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# Effect of Various Parameter on Perihelion Shift and Deflection Angle in Reissner-Nordström Spacetime

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Abstract: Study of perihelion shift of Mercury and deflection angle are important aspects for the verification of general relativity. For Schwarzschild geometry both the perihelion shift and weak deflection angle were independent of charge whereas for Reissner-Nordström space-time charge should effect on both the perihelion shift and deflection angle. This paper shows the perihelion shift decreases with the increases of charge.Effect of other parameters like angular momentum, mass parameter, distance of closest approach on perihelion precession and deflection angle has also discussed. This paper also includes the graphical behavior which verifes our theoretical calculations.

Keywords: perihelion shift; bending of light; Reissner-Nordström Black Hole.

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## Contents



# 1 Introduction

To verify the correctness of general relativity, the perihelion precession of planetary orbits such as Mercury and deflection of light by a massive star in the solar system are two well-known phenomena [1–5]. The first test of general relativity is perihelion precession of Mercury whereas deflection of light is the second famous test of general theory of relativity. In 1919, the first measurement was performed by Eddington and his collaborators. Lately, we know that the establishment of general relativity is a very important event in modern physics. General theory of relativity gives new insights into our understanding of gravity and nowadays it is used as a basic theory of modern cosmology [1]. As the perihelion precession and the deflection of light ray are usually liable event in the solar system or some planet such as Mercury [1–3], so it will be more important to study the perihelion precession and the deflection of light in the more general case [10]. In this paper, we have considered the perihelion precession and deflection of light in the 4-dimensional (4D)

general spherically symmetric space-time. This paper also includes a general formalism based on the perturbative method to get the time-like or null geodesics in space-time and calculate the perihelion precession and deflection angle [3, 7]. It is notified that, due to the generalization of the Birkhoff's theorem in Einstein's general theory of ralativity, we get the general spherically symmetric space-times in Einstein's general theory of relativity. The unique spherically symmetric solution of the Einstein-Maxwell equations with non-constant area radial function (r) is the Reissner-Nordström solution [1, 6]. Therefore, we take the Reissner-Nordstörm solution in Einstein's general theory of relativity to discuss the corresponding perihelion precession and deflection of light.
For the Reissner-Nordstörm space-time with the electric charge (Q), we observe that the electric charge can affect both the perihelion precession and deflection of light, while the angular momentum can only affect the perihelion precession of Mercury. Our results further reveal that for R-N spacetime the perihelion shift and amount of deflection decreases with increase in electric charge, while for a Schwarzschild space-time it is independent of electric charge [7, 8].

This paper is presented in six parts. In section 2, using the peturbative method, we evaluate the amount of perihelion precession. Here the possibility of planner motion in this spacetime has also been discussed. By doing graphical analysis, the effects of various parameters on the perihelion shift are studied in section 3. In section 4, we derive deflection angle. Similar to the perihelion precession, the graphical analyses to study the effects of various parameters on deflection angle are also presented in section 5. Finally, we conclude our results and make a final remarks in section 6.

## 2 The Precession Of The Perihelion Motion Of Mercury

In case of weak field limit, general theory of relativity behaves like Newtonian theory of gravity ,i.e, any observation which is consistent with Newtonian theory of gravity, gives indirect evidence to general theory of relativity. But some phenomena which are not fully explained by Newtonian theory, can be explained by general theory of relativity.
Let us consider the motion of a massive test particle (in our case it's Mercury) in Reissner-Nordström space-time. It's known that the massive particles moves along time-like geodesic. It is very much similar to the perihelion precession in Schwarzschild geometry [8]. Line element in Reissner-Nordström (R-N) space-time is [11]

$$ds^2 = -(1 - \frac{2m}{r} + \frac{Q^2}{r^2})dt^2 + \frac{dr^2}{(1 - \frac{2m}{r} + \frac{Q^2}{r^2})} + r^2 d\Omega_2^2 \tag{2.1}$$

Therefore,

$$2\mathcal{L} = -(1 - \frac{2m}{r} + \frac{Q^2}{r^2})\dot{t}^2 + \frac{\dot{r}^2}{(1 - \frac{2m}{r} + \frac{Q^2}{r^2})} + r^2 d\Omega_2^2 = -1 \tag{2.2}$$

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where $\dot{t}$ is the derivative with respect to proper time $\tau$. Let's consider $x^0 = t$, $x^1 = r$, $x^2 = \theta$, $x^3 = \phi$. Now, Remembering well known Euler-Lagrangian equation :

$$\frac{d}{d\tau}(\frac{\partial \mathcal{L}}{\partial \dot{x}^f}) - \frac{\partial \mathcal{L}}{\partial x^f} = 0$$

Euler-Lagrangian equation for f = 0, f = 1 and f = 2, respectively, takes the following form :

$$\frac{d}{d\tau}[(1 - \frac{2m}{r} + \frac{Q^2}{r^2})\dot{t}] = 0 \tag{2.3}$$

$$\frac{d}{d\tau}(r^2\dot{\theta}) - r^2 sin\theta cos\theta\dot{\phi}^2 = 0 \tag{2.4}$$

$$\frac{d}{d\tau}(r^2 sin^2\theta\dot{\phi}) = 0 \tag{2.5}$$

It is not worthy to take the case for f = 1, which corresponds to coordinate r. Since time-like geodesic is charecterized by four equations, $t = t(\tau)$, $r = r(\tau)$, $\theta = \theta(\tau)$ and $\phi = \phi(\tau)$, also Eq. (2) and Euler-Lagrangian equation gives enough information. Since differentiating Eq. (2) with respect to $r$ will clearly make the bigger problem, so we can neglect f = 1 case.

