indeed measure them. The esoteric physical existence of unmeasured magnitudes should not be confused with plain determinism. They are unnecessary because, as we have seen in sect. 3.2, they are employed neither to prove quantum nonlocality nor to derive the Bell inequality [22].

Bohr attacked the reality criterion [29]. Einstein did not write the EPR paper, and he did not like how it came out. In a letter to Schrödinger he wrote

> But still it has not come out as well as I really wanted; on the contrary, the main point was, so to speak, buried by erudition [35]

Einstein based his argument for incompleteness in his separation principle and avoided any reference to the reality criterion. Thus, it is worth noticing that Einstein and Bell distanced themselves from the original EPR elements of physical reality criterion. Even in writings where Bell referenced the EPR article, he never mentioned the elements of physical reality. Neither he recoursed to fanciful cognate ideas such as incompatible experiments or counterfactual definiteness. Claims to the contrary are unjustified fabrications unfairly attributed to John Bell. He never went beyond determinism which is quite different from assuming the physical existence of ghostly entities.

So, it is possible to address nonlocality and the Bell inequality in more rational terms without recursing metaphysical and fanciful concepts.

As Van Fraassen wisely observed in 1982:

> A reader as yet unfamiliar with the literature will be astounded to see the incredible metaphysical extravaganzas to which this subject has led. [18]

Notably, those metaphysical extravaganzas have not subsided since Van Fraassen's observation. But seem to have multiplied.

4 Quantum Locality

We shall review three counterarguments that justify considering quantum mechanics as a local theory. Only one of them, the third one, contemplates the use of the Bell inequality and is related to the Bell theorem.

4.1 Rejecting Local Causality

Jarrett [36] helped clarify the nature of local causality by decomposing it into the conjunction of two different conditions, which Shimony [37] respectively called parameter independence (PI) and outcome independence (OI)

$$LC \equiv PI \wedge OI \tag{17}$$

Shimony also proposed the more picturesque expressions controllable and uncontrollable nonlocality, respectively. Appendix 8 contains a brief introduction to these concepts.

Jarret proved that a theory complying with PI is no-signaling. He also showed quantum mechanics respects PI, hence is no-signaling. However, quantum mechanics violates OI, thus violating LC.

We can effectively block the argument in favor of quantum nonlocality by adopting parameter independence as the appropriate concept for locality and rejecting outcome independence as a necessary condition. Of course, those who claim quantum mechanics is not local will not accept the definition. However, more rational discussions are possible by explicitly acknowledging the different criteria.

In summarizing, by accepting no-signaling (parameter independence) as a sufficient criterion for locality, we reject the more stringent condition of local causality, turning ordinary quantum mechanics into a local theory.

4.2 Rejecting Realism

Local causality does not rely on classical assumptions, then why is it possible to recover locality by rejecting realism?

The various and sometimes obscure ponderings about realism notwithstanding [26–28, 38], there is a clear meaning for which its rejection would allow saving locality. Van Fraassen [18] expressed this view as

> ...empirical adequacy of a theory consists in it having a model that all the (models of) actual phenomena will fit into. In some cases, the methodological tactic of developing a causal theory will achieve this aim of empirical adequacy, in other case it will not, and that is just the way the world is. The causal terminology is descriptive, in any case, not of the (models of the) phenomena, but of the proffered theoretical models. So pervasive has been the success of causal models in the past, especially in a rather schematic way at a folk-scientific level, that a mythical picture of causal processes got a grip on our imagination

Van Fraassen's paper title eloquently begins with "The Charybdis of Realism". We can understand it as a rejection of Reichenbach's common cause principle [31] and, consequently, is yet another form of rejecting local causality as defined by Bell.

Howard Wiseman expressed this in more colloquial terms saying that *one could simply "refuse to consider the correlations mysterious"* [39]. There is no action at a distance simply because there is no need for a causal explanation.

Therefore, a coherent rejection of realism as a "causal explanation" has nothing to do with the infamous elements of physical reality or logically ill-conceived machinations such as counterfactual definiteness, incompatible experiments, or joint probabilities [22].

4.3 Completing Quantum Mechanics

This approach is different from the former two because it implies going beyond orthodox quantum mechanics by accepting local causality as the correct locality concept, thus recovering a causal explanation.

