Article

Telling the Wave Function: An Electrical Analogy

Leonardo Chiatti[1,*]

[1] Medical Physics Laboratory, ASL VT, Via Enrico Fermi 15, 01100 Viterbo (Italy)
* Correspondence: leonardo.chiatti@asl.vt.it

Abstract: The double nature, wave and corpuscular, of the material particles is usually considered incomprehensible as it cannot be represented visually. It is proposed to the student, in the introductory courses, as a fact justified by quantum interference experiments of which, however, no further analysis is possible. In this note we propose a description of the wave function in terms of a simple electrical analogy, which reproduces at least some of its essential properties. The aim we propose is to provide a cognitive representation of an analogical type: starting from a classical context (electrical circuits) and introducing in an appropriate way the notions of "wave" and "particle", we show how typically quantum properties such as delocalization and entanglement emerge in a natural, understandable and intuitive way.

Keywords: Quantum mechanics; particle wave duality; quantum jump; quantum entanglement

1. Introduction

The wave nature of material particles, conceived by de Broglie in 1923 [1], still appears strongly counterintuitive today although confirmed by all the experiments carried out to verify it. There are at least two aspects of the particle wave duality that seem contrary to common sense. The first is the "delocalization" of the corpuscle, *ie* the fact that an individual entity is somehow simultaneously present on the entire spatial volume occupied by the wave. Secondly, if we accept, as we will do, the standard formulation of quantum mechanics and therefore the projection postulate [2], then in a "quantum jump" originating from the interaction with other particles or fields the corpuscle abruptly changes its state of delocalization. As a particular ideal limit case, the corpuscle can undergo a precise localization in space acquiring, at least in an ephemeral way, an attribute of position. That same attribute that is permanently possessed by a "classical" corpuscle.

The difficulties, however, do not end here. Amplitude interference experiments with single particles clearly demonstrate [3,4] the absence of trajectories attributable to quantum "corpuscles". These experiments have become, since at least the 1960s, the basis of the didactic presentation of the particle wave duality [5]. A widely used example is that of Young's interferometer with double slit. In this device, the particle, which manifests itself as a localized corpuscle in its impact on the rear screen, interferes with itself as the slit it crosses is not defined.

A similar phenomenon occurs with two identical particles emitted in a coherent way, which can hit two distinct detectors (intensity interference). In this case there may be a situation H_1 in which the particle emitted by the source S_1 hits the detector D_1 and the particle emitted by the source S_2 hits the detector D_2; or there may be a situation H_2 in which the particle emitted by the source S_1 hits the detector D_2 and the particle emitted by the source S_2 hits the detector D_1. The absence of trajectories leads to the undecidability between H_1 and H_2, and therefore to an interference connected with this undecidability. This interference is a particular effect of a general phenomenon correlated with the non-factorizability of the wave function of the system consisting of the two particles. This phenomenon is the entanglement [6], and it constitutes a further aspect of the quantum domain that does not seem to admit a classical representation.

We are thus faced with the problem of how to find a viewable representation of the quantum phenomena that we have briefly mentioned. In our opinion, this problem has a well-known historical precedent in the development of non-Euclidean geometries, in particular the elliptical geometry of Riemann [7] and the hyperbolic geometry of Bólyai and Lobačevskij [8]. In an attempt to construct a geometry that denies the parallel postulate, we are faced with spaces that are impossible to visualize. Certainly, no one is able to visualize a space where, given a straight line and a point external to it, no parallel to the given straight line passes through that point. Or an infinite number of parallels pass through it. This impossibility, however, persists only as long as we keep the original Euclidean notions of "line" and "point". It is in fact well known that Riemann's elliptical (two-dimensional) geometry admits a representation on the surface of an Euclidean sphere when appropriate redefinitions of the terms are performed; for example, if the lines are interpreted as maximum circles of the Euclidean narrative [9]. In the same way it is possible to represent the hyperbolic (plane) geometry on a portion of the Euclidean plane through the construction of Klein [9]. In the simplest version of this construction, space is the internal region of a circumference (understood in the Euclidean sense); the "straight lines" are (in Euclidean terms) the segments inside the circumference that secate it in two points, and so on. In summary, it is possible to represent the unrepresentable by attributing a new meaning to the terms. By carrying out this operation with care, the "new" can be represented, in a fully viewable way, in the same environment as the "old". It should be emphasized that this operation is analogical, and numerous alternative representations of the same represented structure are possible.