Now, as like the Newtonian and Schwarzschild case, we will must establish whether planner motion in this space-time is possible or not. We will consider the motion in equatorial plane $\theta = \frac{\pi}{2}$. Therefoe equation (2.4) becomes

$$r^2\ddot{\theta} + 2r\dot{r}\dot{\theta} - r^2 sin\theta cos\theta\dot{\phi}^2 = 0$$

so

$$\ddot{\theta} = 0$$

this indicates that if initially $\theta$ takess the constant value $\frac{\pi}{2}$, then throughout the geodesic it will take the same and almost constant value. So there must remains two space coordinates r and $\phi$. Hence planner motion is possible in general relativity. In other words, here also geodesic is confined to a single plane as in Newtonian mechanics and Schwarzschild geometry.

By iIntegrating of equation (2.5) gives

$$r^2\dot{\phi} = h \tag{2.6}$$

where h is constant and represents angular momentum. Again integrating equation (2.3) we obtain,

$$(1 - \frac{2m}{r} + \frac{Q^2}{r^2})\dot{t} = k = const. \tag{2.7}$$

Now, in equatorial plane with the help of equations (2.2) and (2.7),

$$k^2(1-\frac{2m}{r}+\frac{Q^2}{r^2})^{-1}-(1-\frac{2m}{r}+\frac{Q^2}{r^2})^{-1}\dot{r}^2-r^2\dot{\phi}^2=1 \tag{2.8}$$

By considering $u=\frac{1}{r}$, from equations (2.6) and (2.8), we finally get,

$$\frac{k^2}{(1-2mu+Q^2u^2)}=\frac{h^2(\frac{du}{d\phi})^2}{(1-2mu+Q^2u^2)}-h^2u^2=1$$

$$\frac{k^2}{h^2}-\left(\frac{du}{d\phi}\right)^2-h^2u^2(1-2mu+Q^2u^2)=\frac{(1-2mu+Q^2u^2)}{h^2}$$

Now, we get a first order differential equation for Mercury's motion in R-N space-time as following,

$$\left(\frac{du}{d\phi}\right)^2+u^2=\frac{k^2-1}{h^2}+\frac{2m}{h^2}u-\frac{Q^2}{h^2}u^2+2mu^3-Q^2u^4 \tag{2.9}$$

this first order differential equation has a degree = 4. It determine the orbit of a time-like particle. This trajectory lies in the hypersurface t = constant and $\theta$ is constant, only variable is r and $\phi$. Therefore we can say that the trajectory is not confined in 4-dimensional plane rather it's confined in r and $\phi$ surface.

Now, we obtain the second order differential equation for Mercury's motion in R-N space-time as,

$$\frac{d^2u}{d\phi^2}+u=\frac{m}{h^2}-\frac{Q^2}{h^2}u+3mu^2-2Q^2u^3 \tag{2.10}$$

This equation closely resembles the differential equation in Schwarzschild case, except there are additional two terms $-\frac{Q^2}{h^2}u$ and $-2Q^2u^3$. This additional term appears due to the presence of charge (Q) in this space-time. The above equation can return to the well known Schwarzschild case when charge term will be absent.

We can solve the above equation (2.10) using a perturbative method. Let's expressing the differentiation with respect to $\phi$ with a 'prime' and rewritting the above equation (2.10) by introducing the parameter $\epsilon_1$ = -$\frac{Q^2}{h^4}$u, $\epsilon_2=\frac{3m^2}{h^2}$ and $\epsilon_3$ = -$\frac{2Q^2m}{h^2}$ as,

$$u''+u=\frac{m}{h^2}+\epsilon_1\frac{h^2u}{m}+\epsilon_2\frac{h^2u^2}{m}+\epsilon_3\frac{h^2u^3}{m} \tag{2.11}$$

Let's consider the solution has a form : $u=u_0+\epsilon_1u_1+\epsilon_2u_2+\epsilon_3u_3+O(\epsilon_1^2,\epsilon_2^2,\epsilon_3^2)$. Then by differentiating this solution twice, we will substitute it in equation (2.11) and after rearranging we get;

$$u_0'' + u_0\frac{m}{h^2} + \epsilon_1(u_1'' + u_1 - \frac{h^2u_0}{m}) + \epsilon_2(u_2'' + u_2 - \frac{h^2u_0^2}{m}) + \epsilon_3(u_3'' + u_3 - \frac{h^2u_0^3}{m})$$
$$+O(\epsilon_1^2, \epsilon_2^2, \epsilon_3^2, \epsilon_1\epsilon_2, \epsilon_1\epsilon_3, \epsilon_2\epsilon_1, \epsilon_2\epsilon_3, \epsilon_3\epsilon_1, \epsilon_3\epsilon_2) = 0 \quad (2.12)$$

Now, equate the coefficient of $\epsilon_1$, $\epsilon_2$, $\epsilon_3$, $\epsilon_1^2$, $\epsilon_2^2$, $\epsilon_3^2$, $\cdots$ to zero. Then $u_0 = \frac{m}{h^2}(1 + ecos\phi)$ is the zeroth order solution (Newtonian solution) of equation (2.12). This solution can be justified by differentiating twice and substitute into equation(2.12). For $u = \frac{1}{r}$, this solution agrees with the Newtonian solution.