We can complete quantum mechanics with local hidden variables if we reject statistical independence. The 1975 version of the Bell theorem is given by (13), $LC \wedge SI \rightarrow Bell \quad inequaqlity$. Thus, we can retain local causality in a hidden variable theory by rejecting statistical independence. Indeed, well-known local hidden variables models exist reproducing the singlet correlations violating statistical independence [40, 41].

Whether statistical independence is a necessary physical condition is a contentious issue. According to some physicists, its rejection is a rational position [42–44]. Others, including John Bell [45], sustain its rejection as

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inadmissible since it purportedly compromises the experimental freedom implying unreasonable conspiracies.

5 Conclusions

Rationalizing the current controversy around the nonlocal character of quantum mechanics requires a radical change in the way localists and non-localists interpret the Bell theorem and the Bell inequality violations.

A stance we deem as radical non-localist [10, 14, 16, 46] interprets the Bell inequality violations as definitive proofs of quantum nonlocality. That interpretation, however, is controversial, poorly supported, and does not reflect Bell's clear line of reasoning. It spawns a radical localist stance [6, 9, 13, 15] asserting there exist no solid arguments sustaining the nonlocal character of quantum mechanics. Radical localists base their claims on trivial truisms such as

> Thus the usual derivations of CHSH and other Bell inequalities employ classical physics to discuss quantum systems, so it is not surprising when these inequalities fail to agree with quantum predictions, or the experiments that confirm these predictions. [15]

In fact, since all deterministic derivations of Bell-type inequalities are not based on quantum formalism, the former claim is arguably well justified, notwithstanding radical non-localists' complaints [14,33]. On the other hand, stochastic derivations of the Bell inequality assume hidden parameters not present in orthodox quantum mechanics. Therefore, they are also "classical" derivations.

The current impasse arises from the incorrect mixing of two different issues, the arguments for quantum nonlocality and quantum completion. We propose a clear separation of those arguments that imply changing the usual interpretation of the Bell theorem as a quantum nonlocality theorem. Our proposed reinterpretation, however, is far from being original. It is already there explicit and well documented in the writings that John Bell has left us.

In 1964, he accepted the EPR reasoning and already assumed nonlocality from the beginning. Then, in 1975 [30] and again in 1990 [47], he argued that quantum mechanics is not locally causal before setting out his inequality.[6]

[6] Unfortunately, Bell did not make this point explicitly clear in one of his most celebrated papers, "Bertlmann's socks and the nature of reality" [45].

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In all cases, the conclusion he drew from his line of reasoning, i.e., the thesis of his theorem, was:

> Quantum mechanics cannot be embedded in a locally causal theory. [47]

Because the alleged locally causal theory is classical, it is not clear that quantum nonlocality should follow from the Bell theorem. John Bell never made such a claim.[7]

On the other hand, his non-localist followers reverse Bell's reasoning claiming that quantum nonlocality follows from the Bell inequality violations [10, 14, 16, 46, 48] rendering a convenient excuse to localists for readily dismissing the awkward nonlocal character of quantum mechanics [6, 9, 12].

We urge the scientific community to vindicate John Bell and not to distort his clear line of reasoning as he laid it out from 1975 onwards. The allegedly not local character of orthodox quantum mechanics arises because it violates local causality (equivalently the common cause principle), not because it violates Bell-type inequalities.

It could be confusing that since local causality is a hypothesis of the Bell inequality in (13), wouldn't the inequality violation by quantum mechanics imply that it violates local causality? The answer is negative because the target of the inequality is "classical theories", i.e., those theories containing parameters that are not present in orthodox quantum mechanics.

Thus, quantum mechanics violation of local causality requires independent proof as given by (12), and the inequality violations were never an issue for quantum nonlocality. They signal the impossibility of local hidden variables when statistical independence is assumed. Despite non-localists' strong complaints, *the Bell theorem is a no-local-hidden-variables theorem, not a quantum nonlocality theorem. John Bell never claimed otherwise.*

6 Final Remarks

Simultaneity and nonlocality are closely related concepts. Both lack direct and clear-cut physical determination. Admitting a certain degree of convention is necessary if we want to maintain a coherent level of discourse.