Returning to our specific problem, the application of this type of strategy first of all involves the identification of a specific classical setting within which to operate the redefinition of the terms "particle", "wave", "quantum jump". It is then necessary to show that the entities thus redefined (but easily visualized in the chosen classical setting) behave like the corresponding quantum entities in the context of their relations. The intuitive comprehensibility of the redefined entities is guaranteed by their coexisting “narration” in classical terms.

In attempting to search for a representation of quantum phenomena intended in this way, it seems inevitable to start with the concept of "particle". It is immediately evident that the usual notion of localized entity is unsuitable because it conflicts with delocalization and the absence of trajectories; therefore something geometrically less constrained is needed. We choose to assimilate the particle to an electromechanical actuator (a sort of relays) which exchanges energy elements associated with a charge. Recall that a particle can carry different types of charge: certainly a gravitational charge; possibly a weak, strong or electric charge. We will deal with the concept of charge assimilating it to that of electric charge, in any case keeping in mind the more general meaning of the term.

The next problem is how to introduce a spatially delocalized charge. The first idea that arises to mind is that in each point of empty space there is a "charge reservoir" (*ie* a system of capacitors) which exchanges elements of energy with the actuator, *ie* with the particle. The charge and energy of the particle are thus spatially delocalized, because they derive from the contribution of the capacitors associated with each point in space. The wave function of the particle then measures the local contribution to the total charge of the particle.

Invading empty space with circuits of electric capacitors may appear as an annoying reminiscence of 19th century English physics with its "ether of space". It is therefore appropriate to repeat that the meaning of our proposal is not to specify what a quantum particle is, but how it can be told using the language of classical physics. Our "ether of capacitors" is fictional but functional for that purpose. Its state of charge allows us to define the wave function of the particle.

Having in mind these reasons and this strategy, in Section 2 we pass to examine the basic ideas of our representation and then introduce, in the subsequent Section 3, the

Preprints (www.preprints.org) | NOT PEER-REVIEWED | Posted: 5 July 2022 doi:10.20944/preprints202207.0066.v1

relevant definition of wave function. An electrical analogy of the quantum jump is presented in Section 4, which also discusses the reason why the wave function "lives" in the enlarged configurational space and not in ordinary spacetime. The notion of a multi-particle wave function is thus introduced and the meaning of entanglement is illustrated. In Section 5 some issues related to the wave-particle dualism are specified, in particular the notions of particle and corpuscle in the context of the present representation. The inclusion of spin is discussed in Section 6. Section 7 summarizes the conclusions.

2. Basic ideas

Let us now see how to express in a more precise way the ideas illustrated in Section 1. For the moment we limit ourselves to considering the case of the propagation of a single particle of mass m and only subsequently we will consider the more general case of a system with several particles.

We assume that the propagation of the particle in four-dimensional spacetime is associated with a phenomenon of polarization of the vacuum, structured as follows. We denote by $\boldsymbol{x}$ a generic point of the four-dimensional spacetime whose coordinates in the reference of rest of the particle are (x, y, z, ct) and we consider the two regions (past and future) of the light cone having vertex in $\boldsymbol{x}$ and extension $\pm L/c$ in t. There are no stringent indications on the value of L; we will assume that $L = \hbar/mc$, the Compton length of the particle. This assumption seems plausible, because it is below this spatial scale that the particle is dissociated into particle-antiparticle pairs and therefore the polarization effects are manifested; however, any other physically reasonable choice of L is just as good.