Now, let's examine the coefficient of $\epsilon_1$ in equation (2.12),

$$u_1'' + u_1 = \frac{h^2u_0}{m}$$

$$u_1'' + u_1 = (1 + ecos\phi) \quad (2.13)$$

comparing the coefficient of $\epsilon_2$ in equation (2.12) is

$$u_2'' + u_2 = \frac{h^2u_0^2}{m} = \frac{m}{h^2}(1 + ecos\phi)^2 = \frac{m}{h^2}(1 + 2ecos\phi + e^2cos^2\phi)$$

$$u_2'' + u_2 = \frac{m}{h^2}(1 + \frac{e^2}{2}) + \frac{2me}{h^2}cos\phi + \frac{me^2}{2h^2}cos2\phi \quad (2.14)$$

and comparing the coefficient of $\epsilon_3$ in equation (2.12),

$$u_3''+u_3 = \frac{h^2u_0^3}{m} = \frac{m^2}{h^4}(1+ecos\phi)^3 = \frac{m^2}{h^4}\left[1+3ecos\phi+3e^2\frac{(1 + cos2\phi)}{2}+\frac{e^3}{4}(cos3\phi+3cos\phi)\right]$$

Neglecting $O(e^3)$ we get,

$$u_3'' + u_3 = \frac{m^2}{h^4}(1 + \frac{3e^2}{2}) + \frac{m^2}{h^4}3ecos\phi + \frac{3e^2m^2}{2h^4}cos2\phi \quad (2.15)$$

Let's try a general solution

$$u_1 = A_1 + B_1\phi sin\phi + C_1cos2\phi \quad (2.16)$$

$$u_2 = A_2 + B_2\phi sin\phi + C_2cos2\phi \quad (2.17)$$

and

$$u_3 = A_3 + B_3\phi sin\phi + C_3cos2\phi \quad (2.18)$$

Now, solve for the coefficient of $u_1$,$u_3$ and $u_3$ we get respectively,

$$u_1'' + u_1 = (A_1) + (2B_1)cos\phi + (-3C_1)cos2\phi \quad (2.19)$$

$$u_2'' + u_2 = (A_2) + (2B_2)cos\phi + (-3C_2)cos2\phi \tag{2.20}$$

and

$$u_3'' + u_3 = (A_3) + (2B_3)cos\phi + (-3C_3)cos2\phi \tag{2.21}$$

Comparing equations (2.13) and (2.19), we can get at the following value of $A_1$, $B_1$ and $C_1$:

$$A_1 = 1$$

$$B_1 = \frac{e}{2}$$

and

$$C_1 = 0$$

Similarly comparing equations (2.14) and (2.20), we can get the following value of $A_2$, $B_2$ and $C_2$:

$$A_2 = \frac{m}{h^2}(1 + \frac{e^2}{2})$$

$$B_2 = \frac{me}{h^2}$$

and

$$C_2 = -\frac{me}{6h^2}$$

Also comparing equations (2.15) and (2.21), we can get the following value of $A_3$, $B_3$ and $C_3$:

$$A_3 = \frac{m^2}{h^4}(1 + \frac{3e^2}{2})$$

$$B_3 = \frac{3}{2}\frac{m^2e}{h^4}$$

and

$$C_3 = -\frac{m^2e^2}{2h^4}cos2\phi$$

Hence,

$$u_1 = (1 + \frac{e}{2}\phi sin\phi)$$

$$u_2 = \frac{m}{h^2}\left[1 + e\phi sin\phi + e^2(\frac{1}{2} - \frac{1}{6}cos2\phi)\right]$$

and

$$u_3 = \frac{m^2}{h^4}\left[1 + \frac{3}{2}e\phi sin\phi + 3e^2(\frac{1}{2} - \frac{1}{6}cos2\phi)\right]$$

Therefore, finally the general solution to first order is [9],

$$u \approx u_0 + \epsilon_1 u_1 + \epsilon_2 u_2 + \epsilon_3 u_3 + O(\epsilon_1^2, \epsilon_2^2, \epsilon_3^2)$$

$$u \approx u_0 - \frac{Q^2 m}{h^4}(1 + \frac{e}{2}\phi sin\phi) + \frac{3m^3}{h^4}\left[1 + e\phi sin\phi + e^2(\frac{1}{2} - \frac{1}{6}cos2\phi)\right]$$
$$-\frac{2Q^2m^3}{h^6}\left[1 + \frac{3}{2}e\phi sin\phi + 3e^2(\frac{1}{2} - \frac{1}{6}cos2\phi)\right] \qquad (2.22)$$

Verifying the correction term in equation (2.22), we find that the $e\phi sin\phi$ term increases after each revolution, and hence it becomes dominant. Neglecting the other terms in the correction we get a simplified version of (2.22) as follows,

$$u \approx u_0 - \frac{Q^2 m}{h^4}\frac{e}{2}\phi sin\phi + \frac{3m^3}{h^4}e\phi sin\phi - \frac{3Q^2m^3}{h^6}e\phi sin\phi \qquad (2.23)$$

where $u_0 = \frac{m}{h^2}(1 + ecos\phi)$ is an analytical elliptical solution which has already been found in Newtonian theory and e represents the orbital ecentricity which has been considered as a vary small quantity.