[7] We avoid speculating on what Bell might have thought or believed. We base our assertion on what he has written in his published papers.

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Although the locality problem will remain controversial, it is essential to recognize its contentious nature for the correct motives instead of incorrect or obscure reasonings.

The quantum nonlocality problem cannot be summarily dismissed by looking for defects or trivial conceptual loopholes within the Bell-type inequalities and Bell's arguments. Quantum mechanics may require a revision of our notion of causality, just as relativity prompted us to revise our concept of simultaneity. The other possibility is that quantum mechanics is emergent and, because of Bell's theorem, that would require the acceptance of superdeterminism [42–44]. These options are still valid open questions, and pretending they are closed or inexistent is not the best scientific attitude.

APPENDIX

7 Common Causes and the Quantum State

Some researchers find it perplexing that the quantum state $|\psi\rangle$ can be considered a common cause in the definition of local causality. That is so owed to the metaphysical meaning usually attached to the λ variables as EPR elements of physical reality or a necessarily classical concept.

Sustaining that λ is by necessity an element foreign to quantum mechanics amounts to forbidding the application of the local causality concept to quantum mechanics. It is particularly convenient for summarily dismissing its annoying nonlocal character decreeing it local by construction [6]. However, the physical meaning of λ is not limited to classical or metaphysical concepts of any sort other than representing local common causes.

Although Bell might not have explicitly mentioned the case $\lambda = |\psi\rangle$, it is necessary to consider such a case to formalize Bell's (and Einstein's) qualitative arguments of quantum nonlocality according to the rigorous definition of local causality.[8] The inclusion of the quantum state as a λ variable was highlighted, for instance, by Nicolas Gisin [27]. This step is in complete agreement with viewing the quantum state as already containing the local common cause capable of explaining the quantum correlations. At least if

[8]When investigating the Bell inequality, after the nonlocal character of quantum mechanics was already established, we may consider the λ variables as additional parameters not already present in quantum mechanics. That seems to be the only case that Bell considered.

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quantum mechanics, as some people claim, is locally causal. *Thus, there is no valid argument against submitting quantum mechanics, within its own rules, to the local causality test.*

To see whether ordinary quantum mechanics complies with the local causality criterion, all we have to do is take $\lambda = \mid \psi\rangle$ in (9) with $\mid \psi\rangle$ given by (10), where $\mid +\rangle$ and $\mid -\rangle$ denote the spin eigenstates in the z-direction. We assume that motion takes place in y direction with setting angles a and b lying the $x-z$ plane measured with respect to the z axis. If $\mid a,+\rangle$ and $\mid a,-\rangle$ are the spin eigenstates in the a direction

$$\mid a,+\rangle = +\cos\frac{a}{2}\mid +\rangle + \sin\frac{a}{2}\mid -\rangle \tag{18}$$

$$\mid a,-\rangle = -\sin\frac{a}{2}\mid +\rangle + \cos\frac{a}{2}\mid -\rangle \tag{19}$$

Analogously for the particle measured at the other laboratory, we have

$$\mid b,+\rangle = +\cos\frac{b}{2}\mid +\rangle + \sin\frac{b}{2}\mid -\rangle \tag{20}$$

$$\mid b,-\rangle = -\sin\frac{b}{2}\mid +\rangle + \cos\frac{b}{2}\mid -\rangle \tag{21}$$

The joint probability according the quantum formalism is

$$P(A,B\mid a,b,\mid\psi\rangle) = \langle\psi\mid(\mid a,A\rangle\otimes\mid b,B\rangle\langle a,A\mid\otimes\langle b,B\mid)\mid\psi\rangle \tag{22}$$

Letting $A=+1$, $B=-1$ according to (10), (18) and (21)

$$\begin{aligned} P(+1,-1\mid a,b,\mid\psi\rangle) &= \langle\psi\mid(\mid a,+\rangle\otimes\mid b,-\rangle\langle a,+\mid\otimes\langle b,-\mid)\mid\psi\rangle & (23)\\ &= \frac{1}{\sqrt{2}}(\langle +\mid a,+\rangle\langle -\mid b,-\rangle - \langle -\mid a,+\rangle\langle +\mid b,-\rangle)\frac{1}{\sqrt{2}}()^* \\ &= \frac{1}{2}(\cos\frac{a}{2}\cos\frac{b}{2} + \sin\frac{a}{2}\sin\frac{b}{2})^2 & (24)\\ &= \frac{1}{2}\cos^2(\frac{a-b}{2}) & (25)\end{aligned}$$