As a consequence of the vacuum polarization associated with the particle, two opposite charges $+Q_1(\boldsymbol{x})$, $-Q_1(\boldsymbol{x})$ will be induced in the future light cone of $\boldsymbol{x}$ ($t < t' < t + L/c$), while two opposite charges $+Q_2(\boldsymbol{x})$, $-Q_2(\boldsymbol{x})$ will be induced in the past light cone of $\boldsymbol{x}$ ($t - L/c < t' < t$). We pose $Q_1, Q_2 \geq 0$ (the opposite choice is just as good). If we admit the existence of a "vacuum capacity" C, dependent only on the type of particle (electron, muon, etc.), these two charges correspond to two energies $Q_i^2/2Cn$, i = 1, 2. The total energy $(Q_1^2 + Q_2^2)/2Cn = Q^2/2Cn$ is that of a battery of $n(\boldsymbol{x})$ capacitors in parallel of the same capacity C brought to the common voltage V. The voltage V is assumed to be independent of $\boldsymbol{x}$. The charge $q = CV$ then depends on the type of particle. From the usual formalism of capacitors in parallel we have $Q^2/2Cn = nCV^2/2$ from which the relation $Q = nq$ follows.

A battery of capacitors in parallel is therefore associated with each point-event $\boldsymbol{x}$. The key assumption is that this battery contributes, with a part of its charge Q, to the total charge of an actuator (the particle) which we will assume to be q.

If the particle were exactly localized in $\boldsymbol{x}$, its charge would be integrally supplied by the capacitor battery present in $\boldsymbol{x}$. In this case, the total energy of the battery would vary by an amount of $\pm q^2/2Cn$. The negative sign corresponds to the transfer, by the battery, of a charge q to the particle; the positive sign corresponds to the transfer, by the particle, of a charge q to the battery. In the hypothesis of perfect localization of the particle in $\boldsymbol{x}$, the number of possible energy elements that can be exchanged between battery and particle is given by the ratio $(Q^2/2Cn)/(q^2/2Cn) = (Q/q)^2 = n^2$.

This suggests how to deal with the more general case of a delocalized particle. The charge exchanged between battery and particle is, in this case, a fraction (possibly infinitesimal) of q. We can therefore assume $\rho(\boldsymbol{x})dxdydz = dn^2(\boldsymbol{x})/A$ as the probability of the presence of the particle in the neighborhood d$xdydz$ of $\boldsymbol{x}$. The dimensionless normalization constant A can be determined according to the relation:

$$\int_{Vol} \rho(x,y,z,t)\, dxdydz = 1 \qquad (1)$$

Preprints (www.preprints.org) | NOT PEER-REVIEWED | Posted: 5 July 2022 doi:10.20944/preprints202207.0066.v1

The contribution to q^2 of the element $dxdydz$ around $\mathbf{x}$ is then $q^2\rho(\mathbf{x})dxdydz = q^2dn^2/A = dQ^2$. Each point of space contributes, with its own battery of capacitors in parallel, to the total charge q of the particle. This result constitutes a description of the delocalization of the particle in classical terms and we will return to it later. At the Compton scale (the minimum scale at which the wave function is defined) the relation $q^2\rho = dQ^2/(dxdydz)$ becomes $q^2\rho \sim Q^2/L^3$. Since q does not depend on $\mathbf{x}$, the density ρ is locally proportional to Q^2.

We must now see how the other actor appears, namely the wave function of the particle.

3. Positional and impulse representation

Let us now consider the two complex conjugate functions:

$$\psi = \frac{(Q_1+Q_2)}{2} + i\frac{(Q_1-Q_2)}{2} \tag{2a}$$

$$\psi^* = \frac{(Q_1+Q_2)}{2} - i\frac{(Q_1-Q_2)}{2} \tag{2b}$$

As we have seen, the probability density ρ is proportional to $Q^2 = Q_1^2 + Q_2^2$ since q is independent on $\mathbf{x}$. Therefore ρ is proportional to $\psi\psi^*$. We interpret functions (2a), (2b) as the two wave functions, delayed and anticipated, of the particle. We note that:

1) If the charges $Q_{1,2}$ are multiplied by a real common factor k, the probability density (not normalized) is multiplied by k^2, while the (2a), (2b) are multiplied by k.