Therefore, equation (2.23) can be further reduced by substituting $u_0$ and by neglecting higher order terms we get:

$$u \approx \frac{m}{h^2}(1 + ecos\phi) - \frac{Q^2 m}{h^4}\frac{e}{2}\phi sin\phi + \frac{3m^3}{h^4}e\phi sin\phi \approx \frac{m}{h^2}\left[1 + e\{cos\phi + \epsilon\phi sin\phi\}\right] \qquad (2.24)$$

where $\epsilon = (\frac{3m^2}{h^2} - \frac{Q^2}{2h^2})$. As a result,

$$u \approx \frac{m}{h^2}\left[1 + ecos\{\phi(1-\epsilon)\}\right] \qquad (2.25)$$

The above equation (2.25) indicates that Mercury's orbit is no longer is an ellipse. For the perihelion of $r(\phi)$, it satisfies : $cos\{\phi(1-\epsilon)\} = 1$. Therefore, the precession angle of perihelion is

$$\delta\phi = 2\pi\epsilon$$
$$\delta\phi = \frac{6\pi m^2}{h^2} - \frac{Q^2\pi}{h^2} \qquad (2.26)$$

We see that the perihelion shift depends on mass parameter (m), charge (Q) and angular momentum (h). Compared with the case of Schwarzschild space-time $\delta\phi = \frac{6\pi m^2}{h^2}$, an additional term $-\frac{Q^2\pi}{h^2}$ comes from the electric charge contribution. The above equation (2.26) can return into the well known Schwarzschild case when charge term will be absent.

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# 3 Graphical analysis

## 3.1 Effect of angular momentum (h) on perihelion shift ($\delta\phi$)

To discuss the effect of angular momentum on perihelion shift, we plot figure 1. Here,

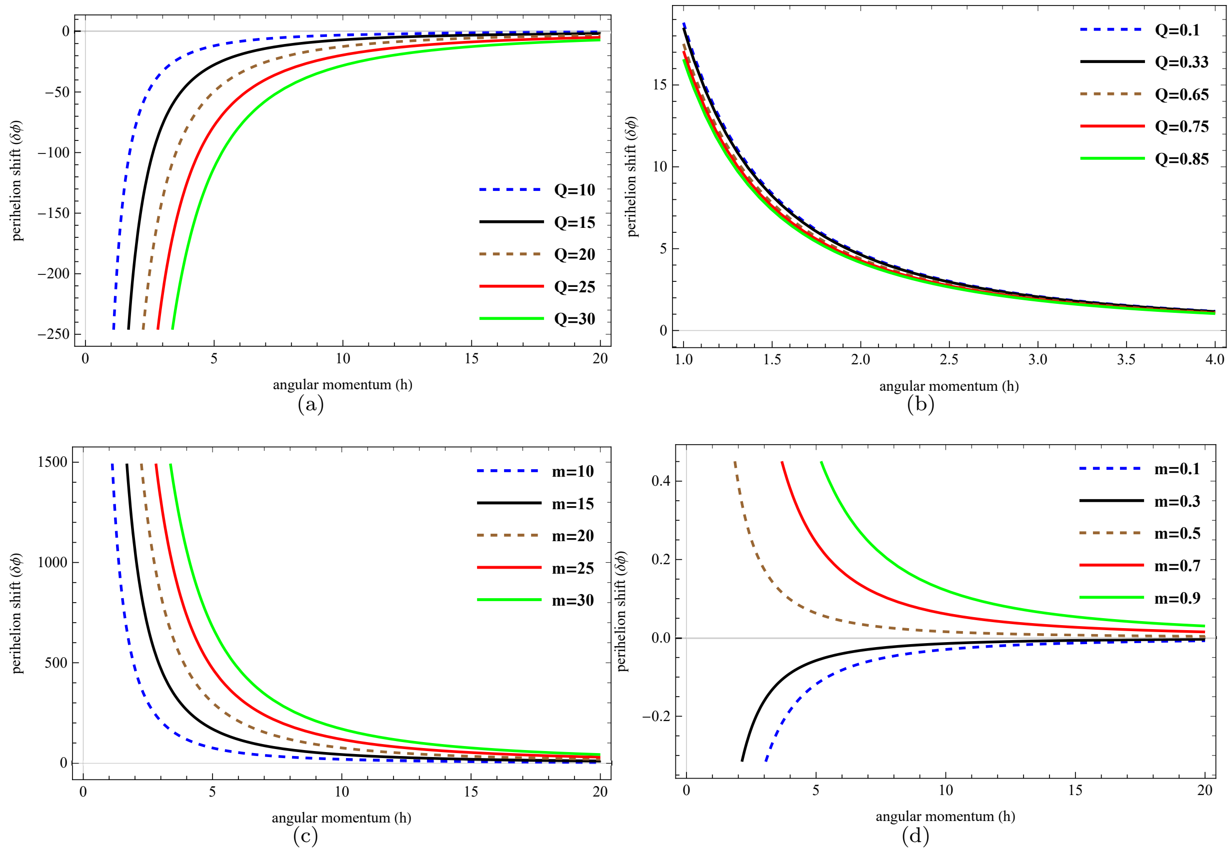


**Figure 1**. In 1(a) and 1(b), the behavior of perihelion shift ($\delta\phi$) with respect to angular momentum (h) for varying $Q$ but fixed $m = 1$ In 1(c) and 1(d), the behavior of $\delta\phi$ with respect to $h$ for varying $m$ but fixed $Q = 1$.