Where $()^*$ represents the complex conjugate of the first factor in parenthesis. If we further assume $a=b$, (25) gives

$$P(+1,-1\mid a,a,\mid\psi\rangle) = \frac{1}{2} \tag{26}$$

When we perform a measurement only in Alice's laboratory, the quantum formalism prescribes

$$\begin{aligned}
P(+1, a, \mid \psi\rangle) &= \langle\psi \mid (\mid a,+\rangle\langle a,+\mid \otimes I) \mid \psi\rangle && (27)\\
&= \langle\psi \mid [(\mid a,+\rangle\langle a,+\mid \otimes I) \mid \psi\rangle] && (28)\\
&= \langle\psi \mid \left[\frac{1}{\sqrt{2}}(\mid a,+\rangle\langle a,+\mid +\rangle\otimes \mid -\rangle - \mid a,+\rangle\langle a,+\mid -\rangle\otimes \mid +\rangle)\right] &&\\
&= \langle\psi \mid \left[\frac{1}{\sqrt{2}}(\cos\frac{a}{2} \mid a,+\rangle\otimes \mid -\rangle - \sin\frac{a}{2} \mid a,+\rangle\otimes \mid +\rangle)\right] && (29)\\
&= \frac{1}{2}\left[\cos\frac{a}{2}\langle + \mid a,+\rangle + \sin\frac{a}{2}\langle - \mid a,+\rangle\right] && (30)\\
&= \frac{1}{2}\left[\cos^2\frac{a}{2} + \sin^2\frac{a}{2}\right] && (31)\\
&= \frac{1}{2} && (32)
\end{aligned}$$

Where $I = \mid +\rangle\langle + \mid + \mid -\rangle\langle - \mid$ is the identity operator in the one particle two-dimensional Hilbert-space. In a similar way, performing a measurement only on Bob's laboratory we find

$$\begin{aligned}
P(-1, b, \mid \psi\rangle) &= \langle\psi \mid (I\otimes \mid b,-\rangle\langle b,-\mid) \mid \psi\rangle && (33)\\
&= \frac{1}{2} && (34)
\end{aligned}$$

From (26), (32), and (34), we obtain (12) formally proving that ordinary quantum mechanics lacks a local common cause explanation for its correlations.

As we explain in the main text and contrary to widespread beliefs, Bell inequality violation does not tell us that quantum mechanics is not local. Nonlocality follows from quantum formalism itself. John Bell (and Einstein) never claimed otherwise. What is puzzling about the Bell theorem is that we cannot complete quantum mechanics with additional parameters under reasonable assumptions, proving that the experimentally tested quantum predictions seem to be hopelessly nonlocal. Probably that would have disappointed Einstein.

A similar argument for quantum nonlocality using the concept of local causality alone was also made, for instance, by Norsen [49]. However, our disagreement consists in that he ultimately presents the CHSH inequality as a quantum nonlocality proof when, after taking for granted statistical independence, he declares

> ...the empirically violated Clauser-Horne-Shimony-Holt inequality can be derived from Bell's concept of local causality alone, without the need for further assumptions involving determinism, hidden variables, "realism," or anything of that sort.

In our opinion, that move is unwarranted and justifies the opposite stance held by localists. Indeed, the CHSH inequality cannot be formulated without hidden variables or common causes not present in quantum mechanics precisely because quantum mechanics violates (9), as proved by (12). Certainly, proving Bell-type inequalities require writing joint probabilities as

$$P(A, B \mid a, b) = \int P(A \mid a, \lambda) P(B \mid b, \lambda) P(\lambda) d\lambda \qquad (35)$$

which is impossible without going beyond quantum mechanics. Admittedly, it is a trivial logical loophole. However, an endemic loophole that is frequently exploited by localists to debunk even the most lucid quantum nonlocality presentations like the one by Brunner et al. [46] where again (9) is correctly explained but finally (35) is highlighted as the "locality constraint", declaring

> This is the content of Bell's theorem, establishing the nonlocal character of quantum theory and of any model reproducing its predictions.