2) If k is complex, the (non-normalized) probability density is multiplied by kk^*, while (2a), (2b) are multiplied by k. This implies that the modulus of (2a), (2b) is multiplied by the modulus of k, while the two functions are rotated around the origin of the complex plane by an angle equal to the argument of k.

3) From the proportionality of (2a), (2b) to the capacitor charges and the additive nature of these latter, it follows that functions of this type can be summed generating interference effects.

4) The time inversion $t \to -t$ implies the exchange $Q_1 \leftrightarrow Q_2$, and then $\psi \leftrightarrow \psi^*$.

5) The functions (2a), (2b) have, of course with reference to the representation discussed here, a clear ontic meaning as charge states of the network of batteries of capacitors associated with the particle.

On the other hand, it is possible to derive the (2a), (2b) with respect to the spacetime coordinates. Dimensionally, the quantities $I_{1,2} = c\partial_\mu Q_{1,2}$, where c is the maximal speed and $\mu = 0,1,2,3$ is the spacetime coordinate index, are currents. Among the functions (2a), (2b), those which are eigenfunctions of $c\partial_\mu$ are also eigenfunctions of the quadrimpulse $-i\hbar\partial_\mu$. It therefore becomes possible to replace (2a), (2b) with analogous complex functions containing currents instead of charges, thus passing to the impulse representation:

$$\varphi = \frac{(I_1+I_2)}{2} + i\frac{(I_1-I_2)}{2} \tag{3a}$$

Preprints (www.preprints.org) | NOT PEER-REVIEWED | Posted: 5 July 2022 doi:10.20944/preprints202207.0066.v1

$$\varphi^* = \frac{(I_1+I_2)}{2} - i\frac{(I_1-I_2)}{2} \tag{3b}$$

In general, (3a), (3b) can be modeled as a system of n inductors in parallel, of individual inductance M, through which the current $I = (I_1^2 + I_2^2)^{1/2}$ flows. The energy of the single inductor is $Mi^2/2$, being $i = I/n$ the current flowing in it. The total inductance of the system is $M_T = M/n$. We therefore have:

$$\frac{M_T I^2}{2} = \frac{MI^2}{2n} = \frac{M}{n}\frac{n^2 i^2}{2} = \frac{nMi^2}{2} \tag{4}$$

The representation in terms of inductances is completely specular to that in terms of capacitors, and in this paper we choose to focus on the latter.

4. Multi-particle systems

The independent variable $\boldsymbol{x}$ of the functions $Q_1(\boldsymbol{x})$, $Q_2(\boldsymbol{x})$ labels a battery of capacitors connected in parallel. Therefore there is a continuous quadruple infinity of these batteries. In a quantum jump the functions (2a), (2b) are zeroed and new ϕ functions of the same type are generated at the output. This means that the capacitors associated with the labels $\boldsymbol{x} = (x, y, z, ct)$, with t = instant of the jump, are discharged and new capacitors, associated with new labels of the same type, are charged according to the functions ϕ. It is possible to represent the quantum jump $\psi \rightarrow \phi$ with the electrical diagram in Figure 1.

Each point-event of spacetime corresponds to a battery of capacitors associated with the incoming function ψ and a battery of capacitors associated with the outgoing wave function ϕ. There are therefore batteries $\psi(\boldsymbol{x}_i)$ paired with batteries $\phi(\boldsymbol{y}_i)$ as in Figure 1, where the labels $\boldsymbol{x}_i$, $\boldsymbol{y}_i$ (i = 1, 2, ...) represent the same point-event: same spatial position and same instant in time coinciding with that of the jump.

The homologous ends of all batteries $\psi(\boldsymbol{x}_i)$ are connected to the corresponding two ends of the switch a. Closing a involves short-circuiting all the batteries of capacitors associated with function ψ. The total energy of all the batteries is dissipated in the internal resistance of the circuit and ψ is canceled. The closing of a also implies, as an automatic consequence, the closing of the switches such as b. The latter, in turn, implies the charging of all the batteries of capacitors associated with the function ϕ.