from the plots 1(a) and 1(b), it is clear that the perihelion shift decreases with the angular momentum for very small $Q$ but remains positive. In contrast, for large $Q$, the perihelion shift increases with $h$ but takes negative values only.
However, from figures 1(c) and 1(d), we see that the perihelion shift increases with the angular momentum for small $m$ and remains positive valued. For larger black hole mass, the perihelion shift is asymptotically decreasing function but remains positively valued.

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### 3.2 Effect of charge (Q) on perihelion shift ($\delta\phi$)

To study the effect of electric charge Q on perihelion shift, we plot figure 2. From the plot, we observe that the perihelion shift is a decreasing function of charge Q. The value of perihelion shift becomes more negative when angular momentum h decreases.

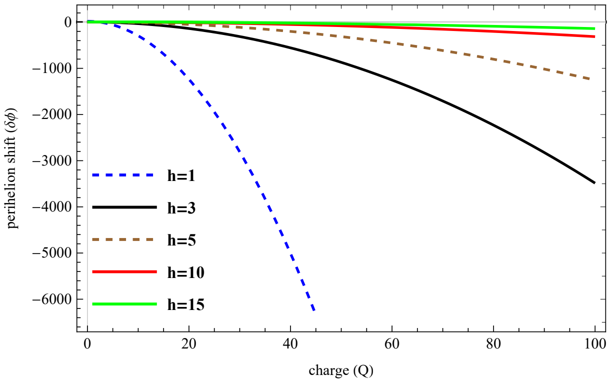


**Figure 2**. Variation of perihelion shift with charge for fixed m = 1.

### 3.3 Effect of mass parameter (m) on perihelion shift ($\delta\phi$)

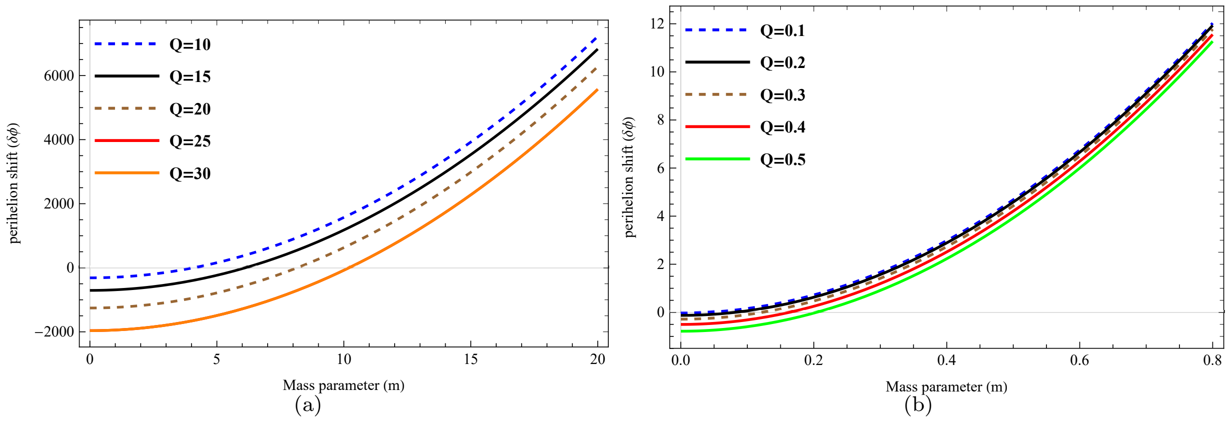


**Figure 3**. The behavior of perihelion shift ($\delta\phi$) with respect to m by changing $Q$ but fixed $h = 1$.

To study the effect of the mass parameter (m) on perihelion shift, we plot figure 3. The plot tells that the perihelion shift is an increasing function of the mass parameter. There is a critical value for perihelion shift that does not depend on the value of Q. For large Q, when the value of Q increases then perihelion shift decreases for massive black holes and it takes a negative value for small m.

However, In 3(b), we see that for very small Q, the perihelion shift takes a positive value for very small m. As like 3(a), if the value of Q increases then perihelion shift decreases for massive black holes.