The counterargument states that since the inequality based on (35) is not about quantum mechanics, it signals the nonlocality of something else. That is why the Bell theorem concerns the impossibility of a local completion of quantum mechanics. It does not justify (convincingly?) orthodox quantum mechanics' nonlocal character.

8 PI and OI

In 1984 Jarrett noticed that local causality (9) is equivalent to the conjunction of two conditions, parameter independence (PI) and outcome independence (OI). Relativistic locality requires that the state of affairs on each laboratory be completely independent of any spacelike separated event at the other laboratory. Thus, on closer analysis, LC requires that the factors on the r.h.s. of (9) describe two different situations

$$P(A \mid a, \lambda) = \begin{cases} P(A, * \mid a, *, \lambda) \\ \sum_B P(A, B \mid a, b, \lambda) & , B \in \{-1, +1\} \end{cases} \qquad (36)$$

$P(A, * \mid a, *, \lambda)$ is Alice's probability of finding A when her measuring apparatus is set to a and Bob does not perform any measurement at all.[9]
$\sum_B P(A, B \mid a, b, \lambda)$ is Alice's marginal probability of finding A when measuring in direction a and Bob measures in the b direction.

Similarly, for Bob

$$P(B \mid b, \lambda) = \begin{cases} P(*, B \mid *, b, \lambda) \\ \sum_A P(A, B \mid a, b, \lambda) & , A \in \{-1, +1\} \end{cases} \tag{37}$$

Eliminating $P(A \mid a, \lambda)$ and $P(B \mid b, \lambda)$ from (36) and (37), we get parameter independence

$$\begin{array}{rcl} P(A, * \mid a, *, \lambda) & = & \sum_B P(A, B \mid a, b, \lambda) \\ P(*, B \mid *, b, \lambda) & = & \sum_A P(A, B \mid a, b, \lambda) \end{array} \tag{38}$$

Note that PI means Alice's (Bob's) probability is not influenced by Bob's (Alice's) setting nor by the fact he (she) decides not to perform any measurement at all.

Jarret showed that a theory complying with PI is no-signaling. Putting $\lambda =\mid \psi\rangle$ in (38) and following the same method as in Appendix 7, we can readily check that quantum mechanics satisfies the PI condition, therefore is no-signaling.

However, PI alone is not sufficient to satisfy LC. PI says nothing about the joint probability AB of Alice's and Bob's outcomes. Reichenbach principle of common causes [31,32] also requires the outcome independence condition

$$P(A, B \mid a, b, \lambda) = \sum_B P(A, B \mid a, b, \lambda) \sum_A P(A, B \mid a, b, \lambda) \tag{39}$$

Replacing PI (38) in OI (39), we have

$$P(A, B \mid a, b, \lambda) = P(A, * \mid a, *)P(*, B \mid *, b) \tag{40}$$

Jarret defined Bell's local causality by (40). Thus we have that

$$PI \wedge OI \implies LC \tag{41}$$

[9]In Appendix 7 we use this meaning of $P(A \mid a, \lambda)$ to prove that quantum mechanics violates local causality.

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On the other hand, we can obtain PI (38) from LC by adding with respect to outcomes in (40)

$$\sum_B P(A,B \mid a,b,\lambda) = P(A,* \mid a,*) \underbrace{\sum_B P(*,B \mid *,b)}_{=1} \tag{42}$$

$$\sum_B P(A,B \mid a,b,\lambda) = P(A,* \mid a,*) \tag{43}$$

Analogously adding with respect to A in (40)

$$\sum_A P(A,B \mid a,b,\lambda) = P(B,* \mid b,*) \tag{44}$$

So, $LC \implies PI$ and replacing the first terms of (43) and (44) back in (40) we also recover OI (39) and $LC \implies OI$. Thus we have

$$LC \implies PI \wedge OI \tag{45}$$

From (41) and (45) we get the equivalence between LC and $PI \wedge OI$

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