The function of the electromechanical actuator constituting the particle is to keep the circuit associated with function ψ open, that is, to keep the switch a open. The energy accumulated in the actuator is therefore the potential energy of the switch a open. The interaction responsible for the quantum jump provides the energy needed to overcome this potential barrier and close a. At the end of the jump, a is closed and its potential energy is zero.

It can be seen from the diagram in Figure 1 that each value of the $\boldsymbol{x}$ ($\boldsymbol{y}$) argument represents a mesh and therefore a discharge (charge) line. These labels are unique to the network of batteries of capacitors that is discharged or charged. If the network corresponds to a particle, in the sense that it exchanges charge with that particle (actuator) only, then the label is shared by that particle. Two distinct particles A, B then have distinct spacetime labels $\boldsymbol{x}_A$, $\boldsymbol{x}_B$. The wave functions associated with them are respectively $\phi(\boldsymbol{x}_A)$, $\varphi(\boldsymbol{x}_B)$ and each describes the state of charge of the network associated with the corresponding particle. The overall state of charge of the two quadruple infinity of batteries of capacitors associated with the two particles will be represented by the product $\phi(\boldsymbol{x}_A)\varphi(\boldsymbol{x}_B)$. Normally, in this product the two functions are considered at the same instant in time.

Preprints (www.preprints.org) | NOT PEER-REVIEWED | Posted: 5 July 2022 doi:10.20944/preprints202207.0066.v1

On the other hand, it is possible to imagine multiple networks and an equal number of actuators that: 1) exchange charge with each network; 2) contribute to the potential energy related to the opening of the circuit of each network. In this case, the spatial labels of *all* networks must be simultaneously assigned to the single actuator (particle). An example can be made by returning to the previous case of the two networks A, B. Imagine two actuators (particles) to whose charge both networks contribute and each of which contributes to the opening of both circuits A, B. In this case it is not possible distinguish the two particles through their network and one can have an entangled state such as, for example, $\phi(x_A)\varphi(x_B) \pm \phi(x_B)\varphi(x_A)$. The entanglement describes the contributions of the two networks to the opening of the two switches *a* inserted respectively on network A and network B.

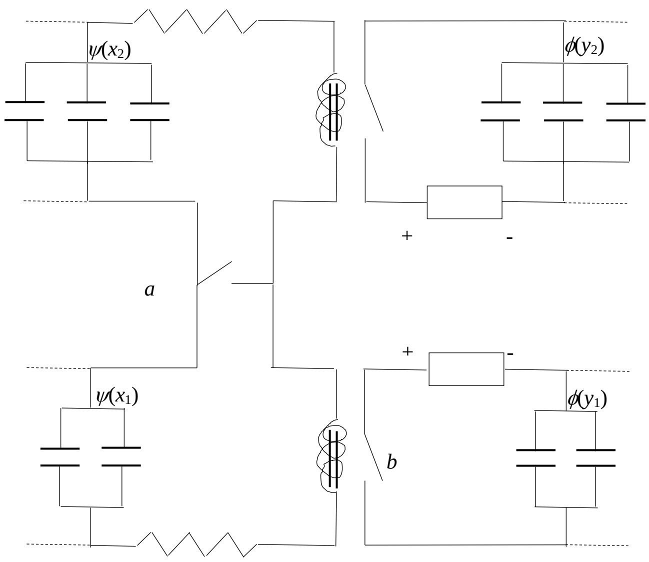

Figure 1. Electrical diagram of the quantum jump $\psi \rightarrow \phi$ (for simplicity, only two pairs of opposing meshes are represented)

The proposed representation therefore allows us to define both single-particle and multi-particle wave functions, the latter both factored and entangled. It also supports a viewable model of quantum jump.

5. Corpuscle wave dualism

It is possible to assume that the energy of the actuator constituting the particle associated with the wave function ψ is $q^2/2C$, with the meaning of the symbols already seen in the previous Sections. In this hypothesis, this is the minimum energy required to keep the switch *a* open and thus allow the propagation of ψ. It is therefore natural to suppose that for a particle of mass m is $q^2/2C = mc^2$, the rest energy of the particle. This is in fact the minimum energy required for an interaction to create an outgoing state containing that particle. The delocalization of the particle described in Sections 2-3 then corresponds, physically, to the delocalization of its rest energy; the energy stored in each capacitor battery represents the local contribution to the rest energy.