## 4 Bending Of Light

Consider a spherically symmetric point source of gravity in this space-time. A light particle moves in the gravitational field of that point source of gravity. Since light particles moves along null geodesics, so $ds^2 = 0$. Therefore,

$$2\mathcal{L} = -(1 - \frac{2m}{r} + \frac{Q^2}{r^2})\dot{t}^2 + \frac{\dot{r}^2}{(1 - \frac{2m}{r} + \frac{Q^2}{r^2})} + r^2 d\Omega_2^2 = 0 \tag{4.1}$$

Euler-Lagrangian equation will be same as in equations (2.3), (2.4) and (2.5). At the equatorial plane, i.e, $\theta = \frac{\pi}{2}$ equation (4.1) becomes,

$$\frac{k^2}{(1 - 2mu + Q^2u^2)} = \frac{h^2(\frac{du}{d\phi})^2}{(1 - 2mu + Q^2u^2)} - h^2u^2 = 0$$

Multiply by $\frac{(1-2mu+Q^2u^2)}{h^2}$ we obtain

$$\left(\frac{du}{d\phi}\right)^2 + u^2 = \frac{k^2}{h^2} + 2mu^3 - Q^2u^4 \tag{4.2}$$

Differentiating with respect to $\phi$

$$\frac{d^2u}{d\phi^2} + u = 3mu^2 - 2Q^2u^3 \tag{4.3}$$

**In the case of special theory of relativity**$(m \to 0,\ Q \to 0)$ :

$$\frac{d^2u}{d\phi^2} + u = 0$$

it's solution

$$u = \frac{1}{D} sin(\phi - \phi_0) \tag{4.4}$$

it is the equation of straight line, so in the limit of special relativity light ray follows a straight line path and here D is the distance of closest approach to the origin.

**In case of general relativity**$(m \neq 0,\ Q \neq 0)$ : Assuming the solution of the equation (4.3) to be in the form

$$u = u_0 + \epsilon_1 u_1 + \epsilon_2 u_2 \cdots$$

will gives us to approximate the solution to arbitrary order of $\epsilon_1$ and $\epsilon_2$.

$$u \approx u_0 + 3mu_1 - 2Q^2u_2 \tag{4.5}$$

since mu and $Q^2u$ is very small, so we can neglect the higher order term of this quantity. Substitute equation(4.5) in equation (4.3) we get,

$$u_0''+u_0+3m\left(u_1''+u_1\right)-2Q^2\left(u_2''+u_2\right) = 3m\left[u_0^2+m^2u_1^2+2mu_0u_1-2Q^2mu_1u_2-2Q^2u_0u_2+Q^4u_2^2\right]$$
$$-2Q^2\left[u_0^3+3u_0^2(mu_1-Q^2u_2)+3u_0(mu_1-Q^2u_2)^2+3(mu_1-Q^2u_2)^3\right] \tag{4.6}$$

Equating the coefficients on the both sides of above equation (4.6),

$$u_0''+u_0=0$$

Assuming $\phi_0 = 0$

$$u_0 = \frac{1}{D}sin\phi$$

$$u_1''+u_1 = u_0^2 = \frac{sin^2\phi}{D^2}$$

and

$$u_2''+u_2 = u_0^3 = \frac{sin^3\phi}{D^3}$$

The approximation value of $u_1$ and $u_2$ in equation (4.5) are [8, 10]

$$u_1 \approx \frac{1}{3D^2}(1+Ccos\phi+cos^2\phi)$$

$$u_2 \approx \frac{1}{D^3}(-\frac{3}{8}\phi cos\phi+\frac{1}{32}sin3\phi)$$

where C is an arbitrary integration constant. Therefore, the full solution up to first order corrections is :

$$u \approx \frac{sin\phi}{D} + \frac{m}{D^2}(1+Ccos\phi+cos^2\phi) - \frac{2Q^2}{D^3}(-\frac{3}{8}\phi cos\phi+\frac{1}{32}sin3\phi) \tag{4.7}$$

compared with the case in Schwarzschild spece-time $u \approx \frac{sin\phi}{D} + \frac{m}{D^2}(1+Ccos\phi+cos^2\phi)$, there is an additional term $-\frac{2Q^2}{D^3}(-\frac{3}{8}\phi cos\phi+\frac{1}{32}sin3\phi)$ which comes from electric charge

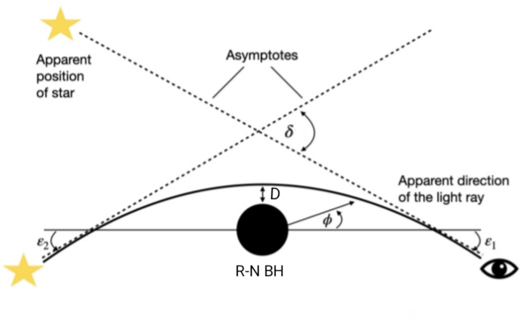


**Figure 4**. Deflection of light ray in the gravitational field of R-N space-time.

(Q) contribution. If we put Q = 0 in the above equation then it can return to the well known Schwarzschild case. In STR limit the path of light beam was straight line while in GTR case, the path of light beam is deviated by two small term $\frac{m}{D^2}(1 + Ccos\phi + cos^2\phi)$ and $-\frac{2Q^2}{D^3}(-\frac{3}{8}\phi cos\phi + \frac{1}{32}sin3\phi)$. So it's no longer be a straight line path.