In summary: the quantum wave is the state of charge of capacitor networks. Each network contributes to the charge and energy of an actuator which is the particle. In this sense, the particle is "delocalized" on the network. The wave function defines at the same time the state of charge of the network and the delocalization of the particle. Different

Preprints (www.preprints.org) | NOT PEER-REVIEWED | Posted: 5 July 2022 doi:10.20944/preprints202207.0066.v1

particles can correspond to different networks, and in this case the state of charge of the networks is described by a factorizable wave function. On the other hand, when different networks contribute to powering the same actuator and the latter acts on all the networks that feed it, entanglement occurs. In both cases the wave function depends on the spatial coordinates of all the particles involved and not on the generic spacetime coordinates (as it would be for a classical field). The ambient space of the wave function is therefore the enlarged space of configurations, not the ordinary four-dimensional space where an observer coordinates the interaction events.

In a quantum jump there is a sudden change in the wave function: the wave function entering the jump is zeroed by short-circuiting the network of capacitors associated with it; a new capacitor network, associated with a new wave function coming out of the jump, is charged. The net charge q of the particle - if it is a conserved quantity - is transferred from the incoming function to the outgoing one and becomes an additive contribution to the total charge of the particles it describes. This transfer is what we normally refer to as the term "corpuscle". This definition captures a feature of the corpuscular aspect of matter that is well known from quantum experiments: its ephemeral nature. In other words, the corpuscle is an event (such as, for example, the impact of an electron on a photographic plate), not a persistent object that can be recognized and traced within the wave function.

The analogical representation proposed here applies to the wave function of systems of one or more particles. It is not applicable to the wave function of idealized systems such as the rotator, the vibrator, etc. because for these systems the concept of charge associated with a (virtual) polarization of the vacuum loses meaning. It must however be considered that the real systems corresponding to these idealizations can in any case be described as aggregates of interacting particles and therefore their exact treatment leads back to the case examined here.

6. Spin

It is possible to construct row or column vectors whose components are different functions of the type (2a), (2b) or, respectively, of the type (3a), (3b). Each component of these vectors is associated with a different network of batteries of capacitors in parallel or, respectively, of inductances in parallel, afferent to a single actuator. If the relativistic covariance rules are applied to these vectors, they become spinors. We then have the Pauli spinors in the non-relativistic limit, and the Dirac spinors in the general relativistic case. It thus becomes possible, in principle, to extend the present electrical analogy to include the spin of elementary particles and their internal degrees of freedom such as isospin or strangeness. Here we limit ourselves to an observation on spin.

A generic wave function of spin ½ takes, in the Pauli algebra, the following form:

$$\psi(x,t) = \alpha(x,t)\psi_z^+ + \beta(x,t)\psi_z^- \; ; \qquad |\alpha\alpha^*| + |\beta\beta^*| = 1 . \tag{5}$$

where the spinors ψ_z^+, ψ_z^- represent fully polarized beams along the z axis, corresponding to the two eigenvalues of the spin projection along that axis. The meaning of (5) in the present representation is that a single actuator (particle) exchanges charge with two distinct networks, each of which associated with one of the two projections, and keeps them both open. The (complex) coefficients α, β measure the relative intensity of the exchange with the relevant network, in the terms seen in Section 3. The fact that by varying the z axis, the coefficients α, β change, means that the exchange depends on the choice of the z axis. It must be borne in mind that the variation of the z axis is always associated with a modification of the experimental situation, such as fixing a direction to a magnetic field or reorienting a polarizer. This modification involves the variation of the charge exchange between the particle and the two networks. The directional dependence of the exchange represents, in a certain sense, a directional structure of the particle itself.

Preprints (www.preprints.org) | NOT PEER-REVIEWED | Posted: 5 July 2022 doi:10.20944/preprints202207.0066.v1

But this structure or internal "direction" of the particle has nothing to do with the rotation of a solid object in three-dimensional space. This is consistent with the non-classical nature of spin [10].