Now, we are more interested in calculating the deflection angle ($\delta$) for a light beam in the appearence of a charge in R-N black hole space-time. Far away from the source in this space-time, i.e, $r \to \infty$ and $u \to 0$, which makes the right hand side of equation (4.7) to vanish. Without loss of any generality, let's consider the values of $\phi$ such that for $r \to \infty$ this angle $\phi$ asymptotes to $-\epsilon_1$ and $\pi + \epsilon_2$ respectively, shown in figure 4. Expanding equation (4.7) for $-\epsilon_1$ and $-\epsilon_2$, we obtain,

$$0 = -\frac{\epsilon_2}{D} + \frac{m}{D^2}(2 - C) - \frac{2Q^2}{D^3}\left(\frac{3}{8}(\pi + \epsilon_2)\right)$$

and

$$0 = -\frac{\epsilon_1}{D} + \frac{m}{D^2}(2 + C) - \frac{2Q^2}{D^3}(\frac{3}{8}\epsilon_1)$$

Taking these two equations together, we find the total deflection angle,

$$\delta = \epsilon_1 + \epsilon_2 = \frac{4m}{D} - \frac{3\pi Q^2}{4D^2} \quad (4.8)$$

Here, it is evident that the deflection angle of Reissner-Nordström black hole depends on the various parameters like distance of closest approach D, charge Q and the mass parameter m. This equation very closely resembles the deflection angle encountered in Schwarzschild space-time, except there is an additional term $-\frac{3\pi Q^2}{4D^2}$ in the above equation which comes from the presence of cahrge Q.

# 5 Graphical analysis

## 5.1 Effect of distance of closest approach (D) on deflection angle ($\delta$):

To study the behavior of deflection angle and its dependence on distance of closest approach for varying Q and m, we plot figure 5. From these plots, we see that the deflection angle is an exponentially decreasing function of D for large m and very small Q but takes always a positive value. However, $\delta$ is an exponentially increasing function of D for large Q and very small m but takes always negative values. For large values of D, the deflection angle always saturates.

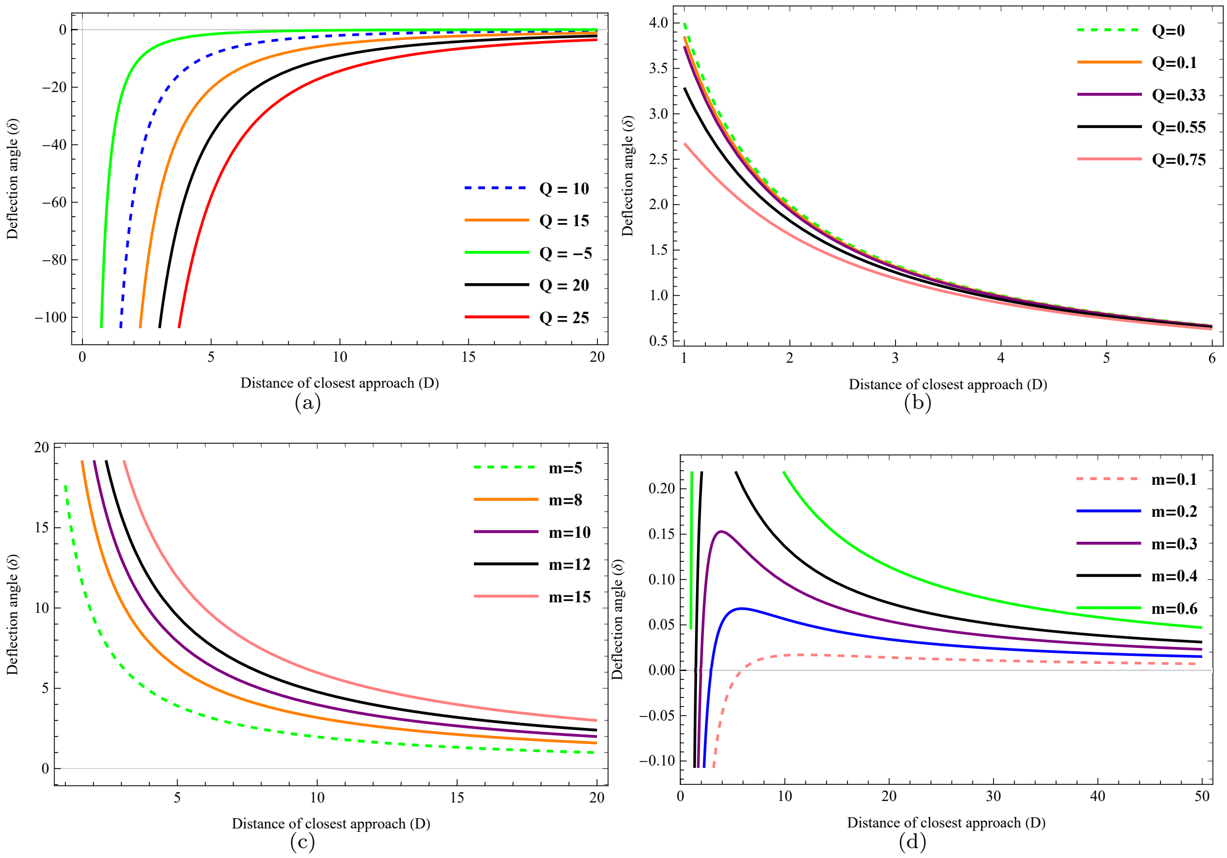


**Figure 5**. In 5(a) and 5(b), the behavior of deflection angle ($\delta$) with respect to distance of closest approach (D) for varying $Q$ but fixed $m = 1$ In 5(c) and 5(d), the behavior of $\delta$ with respect to $D$ for varying $m$ but fixed $Q = 1$.

### 5.2 Effect of charge (Q) on deflection angle ($\delta$):

Figure 6 depicts how the deflection angle depends on charge Q. we see that the deflection angle is a decreasing function of Q and becomes more negative for smaller values of D .

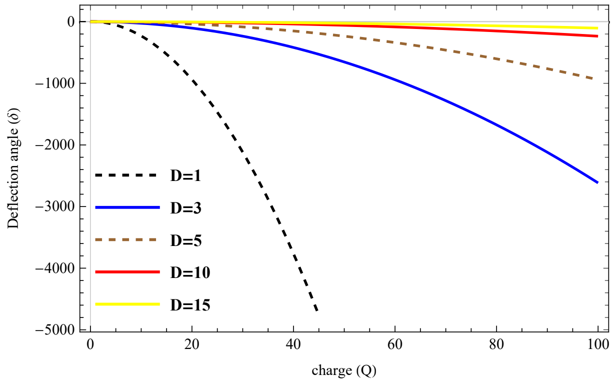


**Figure 6**. The behavior of deflection angle ($\delta$) with respect to charge Q by changing D.Here $m = 1$.

### 5.3 Effect of mass parameter (m) on deflection angle ($\delta$)

The dependence of defection angle $\delta$ on mass parameter m by varying charge Q is depicted in figure 7. The deflection angle is an increasing function of mass parameter m. In plot 7(a), for smaller Q values the deflection angle curve increases sharply and for small m and higher Q, deflection angle is negative valued. From figure 7(b), for small m and very small Q, deflection angle is positive valued.

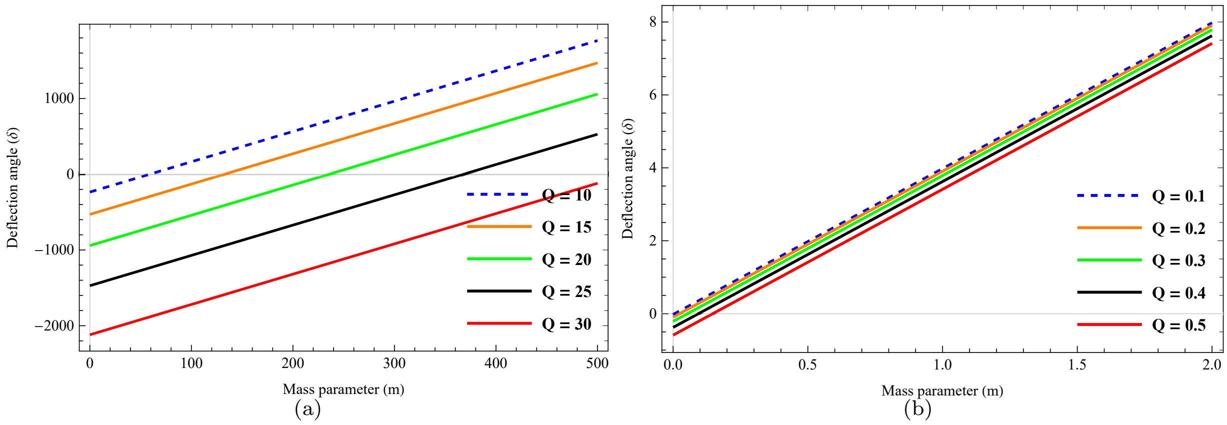


**Figure 7**. In 7(a) and 7(b), the behavior of deflection angle ($\delta$) with respect to m by changing $Q$ but fixed $D = 1$.

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## 6 Conclusion and remarks

In this paper, the perihelion precession and deflection of light have been investigated in the 4-dimensional general spherically symmetric R-N black hole spacetime. The perihelion precession and deflection of light can be treated, respectively, as the time-like and null geodesic in spacetime. Moreover, the effects from the charge on both the perihelion precession and deflection of light has been investigated. We find that the electric charge can affect both the perihelion precession and the deflection of light.

We have considered a charged Reissner-Nordström black hole solution in the background spacetime and with the help of perturbation method we have calculated perihelion shift. Here, we have found that except the charge the perihelion shift of charged Reissner-Nordström black hole depends on the various parameters like angular momentum, the mass parameter and graviton mass. Likewise, except the charge the deflection angle of charged Reissner-Nordström black hole depends on the various parameters like distance of closest approach, the mass parameter and graviton mass. To check the dependency of the perihelion shift and deflection angle on these parameters, we have done a graphical analysis. The graphs for perihelion shift declared that it decreases and increases with the angular momentum for very small and large values of charge, receptively. However, the perihelion shift increases and decreases along with the angular momentum for small and large masses, respectively. similarly,The graphs for deflection angle declared that it decreases and increases with the distance of closest approach for very small and large values of charge, receptively. Moreover, the deflection angle increases and decreases along with the distance of closest approach for small and large masses, respectively. We also found that the perihelion shift and deflection angle is a decreasing and increasing function of charge and mass of black hole, respectively.

More importantly, When mercury can make the rotation around a R-N black hole spacetime then effectively both the perihelion shift and deflection angle becomes smaller for the higher valus of charge. Mercury itself is uncharged but this is happened bacause of the fact that if any spacetime contains charge, then the force due to charge and force due to mass will be opposite to each other and as a result, reduces the effective gravity.

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