7. Conclusions

In these concluding notes we would like to try to put this contribution in context. In our opinion, the non-viewability of quantum processes has generated the widespread belief that they are inherently incomprehensible. Quantum theories are often assimilated to formal recipes that are very effective on the predictive level, but whose connection with any intelligible ontology remains obscure. Following the historical precedent of non-Euclidean geometries, we have tried to circumvent these obscurities by providing a model of quantum behavior in a classical context: that of electrical circuits. As in the case of non-Euclidean geometries, we had to redefine the notions of particle, wave and corpuscle in this context, moving away from their original classical meaning. By paying this price we have obtained the visualization of quantum entities and processes, guaranteed by the classical nature of the context in which their redefinition is carried out.

We believe that this attempt is located in a sort of middle land between the choice of surrendering to non-visualization (with the consequent problems of conceptual opacity) and the strong choice of determining an ontology of elementary processes, which is the goal of any physical interpretation of the quantum formalism. Although, in choosing our representative model, we tried to adhere to criteria of "sound physics", we find it difficult to seriously believe that space is the equivalent of a cabinet of electric capacitors. Our representation is therefore less than a *sensu strictu* interpretation of quantum formalism such as, for example, Bohm's and relative states interpretations. At the same time, however, it is more than just a surrender to mystery and allows for an analogical narrative of concepts such as the wave function or the particle wave dualism. Our aim is to facilitate communication relating to quantum processes, through images that can be understood by anyone familiar with the basics of classical physics concerning electrical circuits.

This objective responds to a pedagogical need which has been our strongest motivation and which seems particularly urgent to us in a moment like the present one, in which quantum theories have become part of the educational background of the new generations of engineers and technologists, engaged in development of the amazing technologies that the quantum nature of reality makes possible.

Funding: This research received no external funding.

Institutional Review Board Statement: Not applicable.

Informed Consent Statement: Not applicable.

Data Availability Statement: Not applicable.

Acknowledgments: The author wishes to thank the colleague I. Licata for the critical revision of the manuscript and many interesting discussions.

Conflicts of Interest: The author declares no conflict of interest.

References

1. de Broglie, L. Recherches sur la théorie des quanta. *Ann. de Phys.* **1925**, 10 (3), 22
2. Von Neumann, J. *Mathematical Foundations of Quantum Mechanics*; Princeton University Press: Princeton, USA, 1932

3. Merli, P.G.; Missiroli, G.F.; Pozzi, G. Electron interferometry with the Elmiskop 101 electron microscope. *Journ. Phys. E: Scientific Instruments* **1974**, **7,** 729–732
4. Merli, P.G.; Missiroli, G.F.; Pozzi, G. On the statistical aspect of electron interference phenomena. *Am. Journ. Phys.* **1976**, 44**,** 306–307
5. Feynman, R.; Leighton, R.; Sands, M. *The Feynman lectures on physics (Vol.III)*; California Institute of Technology: Berkeley, USA, 1965
6. Horodecki, R.; Horodecki, P.; Horodecki, M.; Horodecki, K. Quantum entanglement. *Rev. Mod. Phys.* **2009**, 81, 865
7. Gray, J. Bernhard Riemann, posthumous thesis 'On the hypotheses which lie at the Foundation of Geometry' (1867). In *Landmark writings in Western mathematics 1640-1940*; Grattan-Guinness, I., Corry, L., Guicciardini, N., Cooke, R., Crépel, P., Eds.; Elsevier, 2005; pp. 506-520.
8. Lobačevskij, N.I. *New Principles of Geometry with Complete Theory of Parallels. Complete Collected Works (Vol II)*; Kagan, V.F. Ed.; GITTL: Moscow–Leningrad, USSR, 1951
9. Non Euclidean geometry. Britannica. https://www.britannica.com/science/non-Euclidean-geometry
10. Aerts, D.; Sassoli de Bianchi, M. Do spins have directions? *Soft Computing* **2017**, 21 (6), 1483-1504